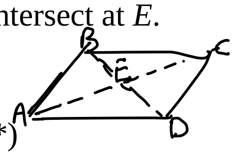
YISHUN JUNIOR COLLEGE
2015 JC2 PRELIMINARY EXAM PAPER 1
H2 MATHEMATICS
SOLUTION

Qn	Solution
1	Let x, y, and z be the number of trays of blueberry, strawberry and chocolate cupcakes respectively. Time: $8x+7y+6z=17\times 60=1020$ Amt: $0.6x+0.6y+0.8z=96$ Price: $12x(1)+12y(0.9)+12z(0.8)=1572$ Using GC, $x=50,\ y=50,\ z=45$
2(a)	If $\mathbf{a},\mathbf{b}\neq\mathbf{0}$ and , then $\mathbf{a}\cdot\mathbf{b}=0$ implies that the two vectors $\mathbf{a}$ and $\mathbf{b}$ are perpendicular to each other i.e. $\mathbf{a}\perp\mathbf{b}$.
2(b)	If $\mathbf{a}$ lies in $x-$axis and $\mathbf{m}\times\mathbf{a}=\mathbf{0}$, then $\mathbf{m}$ is parallel $\mathbf{a}$, and hence is parallel to $\mathbf{i}$. Since $\lvert\mathbf{m}\rvert=1$ then $\mathbf{m}=\mathbf{i}$ or $-\mathbf{i}$ (just one will do)
2(c)	<u>**Method 1**</u>: Let the diagonals BD and AC intersect at E. Given DE=EB -------(1) and AE=EC-----------(2)  $\overrightarrow{AB}=\overrightarrow{AE}+\overrightarrow{EB}$ (*) $\overrightarrow{DC}=\overrightarrow{DE}+\overrightarrow{EC}$ (*) $=\overrightarrow{EB}+\overrightarrow{AE}$ [from (1) and (2)] $=\overrightarrow{AB}$ $\overrightarrow{AB}=\overrightarrow{DC}$ i.e. $AB=DC$ and $AB//DC$ Therefore $ABCD$ is a parallelogram. (Proven) <u>**Method 2**</u>: Using Ratio Theorem, $\overrightarrow{OE}=\frac{1}{2}\left(\overrightarrow{OA}+\overrightarrow{OC}\right)=\frac{1}{2}\left(\overrightarrow{OD}+\overrightarrow{OB}\right)$ $\overrightarrow{OA}+\overrightarrow{OC}=\overrightarrow{OD}+\overrightarrow{OB}$ $\overrightarrow{OB}-\overrightarrow{OA}=\overrightarrow{OC}-\overrightarrow{OD}$ $\overrightarrow{AB}=\overrightarrow{DC}$ i.e. $AB=DC$ and $AB//DC$ Therefore $ABCD$ is a parallelogram. (Proven)
2(d)	$\overrightarrow{OP}=p\mathbf{x}$, $\overrightarrow{OQ}=q\mathbf{y}$ and $\overrightarrow{OR}=r\mathbf{x}+s\mathbf{y}$ Since P, Q, and R are collinear, $\overrightarrow{PQ}=k\overrightarrow{PR}$ for some $k\in\mathbb{R}$. $q\mathbf{y}-p\mathbf{x}=k\left(r\mathbf{x}+s\mathbf{y}-p\mathbf{x}\right)=k\left(r-p\right)\mathbf{x}+ks\mathbf{y}$ $k(r-p)=-p$ ----- (1) $ks=q$ ----- (2)

$(1)\div(2): \dfrac{r-p}{s}=\dfrac{-p}{q}$

$rq-pq=-ps$

Therefore $ps+rq=pq$ (Shown)

3

External volume = $\pi(r+1)^2(h+1)$

Internal volume= $\pi r^2 h=1000 \quad \Rightarrow h=\dfrac{1000}{\pi r^2}$

$V=\pi(r+1)^2(h+1)-1000$

$V=\pi(r+1)^2\left(\dfrac{1000}{\pi r^2}+1\right)-1000$ (Shown)

$$\frac{\mathrm{d}V}{\mathrm{d}r}=2\pi(r+1)\left(\frac{1000}{\pi r^2}+1\right)+\pi(r+1)^2\left(-\frac{2000}{\pi r^3}\right)$$

