2014 Sec 4 Amath

EXAMGURU

1	Anglican High School
2	Chung Cheng High School
3	Crescent Girls' School
4	Dunman Secondary School
5	Fairfield Methodist School
6	Geylang Methodist School
7	Holy Innocent High School
8	Nan Chiau School
9	Nan Hua High School
10	Paya Lebar Methodist School
11	St. Nicholas School
12	STJI School
13	Tanjong Katong Girls School
14	Victoria School

Name : ______________________ Sec 4 ____ (   )

ANGLICAN HIGH SCHOOL
PRELIMINARY EXAMINATION 2014
SECONDARY FOUR

Additional Mathematics — 4047/01

Paper 1 — 31 July 2014

2 hours

Additional Materials: Writing paper (8 sheets)
Graph paper (1 sheet)

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together and attach the question paper on top of the scripts.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

For Examiners' Use													
Question	1	2	3	4	5	6	7	8	9	10	11	12	13
Marks													
											80		

Parent's signature:
(date):

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\text{Area of } \Delta = \frac{1}{2} ab \sin C$$

1 (a) Find the smallest value of the integer a for which $ax^2 - 8x + 3$ is positive for all values of x. [3]

(b) The equation of a curve is $y = x^3 + \frac{5}{2}x^2 - 2x + 1$.

Find the set of values of x for which y is a decreasing function. [4]

2 **Without using a calculator,**

(i) show that $\tan\left(\frac{5\pi}{12}\right) = \frac{1+\sqrt{3}}{\sqrt{3}-1}$, [2]

(ii) express $\tan\left(\frac{5\pi}{12}\right)\sin\left(\frac{\pi}{6}\right)$ in the form $k(2+\sqrt{3})$, where k is a constant. [4]

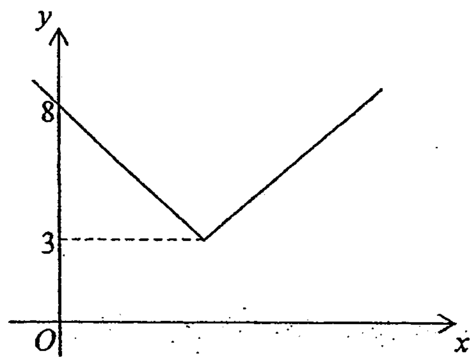
3 Show that $(\tan A + \cot A)\sec A = \frac{1}{\sin A - \sin^3 A}$ [4]

4 In the expansion of $\left(ax + \frac{b}{2x}\right)^n$, the fourth term is $-8437\frac{1}{2}$ and the coefficient of x^2 is $7593\frac{3}{4}$, where a and b are constants.

(i) Write down the value of n. [1]

(ii) Find the value of the positive constant a. [5]

5



The diagram above shows part of the graph of $y = |2x - p| + q$. The y–intercept of the graph is 8 and its minimum y value is 3.

(i) State the value of q. [1]

(ii) Find the value of p. [3]

Determine the number of intersections of the line $y = -2x + 10$ with the graph $y = |2x - p| + q$, justifying your answer. [1]

4

6 Express $\dfrac{3x(2x^2+3)-4}{3x^3-x^2}$ in partial fraction.

7 A population of a certain type of bacteria, P, at time t hours, is given by $P = P_0$ … P_0 is the initial number of bacteria and k is a constant.
The bacteria doubles in population every 7.5 hours. Given that there were appr… 100 bacteria to start with, find

(i) the value of k,

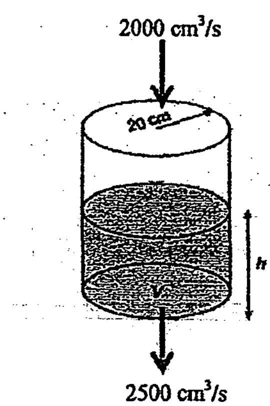
(ii) the approximate number of bacteria that will there be in 2.5 days.

8 The roots of the equation $x^2 - 2x + 49 = 0$ are α^2 and β^2 where $\alpha > 0$ and … Given that the roots of $x^2 + px + q = 0$ are α^3 and β^3, find the value of p … p and q are constants.

9 (i) Differentiate $\dfrac{\sin 2x}{e^{2-x}}$. Hence find, in terms of π, the equation of the tang… curve $y = \dfrac{\sin 2x}{e^{2-x}}$ at the point when $x = \pi$.

(ii) Find the equation of the curve which passes through the point $(2, \pi)$ … gradient function of $\dfrac{dy}{dx} = \tan^2 x + 1$.

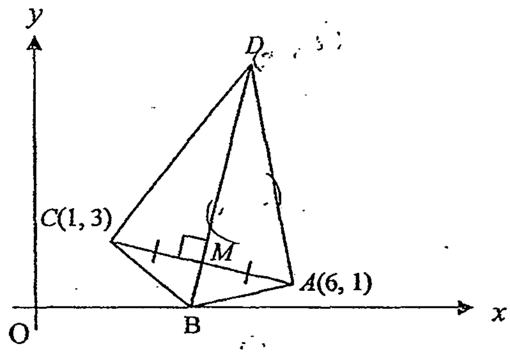
10 Water is leaking from a cylindrical tank of radius 20 cm at the rate of 2500 cm… water is added at the rate of 2000 cm³/s. Find the rate at which the water level … is decreasing.



11 **(i)** Find the value of k for which the line $y+4x=k$ is a tangent to the curve $y=x^4-k$. [4]

(ii) A curve has the equation $y=\cos 2x-3\sin x$ where $0\le x\le \pi$.
Find the x-coordinates of the stationary points. [3]

12 **Solutions to this question by accurate drawing will not be accepted.**
The diagram shows a quadrilateral $ABCD$ where A is (6, 1), B is on the x-axis and C is (1, 3). The diagonal BD bisects AC at right angles at M and $BD=\frac{7}{2}BM$. Find



(i) the equation of BD, [3]

(ii) the x-coordinate of B, [1]

(iii) the coordinates of D. [3]

13 The table shows experimental values of two variables, x and y, which are connected by an equation of the form $y=ab^x$, where a and b are constants.

x	1	2	3	4	5
y	42	120	365	920	2700

(i) Using graph paper, draw the graph of $\lg y$ against x. [3]

(ii) Use your graph to estimate the value of a and of b. [4]

(iii) By drawing a suitable straight line on your graph, solve the equation $10^{2-2x}=ab^x$. [2]

End Of Paper

Name: ____________________ (   ) Class: 4 ____

Anglican High School

Secondary Four Preliminary Examination

Additional Mathematics Paper 2 (4047/02)

Additional Materials: 8 writing papers   05 August 2014   2 hours 30 minutes

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers. Omission of essential working will result in loss of marks.

At the end of the examination, fasten all your work securely together and attach the question paper on top of the scripts.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

For Examiners' Use

Question	1	2	3	4	5	6	7	8
Marks								
Question	9	10	11	12	13	Total		
Marks							100	

Parent's Name and Signature ____________________   Date ________

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$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

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$$a^2 = b^2 + c^2 - 2bc\cos A$$
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Answer ALL Questions

1. The line $5y = 6x + 2$ meets the curve $3x^2 + 4xy - 4y^2 = 0$ at the points A and B. Find the coordinates of A and B. [5]

2. Solve the equation $3^{1-x} + 1 = 10(3^x)$. [5]

3. (i) Show that $\frac{d}{dx}\ln\left(\frac{x^2}{\sin x}\right) = \frac{2}{x} - \cot x$. [2]

(ii) Hence evaluate $\int_1^2 \left(\frac{3}{4}\cot x\right) dx$. [3]

4. Given that $(x - 1)$ and $(x + 2)$ are both factors of the expression $f(x)$ where $f(x) = 2x^3 + mx^2 - 7x + n$, find the value of m and of n. Factorise $f(x)$ completely. [5]
Hence, solve the equation $f(x) = 2(x - 1)(x + 2)$. [3]

5. Solve the following equations, where $0° < x < 360°$.

i) $2\operatorname{cosec}^2\frac{x}{2} = \sin\frac{x}{2}$ [4]

ii) $5\sin^2 x + 9\cos x - 9 = 0$ [5]

6. (a) If $\log_x \frac{p}{\sqrt{q}} - 3\log_x \sqrt{q} = \log_x(p - q)$, express p in terms of q. [3]

(b) Given that $a = \log_2 3$ and $b = \log_2 7$, express $\log_2 21 + \log_4 \frac{16}{7}$ in terms of a and b. [4]

(c) **Without using the calculator**, simplify the following

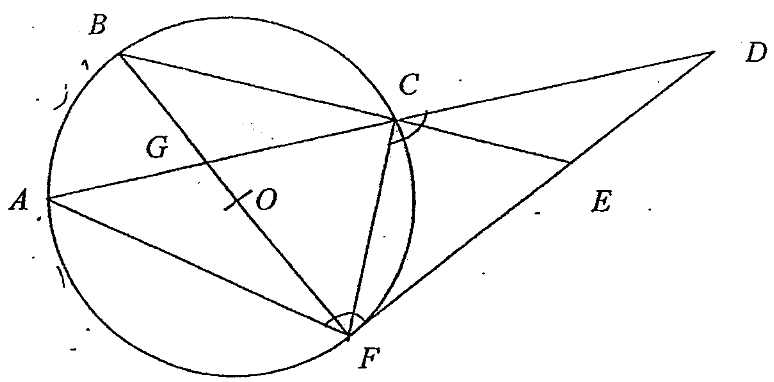
$$\frac{(\sqrt[10]{x} + 1)(x^{\frac{21}{10}} - x^2)}{\sqrt[5]{x} - 1}$$ [3]

7. In the diagram, not drawn to scale, A, B, C and F lie on a circle with centre, O. $BGOF$, $AGCD$, BCE and FED are straight lines where FED is a tangent to the circle at F.

Prove that

(a) $\angle FCD = \angle AFD$, and [3]

(b) $FC \times FE = BF \times CE$ [3]



8. On the same diagram, sketch the graphs of $y = \frac{1}{16}\sqrt{x^5}$ and $y = \frac{4}{\sqrt{x}}$ for $x \geq 0$. [4]

(i) Find the point of intersection of the graphs. [2]

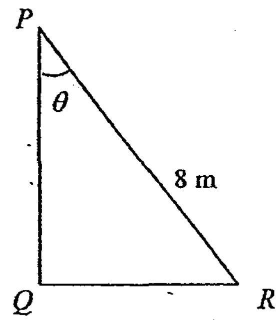
(ii) Calculate the area enclosed by the curve $y = \frac{1}{16}\sqrt{x^5}$, $y = 3$, $x = 5$ and $x = 8$. [4]

9. (a) The function f is defined by $f(x) = c - a \sin bx$.
Given that the period of $f(x)$ is 720° and $-1 \leq f(x) \leq 3$, state the value of a, of b and of c.

(b) Sketch the graphs of $y = |3 \cos 2x|$ and $y = 2 - \frac{2}{\pi}x$ on the same axes for $0 \leq x \leq \pi$.
Hence, state the number of solution for $|3\pi \cos 2x| + 2x = 2\pi$. [4]

10. Given that the gradient of the tangent to the circle, C_1, at $A(5, 2)$ is $-\frac{4}{5}$, find the equation of the normal to the circle at A.
The circle, C_1, also passes through the point $B(5, 6)$.
Find the centre of the circle, C_1. [3]
Hence, find the equation of circle, C_1. [3]
A circle, C_2, is formed when the circle, C_1, is reflected along the line $y = 0$.
Write down the equation of the circle, C_2. [1]

11. A gardener is going to fence off part of his garden by using a piece of 19 metres long fence. The enclosed area is in the shape of a right-angled triangle PQR with $PR = 8$ m.
Given that θ is the included angle between the sides PQ and PR,



i) show that $8 \cos \theta + 8 \sin \theta = 11$. [2]

Hence, express the equation in the form $R \cos(\theta - \alpha) = C$, where C is an integer, $R > 0$ and $0° < \alpha < 90°$. [2]

ii) find the possible values of θ. [3]

12. A particle moves in a straight line with velocity, v m/s, given by $v = t^2 - 8t + 15$ where t is the time in seconds, measured from the start of the motion. Its initial displacement from a fixed point O is -6 metres.

(a) Find

(i) the values of t for which the particle is at instantaneous rest [2]

(ii) the minimum velocity of the particle [2]

(b) Determine the average speed of the particle in the first 7 seconds. [5]

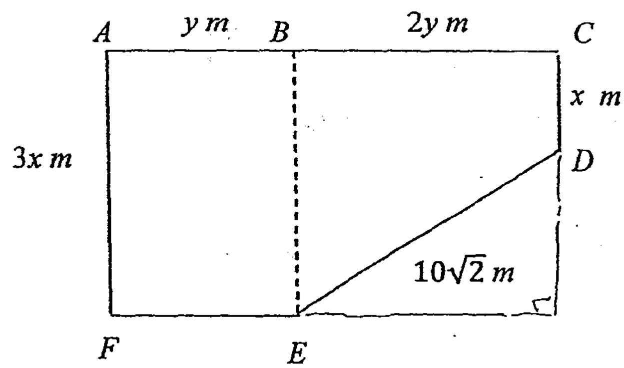
(c) Show that the particle does not return to its starting point. [1]

13. The diagram, not drawn to scale, shows the plan of a park, $ABCDEF$. $BCDE$ ... trapezium, with CD parallel to BE. $ABEF$ is a rectangle and ABC is a ... Given that $DE = 10\sqrt{2}$ metres, $AB = y$ metres, $BC = 2y$ metres, $CD = x$ metres and $AF = 3x$ metres.

(i) Express y in terms of x.

(ii) Show that the area of the park, A m², is given by $A = 7x\sqrt{50 - x^2}$

(iii) Determine the value of x that will give the maximum area.



***** END OF PAPER *****

Name : ______________________________________ Sec 4 _____ (   )

ANGLICAN HIGH SCHOOL
PRELIMINARY EXAMINATION 2014
SECONDARY FOUR

Additional Mathematics 4047/01

Paper 1

31 July 2014

2 hours

Additional Materials: Writing paper (10 sheets)

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Marks													
												/ 80	

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(date): ______________

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$$\text{Area of } \Delta = \frac{1}{2} ab \sin C$$

1 **(a)** Find the smallest value of the integer a for which $ax^2 - 8x + 3$ is positive for all values of x. [3]

(b) The equation of a curve is $y = x^3 + \frac{5}{2}x^2 - 2x + 1$.

Find the set of values of x for which y is a decreasing function. [4]

(a) $ax^2 - 8x + 3 > 0$

$$(-8)^2 - 12a < 0$$
$$12a > 64$$
$$a > 5\frac{1}{3}$$

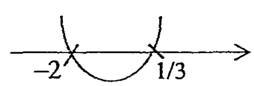
The smallest integer a is 6.

(b) $y = x^3 + \frac{5}{2}x^2 - 2x + 1$

$$\frac{dy}{dx} = 3x^2 + 5x - 2$$

For decreasing function, $\frac{dy}{dx} < 0$

$$3x^2 + 5x - 2 < 0$$
$$(3x-1)(x+2) < 0$$



$$-2 < x < \frac{1}{3}$$

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2 **Without using a calculator,**

(i) show that $\tan\left(\frac{5\pi}{12}\right) = \frac{1+\sqrt{3}}{\sqrt{3}-1}$, [2]

(ii) express $\tan\left(\frac{5\pi}{12}\right)\sin\left(\frac{\pi}{6}\right)$ in the form $k(2+\sqrt{3})$, where k is a constant. [4]

(i) $\tan\left(\frac{5\pi}{12}\right) = \tan\left(\frac{2\pi+3\pi}{12}\right)$

$$= \tan\left(\frac{\pi}{6} + \frac{\pi}{4}\right)$$
$$= \frac{\tan\frac{\pi}{6} + \tan\frac{\pi}{4}}{1 - \tan\frac{\pi}{6} \times \tan\frac{\pi}{4}}$$
$$= \frac{\frac{1}{\sqrt{3}} + 1}{1 - \frac{1}{\sqrt{3}}}$$
$$= \frac{1+\sqrt{3}}{\sqrt{3}-1}$$

(ii) $\tan\left(\frac{5\pi}{12}\right)\sin\left(\frac{\pi}{6}\right) = \frac{1+\sqrt{3}}{\sqrt{3}-1} \times \frac{1}{2}$

$$= \frac{1}{2} \times \frac{(1+\sqrt{3})(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$$

$$= \frac{1}{2} \times \frac{\sqrt{3}+1+3+\sqrt{3}}{3-1}$$

$$= \frac{1}{4}(4 + 2\sqrt{3})$$

$$= \frac{1}{2}(2 + \sqrt{3})$$

---

3 Show that $(\tan A + \cot A)\sec A = \frac{1}{\sin A - \sin^3 A}$ [4]

$$(\tan A + \cot A)\sec A = \left(\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}\right) \times \frac{1}{\cos A}$$

$$= \frac{\sin^2 A + \cos^2 A}{\sin A \cos A} \times \frac{1}{\cos A}$$

$$= \frac{1}{\sin A \cos A} \times \frac{1}{\cos A}$$

$$= \frac{1}{\sin A \cos^2 A}$$

$$= \frac{1}{\sin A(1-\sin^2 A)}$$

$$= \frac{1}{\sin A - \sin^3 A}$$

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4 In the expansion of $\left(ax + \frac{b}{2x}\right)^n$, the fourth term is $-8437\frac{1}{2}$ and the coefficient of x^2 is $7593\frac{3}{4}$, where a and b are constants.

(i) Write down the value of n. [1]

(ii) Find the value of the positive constant a. [5]

(i) 4^{th} term, $r = 3$, $n = 6$

or $T_{r+1} = \binom{n}{r}(ax)^{n-r}\left(\frac{b}{2x}\right)^r$

$T_4 = \binom{n}{3}(ax)^{n-3}\left(\frac{b}{2x}\right)^3$

$n - 3 - 3 = 0$

$\therefore n = 6$

(ii)

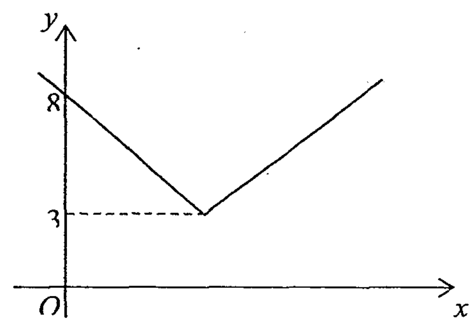
$$\binom{6}{3}(ax)^3\left(\frac{b}{2x}\right)^3 = -8437\frac{1}{2}$$
$$20a^3\left(\frac{b^3}{8}\right) = -8437\frac{1}{2}$$
$$a^3b^3 = -3375$$
$$ab = -15$$

x^2 term: $6 - r - r = 2$
$$r = 2$$

$$\binom{6}{2}(ax)^4\left(\frac{b}{2x}\right)^2 = 7593\frac{3}{4}\,x^2$$
$$15a^4\left(\frac{b^2}{4}\right) = 7593\frac{3}{4}$$
$$a^4b^2 = 2025$$
$$a^2(-15)^2 = 2025$$
$$a^2 = 9$$

$\therefore a = 3$ or $a = -3$ (reject, $\because a > 0$)

5



The diagram above shows part of the graph of $y = |2x - p| + q$. The y–intercept of the graph is 8 and its minimum y value is 3.

(i) State the value of q. [1]

(ii) Find the value of p. [3]

Determine the number of intersections of the line $y = -2x + 10$ with the graph $y = |2x - p| + q$, justifying your answer. [1]

(i) $q = 3$

(ii) when $x = 0, y = 8$

$$8 = |0 - p| + 3$$
$$|-p| = 5$$
$$p = 5 \text{ or } p = -5\text{(reject)}$$

$y = -2x + 10$ is parallel to the Left-hand arm and y–intercept is $10 > 8$, it intersects RH arm once, $\therefore$ there is only 1 intersection.