OR $V=\pi r^2+2\pi r+\pi+\dfrac{2000}{r}+\dfrac{1000}{r^2}\Rightarrow\dfrac{\mathrm{d}V}{\mathrm{d}r}=2\pi+2\pi r-\dfrac{2000}{r^2}-\dfrac{2000}{r^3}$

For stationary V, $\dfrac{\mathrm{d}V}{\mathrm{d}r}=0$

$\pi(r+1)\left(\dfrac{2000}{\pi r^2}+2-\dfrac{2000}{\pi r^2}-\dfrac{2000}{\pi r^3}\right)=0$ OR $2\pi+2\pi r-\dfrac{2000}{r^2}-\dfrac{2000}{r^3}=0$

i.e. $\pi(r+1)\left(2-\dfrac{2000}{\pi r^3}\right)=0$ OR $\pi r^4+\pi r^3-1000r-1000=0\Rightarrow(r+1)\left(\pi r^3-1000\right)=0$

Since $r+1\neq 0\Rightarrow\dfrac{2000}{\pi r^3}=2$ OR $r+1\neq 0\Rightarrow\pi r^3=1000$

$r=\sqrt[3]{\dfrac{1000}{\pi}}=\dfrac{10}{\sqrt[3]{\pi}}$ and $h=\dfrac{1000}{\pi r^2}=\dfrac{1000}{\pi}\times\dfrac{\pi^{\frac{2}{3}}}{100}=\dfrac{10}{\sqrt[3]{\pi}}$

$\dfrac{\mathrm{d}^2V}{\mathrm{d}r^2}=\pi\left(2-\dfrac{2000}{\pi r^3}\right)+\pi(r+1)\left(\dfrac{6000}{\pi r^4}\right)=21.6105>0$ when $r=\dfrac{10}{\sqrt[3]{\pi}}$

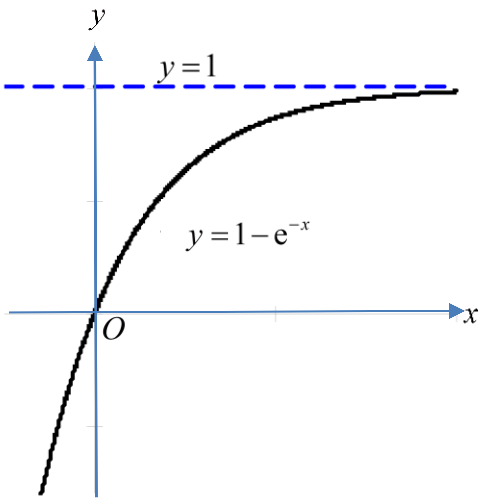
OR $\dfrac{\mathrm{d}^2V}{\mathrm{d}r^2}=2\pi+\dfrac{4000}{r^3}+\dfrac{6000}{r^4}>0$, $\because r>0$

$\Rightarrow$ minimum for $\forall\, r\in\mathbf{R}^+$

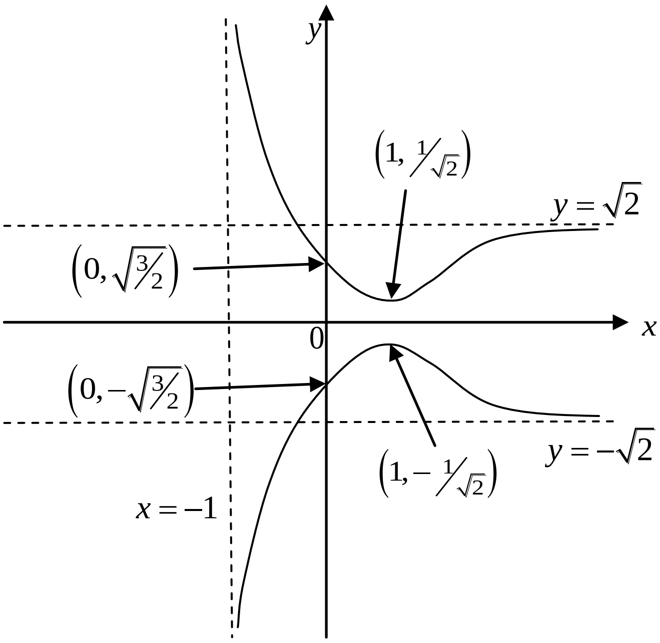
OR

r	$\left(\dfrac{10}{\sqrt[3]{\pi}}\right)^-$	$\dfrac{10}{\sqrt[3]{\pi}}$	$\left(\dfrac{10}{\sqrt[3]{\pi}}\right)^+$
$\dfrac{\mathrm{d}V}{\mathrm{d}r}$	negative	0	Positive
Slope	\	—	/

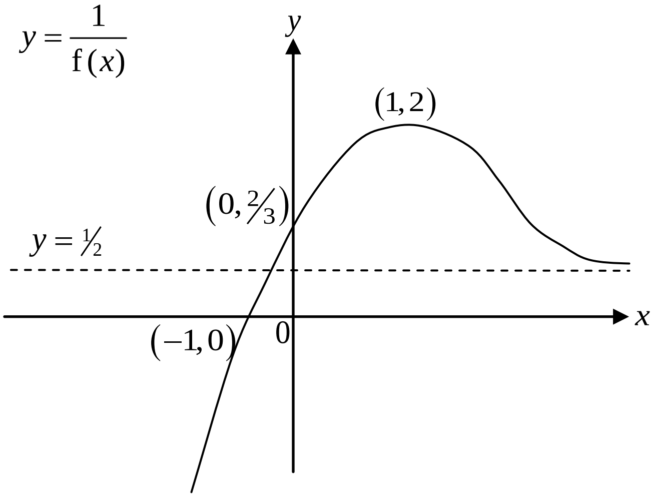
Therefore V is minimum when $r=h=\dfrac{10}{\sqrt[3]{\pi}}$ cm.

4	$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = -\frac{\mathrm{d}y}{\mathrm{d}x}$ $\frac{\mathrm{d}y}{\mathrm{d}x} = -\int \frac{\mathrm{d}y}{\mathrm{d}x}\,\mathrm{d}x$ $\frac{\mathrm{d}y}{\mathrm{d}x} = -y + C,\ C \in \mathbb{R}$ $\frac{\mathrm{d}x}{\mathrm{d}y} = \frac{1}{C-y} \Rightarrow x = -\ln\lvert C-y\rvert + D,\ D \in \mathbb{R}$ $\lvert C-y\rvert = \mathrm{e}^{-x+D} = A\mathrm{e}^{-x},\ \mathrm{e}^{D} = A \in \mathbb{R}^{+}$ $C - y = B\mathrm{e}^{-x},\ B \neq 0$ $y = C - B\mathrm{e}^{-x}$ When $x = 0$, $y = 0$ and $\frac{\mathrm{d}y}{\mathrm{d}x} = 1 \Rightarrow 0 = C - B$ i.e. $C = B$ and $1 = 0 + C \Rightarrow C = 1$ Therefore $y = \mathrm{f}(x) = 1 - \mathrm{e}^{-x}$
4	