6 Express $\frac{3x(2x^2+3)-4}{3x^3-x^2}$ in partial fraction. [7]

$$\frac{3x(2x^2+3)-4}{3x^3-x^2} = \frac{6x^3+9x-4}{3x^3-x^2}$$
$$= \frac{6x^3-2x^2+2x^2+9x-4}{3x^3-x^2}$$
$$= 2 + \frac{2x^2+9x-4}{x^2(3x-1)}$$

or

$$3x^3 - x^2 \overline{)6x^3 + 0 + 9x - 4} \quad \text{quotient } 2$$
$$6x^3 - 2x^2$$
$$+2x^2 + 9x - 4$$

Let $\frac{2x^2+9x-4}{x^2(3x-1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{3x-1}$

$2x^2 + 9x - 4 = Ax(3x-1) + B(3x-1) + Cx^2$

$= (3A + C)x^2 + (-A + 3B)x - B$

By comparison: $B = 4$

$-A + 12 = 9, \quad \therefore A = 3$

$9 + C = 2, \quad \therefore C = -7$

or

when $x = 0$, $B = 4$

when $x = \frac{1}{3}$, $\frac{2}{9} + 3 - 4 = \frac{C}{9}$

$\therefore C = -7$

when $x = 1$, $7 = 2A + 8 - 7$

$\therefore A = 3$

Therefore $\frac{3x(2x^2+3)-4}{3x^3-x^2} = 2 + \frac{3}{x} + \frac{4}{x^2} - \frac{7}{3x-1}$

---

7 A population of a certain type of bacteria, P, at time t hours, is given by $P = P_o e^{kt}$, where P_o is the initial number of bacteria and k is a constant.
The bacteria doubles in population every 7.5 hours. Given that there were approximately 100 bacteria to start with, find

(i) the value of k, [2]

(ii) the approximate number of bacteria that will there be in 2.5 days. [3]

(i)
$$P = P_o e^{kt}$$
$$200 = 100e^{7.5k}$$
$$e^{7.5k} = 2$$
$$7.5k = ln2$$
$$k = 0.092419 \approx 0.0924$$

(ii)
$$P = 100e^{0.092419\times 60}$$
$$= 25599$$
$$\approx 25600$$

There were approximately 25600 bacteria in 2.5 days.

8 The roots of the equation $x^2 - 2x + 49 = 0$ are α^2 and β^2 where $\alpha > 0$ and $\beta > 0$. Given that the roots of $x^2 + px + q = 0$ are α^3 and β^3, find the value of p and of q where p and q are constants. [5]

Product of roots: $\alpha^2\beta^2 = 49$

$\alpha\beta = 7$ or $\alpha\beta = -7$ (reject)

Sum of roots: $\alpha^2 + \beta^2 = 2$

$$(\alpha+\beta)^2 = \alpha^2 + \beta^2 + 2\alpha\beta$$
$$= 2 + 2(7)$$
$$(\alpha+\beta)^2 = 16$$
$$\alpha + \beta = 4 \quad \text{or} \quad \alpha+\beta = -4 \text{ (reject)}$$

When $\alpha + \beta = 4$ and $\alpha\beta = 7$

$$\alpha^3 + \beta^3 = (\alpha+\beta)(\alpha^2 - \alpha\beta + \beta^2)$$
$$= 4(2-7)$$
$$= -20$$

$\alpha^3\beta^3 = 7^3 = 343$

New equationn: $x^2 + 20x + 343 = 0$

$\therefore p = 20,\ q = 343.$

---

9 (i) Differentiate $\frac{\sin 2x}{e^{2-x}}$. Hence find, in terms of π, the equation of the tangent to the curve $y = \frac{\sin 2x}{e^{2-x}}$ at the point when $x = \pi$. [4]

(ii) Find the equation of the curve which passes through the point $(2, \pi)$ and has a gradient function of $\frac{dy}{dx} = \tan^2 x + 1$. [3]

(i) $$\frac{dy}{dx} = \frac{2e^{2-x}\cos 2x + (\sin 2x)e^{2-x}}{\left(e^{2-x}\right)^2}$$
$$= \frac{2\cos 2x + \sin 2x}{e^{2-x}}$$

When $x = \pi$, $y = \frac{\sin 2\pi}{e^{2-\pi}} = 0$

Gradient of tangent $= \frac{2\cos 2\pi + \sin 2\pi}{e^{2-\pi}}$

$$= \frac{2}{e^{2-\pi}}$$
$$= 2e^{\pi-2}$$

Equation of tangent: $y - 0 = 2e^{\pi-2}(x - \pi)$

$y = 2xe^{\pi-2} - 2\pi e^{\pi-2}$ o.e.

9 (ii) $\frac{dy}{dx} = \tan^2 x + 1$

$= \sec^2 x$

$y = \int \sec^2 x \, dx$

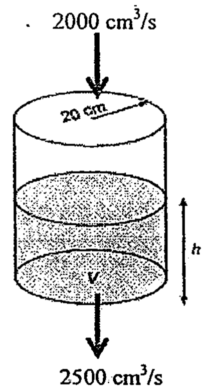
$= \tan x + c$

At point $(2, \pi)$: $\pi = \tan 2 + c$

$c = \pi - \tan 2$ or $c = 5.32663\ldots$

$\therefore \quad y = \tan x + \pi - \tan 2$ or $y = \tan x + 5.33$

10 Water is leaking from a cylindrical tank of radius 20 cm at the rate of 2500 cm^3/s and fresh water is added at the rate of 2000 cm^3/s. Find the rate at which the water level in the tank is decreasing. [5]



Nett $\frac{dV}{dt} = 2000 - 2500$

$= -500 \quad cm^3/s$

Volume of water: $V = \pi r^2 h$

Since $r = 20$, $V = \pi(20)^2 h$

$= 400\pi h$

$\frac{dV}{dh} = 400\pi$

$\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$

$-500 = 400\pi \times \frac{dh}{dt}$

$\frac{dh}{dt} = -0.39788$

$\frac{dh}{dt} = -0.398$ cm/s

Ans: Water level in the tank is falling at 0.398 cm/s.

11 **(i)** Find the value of k for which the line $y+4x=k$ is a tangent to the curve $y=x^4-k$. [4]

(ii) A curve has the equation $y=\cos 2x-3\sin x$ where $0\le x\le \pi$.
Find the x-coordinates of the stationary points. [3]

(i)

(i) Given the curve $y=x^4-k$ ……… ①

$$\frac{dy}{dx}=4x^3$$

Given tangent of the curve: $y=-4x+k$ …….. ②
Gradient of tangent $=-4$

$$\therefore \quad 4x^3=-4$$
$$x=-1$$

When $x=-1$,
①=②, $(-1)^4-k=-4(-1)+k$
$$2k=-3$$
$$k=-\frac{3}{2}$$

(ii)

(ii) Given $y=\cos 2x-3\sin x$

$$\frac{dy}{dx}=-2\sin 2x-3\cos x$$

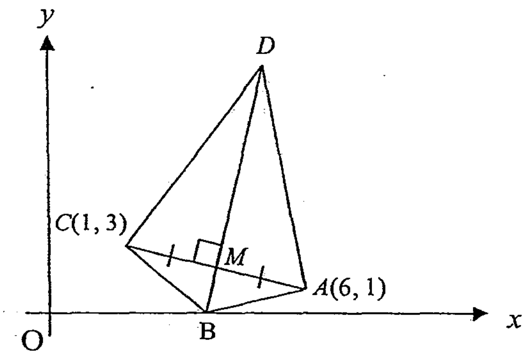
For stationary points, $\frac{dy}{dx}=0$

$$-2\sin 2x-3\cos x=0$$
$$4\sin x\cos x+3\cos x=0$$
$$\cos x(4\sin x+3)=0$$
$$\cos x=0 \quad \text{or} \quad (4\sin x+3)=0$$
$$x=\frac{\pi}{2} \quad \text{or} \quad \sin x=-\frac{3}{4}\text{(reject)}$$

12 **Solutions to this question by accurate drawing will not be accepted.**

10

The diagram shows a quadrilateral $ABCD$ where A is (6, 1), B is on the x-axis and C is (1, 3). The diagonal BD bisects AC at right angles at M and $BD = \frac{7}{2}\ BM$. Find



(i) the equation of BD, [3]

(ii) the x-coordinate of B, [1]

(iii) the coordinates of D. [3]

(i) Coordinates of $M = \left(\frac{1+6}{2}, \frac{3+1}{2}\right) = \left(\frac{7}{2}, 2\right)$

Gradient of $AC = \frac{3-1}{1-6} = -\frac{2}{5}$

Gradient of $BD = \frac{5}{2}$

Equation of BD: $y - 2 = \frac{5}{2}\left(x - \frac{7}{2}\right)$

$y - 2 = \frac{5}{2}x - \frac{35}{4}$

$4y = 10x - 27$

(ii) When $y = 0$, $0 = 10x - 27$

$x = 2\frac{7}{10}$

(iii) $\overrightarrow{BD} = \frac{7}{2}\overrightarrow{BM}$

$\overrightarrow{OD} - \overrightarrow{OB} = \frac{7}{2}\overrightarrow{OM} - \frac{7}{2}\overrightarrow{OB}$

$\overrightarrow{OD} = \frac{7}{2}\overrightarrow{OM} - \frac{5}{2}\overrightarrow{OB}$

$= \frac{7}{2}\begin{pmatrix} \frac{7}{2} \\ 2 \end{pmatrix} - \frac{5}{2}\begin{pmatrix} \frac{27}{10} \\ 0 \end{pmatrix}$

$= \begin{pmatrix} 5\frac{1}{2} \\ 7 \end{pmatrix}$

$\therefore$ D is $\left(5\frac{1}{2}, 7\right)$

`Alternative by similar triangle:

Let the coordinates of D be (a, b).

$\frac{b}{2} = \frac{7}{2}$

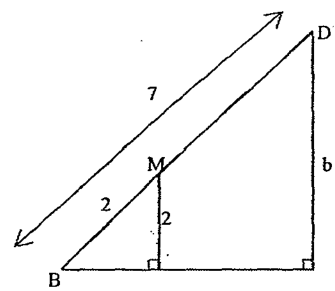
$\therefore$ $b = 7$

$4y = 10x - 27$

$28 = 10a - 27$

$10a = 55$

$a = 5\frac{1}{2}$



$\therefore$ D is $\left(5\frac{1}{2}, 7\right)$ o.e.

---

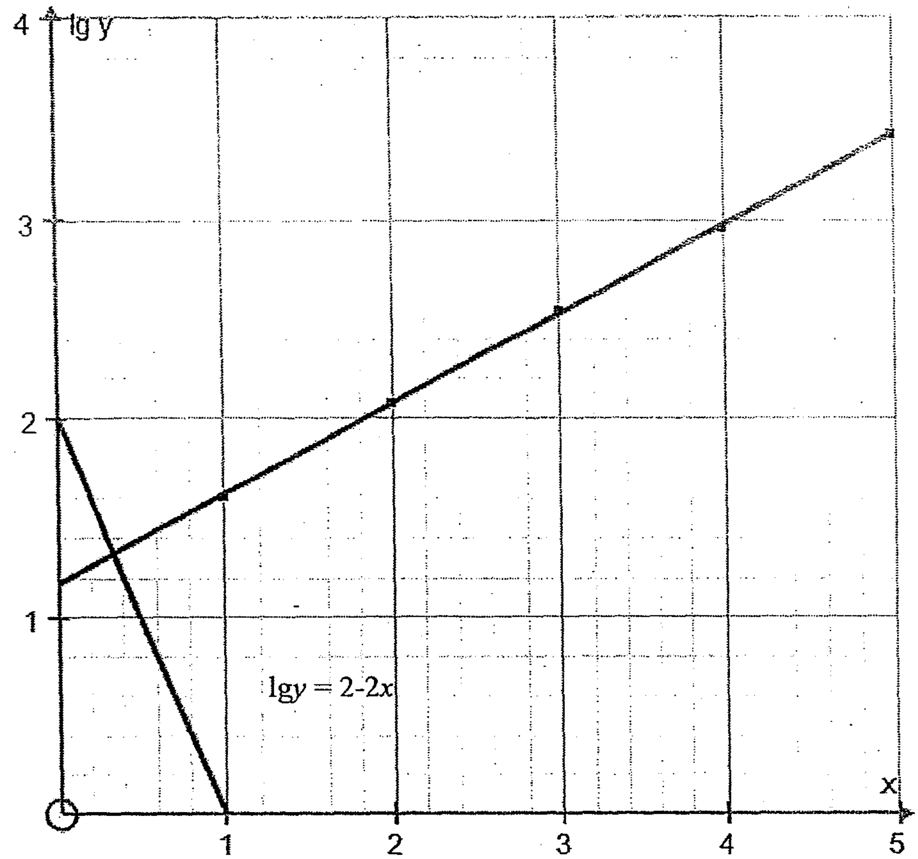
13 The table shows experimental values of two variables, x and y, which are connected by an equation of the form $y = ab^x$, where a and b are constants.

x	1	2	3	4	5
y	42	120	365	920	2700

(i) Using graph paper, draw the graph of $\lg y$ against x. [3]

(ii) Use your graph to estimate the value of a and of b. [4]

(iii) By drawing a suitable straight line on your graph, solve the equation $10^{2-2x} = ab^x$. [2]



$y = ab^x$
$\lg y = \lg a + x \lg b$

Plot lg y against x.

x	1	2	3	4	5
y	1.62	2.07	2.56	2.96	3.43

From the graph,

$\lg a = 1.15$
$a = 14.1$

$$\lg b = \frac{2.96 - 1.15}{4 - 0}$$
$= 0.4525$
$b = 2.83$

$y = 10^{2-2x}$
$\lg y = 2 - 2x$

From the graph, $x = 0.3$

End Of Paper

ANGLICAN HIGH SCHOOL
SECONDARY FOUR PRELIMINARY EXAMINATION
Additional Mathematics Paper 2 (4047/02)

Additional Materials: 8 writing papers | 05 August 2014 | 2 hours 30 minutes

Solutions

1. The line $5y = 6x + 2$ meets the curve $3x^2 + 4xy - 4y^2 = 0$ at the points A and B. Find the coordinates of A and B. [5]

Solution:

Given: $5y = 6x + 2$

$$y = \frac{6}{5}x + \frac{2}{5} \quad \text{............①}$$

sub ① into $3x^2 + 4xy - 4y^2 = 0$

$$3x^2 + 4x(\tfrac{6}{5}x + \tfrac{2}{5}) - 4(\tfrac{6}{5}x + \tfrac{2}{5})^2 = 0$$

$$3x^2 + \frac{24}{5}x^2 + \frac{8}{5}x - 4(\frac{36}{25}x^2 + \frac{24}{25}x + \frac{4}{25}) = 0$$

$$3x^2 + \frac{24}{5}x^2 + \frac{8}{5}x - \frac{144}{25}x^2 - \frac{96}{25}x - \frac{16}{25} = 0$$

$$\frac{51}{25}x^2 - \frac{56}{25}x - \frac{16}{25} = 0$$

$$51x^2 - 56x - 16 = 0$$

$$x = \frac{-(-56) \pm \sqrt{(-56)^2 - 4(51)(-16)}}{2(51)}$$

$$= \frac{56 + 80}{102} \quad \text{or} \quad = \frac{56 - 80}{102}$$

$$= \frac{136}{102} \quad \text{or} \quad = -\frac{24}{102}$$

$$= \frac{4}{3} \quad \text{or} \quad = -\frac{4}{17}$$

Sub $x = \frac{4}{3}$ into①

$$y = \frac{6}{5}(\frac{4}{3}) + \frac{2}{5}$$
$$= 2$$

or sub $x = -\frac{4}{17}$ into①

$$y = \frac{6}{5}(-\frac{4}{17}) + \frac{2}{5}$$
$$= \frac{2}{17}$$

Ans: $A(\frac{4}{3}, 2)$ and $B(-\frac{4}{17}, \frac{2}{17})$

Accept $A(1.33, 2)$ and $B(-0.235, 0.118)$

2. Solve the equation $3^{1-x}+1=10(3^x)$. [5]

Solution

$$3^{1-x}+1=10(3^x)$$
$$\frac{3}{3^x}+1=10\ (3^x)$$
$$3+1(3^x)=10\ (3^x)(3^x)$$
$$3+3^x=10\ (3^{2x})$$
$$10\ (3^{2x})-3^x-3=0$$

Let $u=3^x$

$$10u^2-u-3=0$$
$$(5u-3)(2u+1)=0$$

$$u=\frac{3}{5}$$
$$3^x=\frac{3}{5}$$
$$\ln 3^x=\ln\frac{3}{5}$$
$$x\ln 3=\ln\frac{3}{5}$$
$$x=\ln\frac{3}{5}\div\ln 3$$
$$=-0.46497$$
$$=-0.465$$

$$u=-\frac{1}{2}$$
$$3^x=-\frac{1}{2}\ \text{(No solution)}$$

3. (i) Show that $\frac{d}{dx}\ln\left(\frac{x^2}{\sin x}\right)=\frac{2}{x}-\cot x$. [2]

(ii) Hence evaluate $\int_1^2 \frac{3}{4}\cot x\ dx$. [3]

Solution

(i)

$$\frac{d}{dx}\ln\left(\frac{x^2}{\sin x}\right)=\frac{d}{dx}\left(\ln x^2-\ln\sin x\right)$$
$$=\frac{d}{dx}\left(2\ln x-\ln\sin x\right)$$
$$=2\left(\frac{1}{x}\right)-\frac{1}{\sin x}(\cos x)$$
$$=\frac{2}{x}-\cot x$$

ALTERNATIVE

$$\frac{d}{dx}\ln\left(\frac{x^2}{\sin x}\right)=\frac{1}{\left(\frac{x^2}{\sin x}\right)}\left(\frac{(\sin x)(2x)-(x^2)(\cos x)}{(\sin x)^2}\right)$$
$$=\frac{\sin x}{x^2}\left(\frac{2x\sin x-x^2\cos x}{\sin^2 x}\right)$$
$$=\frac{\sin x}{x^2}\left(\frac{2x\sin x}{\sin^2 x}\right)-\frac{\sin x}{x^2}\left(\frac{x^2\cos x}{\sin^2 x}\right)$$
$$=\frac{2}{x}-\frac{\cos x}{\sin x}$$
$$=\frac{2}{x}-\cot x$$

(ii)

$$\int_1^2\left(\frac{2}{x}-\cot x\right)dx=\left[\ln\frac{x^2}{\sin x}\right]_1^2$$
$$\int_1^2\frac{2}{x}\,dx-\int_1^2\cot x\,dx=\left(\ln\frac{(2)^2}{\sin 2}\right)-\ln\frac{(1)^2}{\sin 1}$$
$$\left[2\ln x\right]_1^2-\int_1^2\cot x\,dx=1.3087$$
$$(2\ln 2-2\ln 1)-\int_1^2\cot x\,dx=1.3088$$
$$\int_1^2\cot x\,dx=2\ln 2-1.3087$$
$$=0.077594$$

$$\int_1^2\frac{3}{4}\cot x\,dx=\frac{3}{4}\times 0.077594$$
$$=0.0581955$$
$$=0.0582$$

4. Given that $(x-1)$ and $(x+2)$ are both factors of the expression $f(x)$ where $f(x)=2x^3+mx^2-7x+n$, find the value of m and of n. Factorise $f(x)$ completely. [5]

Hence, solve the equation $f(x)=2(x-1)(x+2)$. [3]

Solution:

Let $2x^3+mx^2-7x+n=(x-1)(x+2)(2x+c)$

$=(x^2+x-2)(2x+c)$

coeff of x: $-7=c-4$

$c=-3$

coeff of x^2: $m=c+2$

$=-3+2$

$=-1$

constant: $n=-2c$

$=-2(-3)$

$=6$

Therefore,

$f(x)=(x-1)(x+2)(2x-3)$

OR

Given $f(x)=2x^3+mx^2-7x+n$

and $f(1)=0$

$2(1)^3+m(1)^2-7(1)+n=0$

$m+n=5$ ....①

Given $f(-2)=0$

$2(-2)^3+m(-2)^2-7(-2)+n=0$

$4m+n=2$ ....②

②–①, $3m=-3$

$m=-1$

Sub $m=-1$ into ①,

$-1+n=5$

$n=6$

By inspection

$\therefore f(x)=2x^3-x^2-7x+6$

$=(x-1)(x+2)(2x-3)$

Given: $f(x)=2(x-1)(x+2)$

$$(x-1)(x+2)(2x-3)=2(x-1)(x+2)$$

$$(x-1)(x+2)(2x-3)-2(x-1)(x+2)=0$$

$$(x-1)(x+2)[(2x-3)-2]=0$$

$$(x-1)(x+2)(2x-5)=0$$

$$(x-1)=0\ ,\quad (x+2)=0\ ,\quad (2x-5)=0$$

$$x=1\ ,\qquad x=-2\ ,\qquad x=\tfrac{5}{2}$$

5. Solve the following equations, where $0^\circ < x < 360^\circ$.

i) $2\ \text{cosec}^2 \frac{x}{2} = \sin \frac{x}{2}$ [4]

ii) $5 \sin^2 x + 9 \cos x - 9 = 0$ [5]