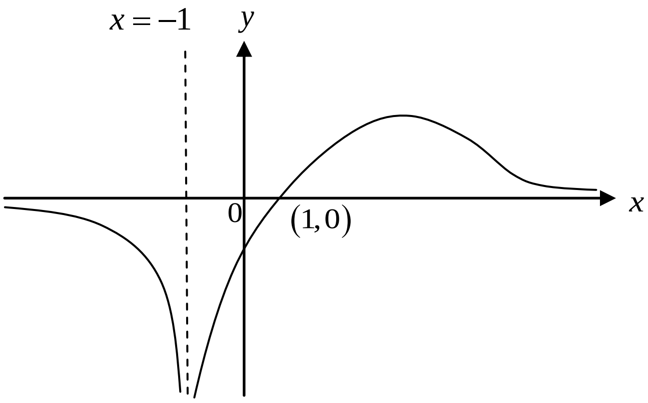
5i $y^2 = f(x)$



5ii $y = \dfrac{1}{f(x)}$



5iii $y = f'(x)$



6a	$x=\frac{1}{2}(1+\sin\theta) \Rightarrow \frac{dx}{d\theta}=\frac{1}{2}\cos\theta \Rightarrow ...dx=...\frac{1}{2}\cos\theta d\theta$ When $x=\frac{3}{4}$ then $1+\sin\theta=\frac{3}{2}$ and $\sin\theta=\frac{1}{2}$ $\therefore \theta=\frac{1}{6}\pi$ When $x=\frac{1}{4}$ then $1+\sin\theta=\frac{1}{2}$ and $\sin\theta=-\frac{1}{2}$ $\therefore \theta=-\frac{1}{6}\pi$ $\int_{\frac{1}{4}}^{\frac{3}{4}} \frac{x}{\sqrt{x-x^2}}\ dx$ $=\int_{-\frac{1}{6}\pi}^{\frac{1}{6}\pi} \frac{\frac{1}{2}(1+\sin\theta)}{\sqrt{\frac{1}{2}(1+\sin\theta)-\frac{1}{4}(1+\sin\theta)^2}} \times \frac{1}{2}\cos\theta\ d\theta$ $=\frac{1}{2}\int_{-\frac{1}{6}\pi}^{\frac{1}{6}\pi} \frac{\frac{1}{2}(1+\sin\theta)\cos\theta}{\sqrt{\frac{1}{4}(1+\sin\theta)\{2-(1+\sin\theta)\}}}\ d\theta$ $=\frac{1}{2}\int_{-\frac{1}{6}\pi}^{\frac{1}{6}\pi} \frac{\frac{1}{2}(1+\sin\theta)\cos\theta}{\frac{1}{2}\sqrt{(1+\sin\theta)(1-\sin\theta)}}\ d\theta$ $=\frac{1}{2}\int_{-\frac{1}{6}\pi}^{\frac{1}{6}\pi} \frac{(1+\sin\theta)\cos\theta}{\sqrt{1-\sin^2\theta}}\ d\theta$ $=\frac{1}{2}\int_{-\frac{1}{6}\pi}^{\frac{1}{6}\pi} \frac{(1+\sin\theta)\cos\theta}{\cos\theta}\ d\theta$ $=\frac{1}{2}\int_{-\frac{1}{6}\pi}^{\frac{1}{6}\pi} (1+\sin\theta)\ d\theta$ Therefore $\int_{\frac{1}{4}}^{\frac{3}{4}} \frac{x}{\sqrt{x-x^2}}\ dx=\frac{1}{2}[\theta-\cos\theta]_{-\frac{1}{6}\pi}^{\frac{1}{6}\pi}$ $=\frac{1}{2}\left[\frac{1}{6}\pi-\cos\frac{1}{6}\pi-\left(-\frac{1}{6}\pi-\cos\frac{1}{6}\pi\right)\right]$ $=\frac{1}{6}\pi$
6bi	$\int_0^m xe^{-3x}\ dx=\left[-\frac{1}{3}xe^{-3x}\right]_0^m-\int_0^m -\frac{1}{3}e^{-3x}\ dx$ $=\frac{1}{3}\left[-xe^{-3x}-\frac{1}{3}e^{-3x}\right]_0^m$ $=\frac{1}{3}\left[-me^{-3m}-\frac{1}{3}e^{-3m}+\frac{1}{3}e^{0}\right]$ $=\frac{1}{9}\left(1-3me^{-3m}-e^{-3m}\right)$
ii	Hence $\int_0^\infty xe^{-3x}\ dx\ =\frac{1}{9}\lim_{m\to\infty}\left(1-3me^{-3m}-e^{-3m}\right)$ $=\frac{1}{9}$

	OR As $m \to \infty,\ e^{-3m} \to 0$ and $\frac{3m}{e^{3m}} \to 0$ then $\frac{1}{9}\left(1-3me^{-3m}-e^{-3m}\right) \to \frac{1}{9}$
7a	$\frac{1}{1+(n-1)a}-\frac{1}{1+na}=\frac{a}{[1+(n-1)a](1+na)}$ $\text{LHS}=\frac{1+na-1-(n-1)a}{\{1+(n-1)a\}(1+na)}$ $=\frac{1+na-1-na+a}{[1+(n-1)a](1+na)}=\frac{a}{[1+(n-1)a](1+na)}$ $=\text{RHS}$ $\sum_{n=1}^{N}\frac{a}{[1+(n-1)a](1+na)}$ $=\sum_{n=1}^{N}\left[\frac{1}{1+(n-1)a}-\frac{1}{1+na}\right]$ $=\begin{bmatrix} \frac{1}{1} & - & \frac{1}{1+a} \\ +\frac{1}{1+a} & - & \frac{1}{1+2a} \\ +\frac{1}{1+2a} & - & \frac{1}{1+3a} \\ \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots \\ +\frac{1}{1+(N-2)a} & - & \frac{1}{1+(N-1)a} \\ +\frac{1}{1+(N-1)a} & - & \frac{1}{1+Na} \end{bmatrix}$ $=1-\frac{1}{1+Na}=\frac{1+Na-1}{1+Na}$ $=\frac{Na}{1+Na}$ (Shown) Letting $a=\frac{1}{2}$ then $\sum_{n=1}^{N}\frac{a}{[1+(n-1)a](1+na)}=\frac{1}{2}\sum_{n=1}^{N}\frac{1}{\{1+\frac{1}{2}(n-1)\}(1+\frac{1}{2}n)}$ $=\frac{1}{2}\left\{\frac{1}{(1)(\frac{3}{2})}+\frac{1}{(\frac{3}{2})(2)}+\frac{1}{(2)(\frac{5}{2})}+\cdots+N\text{th term}\right\}$ $=\frac{\frac{1}{2}N}{1+\frac{1}{2}N}=1-\frac{1}{1+\frac{1}{2}N}$

	i.e. $\frac{1}{(1)(\frac{3}{2})}+\frac{1}{(\frac{3}{2})(2)}+\frac{1}{(2)(\frac{5}{2})}+\cdots+N\text{th term}=2-\frac{2}{1+\frac{1}{2}N}$ As $N\to\infty$, $\frac{2}{1+\frac{1}{2}N}\to 0$ then $\frac{1}{(1)(\frac{3}{2})}+\frac{1}{(\frac{3}{2})(2)}+\frac{1}{(2)(\frac{5}{2})}+...$ converges sum to infinity is 2 .
7b	Let P_n be the statement, $\sum_{r=1}^{n}\frac{r(2^r)}{(r+2)!}=1-\frac{2^{n+1}}{(n+2)!}$ for $n\in\mathbb{Z}^+$ When $n=1$, LHS $=\frac{1(2^1)}{(1+2)!}=\frac{2}{6}$ RHS $=1-\frac{2^2}{(1+2)!}=1-\frac{4}{6}=\frac{2}{6}=$ LHS i.e. P_1 is true Assume that P_k is true for some $k\in\mathbb{Z}^+$ i.e. $\sum_{r=1}^{k}\frac{r(2^r)}{(r+2)!}=1-\frac{2^{k+1}}{(k+2)!}$ Show that P_{k+1} is also true i.e. $\sum_{r=1}^{k+1}\frac{r(2^r)}{(r+2)!}=1-\frac{2^{k+2}}{(k+3)!}$ LHS $=\sum_{r=1}^{k}\frac{r(2^r)}{(r+2)!}+\frac{(k+1)2^{k+1}}{(k+3)!}$ $=1-\frac{2^{k+1}}{(k+2)!}+\frac{(k+1)2^{k+1}}{(k+3)!}$ $=1-\frac{(k+3)2^{k+1}-(k+1)2^{k+1}}{(k+3)!}$ $=1-\frac{2^{k+1}(k+3-k-1)}{(k+3)!}$ $=1-\frac{2^{k+1}(2)}{(k+3)!}=1-\frac{2^{k+2}}{(k+3)!}=$ RHS i.e. P_{k+1} is true Therefore by mathematical induction, P_n is true for $n\in\mathbb{Z}^+$
8a	Let a_k be the no. of marbles placed in the k^{th} bag and A.P.: $a_1,a_2,...,a_n$ where $a_1=6$ and $d=a_{k+1}-a_k=6$ Consider $S_n=\frac{n}{2}[2(6)+6(n-1)]\le 1922$ $0\le n\le 24.816$ When $n=24$, $S_{24}=1800$

	Using GC, n | S_n . | … 24 | $1800 \leftarrow$ less than 1922 25 | 1950 . | … 24 bags contain 1800 marbles i.e. **122** marbles were left behind
8b	(see below)

Month	Start ($)	End ($)
1 (Feb)	$A_1 = 10000$	$B_1 = 1.015(10000)$
2	$A_2 = 1.015(10000) - 1200$	$B_2 = 1.015^2(10000) - 1.015(1200)$
3	$A_3 = 1.015^2(10000) - 1.015(1200) - 1200$	$B_3 = 1.015^3(10000) - 1.015^2(1200) - 1.015(1200)$
…	…	
k	$A_k = 1.015^{k-1}(10000) - 1200(1 + 1.015 + 1.015^2 + ... + 1.015^{k-2})$	$B_k = 1.015^k(10000) - 1200(1.015)(1 + 1.015 + ... + 1.015^{k-2})$

Amount owed at end of month k is $B_k = 1.015^k(10000) - 1200(1.015) \times \dfrac{1.015^{k-1} - 1}{1.015 - 1}$

Final payment will be at the start of the month after $B_k \leq 1200$

From GC,

$B_8 = 2345.52 > 1200$

$B_9 = 1162.70 \leq 1200$

Amount of final payment $= \$1162.70$, made on start of 10^{th} month

Final payment of $\$1162.70$ (nearest cents) is paid on 1^{st} November 2015.