Solution:

i) $2\ \text{cosec}^2 \frac{x}{2} = \sin \frac{x}{2}$ $[0^\circ < \frac{x}{2} < 180^\circ]$

$$\frac{2}{\sin^2 \frac{x}{2}} = \sin \frac{x}{2}$$

$$\sin^3 \frac{x}{2} = 2$$

$$\sin \frac{x}{2} = \sqrt[3]{2}$$

no solution $[\because \sqrt[3]{2} > 1]$

ii) $5 \sin^2 x + 9 \cos x - 9 = 0$ $[0^\circ < x < 360^\circ]$

$$5(1 - \cos^2 x) + 9 \cos x - 9 = 0$$

$$5 - 5\cos^2 x + 9 \cos x - 9 = 0$$

$$5 \cos^2 x - 9 \cos x + 4 = 0$$

$$(5\cos x - 4)(\cos x - 1) = 0$$

$\cos x = \frac{4}{5}$ or $\cos x = 1$

$x = 36.869^\circ, 360^\circ - 36.869^\circ$ $x = 0^\circ, 360^\circ$ (all rejected)

$\approx 36.869^\circ, 323.131^\circ$

$\approx 36.9^\circ, 323.1^\circ$

6 (a) If $\log_x \frac{p}{\sqrt{q}} - 3\log_x \sqrt{q} = \log_x (p-q)$, express p in terms of q. [3]

Solution

6a)

$$\log_x \frac{p}{\sqrt{q}} - 3\log_x \sqrt{q} = \log_x (p-q)$$

$$\log_x \frac{p}{q^{\frac{1}{2}}} - \log_x q^{\frac{3}{2}} = \log_x (p-q)$$

$$\log_x \left(\frac{p}{q^{\frac{1}{2}}} \div q^{\frac{3}{2}} \right) = \log_x (p-q)$$

$$\log_x \left(\frac{p}{q^{\frac{1}{2}} \times q^{\frac{3}{2}}} \right) = \log_x (p-q)$$

$$\frac{p}{q^2} = p - q$$

$$p = pq^2 - q^3$$

$$p - pq^2 = -q^3$$

$$p\left(1-q^2\right) = -q^3$$

$$p = \frac{-q^3}{1-q^2}$$

$$= \frac{q^3}{q^2-1}$$

6 (b) Given that $a = \log_2 3$ and $b = \log_2 7$, express $\log_2 21 + \log_4 \frac{16}{7}$ in terms of a and b, [4]

Solution

6b)

$$\log_2 21 + \log_4 \frac{16}{7} = \log_2 (3 \times 7) + \left(\log_4 2^4 - \log_4 7\right)$$

$$= \log_2 3 + \log_2 7 + \left(\frac{\log_2 2^4}{\log_2 4} - \frac{\log_2 7}{\log_2 4}\right)$$

$$= \log_2 3 + \log_2 7 + \left(\frac{4\log_2 2}{\log_2 2^2} - \frac{\log_2 7}{\log_2 2^2}\right)$$

$$= a + b + \left(\frac{4\,(1)}{2\log_2 2} - \frac{b}{2\log_2 2}\right)$$

$$= a + b + \left(\frac{4}{2\,(1)} - \frac{b}{2(1)}\right)$$

$$= a + b + 2 - \frac{1}{2}b$$

$$= 2 + a + \frac{1}{2}b$$

---------------------------------- Or ----------------------------------

$$\log_2 21 + \log_4 \frac{16}{7} = \log_2 21 + \frac{\log_2 \frac{16}{7}}{\log_2 4}$$

$$= \log_2 3 + \log_2 7 + \frac{1}{2}(\log_2 2^4 - \log_2 7)$$

$$= \log_2 3 + \log_2 7 + 2 - \frac{1}{2}\log_2 7$$

$$= \log_2 3 + 2 + \frac{1}{2}\log_2 7$$

$$= a + 2 + \frac{1}{2}b$$

6. (c) **Without using the calculator**, simplify the following

$$\frac{(\sqrt[10]{x}+1)(x^{\frac{21}{10}}-x^2)}{\sqrt[5]{x}-1}$$ [3]

Solution:

6c) $$\frac{(\sqrt[10]{x}+1)(x^{\frac{21}{10}}-x^2)}{\sqrt[5]{x}-1}=\frac{(\sqrt[10]{x}+1)(x^2)(x^{\frac{1}{10}}-1)}{\sqrt[5]{x}-1}$$

$$=\frac{((\sqrt[10]{x})^2-1)(x^2)}{\sqrt[5]{x}-1}$$

$$=\frac{(\sqrt[5]{x}-1)(x^2)}{\sqrt[5]{x}-1}$$

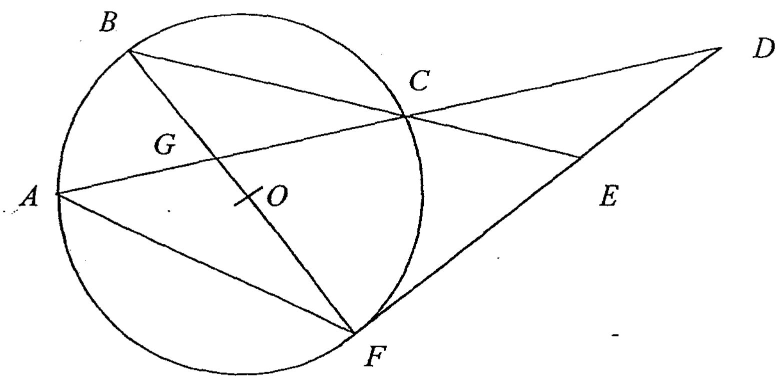
$$=x^2$$

7. In the diagram, not drawn to scale, A, B, C and F lie on a circle with centre, O. $BGOF$, $AGCD$, BCE and FED are straight lines, where FED is a tangent to the circle at F.

Prove that

(a) $\angle FCD = \angle AFD$, and [3]

(b) $FC \times FE = BF \times CE$ [3]



Solution

(a) $\angle DFC = \angle FAD$ (Alternate Segment Theorem / Tangent Chord Theorem)

$\angle CDF = \angle ADF$ (Common angle)

Triangle CDF and triangle FDA are similar (AA property).

Hence $\angle FCD = \angle AFD$.

(b) $\angle BCF = 90°$ (Angle in a semicircle)

$\angle FCE = 90°$ (Sum of angles on a straight line)

$\angle BCF = \angle FCE$

$\angle FBC = \angle EFC$ (Alternate Segment Theorem / Tangent Chord Theorem)

Triangle BFC and triangle FEC are similar (AA property).

$$\frac{BF}{FC} = \frac{FE}{EC}$$

$BF \times EC = FE \times FC$

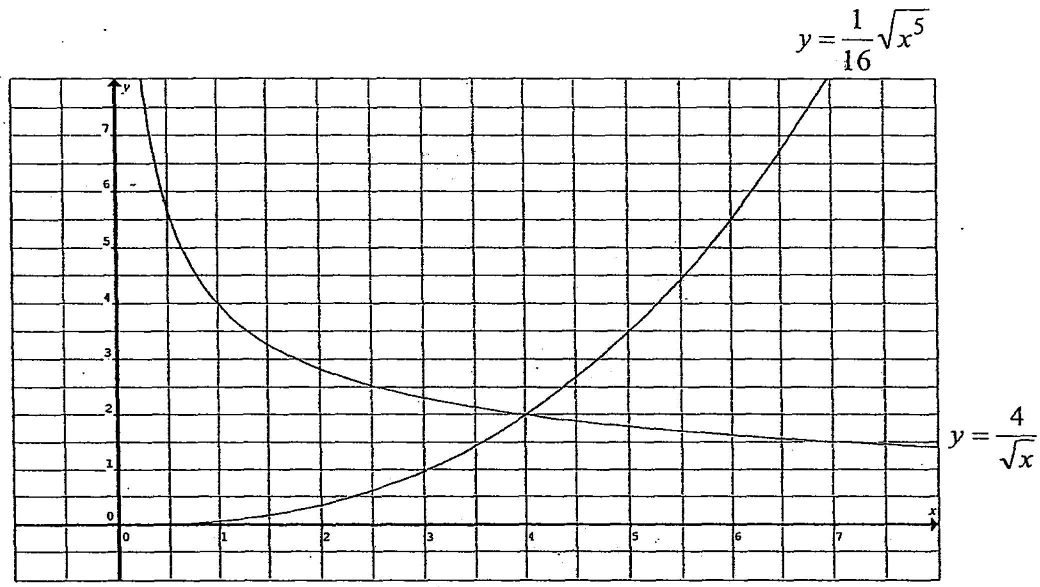
Hence $FC \times FE = BF \times CE$.

8. On the same diagram, sketch the graphs of $y=\frac{1}{16}\sqrt{x^5}$ and $y=\frac{4}{\sqrt{x}}$ for $x \geq 0$. [4]

(i) Find the point of intersection of the graphs. [2]

(ii) Calculate the area enclosed by the curve $y=\frac{1}{16}\sqrt{x^5}$, $y=3$, $x=5$ and $x=8$. [4]

Solution



(i) $\frac{1}{16}\sqrt{x^5}=\frac{4}{\sqrt{x}}$

$$\frac{1}{16}x^{\frac{5}{2}}=4x^{-\frac{1}{2}}$$

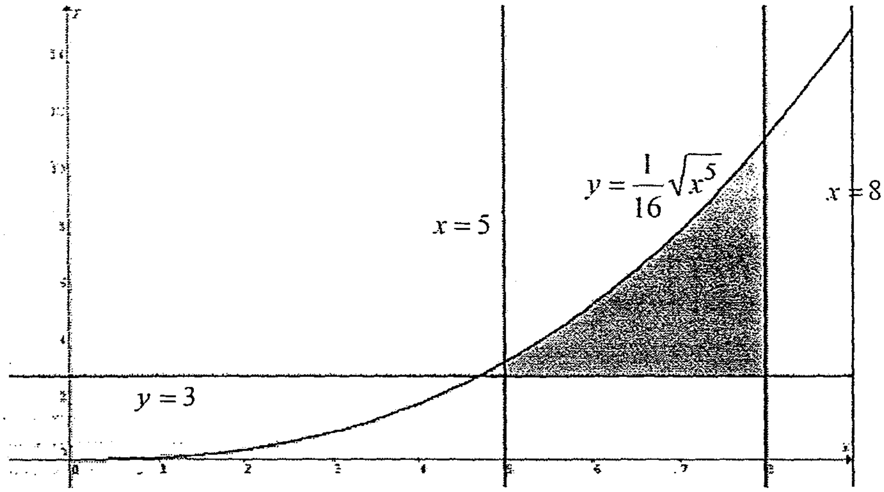
$$\left(x^{\frac{5}{2}}\right)\left(x^{\frac{1}{2}}\right)=(4)(16)$$

$$x^3=4^3$$
$$x=4$$

When $x=4$, $y=\frac{4}{\sqrt{4}}=2$.

Therefore the point of intersection is $(4, 2)$

8(ii)



$$\text{Area} = \int_5^8 \left(\frac{1}{16}x^{\frac{5}{2}} - 3\right)\, dx$$

$$= \left[\frac{1}{16}\left(\frac{2}{7}\right)x^{\frac{7}{2}} - 3x\right]_5^8$$

$$= \left(\frac{1}{56}(8)^{\frac{7}{2}} - 3(8)\right) - \left(\frac{1}{56}(5)^{\frac{7}{2}} - 3(5)\right)$$

$$= 11.868681$$

$$\approx 11.9 \ \text{units}^2$$

Or

$$\text{Area} = \int_5^8 \left(\frac{1}{16}\, x^{\frac{5}{2}}\right)\, dx - 3(8-5)$$

$$= \left[\frac{1}{56}\, x^{\frac{7}{2}}\right]_5^8 - 9$$

$$= \left(\frac{1}{56}(8)^{\frac{7}{2}} - \frac{1}{56}(5)^{\frac{7}{2}}\right) - 9$$

$$= 20.868681 - 9$$

$$= 11.868681$$

$$\approx 11.9 \ \text{units}^2$$

9. (a) The function f is defined by $f(x) = c - a \sin bx$.
Given that the period of $f(x)$ is 720° and $-1 \le f(x) \le 3$.
State the value of a, of b and of c.

(b) Sketch the graphs of $y = |3 \cos 2x|$ and $y = 2 - \frac{2}{\pi}x$ on the same axes.
Hence, state the number of solution for $|3\pi \cos 2x| + 2x = 2\pi$.

Solution:

(a) $a = \pm 2$,

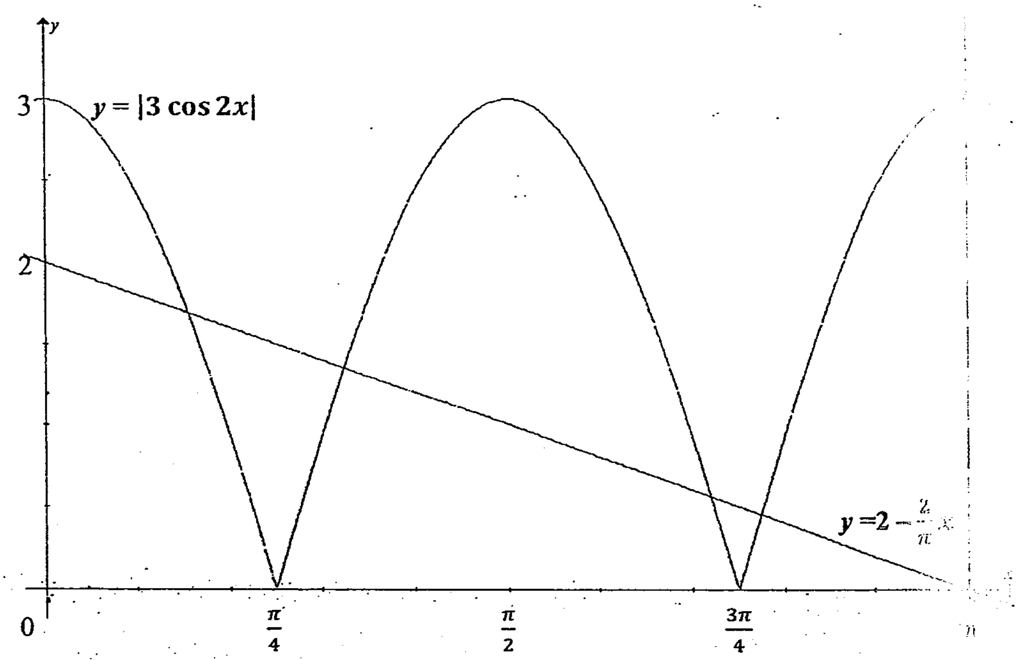
$b = \frac{1}{2}$

$c = 1$

$f(x) = 1 - 2 \sin \frac{x}{2}$

For $f(x) = c - a \sin bx$
Given period of $f(x) = 720^\circ$ $\therefore$ $b = 360^\circ \div 720^\circ$
$= \frac{1}{2}$
Given: $-1 \le f(x) \le 3$. $\therefore$ a = amplitude
$= \frac{1}{2}(3 - (-1))$
$= 2$
c = max value − amplitude
$= 3 - 2$
$= 1$

(b) Given: $y = |3 \cos 2x|$ and $y = 2 - \frac{2}{\pi}x$ for $0 \le x \le \pi$.



$|3\pi \cos 2x| + 2x = 2\pi$.

$|3 \cos 2x| + \frac{2}{\pi}x = 2$

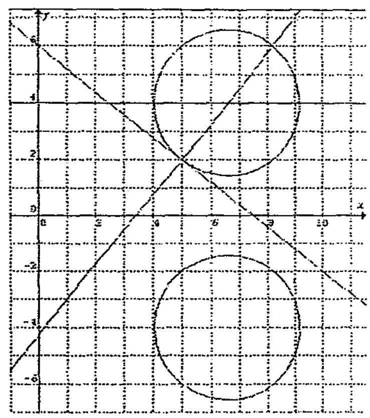
$|3 \cos 2x| = 2 - \frac{2}{\pi}x$

Number of solution = 4

10. Given that the gradient of the tangent to the circle, C_1, at $A(5,2)$ is $-\frac{4}{5}$, find the equation of the normal to the circle at A. [2]
The circle, C_1, also passes through the point $B(5, 6)$. Find the centre of the circle, C_1. [3]
Hence, find the equation of circle, C_1. [3]
A circle, C_2, is formed when the circle, C_1, is reflected along the line $y = 0$.
Write down the equation of the circle, C_2. [1]

Solution:

Gradient of normal $= -\frac{1}{\left(-\frac{4}{5}\right)} = \frac{5}{4}$



Equation of normal:

$$y-2=\frac{5}{4}(x-5)$$

$$y=\frac{5}{4}x-\frac{17}{4}$$

Mid-point of (5, 2) and (5, 6) $= \left(5, \frac{2+6}{2}\right)$

$= (5, 4)$

Perpendicular bisector of AB is $y=4$

The perpendicular bisector will intersect the normal at the centre of the circle.

Consider the intersection of $y=4$ and $y=\frac{5}{4}x-\frac{17}{4}$

$$4=\frac{5}{4}x-\frac{17}{4}$$

$$\frac{5}{4}x=\frac{33}{4}$$

$$x=\frac{33}{5}$$

Centre of the circle is $\left(\frac{33}{5}, 4\right)$

Radius of the circle $=\sqrt{\left(5-\frac{33}{5}\right)^2+(2-4)^2}$

$=\sqrt{\frac{164}{25}}$

Equation of Circle, C_1:

$$\left(x-\frac{33}{5}\right)^2+(y-4)^2=\left(\sqrt{\frac{164}{25}}\right)^2$$

$$\left(x-\frac{33}{5}\right)^2+(y-4)^2=\frac{164}{25}$$

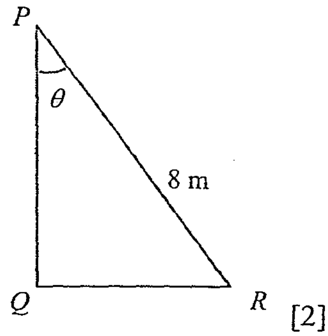
Accept $5x^2+5y^2-66x-40y+265=0$

Equation of Circle, C_2:

$$\left(x-\frac{33}{5}\right)^2+(y+4)^2=\frac{164}{25}$$

Accept $5x^2+5y^2-66x+40y+265=0$

11. A gardener is going to fence off part of his garden by using a piece of 19 metres long fence. The enclosed area is in the shape of a right-angled triangle PQR with $PR = 8$ m.



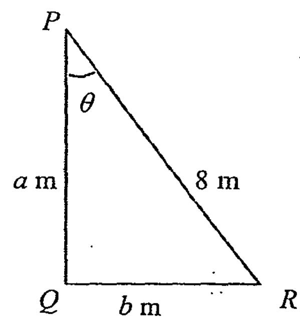
Given that θ is the included angle between the sides PQ and PR,

i) show that $8\cos\theta + 8\sin\theta = 11$. [2]

Hence, express the equation in the form $R\cos(\theta - \alpha) = C$, where C is an integer, $R > 0$ and $0° < \alpha < 90°$. [2]

ii) find the possible values of θ. [3]

Solution:



i) $a = 8\cos\theta$

$b = 8\sin\theta$

$8\cos\theta + 8\sin\theta + 8 = 19$

$8\cos\theta + 8\sin\theta = 11$ (shown)

$(\sqrt{8^2+8^2})\cos(\theta - \tan^{-1}\frac{8}{8}) = 11$ $\quad [-45° < \theta - 45° < 45°]$

$\sqrt{128}\cos(\theta - 45°) = 11$

$8\sqrt{2}\cos(\theta - 45°) = 11$

ii) $\cos(\theta - 45°) = \frac{11}{\sqrt{128}}$

$\theta - 45° \approx -13.524°, 13.524°$

$\theta \approx -13.524° + 45°, 13.524° + 45°$

$\approx 31.476°, 58.524°$

$\approx 31.5°, 58.5°$

Ans: the possible values of $\theta \approx 31.5°, 58.5°$

12 A particle moves in a straight line with velocity, v m/s, given by $v = t^2 - 8t + 15$ where t is the time in seconds, measured from the start of the motion. Its initial displacement from a fixed point O is -6 metres.

(a) Find

(i) the values of t for which the particle is at instantaneous rest [2]

(ii) the minimum velocity of the particle [2]

(b) Determine the average speed of the particle in the first 7 seconds. [5]

(c) Show that the particle does not return to its starting point. [1]

Solution

(a)(i) At instantaneous rest, $v = 0$

$$t^2 - 8t + 15 = 0$$
$$(t-3)(t-5) = 0$$
$$t = 3 \text{ s} \quad \text{or } t = 5 \text{ s}$$

(a)(ii) At maximum / minimum velocity, $a = 0$

$$a = \frac{dv}{dt}$$
$$= 2t - 8$$

$$a = 0$$
$$2t - 8 = 0$$
$$t = 4$$

Check for max / min, $\frac{d^2v}{dt^2} = 2 > 0$, $t = 4$ s gives minimum velocity.