OR

Amount owed at start of month k is $A_k = 1.015^{k-1}(10000) - 1200 \times \dfrac{1.015^{k-1} - 1}{1.015 - 1}$

Final payment is made when $A_k \leq 0$

From GC,

$A_9 = 1145.52$

$A_{10} = -27.30$

Final payment of $-27.30 + 1200 = \$1162.70$ (nearest cents) is paid on 1^{st} November 2015.

9i	$x = 2\sin^3\theta,\ y = \cos^3\theta$ where $0 \le \theta \le \dfrac{\pi}{2}$ $\dfrac{dx}{d\theta} = 6\sin^2\theta\cos\theta,\ \ \dfrac{dy}{d\theta} = -3\sin\theta\cos^2\theta$ $\dfrac{dy}{dx} = -\dfrac{\cos\theta}{2\sin\theta} = -\dfrac{1}{2}\cot\theta$ At $x = 0.25$, $2\sin^3\theta = 0.25 \Rightarrow \sin\theta = 0.5$ Hence $\theta = \dfrac{1}{6}\pi$, $\dfrac{dy}{dx} = -\dfrac{\sqrt{3}}{2}$ and $y = \left(\cos\tfrac{1}{6}\pi\right)^3 = \left(\dfrac{\sqrt{3}}{2}\right)^3$ Tangent: $y - \dfrac{\sqrt{3}^3}{8} = -\dfrac{\sqrt{3}}{2}\left(x - \dfrac{1}{4}\right)$ $y = -\dfrac{\sqrt{3}}{2}x + \dfrac{1}{2}\sqrt{3}$ Normal: $y - \dfrac{\sqrt{3}^3}{8} = \dfrac{2}{\sqrt{3}}\left(x - \dfrac{1}{4}\right)$ $y = \dfrac{2}{\sqrt{3}}x + \dfrac{5\sqrt{3}}{24}$
9ii	At $x = 0.25$, $\dfrac{dy}{dt} = \dfrac{dy}{d\theta} \times \dfrac{d\theta}{dt}$ $= -3\left(\sin\tfrac{1}{6}\pi\cos^2\tfrac{1}{6}\pi\right)\left(\dfrac{1}{18}\right)$ $= -3\left(\dfrac{1}{2} \times \dfrac{3}{4}\right)\left(\dfrac{1}{18}\right) = -\dfrac{1}{16}$ units/sec y is decreasing at $\dfrac{1}{16}$ units/sec.
10i	$y = \cos[\ln(1+x)]$ $\dfrac{dy}{dx} = -\dfrac{1}{1+x}\sin[\ln(1+x)]$ $(1+x)\dfrac{dy}{dx} = -\sin[\ln(1+x)]$ (Shown)
10ii	$(1+x)\dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} = -\dfrac{1}{1+x}\cos[\ln(1+x)]$ $(1+x)^2\dfrac{d^2y}{dx^2} + (1+x)\dfrac{dy}{dx} = -\cos[\ln(1+x)] = -y$ $(1+x)^2\dfrac{d^2y}{dx^2} + (1+x)\dfrac{dy}{dx} + y = 0$ (Shown)

$$(1+x)^2\frac{d^3y}{dx^3}+2(1+x)\frac{d^2y}{dx^2}+(1+x)\frac{d^2y}{dx^2}+\frac{dy}{dx}+\frac{dy}{dx}=0$$

$$(1+x)^2\frac{d^3y}{dx^3}+3(1+x)\frac{d^2y}{dx^2}+2\frac{dy}{dx}=0$$

When $x=0$, $y=\cos(\ln 1)=1$, $\frac{dy}{dx}=-\sin(\ln 1)=0$,

$\frac{d^2y}{dx^2}+0+1=0 \Rightarrow \frac{d^2y}{dx^2}=-1$, $\frac{d^3y}{dx^3}+3(-1)+0=0 \Rightarrow \frac{d^3y}{dx^3}=3$

$$\therefore y=1-\frac{1}{2!}x^2+\frac{3}{3!}x^3+\dots$$

i.e. $y=1-\frac{1}{2}x^2+\frac{1}{2}x^3+\dots$

$$y=\cos\{\ln(1+x)\}$$

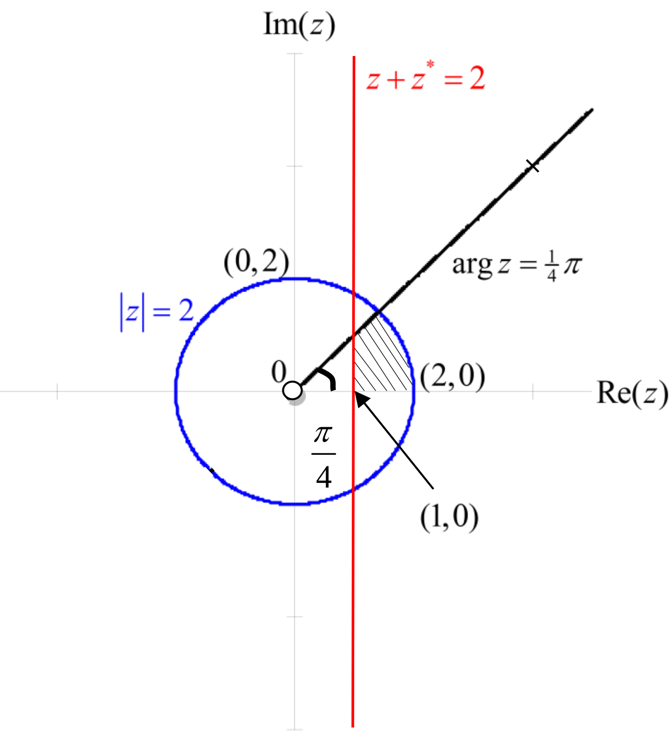
$$=\cos\left\{x-\frac{1}{2}x^2+\dots\right\}$$

$$=1-\frac{1}{2!}\left(x-\frac{1}{2}x^2\right)^2+\cdots$$

$$=1-\frac{1}{2}\left(x^2-x^3+\frac{1}{4}x^4\right)+\cdots$$

$$=1-\frac{1}{2}x^2+\frac{1}{2}x^3+\dots \text{ (Verified)}$$

11a



Shaded region represents the set of required points.

	Greatest value of $\lvert z-4-4\mathrm{i}\rvert = \sqrt{(4-1)^2+4^2} = 5$ Greatest value of $\arg(z-4) = \pi$
11b	$w^3+1=0 \Rightarrow w^3 = -1 = \mathrm{e}^{(2k\pi+\pi)\mathrm{i}}$, where $k \in \mathbb{Z}$ $w = \mathrm{e}^{\frac{1}{3}(2k+1)\pi\mathrm{i}}$, where $k = 0, \pm 1$ $k=0$, $w = \cos\tfrac{1}{3}\pi + \mathrm{i}\sin\tfrac{1}{3}\pi = \dfrac{1}{2} + \mathrm{i}\dfrac{\sqrt{3}}{2}$ $k=-1$, $w = \cos\tfrac{1}{3}\pi - \mathrm{i}\sin\tfrac{1}{3}\pi = \dfrac{1}{2} - \mathrm{i}\dfrac{\sqrt{3}}{2}$ $k=1$, $w = \cos\pi + \mathrm{i}\sin\pi = -1$ <u>Alternative</u> $w^3+1 = (w+1)(w^2-w+1) = 0$ $w = -1,\ \dfrac{1\pm\sqrt{1-4}}{2}$ i.e. $w = -1,\ \dfrac{1\pm \mathrm{i}\sqrt{3}}{2}$ For $\left(\dfrac{z+1}{z}\right)^3 = -1$, let $\dfrac{z+1}{z} = w$, Then $z+1 = wz$ and $z = \dfrac{1}{w-1}$ i.e. $z = \dfrac{1}{\mathrm{e}^{\frac{1}{3}(2k+1)\pi\mathrm{i}}-1}$ where $k = 0, \pm 1$ $z = \dfrac{1}{\frac{1}{2}+\mathrm{i}\frac{\sqrt{3}}{2}-1} = -\dfrac{1}{2} - \mathrm{i}\dfrac{\sqrt{3}}{2}$ $z = \dfrac{1}{\frac{1}{2}-\mathrm{i}\frac{\sqrt{3}}{2}-1} = -\dfrac{1}{2} + \mathrm{i}\dfrac{\sqrt{3}}{2}$ $z = \dfrac{1}{-1-1} = -\dfrac{1}{2}$
12i	$\mathbf{a} = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}$ $\mathbf{a}\times\mathbf{b} = \begin{pmatrix} -1 \\ 4+1 \\ 2 \end{pmatrix} = \begin{pmatrix} -1 \\ 5 \\ 2 \end{pmatrix} = -\mathbf{i} + 5\mathbf{j} + 2\mathbf{k}$ $\lvert\mathbf{a}\times\mathbf{b}\rvert$ is the area of the parallelogram with two adjacent sides OA and OB or twice the area of ΔOAB