Minimum velocity,

$$v = (4)^2 - 8(4) + 15 = -1 \text{ m/s}$$

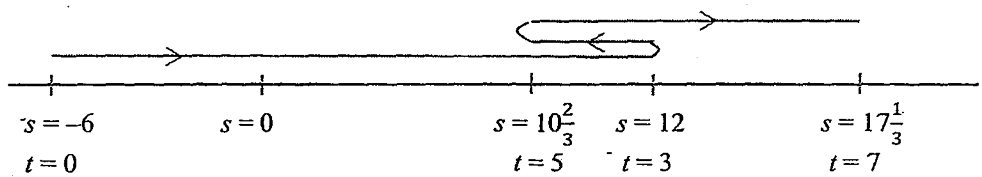
(b) Displacement, $s = \int v \, dt$

$$s = \int (t^2 - 8t + 15) dt$$
$$= \frac{1}{3}t^3 - 4t^2 + 15t + c$$

When $t = 0$, $s = -6$, therefore $-6 = \frac{1}{3}(0)^3 - 4(0)^2 + 15(0) + c$, and $c = -6$

Hence $s = \frac{1}{3}t^3 - 4t^2 + 15t - 6$

Time, t (sec)	Displacement, s (m)	Distance travelled (m)
0	-6	
3	12	$12-(-6)=18$
5	$10\frac{2}{3}$	$12-\left(10\frac{2}{3}\right)=1\frac{1}{3}$
7	$17\frac{1}{3}$	$17\frac{1}{3}-\left(10\frac{2}{3}\right)=6\frac{2}{3}$



$$\text{Total distance travelled} = 18+1\frac{1}{3}+6\frac{2}{3}$$
$$= 26 \text{ m}$$

$$\text{Average Speed} = \frac{26}{7}$$
$$= 3\frac{5}{7} \quad \text{or} \quad 3.7143 \approx 3.71 \text{ m/s}$$

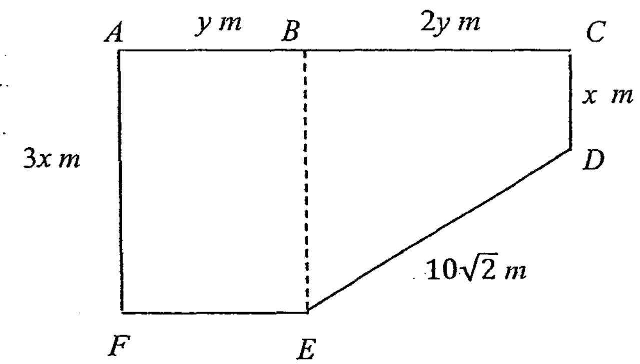
(c) At the second turning point, the displacement, $s=1\frac{1}{3}$ m. The displacement increases after that, and never becomes -6 m. Hence, the particle never returns to its starting point.

13. The diagram, not drawn to scale, shows the plan of a park, $ABCDEF$. $BCDE$ is a trapezium, with CD parallel to BE. $ABEF$ is a rectangle and ABC is a straight line. Given that $DE = 10\sqrt{2}$ metres, $AB = y$ metres, $BC = 2y$ metres, $CD = x$ metres and $AF = 3$ metres.

(i) Express y in terms of x. [2]

(ii) Show that the area of the park, A m^2, is given by $A = 7x\sqrt{50-x^2}$ [2]

(iii) Determine the value of x that will give the maximum area. [5]



Solution

(i) $(2y)^2 + (2x)^2 = (10\sqrt{2})^2$ (Pythagoras' Theorem)

$$4y^2 + 4x^2 = 200$$
$$4y^2 = 200 - 4x^2$$
$$y = \sqrt{50 - x^2} \quad (y > 0)$$

(ii) $A = (3x)(y) + \frac{1}{2}(x + 3x)(2y)$

$$A = 3xy + 4xy$$
$$= 7xy$$
$$= 7x\sqrt{50 - x^2}$$

13(iii) $\frac{dA}{dx} = 7\sqrt{50 - x^2} + 7x\left(\frac{1}{2}\right)\left(50 - x^2\right)^{-\frac{1}{2}}(-2x)$

When $\frac{dA}{dx} = 0$

$$7\sqrt{50 - x^2} + 7x\left(\frac{1}{2}\right)\left(50 - x^2\right)^{-\frac{1}{2}}(-2x) = 0$$

$$7\sqrt{50 - x^2} - \frac{7x^2}{\sqrt{50 - x^2}} = 0$$

$$7\sqrt{50 - x^2} = \frac{7x^2}{\sqrt{50 - x^2}}$$

$$\sqrt{50 - x^2} = \frac{x^2}{\sqrt{50 - x^2}}$$

$$50 - x^2 = x^2$$

$$2x^2 = 50$$

$$x^2 = 25$$

$x = 5$ or $x = -5$ (NA)

Not necessary to prove

x	Slightly < 5	5	Slightly > 5
$\frac{dA}{dx}$	> 0	0	< 0
Or sketch of $\frac{dA}{dx}$	/	—	\

-------- Or ----------

13(iii) $\frac{dA}{dx} = 7\sqrt{50 - x^2} + 7x\left(\frac{1}{2}\right)\left(50 - x^2\right)^{-\frac{1}{2}}(-2x)$

$= 7(50 - x^2)^{-\frac{1}{2}}(50 - 2x^2)$

When $\frac{dA}{dx} = 0$

$7(50 - x^2)^{-\frac{1}{2}}(50 - 2x^2) = 0$

$\Rightarrow \quad (50 - 2x^2) = 0$

$2x^2 = 50$

$x^2 = 25$

$x = 5$ or $x = -5$ (NA)

**** END OF PAPER *****

Calculator Model:

Name:	Class	Class Register Number/ Centre No./Index No.

中正中学

CHUNG CHENG HIGH SCHOOL (MAIN)

PRELIMINARY EXAMINATION 2014
SECONDARY 4

Additional Mathematics — 4047/01

Paper 1

Tuesday 26 August 2014
2 hours

Additional Materials: Answer Paper
Graph Paper

READ THESE INSTRUCTIONS FIRST

Do not open this booklet until you are told to do so.

Write your name, class and index number clearly in the spaces provided at the top of this page.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams, graphs or rough working.
Do not use highlighters or correction fluid.

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The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

/ 80

1. ALGEBRA

Quadratic Equation

For the equation $ax^2+bx+c=0$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Binomial expansion

$$(a+b)^n=a^n+\binom{n}{1}a^{n-1}b+\binom{n}{2}a^{n-2}b^2+\ldots+\binom{n}{r}a^{n-r}b^r+\ldots+b^r,$$

where n is a positive integer and $\binom{n}{r}=\frac{n!}{r!(n-r)!}=\frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A+\cos^2 A=1$$

$$\sec^2 A=1+\tan^2 A$$

$$\operatorname{cosec}^2 A=1+\cot^2 A$$

$$\sin(A\pm B)=\sin A\cos B\pm\cos A\sin B$$

$$\cos(A\pm B)=\cos A\cos B\mp\sin A\sin B$$

$$\tan(A\pm B)=\frac{\tan A\pm\tan B}{1\mp\tan A\tan B}$$

$$\sin 2A=2\sin A\cos A$$

$$\cos 2A=\cos^2 A-\sin^2 A=2\cos^2 A-1=1-2\sin^2 A$$

$$\tan 2A=\frac{2\tan A}{1-\tan^2 A}$$

Formula for ΔABC

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

$$a^2=b^2+c^2-2bc\cos A$$

$$\Delta=\frac{1}{2}bc\sin A$$

1 Given that α and β are the roots of a quadratic equation where $\alpha\beta = 4\frac{1}{4}$ and $\alpha^2 + \beta^2 = -2\frac{1}{4}$.

(i) Find the value of $\alpha + \beta$ where $\alpha + \beta < 0$. [2]

(ii) Show that $\alpha^3 + \beta^3 = 16\frac{1}{4}$. [1]

(iii) Find a quadratic equation whose roots are $\frac{\alpha^2}{\beta}$ and $\frac{\beta^2}{\alpha}$. [3]

2 Given that curve $y = x^2$ lies above the line $y = px - q^2$ for $-2 < p < 2$. Find the value(s) of constant q. [4]

3 The equation of a cuve is $y = 2xe^{3-x^2}$. Find the x-coordinates of the stationary points, leaving your answer in exact form and determine the nature of the stationary points. [5]

4 It is given that $2^{2x+1} + 4^{x-1} = 2\left(3^{1-x}\right)$.

(i) Show that $12^x = 2\frac{2}{3}$. [3]

(ii) Find the value of x correct to 2 decimal places. [1]

5 Given that $\int_0^{\frac{\pi}{4}} \mathrm{f}(x)\,\mathrm{d}x = 5$ and $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \mathrm{f}(x)\,\mathrm{d}x = 2$, find the value of a given that $\int_{\frac{\pi}{2}}^{0} \left(\mathrm{f}(x) + a\sin 2x\right) = 10$. [4]

6 A particle travels in a straight line so that t seconds after passing a fixed point O with a velocity of 15 cms^{-1}, its acceleration a cms^{-2} is given by $a = k - 2t$ where k is a constant. The particle reaches maximum velocity in 1 second. Find

(i) the value of k, [1]

(ii) an expression for the velocity of the particle in terms of t, [2]

(iii) the distance travelled in the first 6 seconds. [4]

7 (i) Show that $\dfrac{2\cos^2\theta - \sin\theta\cos\theta + 1}{\sin^2\theta} = 3\cot^2\theta - \cot\theta + 1$. [3]

(ii) Hence solve the equation $2\cos^2\theta - \sin\theta\cos\theta + 1 = \sin^2\theta$ for $-\pi \le \theta \le \pi$. [4]

8 Variables x and y are connected by the equation $y = ax^2 + b\sqrt{x}$, where a and b are constants. When a graph of $\dfrac{y}{\sqrt{x}}$ against $x\sqrt{x}$ is plotted using experimental values of x and y, a straight line is obtained and passes through the point $(1,\ 5)$. The straight line makes an angle of 45° with the $x\sqrt{x}$-axis.

(i) Find the values of a and of b. [4]

(ii) Find the coordinates of the point on the line at which $y = 3\sqrt{x}$. [3]

9 (i) Sketch the graph $y = |x^2 - 4x|$ indicating the intercepts and coordinates of the turning point. [3]

(ii) In each of the following case, determine the number of solutions of the equation $|x^2 - 4x| = mx + c$ where $0 < c < 4$, justifying your answer.

(a) $m = 0$, [1]

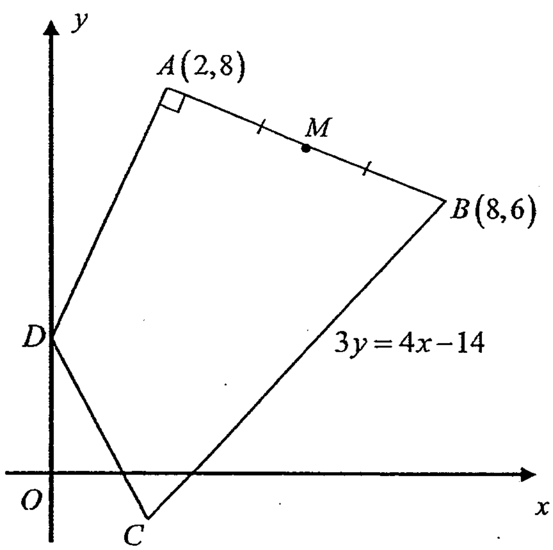
(b) $m = -1$. [2]

10 The diagram shows a quadrilateral $ABCD$ in which $A(2, 8)$ and $B(8,6)$. M is the midpoint of AB and CM is perpendicular to AB. The equation of BC is $3y = 4x - 14$. The point D lies on the y-axis and $\angle DAB = 90°$. Find

(i) the coordinates of D, [2]

(ii) the coordinates of C, [5]

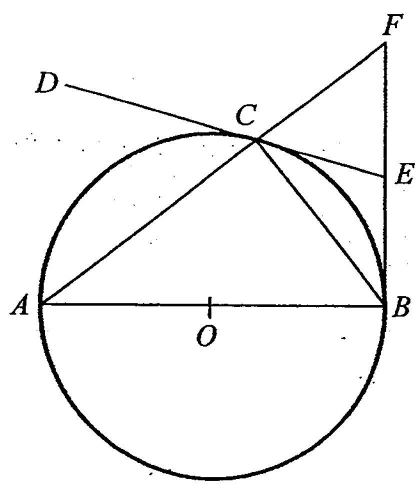
(iii) the ratio of $AD : CM$. [2]



11 In the diagram, AB is a diameter of the circle with centre O. DE and BF are tangents to the circle at C and B respectively. DCE and BEF are straight lines. Prove that

(i) ΔABC and ΔAFB are similar, [3]

(ii) ΔCFE is isosceles. [4]

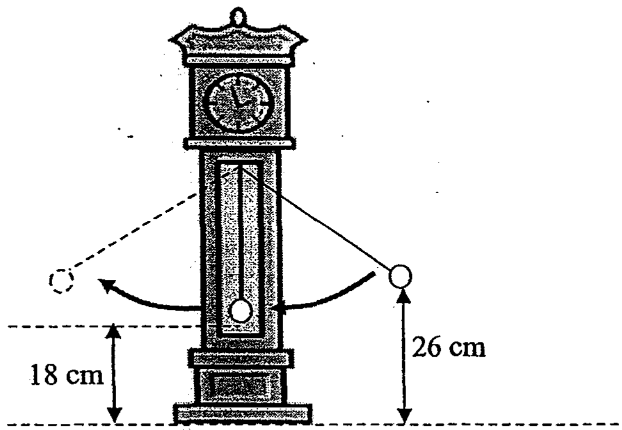


12 Sand is poured onto a flat surface at a rate of 2π cm^{3}s^{-1} and formed a right circular cone. The radius of the cone is always $\frac{1}{3}$ of its height.

(i) Find the rate of change of the radius 3 seconds after the start of pouring. [5]

(ii) State, with a reason, whether this rate will increase or decrease as t increases. [1]

13



The pendulum of a grandfather clock swings back and forth with a periodic motion that can be represented by the equation $h = a\cos kt + b$, where a, k and b are constants, h cm is the height of the pendulum above the base and t is the time in seconds after the pendulum is released 26 cm above the base. At rest, the pendulum is 18 cm above the base. It takes four seconds for a complete swing back and forth.

(i) State the value of a and b. [2]

(ii) Show that the value of k is $\frac{\pi}{2}$ rad/s. [1]

(iii) Sketch the graph of $h = a\cos kt + b$ for $0 \le t \le 6$. [2]

(iv) Find the time interval in which the pendulum is 20 cm above the base during one revolution. [3]

7

Answer Key

1. (i) $-\frac{5}{2}$

   (iii) $x^2 - \frac{65}{17}x + 4\frac{1}{4} = 0$

2. $q = \pm 1$

3. At $x = -\frac{1}{\sqrt{2}}$ or $-\frac{\sqrt{2}}{2}$, the point is a minimum.

   At $x = \frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$, the point is a maximum. ------------ A1

4. (ii) 0.39

5. $a = -17$

6. (i) $k = 2$

   (ii) $v = 2t - t^2 + 15$

   (iii) $62\frac{2}{3}$ cm

7. (ii) $\theta = -1.89$ or 1.25

8. (i) $a = 1,\ b = 4$

   (ii) $(-1,\ 3)$

10. (i) $D(0,2)$

    (ii) $C(2,-2)$

    (iii) $AD : CM = 2 : 3$

12. (i) 0.202 cm/s

13. (i) $a = 4,\ b = 22$

    (iv) $0 \le t \le \frac{2}{3}$ and $\frac{4}{3} \le t \le 2$

Calculator Model:

Name:	Class	Class Register Number/ Centre No./Index No.

中正中学

CHUNG CHENG HIGH SCHOOL (MAIN)

PRELIMINARY EXAMINATION 2014
SECONDARY 4

Additional Mathematics — 4047/01

Paper 1 ***(Marking Scheme)***

26 August 2014

2 hours

Additional Materials: Answer Paper
Graph Paper

READ THESE INSTRUCTIONS FIRST

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Quadratic Equation

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Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

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Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formula for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 Given that α and β are the roots of a quadratic equation where $\alpha\beta = 4\frac{1}{4}$ and $\alpha^2 + \beta^2 = -2\frac{1}{4}$.

(i) Find the value of $\alpha + \beta$ where $\alpha + \beta < 0$. [2]

(ii) Show that $\alpha^3 + \beta^3 = 16\frac{1}{4}$. [1]

(iii) Find a quadratic equation whose roots are $\frac{\alpha^2}{\beta}$ and $\frac{\beta^2}{\alpha}$. [3]

Marking Scheme

(i) $(\alpha+\beta)^2 = \alpha^2 + 2\alpha\beta + \beta^2$ --------- M1 c.a.o

$$= -2\frac{1}{4} + 2\left(4\frac{1}{4}\right)$$

$$= \frac{25}{4}$$

$\alpha + \beta = -\sqrt{\frac{25}{4}}$ (given that $\alpha + \beta < 0$)

$= -\frac{5}{2}$ -------- A1

(ii) $\alpha^3 + \beta^3 = (\alpha+\beta)(\alpha^2 - \alpha\beta + \beta^2)$ ---------- B1 c.a.o

$$= \left(-\frac{5}{2}\right)\left(-2\frac{1}{4} - 4\frac{1}{4}\right)$$

$= 16\frac{1}{4}$ -------- (A.G)

(iii) Sum of roots: $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} = \frac{\alpha^3 + \beta^3}{\alpha\beta}$ ------ M1 c.a.o

$$= 16\frac{1}{4} \div 4\frac{1}{4}$$

$$= \frac{65}{17}$$

Product of roots: $\frac{\alpha^2}{\beta} \times \frac{\beta^2}{\alpha} = \alpha\beta$ ------- M1 c.a.o

$$= 4\frac{1}{4}$$

The quadratic equation is: $x^2 - \frac{65}{17}x + 4\frac{1}{4} = 0$ -------- A1

2 Given that curve $y=x^2$ lies above the line $y=px-q^2$ for $-2<p<2$. Find the value(s) of constant q.

Marking Scheme

$x^2 > px-q^2$ ---------- M1 c.a.o

$x^2 - px + q^2 > 0$

For no real roots, Discriminant <0 -------- M1 c.a.o

$(-p)^2 - 4(1)(q^2) < 0$

$p^2 - 4q^2 < 0$

$-2<p<2 \Rightarrow (p-2)(p+2)<0$ -------- M1 c.a.o

$p^2 - 4 < 0$

By comparison,

$4q^2 = 4$

$q = \pm 1$ --------- A1

3 The equation of a cuve is $y = 2xe^{3-x^2}$. Find the x-coordinates of the stationary points, leaving your answer in exact form and determine the nature of the stationary points. [5]

Marking Scheme

$y = 2xe^{3-x^2}$

$\frac{dy}{dx} = 2x\left(-2xe^{3-x^2}\right) + e^{3-x^2}(2)$ -------- M1 c.a.o

$= 2e^{3-x^2}\left(-2x^2 + 1\right)$

At stationary pt, $\frac{dy}{dx} = 0$.

$2e^{3-x^2}\left(-2x^2 + 1\right) = 0$ ---------- M1

$x^2 = \frac{1}{2}$ or $2e^{3-x^2} = 0$ (N.A)

$x = \pm\frac{1}{\sqrt{2}}$ or $\pm\frac{\sqrt{2}}{2}$ ------- M1

x	-1	$-\frac{1}{\sqrt{2}}$	0
$\frac{dy}{dx}$	−14.8	0	40.2

At $x = -\frac{1}{\sqrt{2}}$ or $-\frac{\sqrt{2}}{2}$, the point is a minimum. ------------ A1

x	0	$\frac{1}{\sqrt{2}}$	1
$\frac{dy}{dx}$	40.2	0	−14.8

At $x = \frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$, the point is a maximum. ----------- A1

4 It is given that $2^{2x+1}+4^{x-1}=2\left(3^{1-x}\right)$.

(i) Show that $12^x=2\frac{2}{3}$. [3]

(ii) Hence find the value of x correct to 2 decimal places. [1]

Marking Scheme

(i) $2^{2x+1}+4^{x-1}=2\left(3^{1-x}\right)$

$2^{2x}\times 2+2^{2x}\times 2^{-2}=2\left(3\times 3^{-x}\right)$ --------- M1

$2^{2x}\left(2+\frac{1}{4}\right)=6\times 3^{-x}$ ------------ M1

$4^x\times 3^x=\dfrac{6}{2\frac{1}{4}}$ --------- M1

$12^x=2\frac{2}{3}$ (A.G)

(ii) $\lg 12^x=\lg 2\frac{2}{3}$

$x=\dfrac{\lg 2\frac{2}{3}}{\lg 12}$

$=0.39$ (2 d.p) --------- B1

5 Given that $\int_0^{\frac{\pi}{4}} f(x)\ dx = 5$ and $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} f(x)\ dx = 2$, find the value of a given that

$\int_{\frac{\pi}{2}}^{0} \left(f(x) + a\sin 2x\right) = 10.$ [4]

Marking Scheme

$$\int_{\frac{\pi}{2}}^{0} \left(f(x) + ax\right) = 10$$

$$\int_{\frac{\pi}{2}}^{0} f(x)\ dx + \int_{\frac{\pi}{2}}^{0} a\sin x\ dx = 10$$

$$-\int_0^{\frac{\pi}{2}} f(x)\ dx + \left[-\frac{a\cos 2x}{2}\right]_{\frac{\pi}{2}}^{0} = 10$$ ----- M1 c.a.o , M1c.a.o

$$-(5+2) + \left(-\frac{a}{2} - \frac{a}{2}\right) = 10$$ ----- M1 c.a.o

$$-7 - a = 10$$

$$a = -17$$ -------- A1

6 A particle travels in a straight line so that t seconds after passing a fixed point O with a velocity of 15 cms^{-1}, its acceleration a cms^{-2} is given by $a = k - 2t$ where k is a constant. The particle reaches maximum velocity in 1 second. Find

(i) the value of k, [1]

(ii) an expression for the velocity of the particle in terms of t, [2]

(iii) the distance travelled in the first 6 seconds. [4]

Marking Scheme

(i) At maximum velocity, $a = 0$

$k - 2t = 0$ at $t = 1$

$k - 2(1) = 0$

$k = 2$ ---------- B1

(ii) $v = \int (2 - 2t)\ \mathrm{d}t$

$= 2t - t^2 + c$ where c is a constant ------ M1

Given that $v = 15$ when $t = 0$,

$c = 15$

$v = 2t - t^2 + 15$ --------- A1

(iii) $s = \int (2t - t^2 + 15)\ \mathrm{d}t$

$= t^2 - \frac{t^3}{3} + 15t + c_1$ where c_1 is a constant ------- M1

Since $s = 0$ when $t = 0$,

$c_1 = 0$

$s = t^2 - \frac{t^3}{3} + 15t$

When $v = 0$,

$2t - t^2 + 15 = 0$

$t^2 - 2t - 15 = 0$

$(t - 5)(t + 3) = 0$

$t = 5$ or -3 (N.A) ----------- M1

When $t = 0$, $s = 0$

When $t = 5$, $s = 58\frac{1}{3}$ ----------- M1

When $t = 6$, $s = 54$

Total distance travelled $= 58\frac{1}{3} + \left(58\frac{1}{3} - 54\right)$

$= 62\frac{2}{3}$ cm -------- A1

7 **(i)** Show that $\frac{2\cos^2\theta-\sin\theta\cos\theta+1}{\sin^2\theta}=3\cot^2\theta-\cot\theta+1$. [3]

(ii) Hence solve the equation $2\cos^2\theta-\sin\theta\cos\theta+1=\sin^2\theta$ for $-\pi\le\theta\le\pi$. [4]

Marking Scheme

(i)

$$\frac{2\cos^2\theta-\sin\theta\cos\theta+1}{\sin^2\theta}=\frac{2\cos^2\theta}{\sin^2\theta}-\frac{\sin\theta\cos\theta}{\sin^2\theta}+\frac{1}{\sin^2\theta} \text{ ------ M1}$$

$$=2\cot^2\theta-\cot\theta+\operatorname{cosec}^2\theta \text{ ------- M1}$$

$$=2\cot^2\theta-\cot\theta+\left(1+\cot^2\theta\right) \text{ ------ M1}$$

$$=3\cot^2\theta-\cot\theta+1 \text{ ---- (A.G)}$$

Alternative Method

$$\frac{2\cos^2\theta-\sin\theta\cos\theta+1}{\sin^2\theta}=\frac{2\cos^2\theta-\sin\theta\cos\theta+\sin^2\theta+\cos^2\theta}{\sin^2\theta} \text{ ------ M1}$$

$$=\frac{3\cos^2\theta-\sin\theta\cos\theta+\sin^2\theta}{\sin^2\theta} \text{ --------- M1}$$

$$=\frac{3\cos^2\theta}{\sin^2\theta}-\frac{\sin\theta\cos\theta}{\sin^2\theta}+\frac{\sin^2\theta}{\sin^2\theta} \text{ --------- M1}$$

$$=3\cot^2\theta-\cot\theta+1 \text{ ------ (A.G)}$$

(ii) $2\cos^2\theta-\sin\theta\cos\theta+1=\sin^2\theta$

$$\frac{2\cos^2\theta-\sin\theta\cos\theta+1}{\sin^2\theta}=1$$

$3\cot^2\theta-\cot\theta+1=1$ --------- M1

$3\cot^2\theta-\cot\theta=0$

$\cot\theta(3\cot\theta-1)=0$ ----------- M1

$\cot\theta=0$ (N.A) or $\cot\theta=\frac{1}{3}$

When $\tan\theta=3$, basic $\angle=1.249$ --------- M1

$\theta=-1.89$ or 1.25 -------- A1

8 Variables x and y are connected by the equation $y = ax^2 + b\sqrt{x}$, where a and b are constants. When a graph of $\frac{y}{\sqrt{x}}$ against $x\sqrt{x}$ is plotted using experimental values of x and y, a straight line is obtained and passes through the point $(1,\ 5)$. The straight line makes an angle of $45°$ with the $x\sqrt{x}$-axis.

(i) Find the values of a and of b. [4]

(ii) Find the coordinates of the point on the line at which $y = 3\sqrt{x}$. [3]

Marking Scheme

(i) $y = ax^2 + b\sqrt{x}$

$\frac{y}{\sqrt{x}} = a\frac{x^2}{\sqrt{x}} + b$

$\frac{y}{\sqrt{x}} = ax\sqrt{x} + b$ --------- M1 c.a.o

$\tan 45° = a$

$a = 1$ ----------- A1

Sub. $(1,5)$,

$5 = 1 + b$ ---------- M1

$b = 4$ --------- A1

(ii) $y = 3\sqrt{x} \Rightarrow \frac{y}{\sqrt{x}} = 3$ ------ M1

$3 = x\sqrt{x} + 4$ ------- M1

$x\sqrt{x} = -1$

The point is $(-1,\ 3)$ --------- A1

9 **(i)** Sketch the graph $y=|x^2-4x|$ indicating the intercepts and coordinates of the turning point. [3]

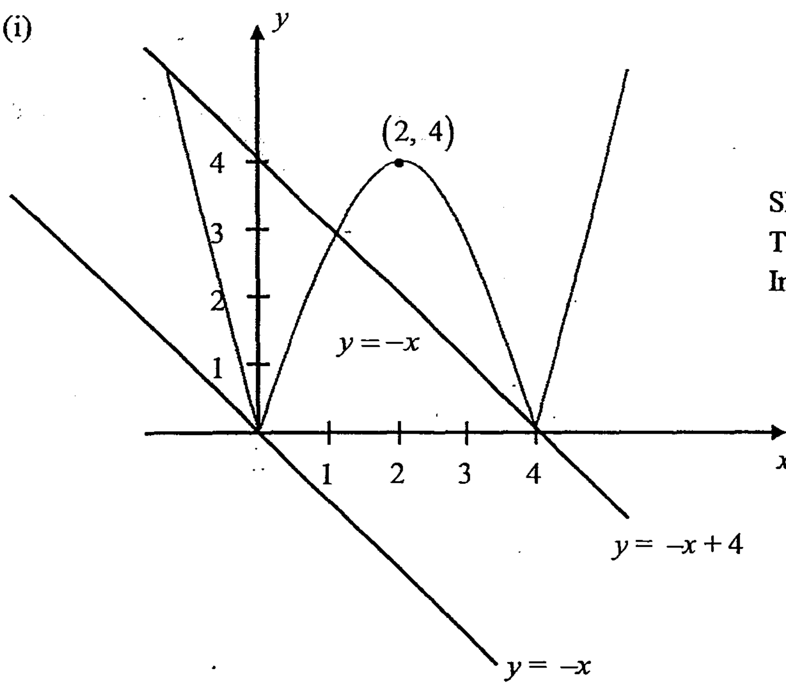
(ii) In each of the following case, determine the number of solutions of the equation $|x^2-4x|=mx+c$ where $0<c<4$, justifying your answer .

(a) $m=0$, [1]

(b) $m=-1$. [2]

Marking Scheme

(i)



Shape ------- B1
Turning pt ------- B1
Intercepts ------- B1

Alternative, draw these two lines to infer ---- M1

(iia) For $0<c<4$, the line $y=c$ cuts the graph $y=|x^2-4x|$ at 4 points, thus there are 4 solutions. --------------- B1

(iib) For $0<c<4$, the line $y=-x$ only intersect the <u>left-hand side half of the graph</u> $y=|x^2-4x|$ at 2 points, thus there are 2 solutions. --------------- A1

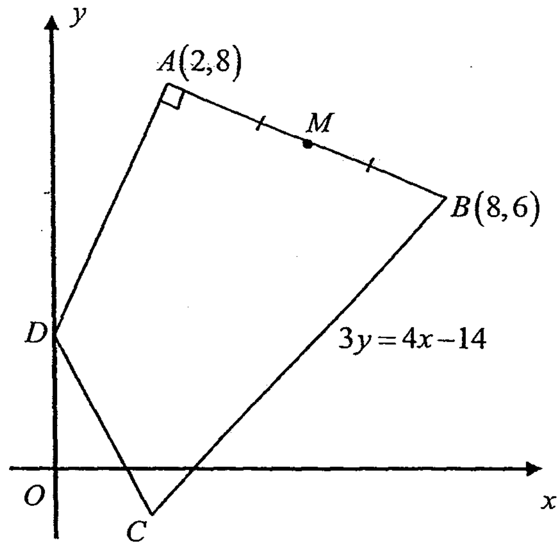
M1

10 The diagram shows a quadrilateral $ABCD$ in which $A(2, 8)$ and $B(8,6)$. M is the midpoint of AB and CM is perpendicular to AB. The equation of BC is $3y = 4x - 14$. The point D lies on the y-axis and $\angle DAB = 90°$. Find

(i) the coordinates of D, [2]

(ii) the coordinates of C, [5]

(iii) the ratio of $AD : CM$. [2]



(i) Let $D(0,\ a)$

Gradient of $AB = \dfrac{8-6}{2-8}$

$= -\dfrac{1}{3}$

Gradient of $AD \times$ Gradient of $AB = -1$

$\dfrac{8-a}{2-0} \times \left(-\dfrac{1}{3}\right) = -1$ -------- M1

$a = 2$

$\therefore D(0,2)$ ------------ A1

(ii) Gradient of $CM = 3$ -------- M1

$M = \left(\dfrac{2+8}{2}, \dfrac{8+6}{2}\right)$ --------- M1 c.a.o

$= (5,\ 7)$

Eq. of CM,

$y - 7 = 3(x - 5)$ ------ M1

$y = 3x - 8$ ------ (1)

Sub. (1) into $3y = 4x - 14$,

$3(3x - 8) = 4x - 14$ ---------- M1

$9x - 24 = 4x - 14$

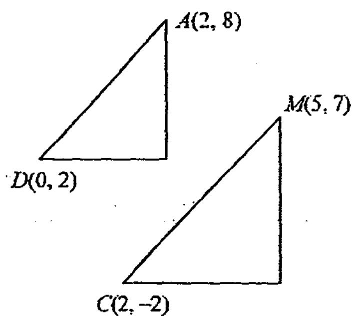
$5x = 10$

$x = 2$

$y = -2$

$\therefore C(2,-2)$ ---------- A1

(iii) Since $AD \parallel CM$, by similar triangles:



------------ M1

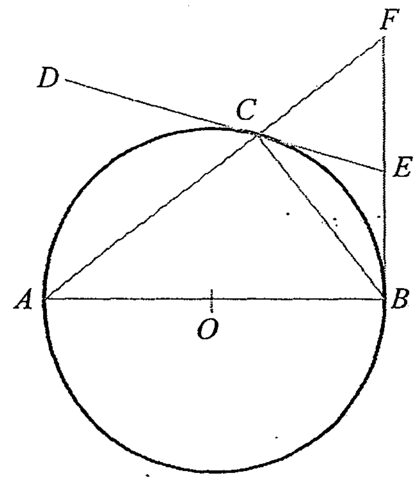
$\dfrac{AD}{CM} = \dfrac{2-0}{5-2}$

$AD : CM = 2 : 3$ ---------- A1

11 In the diagram, AB is a diameter of the circle with centre O. DE and BF are tangents to the circle at C and B respectively. DCE and BEF are straight lines. Prove that

(i) ΔABC and ΔAFB are similar, [3]

(ii) ΔCFE is isosceles. [4]



Marking Scheme

(i) $\angle ACB = 90°$ (rt. $\angle$ in a semi-circle) ---- M1
$\angle FBA = 90°$ (tan. $\perp$ rad.) ----- M1
$\therefore \angle ACB = \angle FBA$
$\angle FAB = \angle CAB$ (common $\angle$) ------ M1
Hence ΔABC and ΔAFB are similar.

(ii) $\angle ABC = \angle DCA$ (tangent-chord theorem) ----- M1
$\angle FCE = \angle DCA$ (vertically opp. $\angle$s) ------ M1
$\angle ABC = \angle CFE$ (similar Δs) ------ M1
Since $\angle CFE = \angle FCE$, ΔCFE is isosceles (base $\angle$s are equal) ----- A1

12 Sand is poured onto a flat surface at a rate of 2π cm^{3}s^{-1} and formed a right circular cone. The radius of the cone is always $\frac{1}{3}$ of its height.

(i) Find the rate of change of the radius 3 seconds after the start of pouring. [5]

(ii) State, with a reason, whether this rate will increase or decrease as t increases. [1]

Marking Scheme

(i) $V=\frac{1}{3}\pi r^2 h$

$=\frac{1}{3}\pi r^2(3r)$

$=\pi r^3$ ----------- M1 c.a.o

After 3 seconds, $\pi r^3=6\pi$

$r=\sqrt[3]{6}$ ------ M1

$\frac{\mathrm{d}V}{\mathrm{d}r}=3\pi r^2$ -------- M1

$\frac{\mathrm{d}r}{\mathrm{d}t}=\frac{\mathrm{d}r}{\mathrm{d}V}\times\frac{\mathrm{d}V}{\mathrm{d}t}$ ------ M1 c.a.o

$=\frac{2}{3r^2}$

$=\frac{1}{3\pi\left(\sqrt[3]{6}\right)^2}\times 2\pi$ at $r=\sqrt[3]{6}$

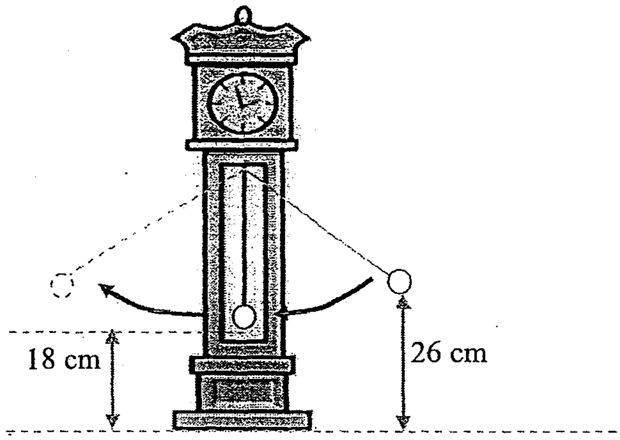
$=0.202$ (3 s.f)

The rate of change of radius is 0.202 cm/s. -------- A1

(ii) As $\frac{2}{3r^2}$ is inversely proportional to r, thus as t increases, r increases and $\frac{2}{3r^2}$ decreases.

This rate will decrease. ------ B1

13



The pendulum of a grandfather clock swings back and forth with a periodic motion that can be represented by the equation $h = a\cos kt + b$, where a, k and b are constants, h cm is the height of the pendulum above the base and t is the time in seconds after the pendulum is released 26 cm above the base. At rest, the pendulum is 20 cm above the base. It takes four seconds for a complete swing back and forth.

(i) State the value of a and b. [2]

(ii) Show that the value of k is $\frac{\pi}{2}$ rad/s. [1]

(iii) Sketch the graph of $h = a\cos kt + b$ for $0 \le t \le 6$. [2]

(iv) Find the time interval in which the pendulum is 20 cm above the base during one revolution. [3]

Marking Scheme

(i) $a = \dfrac{26-18}{2}$

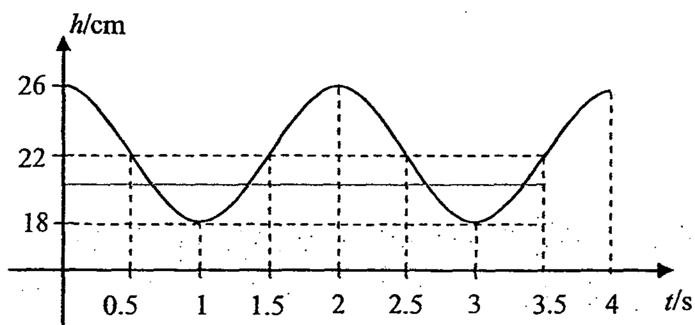
$= 4$ —— B1

$b = 22$ —— B1

(ii) $k = \dfrac{2\pi}{4 \div 2}$ —— B1

$= \pi$ rad/s —— (A.G)

(iii)



B1 -------- 1.5 cycle

B1 ---------- Max. and Min. values

(iii) $h = 4\cos \pi t + 22$

$4\cos \pi t + 22 = 20$

$\cos \frac{\pi}{2} t = -\frac{1}{2}$ -------- M1

Basic $\angle = \frac{\pi}{3}$

$\pi t = \frac{2\pi}{3}$ or $\frac{4\pi}{3}$

$t = \frac{2}{3}$ or $\frac{4}{3}$ ------- M1

Thus the time interval is $0 \le t \le \frac{2}{3}$ and $\frac{4}{3} \le t \le 2$ -------- A1

Calculator Model:

Name:	Class	Class Register Number/ Centre No./Index No.

中正中学

CHUNG CHENG HIGH SCHOOL (MAIN)

PRELIMINARY EXAMINATION 2014
SECONDARY 4

Additional Mathematics 4047/02

Paper 2

Friday, 29th August 2014
2 hours 30 mins

Additional Materials: Answer Paper

READ THESE INSTRUCTIONS FIRST

Do not open this booklet until you are told to do so.

Write your name, class and index number clearly in the spaces provided at the top of this page.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams, graphs or rough working.
Do not use highlighters or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
Calculators should be used where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

/100

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[Turn over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the quadratic equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!\,r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \tfrac{1}{2} bc \sin A$$

1 (i) Calculate the coordinates of the point of intersection of $y=\frac{1}{4}x^{\frac{5}{2}}$ and $y=256x^{-\frac{5}{2}}$, for $x>0$ [2]

(ii) Sketch, on the same diagram, the graphs of $y=\frac{1}{4}x^{\frac{5}{2}}$ and $y=256x^{-\frac{5}{2}}$ for $x>0$, clearly indicating the point of intersection. [3]

(iii) Determine, with explanation, whether the tangents to the two graphs at the point of intersection are perpendicular. [4]

2 (a) The independent term in the expansion of $(2+x)^n$ and the independent term in the expansion of $(2-ax)^{2n+1}$ are in the ratio 1 : 8.

(i) Show that $n=2$. [2]

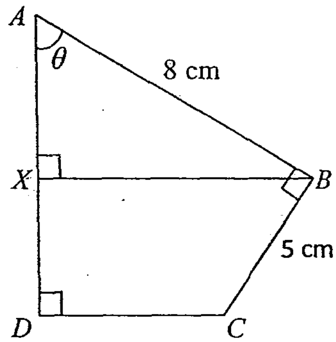
(ii) Given that the coefficient of x^2 in the expansion of $(1+x)(2-ax)^{2n+1}$ is 160, find the value(s) of a. [3]

(b) Given that the coefficient of the $\frac{1}{x^2}$ term in the expansion of $\left(\frac{2}{x}-x\right)^6-\left(1+\frac{2}{x}\right)^n$ is 128, find the value of n. [5]

3 (i) Differentiate $x\cos 3x$ with respect to x. [2]

(ii) Using your answer to part (i), show that $\int_0^{\frac{\pi}{3}} x\sin 3x\,dx=\frac{\pi}{9}$. [4]

4 The diagram shows a quadrilateral $ABCD$ with $AB = 8$ cm, $BC = 5$ cm. $\angle ABC = 90°$ $\angle ADC = 90°$ and $\angle BAD = \theta$ where $0° < \theta < 90°$. BX is perpendicular to AD.