	Area of $\Delta OAB = \frac{1}{2}\lvert \mathbf{a}\times\mathbf{b}\rvert = \frac{1}{2}\sqrt{1^2+5^2+2^2}$ $=\frac{1}{2}\sqrt{30}$ sq units
12ii	$\mathbf{a}-\mathbf{b}=\begin{pmatrix}1\\-1\\3\end{pmatrix}$ Eqn. of line through pts A and B: $\mathbf{r}=\begin{pmatrix}2\\0\\1\end{pmatrix}+\lambda\begin{pmatrix}1\\-1\\3\end{pmatrix}$ where $\lambda\in\mathbb{R}$ Or $\mathbf{r}=\begin{pmatrix}1\\1\\-2\end{pmatrix}+\lambda\begin{pmatrix}1\\-1\\3\end{pmatrix}$ where $\lambda\in\mathbb{R}$
12iii	$\overrightarrow{OC}=\begin{pmatrix}-13\\2\\3\end{pmatrix}$ $\overrightarrow{OM}=\begin{pmatrix}2\\0\\1\end{pmatrix}+\lambda\begin{pmatrix}1\\-1\\3\end{pmatrix}=\begin{pmatrix}2+\lambda\\-\lambda\\1+3\lambda\end{pmatrix}$ for some $\lambda\in\mathbb{R}$ $\overrightarrow{CM}=\overrightarrow{OM}-\overrightarrow{OC}=\begin{pmatrix}15+\lambda\\-\lambda-2\\3\lambda-2\end{pmatrix}$ $\overrightarrow{CM}\cdot\begin{pmatrix}1\\-1\\3\end{pmatrix}=0\Rightarrow 15+\lambda+\lambda+2+9\lambda-6=0$ $11\lambda=-11\Rightarrow\lambda=-1$ $\therefore\overrightarrow{OM}=\begin{pmatrix}1\\1\\-2\end{pmatrix}=\mathbf{i}+\mathbf{j}-2\mathbf{k}$ (Shown)
12iv	$\mathbf{a}\times\mathbf{b}=\begin{pmatrix}-1\\5\\2\end{pmatrix}$ from (i) Plane OAB: $\mathbf{r}\cdot\begin{pmatrix}-1\\5\\2\end{pmatrix}=0$ (as origin is on the plane) Required equation is $x-5y-2z=0$

	$\overrightarrow{CM} = \begin{pmatrix} 14 \\ -1 \\ -5 \end{pmatrix}$ from (iii) Length of the projection of vector $\overrightarrow{CM}$ onto this plane $= \frac{1}{\sqrt{30}} \left| \overrightarrow{CM} \times \begin{pmatrix} -1 \\ 5 \\ 2 \end{pmatrix} \right|$ $= \frac{1}{\sqrt{30}} \left| \begin{pmatrix} 14 \\ -1 \\ -5 \end{pmatrix} \times \begin{pmatrix} -1 \\ 5 \\ 2 \end{pmatrix} \right| = \frac{23}{\sqrt{30}} \left| \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix} \right|$ $= 23\sqrt{\frac{11}{30}}$
12v	Let θ be the acute angle between line OC and the triangle OAB. $\sin\theta = \frac{\left| \begin{pmatrix} -13 \\ 2 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 5 \\ 2 \end{pmatrix} \right|}{\sqrt{182}\sqrt{30}} = \frac{13+10+6}{\sqrt{5460}}$ Therefore $\theta = 23.1°$