(i) Show that the perimeter of $ABCD$, P cm is given by $P = 13 + 13\sin\theta + 3\cos\theta$. [3]

(ii) Express P in the form $k + R\sin(\theta + \alpha)$, where k is a constant, $R > 0$ and α is an acute angle. [3]

(iii) Find the value of θ for which $P = 25$. [2]

(iv) Find the maximum value of P and its corresponding value of θ. [2]

5 A function is defined by $y = \dfrac{1-4x}{x+1}$ where $x > 0$.

(i) Find an expression for $\dfrac{dy}{dx}$. [2]

(ii) Find the equation of the normal to the curve at the point where the curve crosses the x-axis. [3]

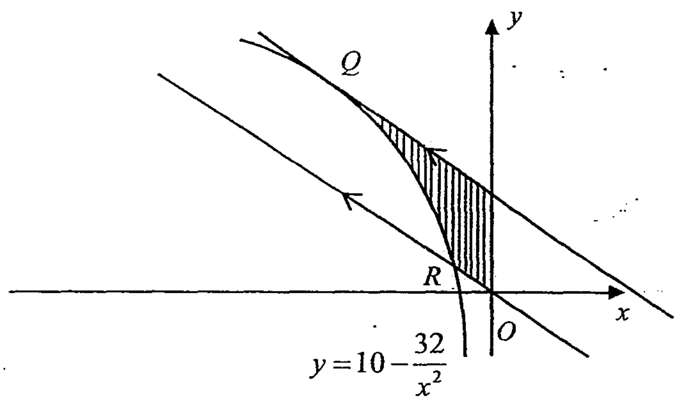
(iii) Determine, with explanation, whether $y = \dfrac{1-4x}{x+1}$ is an increasing or decreasing function. [2]

(iv) Find the range of x for which the gradient of the curve is an increasing function. [3]

6 **(i)** Express $\dfrac{x^3-2x^2-x+3}{x^2-2x+1}$ as partial fractions. [5]

(ii) Hence, find $\displaystyle\int \frac{x^3-2x^2-x+3}{x^2-2x+1}\,dx$. [2]

7



The diagram shows part of the curve $y=10-\dfrac{32}{x^2}$ and two parallel lines OR and PQ. The line OR intersects the curve at the point $R(-2,\ 2)$ and the line PQ is a tangent to the curve at the point Q. Find

(i) the gradient of OR, [1]

(ii) an expression for $\dfrac{dy}{dx}$ and hence, find the coordinates of Q, [3]

(iii) the area of the shaded region $OPQR$. [4]

8 **(a)** **(i)** Prove that $\cot 2A = \dfrac{\operatorname{cosec} A - 2\sin A}{2\cos A}$ [3]

(ii) Hence solve the equation $\dfrac{\operatorname{cosec} A - 2\sin A}{2\cos A} = 1$ for $0 \le A \le 2\pi$, leaving your answers in terms of π. [3]

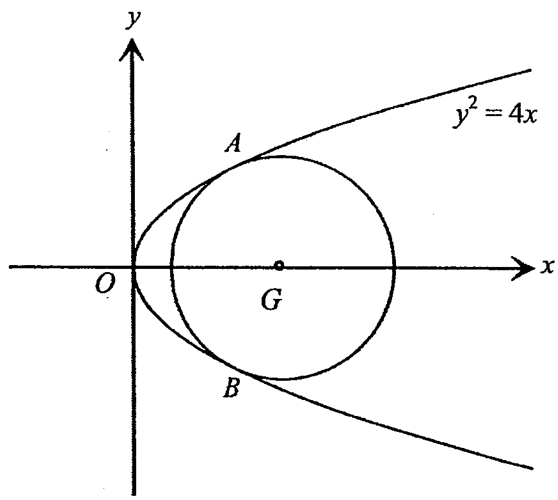
(b) A and B are acute angles such that $\sin(A+B) = \dfrac{7}{9}$ and $\sin A \cos B = \dfrac{4}{9}$. Without using a calculator, find the value of

(i) $\cos A \sin B$, [1]

(ii) $\sin(A-B)$, [1]

(iii) $\dfrac{\tan A}{\tan B}$. [2]

9



In the diagram, the circle with centre G touches the curve $y^2 = 4x$ at the points A and B. The equation of the common tangent to both the circle and the curve at the point A is $y = \dfrac{1}{2}x + 2$. Find

(i) the coordinates of the point A, [3]

(ii) the equation of AG, [2]

(iii) the equation of the circle, [3]

(iv) the equation of a second circle whose centre is G and area is 5 times that of the original circle. [2]

10 **(a)** Solve $\lg(x-8)+2\lg 3=2+\lg\left(\frac{x}{20}\right)$. [3]

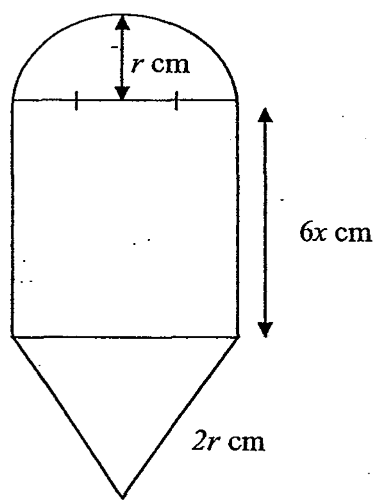
(b) Given that $\log_9 p=x$ and $\log_2 q=y$, express $q\log_3 p$ in terms of x and/or y. [3]

(c) The value, V, of a piece of rare jewel at the beginning of 1990 was \$20000. This value increased continuously such that, after a period of 10 years, the value of the jewel increased exponentially to \$100000. Given that $V=A(1.08)^{kt}$, where A and k are constants and t is the time in years from the beginning of 1990, find

(i) the value of A and k, [3]

(ii) the year in which the value of the stone first reached \$150000. [3]

11 A piece of wire, 400 cm long is cut to form the shape as shown in the diagram below. The shape consists of a semi-circular arc of radius r cm, a rectangle of length of $6x$ cm and an equilateral triangle.



(i) Express x in terms of r. [1]

(ii) Show that the area enclosed by the shape, A cm^2, given by [2]
$A=400r+r^2\left(\sqrt{3}-8-\frac{\pi}{2}\right)$.

(iii) Given that r and x can vary, find the value of r for which A has a stationary value. [3]

(iv) Determine whether this value of r makes A a maximum or a minimum. [2]

~End of Paper~

Answer Key

1. (i) $(4, 8)$

2. (aii) $a=-1$ or 2
   (b) $n=8$

3. (i) $\cos 3x - 3x\sin 3x$

4. (ii) $P = 13 + \sqrt{178}\sin(\theta + 13.0°)$
   (iii) $\theta = 51.1°$
   (iv) Max. value of $P = 26.3$, $\theta = 77.0°$

5. (i) $\dfrac{-5}{(x+1)^2}$
   (ii) $y = \dfrac{5}{16}x - \dfrac{5}{64}$
   (iv) $x > -1$

6. (i) $x - \dfrac{2}{x-1} + \dfrac{1}{(x-1)^2}$
   (ii) $\dfrac{1}{2}x^2 - 2\ln(x-1) - \dfrac{1}{(x-1)} + C$

7. (i) -1
   (ii) $Q(-4,8)$
   (iii) 10units^2

8. (aii) $A = \dfrac{\pi}{8}, \dfrac{5\pi}{8}, \dfrac{9\pi}{8} \; or \; \dfrac{13\pi}{8}$
   (b) (i) $\dfrac{1}{3}$
   (ii) $\dfrac{1}{9}$
   (iii) $\dfrac{4}{3}$

9. (i) $(4, 4)$
   (ii) $y = -2x + 12$
   (iii) $(x-6)^2 + y^2 = 20$
   (iv) $(x-6)^2 + y^2 = 100$

10. (i) $x = 18$
    (ii) $2^{y+1}x$
    (iii) (a) $A = 20000, k = 2.09$
    (b) 2002

11. (i) $x = \dfrac{400 - 8r - \pi r}{12}$

(iii) $r = 25.5$

(iv) A is a maximum when $r = 25.5$

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中正中学

CHUNG CHENG HIGH SCHOOL (MAIN)

PRELIMINARY EXAMINATION 2014

SECONDARY 4

Additional Mathematics 4047/02

Paper 2 *(Marking Scheme)*

Friday, 29th August 2014

2 hours 30 mins

Additional Materials: Answer Paper

READ THESE INSTRUCTIONS FIRST

Do not open this booklet until you are told to do so.

Write your name, class and index number clearly in the spaces provided at the top of this page.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams, graphs or rough working.
Do not use highlighters or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
Calculators should be used where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

/100

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the quadratic equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} .$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n ,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \tfrac{1}{2} bc \sin A$$

1 (i) Calculate the coordinates of the point of intersection of $y = \frac{1}{4}x^{\frac{5}{2}}$ and $y = 256x^{-\frac{5}{2}}$, for $x > 0$ **[2]**

Subst $y = \frac{1}{4}x^{\frac{5}{2}}$ into $y = 256x^{-\frac{5}{2}}$

$$\frac{1}{4}x^{\frac{5}{2}} = 256x^{-\frac{5}{2}}$$

$$x^5 = 1024 \qquad \text{-M1}$$

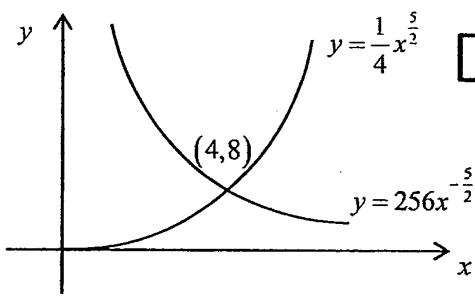
$$x = 4$$

$$y = \frac{1}{4}(4)^{\frac{5}{2}}$$

$$= 8$$

Coordinates of the point of intersection is $(4,\ 8)$. -A1

(ii) Sketch, on the same diagram, the graphs of $y = \frac{1}{4}x^{\frac{5}{2}}$ and $y = 256x^{-\frac{5}{2}}$ for $x > 0$, clearly indicating the point of intersection. **[3]**



B1

B1

Labelling of graphs, axis and intersection – M1 cao

	(iii) Determine, with explanation, whether the tangents to the graphs at the point of intersection are perpendicular.	[4]
	$y=\frac{1}{4}x^{\frac{5}{2}}$ $\frac{d}{dx}\left(\frac{1}{4}x^{\frac{5}{2}}\right)=\frac{1}{4}x^{\frac{3}{2}}\times\frac{5}{2}$ $=\frac{5}{8}x^{\frac{3}{2}}$ –M1 cao $y=256x^{-\frac{5}{2}}$ $\frac{d}{dx}\left(256x^{-\frac{5}{2}}\right)=256x^{-\frac{7}{2}}\times\left(-\frac{5}{2}\right)$ $=-640x^{-\frac{7}{2}}$ –M1 cao When $x=4$, product of gradients $=\frac{5}{8}(4)^{\frac{3}{2}}\times\left[-640(4)^{-\frac{7}{2}}\right]$ -M1 $=5\times(-5)$ $\neq -1$ $\therefore$ The lines are not perpendicular. -A1	
2	**(a)** The independent term in the expansion of $(2+x)^n$ and the independent term in the expansion of $(2-ax)^{2n+1}$ are in the ratio 1 : 8.	
	(i) Show that $n=2$.	[2]
	$\frac{2^n}{2^{2n+1}}=\frac{1}{8}$ --------- M1 c.a.o (Applying ratio) $2^{2n+1-n}=2^3$ ------ M1 By comparison, $n+1=3$ $n=2$ ------- A.G	

	(ii) Given that the coefficient of x^2 in the expansion of $(1+x)(2-ax)^{2n+1}$ is 160, find the value(s) of a.	[3]
	$(1+x)(2-ax)^5 = (1+x)\left[2^5 + \binom{5}{1}(2^4)(-ax) + \binom{5}{2}(2^3)(-ax)^2 + \ldots\right]$ ---- M1 c.a.o $= (1+x)(32 - 80ax + 80a^2x^2 + \ldots)$ $80a^2 - 80a = 160$ ------ M1 $a^2 - a = 2$ $a^2 - a - 2 = 0$ $(a-2)(a+1) = 0$ $a = -1$ or 2 ---------- A1	
	(b) Given that the coefficient of the $\frac{1}{x^2}$ term in the expansion of $\left(\frac{2}{x} - x\right)^6 - \left(1 + \frac{2}{x}\right)^n$ is 128, find the value of n.	[5]
	T_{r+1} of $\left(\frac{2}{x} - x\right)^6 = \binom{6}{r}\left(\frac{2}{x}\right)^{6-r}(-1)^r(x)^r$ Let $\frac{x^r}{x^{6-r}} = x^{-2}$ -------- M1 $x^{2r-6} = x^{-2}$ By comparison, $2r - 6 = -2$ $r = 2$ Coeff. of $\frac{1}{x^2}$ term $= \binom{6}{2}(2)^4(-1)^2$ $= 240$ ------------ M1 c.a.o $\frac{1}{x^2}$ term of $\left(1 + \frac{2}{x}\right)^n = \binom{n}{2}\left(\frac{2}{x}\right)^2$ -------- M1 c.a.o $\therefore \binom{n}{2}(4) = 240 - 128$ $\frac{n(n-1)(4)}{2} = 112$ ---------- M1 c.a.o $n(n-1) = 56$ By inspection: $(8)(7) = 56$ $\therefore n = 8$ ------------ A1	

3 (i) Differentiate $x\cos 3x$ with respect to x.

$$\frac{d}{dx}(x\cos 3x) = \cos 3x + x(-3\sin 3x) \qquad \text{–M1 cao}$$

$$= \cos 3x - 3x\sin 3x \qquad \text{–A1}$$

(ii) Using your answer to part (i), show that $\int_0^{\frac{\pi}{3}} x\sin 3x\,dx = \frac{\pi}{9}$. [4]

$$\frac{d}{dx}(x\cos 3x) = \cos 3x - 3x\sin 3x$$

Integrating both sides,

$$[x\cos 3x]_0^{\frac{\pi}{3}} = \int_0^{\frac{\pi}{3}} \cos 3x - 3x\sin 3x\,dx \qquad \text{–M1 (Integrate both sides)}$$

$$-\frac{\pi}{9} - 0 = \int_0^{\frac{\pi}{3}} \cos 3x\,dx - \int_0^{\frac{\pi}{3}} 3x\sin 3x\,dx \qquad \text{–M1 cao (Subst of values)}$$

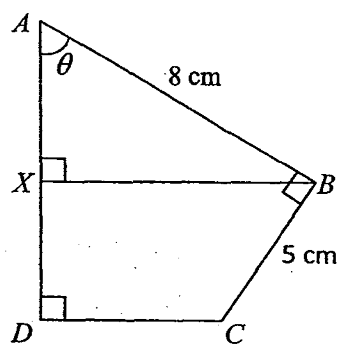
$$\int 3x\sin 3x\,dx = \int_0^{\frac{\pi}{3}} \cos 3x\,dx + \frac{\pi}{9}$$

$$\int 3x\sin 3x\,dx = \left[\frac{1}{3}\sin 3x\right]_0^{\frac{\pi}{3}} + \frac{\pi}{9} \qquad \text{–M1 (Correct integration)}$$

$$= 0 - 0 + \frac{\pi}{9} \qquad \text{–M1 cao (Subst of values)}$$

$$= \frac{\pi}{9} \qquad \text{–AG}$$

4 The diagram shows a quadrilateral $ABCD$ with $AB = 8$ cm, $BC = 5$ cm. $\angle ABC = 90°$ $\angle ADC = 90°$ and $\angle BAD = \theta$ where $0° < \theta < 90°$. BX is perpendicular to AD.



(i) Show that the perimeter of *ABCD*, *P* cm is given by $P=13+13\sin\theta+3\cos\theta$. [3]

$\cos\theta=\frac{AX}{8}$

$AX=8\cos\theta$ –B1 (For AX, DX or BX)

Since $\angle XBC=90°-\angle XAB$

$=90°-(90°-\theta)$

$=\theta$

$\sin\theta=\frac{DX}{5}$

$DX=5\sin\theta$

$\frac{BY}{5}=\cos\theta$

$BY=5\cos\theta$

$\sin\theta=\frac{BX}{8}$

$BX=8\sin\theta$

$DC=8\sin\theta-5\cos\theta$

Perimeter of $ABCD=AD+DC+CB+BA$

$=(8\cos\theta+5\sin\theta)+(8\sin\theta-5\cos\theta)+5+8$ –M1 (Finding AD or DC)

$=13+13\sin\theta+3\cos\theta$ –A1

(ii) Express *P* in the form $k+R\sin(\theta+\alpha)$, where *k* is a constant, $R>0$ and α is an acute angle. [3]

$P=13+13\sin\theta+3\cos\theta$

For $13\sin\theta+3\cos\theta$,

$R=\sqrt{13^2+3^2}$ –M1 cao

$=\sqrt{178}$

$\alpha=\tan^{-1}\frac{3}{13}$ –M1 cao

$=12.995°$ (3dec.pl.)

$\therefore P=13+\sqrt{178}\sin(\theta+13.0°)$ (1dec.pl.) –A1

	(iii) Find the value of θ for which $P = 25$.	[2]
	$P = 25$ $13 + \sqrt{178}\sin(\theta + 12.995°) = 25$ $\sqrt{178}\sin(\theta + 12.995°) = 12$ $\sin(\theta + 12.995°) = \frac{12}{\sqrt{178}}$ –M1 $\theta + 12.995° = 64.084°$ $\theta = 51.1°$ –A1	
	(iv) Find the maximum value of P and its corresponding value of θ.	[2]
	Max. value of $P = 13 + \sqrt{178}$ $= 26.3$ –B1 When $\sin(\theta + 12.995°) = 1$ $\theta + 12.995° = 90°$ $\theta = 77.0°$ (1dec.pl.) –B1	
5	A function is defined by $y = \frac{1-4x}{x+1}$ where $x > 0$.	
	(i) Find an expression for $\frac{dy}{dx}$.	[2]
	$y = \frac{1-4x}{x+1}$ $\frac{dy}{dx} = \frac{-4(x+1)-(1-4x)}{(x+1)^2}$ –M1 cao $= \frac{-5}{(x+1)^2}$ –A1	

	(ii) Find the equation of the normal to the curve at the point where the curve crosses the x-axis.	[3]
	$y=\frac{1-4x}{x+1}$ When $y=0$, $\frac{1-4x}{x+1}=0$ $x=\frac{1}{4}$ —M1 cao When $x=\frac{1}{4}, \frac{dy}{dx}=\frac{-5}{\left(\frac{1}{4}+1\right)^2}$ $=-\frac{16}{5}$ Gradient of normal $=\frac{5}{16}$ —M1 Equation of normal: $y-0=\frac{5}{16}\left(x-\frac{1}{4}\right)$ $y=\frac{5}{16}x-\frac{5}{64}$ —A1	
	(iii) Determine, with explanation, whether $y=\frac{1-4x}{x+1}$ is an increasing or decreasing function.	[2]
	For $x>0$, $\therefore\ (x+1)^2>0$, —M1 $\frac{1}{(x+1)^2}>0$ $\frac{-5}{(x+1)^2}<0$ $\therefore \frac{dy}{dx}<0$ Therefore, $y=\frac{1-4x}{x+1}$ is a decreasing function. —A1	

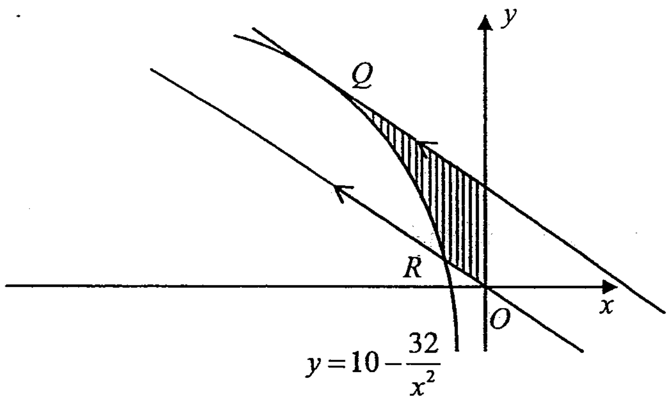
	(iv) Find the range of x for which the gradient of the curve is an increasing function.	[3]
	$\frac{dy}{dx}=\frac{-5}{(x+1)^2}$ $=-5(x+1)^{-2}$ $\frac{d^2y}{dx^2}=-5(x+1)^{-3}\times(-2)$ $=\frac{10}{(x+1)^3}$ –M1 For gradient to be increasing function, $\frac{d^2y}{dx^2}>0$ –M1cao $\frac{10}{(x+1)^3}>0$ $\therefore\ (x+1)^3>0$ $(x+1)(x+1)^2>0$ Since $(x+1)^2>0$, $x+1>0$ $x>-1$ –A1	
6	**(i)** Express $\frac{x^3-2x^2-x+3}{x^2-2x+1}$ as partial fractions.	[5]
	Long division: $(x^3-2x^2-x+3)\div(x^2-2x+1)$, quotient x; $-(x^3-2x^2+x)$; remainder $-2x+3$ –M1 Long Division $\frac{x^3-2x^2-x+3}{x^2-2x+1}=x+\frac{(-2x+3)}{x^2-2x+1}$ Let $\frac{3-2x}{(x-1)^2}=\frac{A}{x-1}+\frac{B}{(x-1)^2}$, where A and B are constants. –M1cao $3-2x=A(x-1)+B$ –M1(*Comparing or substitution*) $-2x+3=Ax+(B-A)$ By comparing coefficients, $A=-2, B=1$ –M1 cao $\therefore \frac{x^3-2x^2-x+3}{x^2-2x+1}=x-\frac{2}{x-1}+\frac{1}{(x-1)^2}$ –A1	

(ii) Hence, find $\int \frac{x^3-2x^2-x+3}{x^2-2x+1}\,dx$. [2]

$$\int \frac{x^3-2x^2-x+3}{x^2-2x+1}\,dx = \int x-\frac{2}{x-1}+\frac{1}{(x-2)^2}\,dx \qquad \text{–M1}$$

$$=\frac{1}{2}x^2-2\ln(x-1)-\frac{1}{(x-1)}+C, \text{ where } C \text{ is a constant.} \qquad \text{–A1}$$

7



The diagram shows part of the curve $y=10-\frac{32}{x^2}$ and two parallel lines *OR* and *PQ*. The line *OR* intersects the curve at the point $R(-2, 2)$ and the line *PQ* is a tangent to the curve at the point *Q*. Find

(i) the gradient of *OR*, [1]

$$\text{Gradient of } OR = \frac{2-0}{-2-0}$$

$$=-1 \qquad \text{–B1}$$

(ii) an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ and hence, find the coordinates of Q, [3]

$$y = 10 - \frac{32}{x^2}$$

$$\frac{dy}{dx} = -\frac{32}{x^3}(-2)$$

$$= \frac{64}{x^3} \qquad \text{–A1}$$

Since $OR // PQ$,

$$\frac{64}{x^3} = -1 \qquad \text{–M1}$$

$$x = -4$$

When $x = -4$, $y = 10 - \frac{32}{(-4)^2}$

$$= 8$$

$$\therefore Q(-4, 8) \qquad \text{–A1}$$

(iii) the area of the shaded region $OPQR$, [4]

Equation of line PQ:

$$y - 8 = (-1)(x + 4)$$

$$y = -x + 4$$

$\therefore P(0,4)$ –M1

Area of trapezium below line $PQ = \frac{1}{2}(4+8)(4)$

$= 24 \text{units}^2$

Area of □ below $OR = \frac{1}{2}(2)(2)$

$= 2 \text{units}^2$

–M1 cao M1 cao

Area below curve $RQ = \int_{-4}^{-2} 10 - \frac{32}{x^2} \, \mathrm{d}x$

$$= \left[10x + \frac{32}{x}\right]_{-4}^{-2}$$

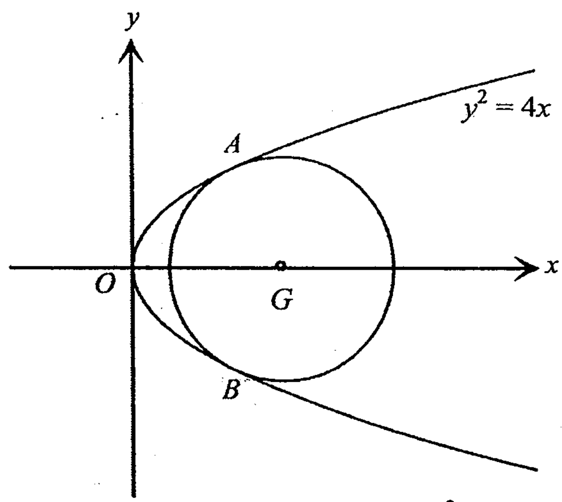
$$= (-20 - 16) - (-40 - 8)$$

$$= 12 \text{units}^2 \qquad \text{–M1}$$

Area of shaded region $= 24 - 2 - 12$

$$= 10 \text{units}^2 \qquad \text{–A1}$$

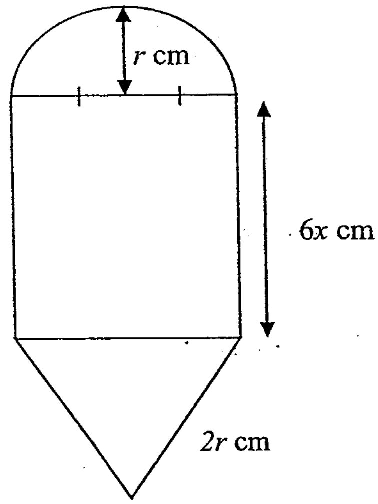
8	(a) (i) Prove that $\cot 2A = \dfrac{\operatorname{cosec} A - 2\sin A}{2\cos A}$.	[3]
	$\text{RHS} = \dfrac{\operatorname{cosec} A - 2\sin A}{2\cos A}$ $= \dfrac{\frac{1}{\sin A} - 2\sin A}{2\cos A}$ –M1 cao (Replace $\operatorname{cosec} A$) $= \dfrac{1 - 2\sin^2 A}{2\sin A\cos A}$ –M1 $\left(\text{Multiply } \dfrac{\sin A}{\sin A}\right)$ $= \dfrac{\cos 2A}{\sin 2A}$ –M1cao $= \cot 2A$ (Proven) –AG	
	(ii) Hence solve the equation $\dfrac{\operatorname{cosec} A - 2\sin A}{2\cos A} = 1$ for $0 \le A \le 2\pi$, leaving your answers in terms of π.	[3]
	$\dfrac{\operatorname{cosec} A - 2\sin A}{2\cos A} = 1$ $\cot 2A = 1$ –M1 cao $\tan 2A = 1$ Basic angle of $2A = \dfrac{\pi}{4}$ –M1 $0 \le A \le 2\pi$ $2A = \dfrac{\pi}{4}, \dfrac{5\pi}{4}, \dfrac{9\pi}{4} \text{ or } \dfrac{13\pi}{4}$ $0 \le 2A \le 4\pi$ $A = \dfrac{\pi}{8}, \dfrac{5\pi}{8}, \dfrac{9\pi}{8} \text{ or } \dfrac{13\pi}{8}$ –A1	
	(b) A and B are acute angles such that $\sin(A+B) = \dfrac{7}{9}$ and $\sin A\cos B = \dfrac{4}{9}$. Without using a calculator, find the value of	
	(i) $\cos A\sin B$,	[1]
	Given $\sin(A+B) = \dfrac{7}{9}$ and $\sin A\cos B = \dfrac{4}{9}$, $\sin A\cos B + \cos A\sin B = \dfrac{7}{9}$ $\dfrac{4}{9} + \cos A\sin B = \dfrac{7}{9}$ $\cos A\sin B = \dfrac{1}{3}$ –B1	

	(ii) $\sin(A-B)$,	[1]
	$\sin(A-B)=\sin A\cos B-\cos A\sin B$ $=\frac{4}{9}-\frac{1}{3}$ $=\frac{1}{9}$ —B1	
	(iii) $\frac{\tan A}{\tan B}$.	[2]
	$\frac{\tan A}{\tan B}=\frac{\sin A\cos B}{\cos A\sin B}$ —M1 cao $=\frac{\frac{4}{9}}{\frac{1}{3}}$ $=\frac{4}{3}$ —A1	
9	 In the diagram, the circle with centre G touches the curve $y^2 = 4x$ at the points A and B. The equation of the common tangent to both the circle and the curve at the point A is $y=\frac{1}{2}x+2$. Find	

	(i) the coordinates of the point A,	[3]
	$y=2\sqrt{x}$ $\frac{dy}{dx}=\frac{1}{\sqrt{x}}$ –M1 cao (Either method) At A, $\frac{dy}{dx}=\frac{1}{2}$ $\frac{1}{\sqrt{x}}=\frac{1}{2}$ –M1cao $x=4$ $\therefore y=4$ Coordinates of A is $(4,\ 4)$. –A1 OR Subst. $y=\frac{1}{2}x+2$ into $y^2=4x$, –M1 cao $\left(\frac{1}{2}x+2\right)^2=4x$ $\frac{1}{4}x^2+2x+4-4x=0$ $x^2-8x+16=0$ $(x-4)^2=0$ – M1 (Factorisation) $x=4$ $\therefore y=4$ Coordinates of A is $(4,\ 4)$. –A1	
	(ii) the equation of AG,	[2]
	Gradient of $AG=-2$ –M1 cao Equation of AG, $\frac{y-4}{x-4}=-2$ $y=-2x+12$ –A1	
	(iii) the equation of the circle,	[3]
	G lies on the line AG and the $x-$axis, hence G is $(6,\ 0)$. –M1 Radius of circle $=\sqrt{(4-6)^2+(4-0)^2}$ –M1 $=\sqrt{20}$ $=2\sqrt{5}$ units Equation of circle: $(x-6)^2+y^2=20$ –A1 *or* $x^2+y^2-12x+16=0$	

	(iv) the equation of a second circle whose centre is G and area is 5 times that of the original circle.	[2]
	Let r cm be the radius of the second circle. Area of second circle = 5 × Area of first circle $\pi r^2 = 5\pi\left(\sqrt{20}\right)^2$ –M1 $r^2 = 100$ Equation of second circle: $(x-6)^2 + y^2 = 100$ –A1	
10	**(i)** Solve $\lg(x-8) + 2\lg 3 = 2 + \lg\left(\frac{x}{20}\right)$.	[3]
	$\lg(x-8) + 2\lg 3 = 2 + \lg\left(\frac{x}{20}\right)$ $\lg(x-8) + \lg 9 = \lg 100 + \lg\left(\frac{x}{20}\right)$ –M1 cao (Power law) $\lg[9(x-8)] = \lg\left[100\left(\frac{x}{20}\right)\right]$ –M1 (Product Law) By comparison, $9x - 72 = 5x$ $4x = 72$ $x = 18$ –A1	
	(ii) Given that $\log_9 p = x$ and $\log_2 q = y$, express $q\log_3 p$ in terms of x and/or y.	[3]
	$\log_2 q = y$ $q = 2^y$ –M1 cao $q\log_3 p = 2^y\left(\frac{\log_9 p}{\log_9 3}\right)$ –M1 cao (for change of base or for forming $\log_9 9$) $= 2^y\left(\frac{x}{\log_9 9^{\frac{1}{2}}}\right)$ $= 2^y(2x)$ $= 2^{y+1}x$ –A1	

	(iii) The value, V, of a piece of rare jewel at the beginning of 1990 was \$20000. This value increased continuously such that, after a period of 10 years, the value of the jewel increased exponentially to \$100000. Given that $V = A(1.08)^{kt}$, where A and k are constants and t is the time in years from the beginning of 1990, find	
	(a) the value of A and k,	[3]
	$V = A(1.08)^{kt}$ When $t = 0$, $V = 20000$ $\therefore A = 20000$ –B1 When $t = 10$, $V = 100000$ $100000 = 20000(1.08)^{10k}$ $(1.08)^{10k} = 5$ $\ln(1.08)^{10k} = \ln 5$ –M1 $10k = \dfrac{\ln 5}{\ln 1.08}$ $k = 2.0912...$ $= 2.09$ (3sig.fig.) –A1	
	(b) the year in which the value of the stone first reached \$150000.	[3]
	$V = 20000(1.08)^{2.0912t}$ When $V = 150000$, $150000 = 20000(1.08)^{2.0912t}$ $(1.08)^{2.0912t} = 7.5$ $\ln(1.08)^{2.0912t} = \ln 7.5$ –M1 $2.0912t = \dfrac{\ln 7.5}{\ln 1.08}$ $t = 12.519$ –M1 Year in which value exceeded \$150000 = 1990+12 = 2002 –A1	

11	A piece of wire, 400 cm long is cut to form the shape as shown in the diagram below. The shape consists of a semi-circular arc of radius r cm, a rectangle of length of $6x$ cm and an equilateral triangle. 	
	(i) Express x in terms of r.	[1]
	Total Length $= 400$ $12x + 8r + \pi r = 400$ $x = \frac{400 - 8r - \pi r}{12}$ –B1	
	(ii) Show that the area enclosed by the shape, A cm^2, given by $A = 400r + r^2\left(\sqrt{3} - 8 - \frac{\pi}{2}\right)$.	[2]
	$A = \frac{1}{2}\pi r^2 + 12rx + \frac{1}{2} \times 2r \times 2r \times \sin 60°$ –M1 cao $= \frac{1}{2}\pi r^2 + 12r\left(\frac{400 - 8r - \pi r}{12}\right) + 2r^2 \times \frac{\sqrt{3}}{2}$ –M1 $= \frac{1}{2}\pi r^2 + 400r - 8r^2 - \pi r^2 + \sqrt{3}r^2$ $= 400r + r^2\left(\sqrt{3} - 8 - \frac{\pi}{2}\right)$ –AG	

	(iii) Given that r and x can vary, find the value of r for which A has a stationary value.	[3]
	$\frac{dA}{dr} = 400 + 2r\left(\sqrt{3} - 8 - \frac{\pi}{2}\right)$ –M1 cao For stationary value, $400 + 2r\left(\sqrt{3} - 8 - \frac{\pi}{2}\right) = 0$ –M1 $r = \frac{400}{2\left(\frac{\pi}{2} + 8 - \sqrt{3}\right)}$ $= 25.5$ –A1	
	(iv) Determine whether this value of r makes A a maximum or a minimum.	[2]
	$\frac{d^2A}{dr^2} = 2\left(\sqrt{3} - 8 - \frac{\pi}{2}\right)$ $= -15.677 < 0$ –M1 cao A is a maximum when $r = 25.5$ –A1	

~End of Paper~

Name:	Register No. :	Class:

CRESCENT GIRLS' SCHOOL
SECONDARY FOUR
PRELIMINARY EXAMINATIONS

ADDITIONAL MATHEMATICS — 4047/01

Paper 1 — **19 Aug 2014**

Additional materials: Answer Paper
Mark Sheet

2 hours

READ THESE INSTRUCTIONS FIRST

Write your name, register number and class in the spaces provided at the top of this page and on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use paper clips, highlighter, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
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[Turn over

Mathematical Formulae

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For the equation $ax^2 + bx + c = 0$,

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$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

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1 The equation of a straight line is $y = 7 + kx$, where k is a constant. Find the value of k for which this straight line is tangent to the curve $\left(\frac{y}{2}\right)^2 = y - x - 1$. [3]

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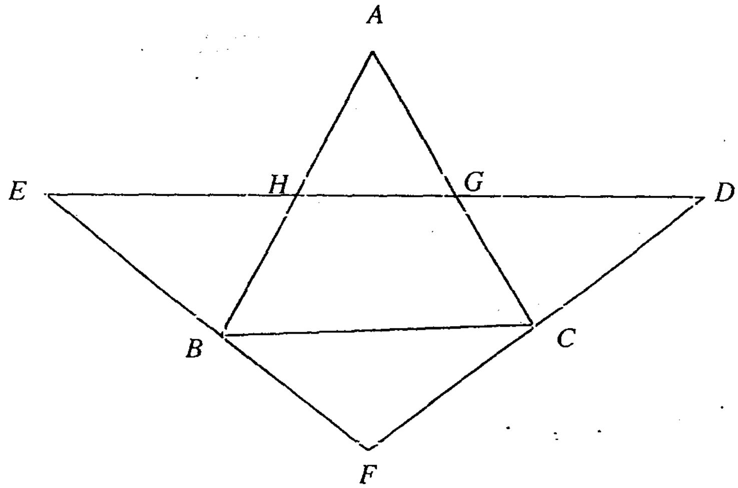

The diagram above shows the triangles ABC and DEF, where $AB = AC$. H and G are the midpoints of AB and AC respectively, while B and C are the midpoints of EF and DF respectively.

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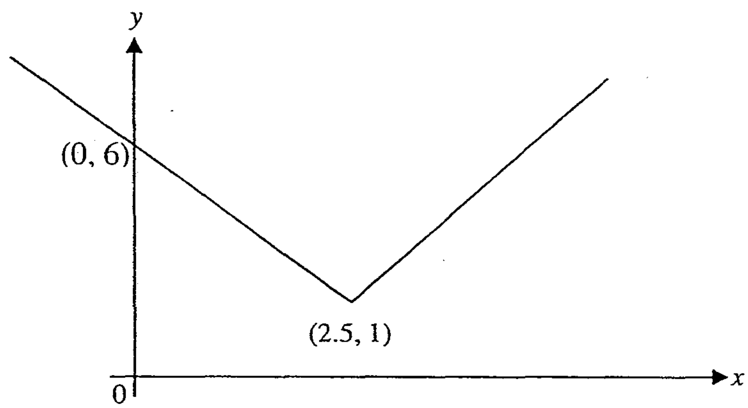
7 **(a)** Solve the equation $x^2-11=|x-9|$. [3]

(b)



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(i) Show that $\frac{dy}{dx} = \frac{-16a^3 x}{(x^2 + 4a^2)^2}$. [2]

A particle moves along the curve. At the point P $(2, -1)$, the y-coordinate of the particle is decreasing at a rate of 0.3 units/sec.

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13 Scientists use trigonometric functions to model populations of predators and prey in the environment. In a particular study which stretched over a period of 4 months, the population of mousedeer is modeled by the equation $P = -300\cos\left(\frac{\pi}{2}t\right) + 500$.

The population of leopards in the same area is modeled by the equation $P = 50\cos\left(\frac{\pi}{2}t\right) + 100$. In each equation, P is the population of the animal and t is the time in months.

(i) On the same axes, sketch the graphs of $P = -300\cos\left(\frac{\pi}{2}t\right) + 500$ and $P = 50\cos\left(\frac{\pi}{2}t\right) + 100$ for $0 \le t \le 4$. [5]

(ii) Using the graphs of these two equations, state the relationship between the population of mousedeer and the population of leopards in the area. [1]

Leopards in the area are classified as vulnerable when their population is 80 or less.

(iii) Find the length of time for which the leopards became vulnerable during the study. [3]

END OF PAPER

Answer Key

1. $k=\frac{-1}{5}$	8. (i) $(\alpha+\beta)^2-4\alpha\beta$
2. (0, –1), point of inflexion	8. (ii) $x^2-44\sqrt{2}x+343=0$
3. (i) (a) 18, (b) $9+3\pi$	9. (i) 54.2°, 144.2°, 234.2°, or 324.2°
3. (ii) $a=\frac{3\pi}{2}, b=\frac{5\pi}{2}$	9. (ii) 12
4. (i) 9	10. (i) $-4.8e^{-0.4t}$
4. (ii) 0.240	10. (ii) 21.3 m
5. Proof	11. (i) $\frac{y}{x}=cx+d$
6. $y=\frac{x}{3}+\frac{11}{6}$	11. (ii) $c=9$, $d=6$
7. (a) 4, –5	11. (iii) $\left(\frac{-12}{17},\frac{-6}{17}\right)$
7. (b) $y=\|2x-5\|+1$, $y=\|5-2x\|+1$	12. (ii) $a=-1$
	12. (iii) –0.6 units/sec
	13. (iii) 1.48 months

Name: MARKING SCHEME	Register No. :	Class:

CRESCENT GIRLS' SCHOOL
SECONDARY FOUR
PRELIMINARY EXAMINATIONS

ADDITIONAL MATHEMATICS 4047/01

Paper 1 **19 Aug 2014**

Additional materials: Answer Paper
Mark Sheet

2 hours

READ THESE INSTRUCTIONS FIRST

Write your name, register number and class in the spaces provided at the top of this page and on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use paper clips, highlighter, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
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$y = 7 + kx$ — ①

$\left(\frac{y}{2}\right)^2 = y - x - 1$ — ②

Sub ① into ②:

$\left(\frac{7+kx}{2}\right)^2 = 7 + kx - x - 1$ [M1]

$49 + 14kx + k^2x^2 = 28 + 4kx - 4x - 4$

$k^2x^2 + 10kx + 4x + 25 = 0$

$b^2 - 4ac = 0$

$(10k+4)^2 - 4k^2(25) = 0$ [M1]

$100k^2 + 80k + 16 - 100k^2 = 0$

$80k + 16 = 0$

$k = -\frac{1}{5}$ [A1]

2 Find the coordinates of the stationary point of the curve $y = \frac{3x^6 - x^3}{x^3}$, and determine the nature of this stationary point. [4]

$\frac{dy}{dx} = 9x^2$ [B1]

when $\frac{dy}{dx} = 0$

$9x^2 = 0$

$\therefore x = 0$
$y = -1$ } [A1]

x	-0.1	0	0.1
$\frac{dy}{dx}$	+	0	+
slope	/	—	/

[M1]

$\therefore (0, -1)$ is a point of inflexion. [A1] must agree with test result

award mark if spelling wrong but obvious what student meant

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(i) (a) $\int_0^{2\pi} |k\sin\theta| \, d\theta = 2\int_0^{\pi} k\sin\theta \, d\theta$

$= 18$ B1

(b) $\int_0^{\pi} (k\sin\theta + 3) \, d\theta = 9 + [3\theta]_0^{\pi}$ M1

$= 9 + 3\pi$ A1

(ii) $a = \frac{3\pi}{2}$, $b = \frac{5\pi}{2}$ B2

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(i) the exact value of 18^{1-x}, [3]

(ii) the value of x. [2]

(i) $3^{2x} = 2^{1-x}$

$3^{2x} \times 9^{1-x} = 2^{1-x} \times 9^{1-x}$ [M1]

$\frac{3^{2x} \times 9}{3^{2x}} = 18^{1-x}$ [M1]

$18^{1-x} = 9$ [A1]

Alt:

$9^{x} = \frac{2}{2^{x}}$

$\frac{1}{2} = 18^{-x}$

} [M1] $2^{x} \cdot 3^{2x} = 18$

$\frac{1}{2} \times 18 = 18^{-x} \times 18$ [M1]

$18^{1-x} = 9$ [A1]

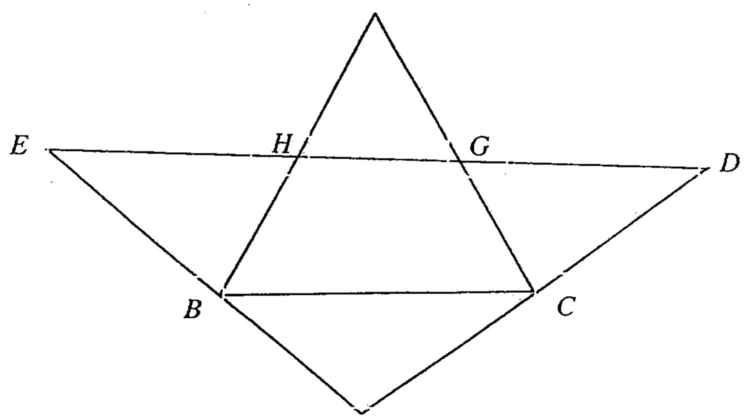
(ii) $18^{1-x} = 9$

$18^{x} = 2$

$x = \frac{\lg 2}{\lg 18}$ [M1]

$= 0.240$ [A1]

5



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(ii) Given that $EH = GD$, show that ΔDEF is isosceles. [6]

(i) $GH = \frac{1}{2} BC$ (midpoint theorem) [B1]

$= \frac{1}{2}\left(\frac{1}{2} DE\right)$ (midpoint theorem) [B1]

$= \frac{1}{4} DE$

$\therefore DE = 4GH$ correct conclusion.

OR

$DE = 2BC$

$= 2(2GH)$

$= 4GH$

(ii) ΔAHG is isosceles ($AH = AG$)

$\angle AHG = \angle AGH$ (base ∡s of isos Δ)

$\angle AHG = \angle EHB$ (vert opp ∡s)
$\angle AGH = \angle DGC$ (vert opp ∡s) } [M1] either

$\therefore \angle EHB = \angle DGC$ [A1]

$EH = GD$ (given) [M1]

$HB = \frac{1}{2} AB$ (H is midpoint)

$= \frac{1}{2} AC$ (given)

$= GC$ (G is midpoint) [M1]

$\therefore \Delta EHB \equiv \Delta DGC$ (SAS) [A1]

$\angle HEB = \angle GDC$ [A1] (corr ∡s of congruent Δs)

$\therefore \Delta DEF$ is isosceles. correct conclusion

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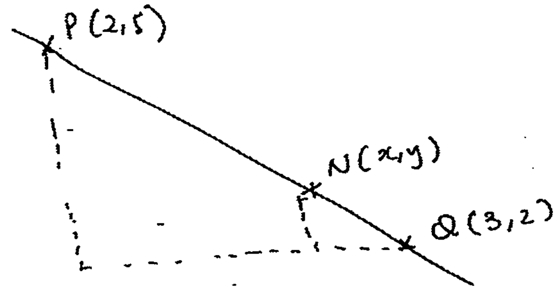
gradient of PQ

$= \frac{5-2}{2-3}$

$= -3$

gradient of $l_1 = \frac{1}{3}$ [M1]

Let the point of intersection between PQ and l_1 be $N(x, y)$



$\frac{PN}{QN} = 3$

$\frac{3-x}{3-2} = \frac{1}{4}$

$3 - x = \frac{1}{4}$

$x = \frac{11}{4}$

[M1] either or

$\frac{y-2}{5-2} = \frac{1}{4}$

$y - 2 = \frac{3}{4}$

$y = \frac{11}{4}$

$\therefore N\left(\frac{11}{4}, \frac{11}{4}\right)$ [A1]

$l_1: \; y - \frac{11}{4} = \frac{1}{3}\left(x - \frac{11}{4}\right)$

$y = \frac{x}{3} - \frac{11}{12} + \frac{11}{4}$

$y = \frac{x}{3} + \frac{11}{6}$ // [A1]

7 (a) Solve the equation $x^2-11=|x-9|$. [3]

$x-9=x^2-11$ or $x-9=-x^2+11$ [M1] 2 cases

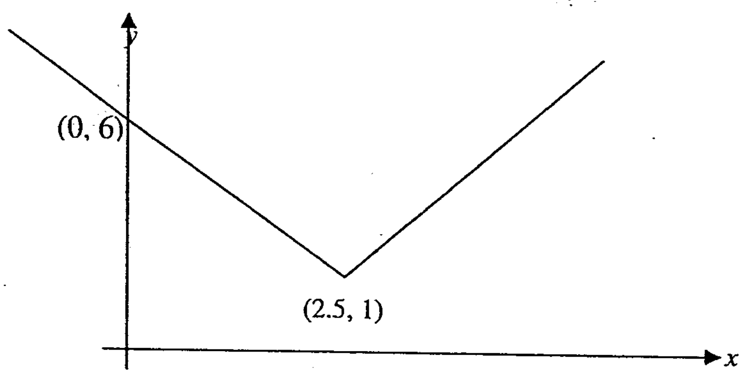
$x^2-x-2=0$ $x^2+x-20=0$

$(x-2)(x+1)=0$ $(x+5)(x-4)=0$ [M1] factorisation

$x=2$ (rej) or $x=-1$ (rej) $x=-5$ or $x=4$

[A1] all four values but reject two

(b)



The diagram above shows part of the graph of $y=|ax+b|+c$, where a, b and c are constants. The coordinates of the y-intercept and vertex are given as (0, 6) and (2.5, 1) respectively. Write down two possible functions of y that can be represented by the graph in the diagram. [3]

$c=1$ [B1]

∴ The two possible functions of y are:

$y=|2x-5|+1$ [B1] or $y=|5-2x|+1$ [B1]

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The roots of the quadratic equation $x^2-2x-7=0$ are α and β, where $\alpha>\beta$.

(ii) Find a quadratic equation whose roots are α^3 and $-\beta^3$. [7]

(i) $(\alpha-\beta)^2 = \alpha^2 - 2\alpha\beta + \beta^2 = (\alpha+\beta)^2 - 4\alpha\beta$ B1

(ii) $\alpha+\beta = 2$, $\alpha\beta = -7$ } B1 both

$\alpha^3-\beta^3 = (\alpha-\beta)(\alpha^2+\alpha\beta+\beta^2)$ B1

$\therefore \alpha^3-\beta^3 = (\alpha-\beta)\left[(\alpha+\beta)^2-\alpha\beta\right]$ M1

$(\alpha-\beta)^2 = (\alpha+\beta)^2 - 4\alpha\beta$

$= 4 + 28$

$= 32$ M1

$\alpha-\beta = 4\sqrt{2}$ $(\because \alpha>\beta)$

$\alpha^3-\beta^3 = 4\sqrt{2}(4+7)$

$= 44\sqrt{2}$ M1

$-\alpha^3\beta^3 = -(\alpha\beta)^3$

$= 343$ M1

$\therefore$ required equation:

$x^2 - 44\sqrt{2}\,x + 343 = 0$ A1

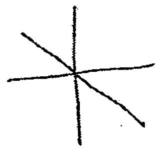
only in exact form

9 (i) Solve the equation $\tan 2A = -3$ for $0° \le A \le 360°$. [4]

(ii) State the number of solutions of the equation $\tan 6A = -3$ for $0° \le A \le 360°$. [1]

(i) $\tan 2A = -3$

Basic ∡ = 71.565° [M1]



$0° \le 2A \le 720°$ [M1]

∴ $2A = 108.435°,\ 288.435°,\ 468.435°,\ 648.435°$ } [M1] dependent on range stated

$A = 54.2°,\ 144.2°,\ 234.2°$ or $324.2°$ [A1] all

(ii) no of solutions $= 4 \times 3$ or 6×2

$= 12$ [B1]

10 A particle moves in a straight line so that, t seconds after leaving a fixed point O, its velocity, v ms^{-1}, is given by $v = 12e^{-0.4t} - 1$. Find

(i) an expression for the acceleration of the particle in terms of t, [2]

(ii) the distance travelled by the particle before it comes to instantaneous rest. [5]

(i) $a = 12e^{-0.4t}(-0.4)$ [M1]

$= -4.8e^{-0.4t}$ [A1]

(ii) when $v = 0$

$12e^{-0.4t} - 1 = 0$ [M1]

$\frac{12}{e^{0.4t}} = 1$

$12 = e^{0.4t}$

$0.4t = \ln 12$

$t = 6.2123$ [A1]

$\therefore$ distance travelled

$= \int_0^{6.2123} (12e^{-0.4t} - 1)\, dt$ [M1]

$= \left[\frac{12e^{-0.4t}}{-0.4} - t\right]_0^{6.2123}$ [M1]

$= \left[-30e^{-0.4t} - t\right]_0^{6.2123}$

$= \left[-8.7123 - (-30)\right]$

$= 21.3$ m // [A1]

OR Alt method:

$s = \int v\, dt$

$= \int 12e^{-0.4t} - 1\, dt$

$= \frac{12e^{-0.4t}}{-0.4} - t + c$ [M1]

$= -30e^{-0.4t} - t + c$

when $t = 0, s = 0$

$\therefore c = 30$ [M1] value of c

$\therefore s = -30e^{-0.4t} - t + 30$

Distance travelled

$= -30e^{-0.4(6.2123)} - 6.2123 + 30$

$= 21.3$ m [A1]

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(iii) Hence, find the coordinates of the point on the straight line at which $2y = x^2$. [3]

(i) $\frac{y}{x} = cx + d$ [B1]

(ii) $c =$ gradient

$= \frac{-21+3}{-3+1} = \frac{-18}{-2}$

$= 9$ // [B1]

At $(-1, -3)$,

$-3 = -9 + d$ [M1]

$d = 6$ // [A1]

(iii) $\frac{y}{x} = 9x + 6$ — (1)

$2y = x^2$

$\frac{y}{x} = \frac{x}{2}$ — (2)

(1) = (2)

$\frac{x}{2} = 9x + 6$ [M1]

$x = 18x + 12$

$17x = -12$

$x = \frac{-12}{17}$ [A1] value of one coordin.

$\therefore \frac{y}{x} = \frac{-12}{17} \div 2$

$= \frac{-6}{17}$

$\therefore$ point is $\left(\frac{-12}{17}, \frac{-6}{17}\right)$ // [A1]

12 The equation of a curve is $y=\dfrac{8a^3}{x^2+4a^2}$, where a is a constant.

(i) Show that $\dfrac{dy}{dx}=\dfrac{-16a^3x}{(x^2+4a^2)^2}$. [2]

$y = 8a^3(x^2+4a^2)^{-1}$

$\dfrac{dy}{dx} = 8a^3(-1)(x^2+4a^2)^{-2}(2x)$ M1

$= \dfrac{-16a^3x}{(x^2+4a^2)^2}$ // A1

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(ii) Find the value of a. [5]

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[Turn over

(ii) Given $\dfrac{dy}{dt} = -0.3$

At P, $-1 = \dfrac{8a^3}{(4+4a^2)}$ M1

$-4-4a^2 = 8a^3$

$2a^3 + a^2 + 1 = 0$ M1

Let $f(a) = 2a^3 + a^2 + 1$

$f(-1) = 0$

∴ By factor theorem,

$a+1$ is a factor of $f(a)$ M1

$(a+1)(2a^2 + Ba + 1) = 2a^3 + a^2 + 1$

By comparing coefficient of a:

$1 + B = 0$

$B = -1$

∴ $f(a) = (a+1)(2a^2 - a + 1)$ M1

when $f(a) = 0$

$a = -1$ or $2a^2 - a + 1 = 0$ A1

(no solution ∵ $b^2 - 4ac < 0$)

(iii) ∴ $y = \dfrac{-8}{x^2+4}$

$\dfrac{dy}{dx} = \dfrac{16x}{(x^2+4)^2}$

$\dfrac{dy}{dt} = \dfrac{dy}{dx} \times \dfrac{dx}{dt}$

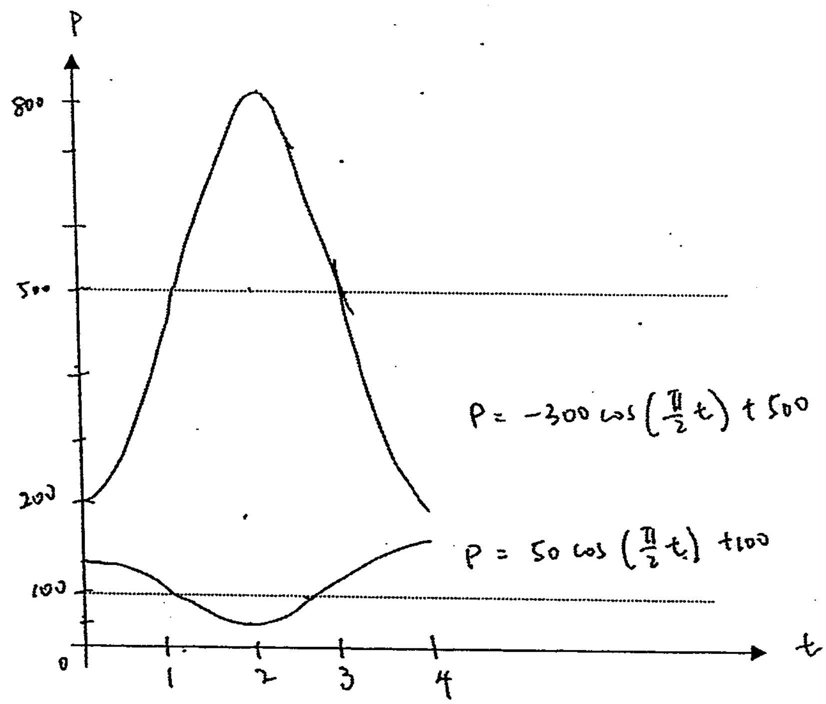
At P,

$-0.3 = 0.5 \times \dfrac{dx}{dt}$ M1

$\dfrac{dx}{dt} = -0.6$ unit/s A1

13 Scientists use sinusoidal functions to model populations of predators and prey in the environment. In a particular study which stretched over a period of 4 months, the population of mousedeer is modeled by the equation $P = -300\cos\left(\frac{\pi}{2}t\right) + 500$. The population of leopards in the same area is modeled by the equation $P = 50\cos\left(\frac{\pi}{2}t\right) + 100$. In each formula, P is the population of the animal and t is the time in months.

(i) On the same axes, sketch the graphs of $P = -300\cos\left(\frac{\pi}{2}t\right) + 500$ and $P = 50\cos\left(\frac{\pi}{2}t\right) + 100$ for $0 \le t \le 4$. [5]



Mousedeer
B1 { max : 800
min : 200

Leopard
B1 { max : 150
min : 50

1 cycle each B1

axes of curves B1

shape is inversion of each other B1

(ii) Using the graphs of these two equations, state the relationship between the population of mousedeer and the population of leopards in the area. [1]

When the population of mousedeer is at its maximum, the number of leopards is at its minimum, and vice versa. [B1] only if description matches (i)

OR

when population of leopards dropped, the population of mousedeer thrived

Leopards in the area are classified as vulnerable when their population is 80 or less.

(iii) Find the length of time for which the leopards became vulnerable during the study. [3]

$p = 50\cos\left(\frac{\pi}{2}t\right) + 100$

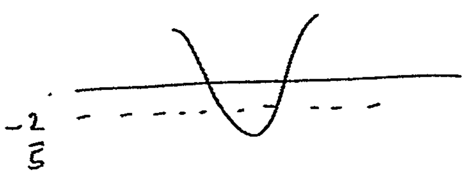
$50\cos\left(\frac{\pi}{2}t\right) + 100 \leq 80$

$\cos\left(\frac{\pi}{2}t\right) \leq -\frac{2}{5}$ [M1]

Basic $\angle$ = 1.1593

$0 \leq t \leq 4$

$0 \leq \frac{\pi}{2}t \leq 2\pi$



$\therefore \frac{\pi}{2}t = 1.9823$ or 4.3008 [M1]

$t = 1.2619$ or 2.7379

$\therefore$ Length of time $= 2.7379 - 1.2619$ $= 1.48$ months [A1]

END OF PAPER

Name:	Register No:	Class:

CRESCENT GIRLS' SCHOOL
SECONDARY FOUR
PRELIMINARY EXAMINATION

ADDITIONALMATHEMATICS4047/02

Paper 2 **21August 2014**

Additional materials: Answer Paper
Mark Sheet

2 hours 30 minutes

READ THESE INSTRUCTIONS FIRST

Write your name, register number and class in the spaces provided at the top of this page and on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use paper clips, highlighter, glue or correction fluid.

Answer all the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
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At the end of the examination, fasten all your work and mark sheetsecurely together.
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The total number of marks for this paper is 100.

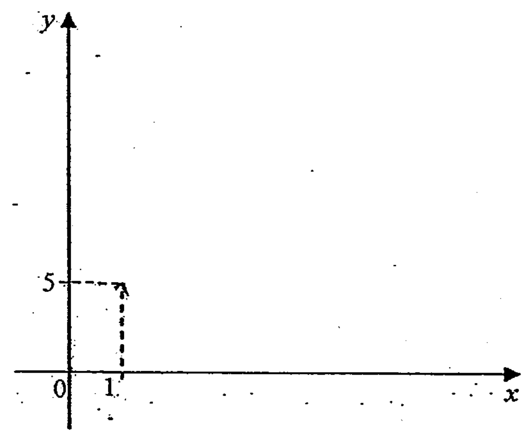
[Turn over

1 The curves $y = m\sqrt[3]{x}$ and $y = \dfrac{2m}{kx^2}$ meet at the point (1, 5), where m and k are constants.

(i) Find the value of m and of k. [2]

(ii) Sketch the two curves on the same axes, for $x > 0$. [3]

(iii) The normal to the curve $y = m\sqrt[3]{x}$ at (1, 5) meet the x-axis at P. Find the coordinates of P. [3]



2 (i) Given that coefficient of x^{-3} in the expansion of $\left(ax^2 - \dfrac{1}{x}\right)^{12}$ is -1760, where a is a constant, find the value of a. [4]

AD

(ii) Using the value of a found in part (i), find the coefficient of x^{-3} in the expansion of

$$\left(1+5x^3\right)\left(ax^2-\frac{1}{x}\right)^{12}.$$ [4]

3 Given the graph $f(x)=hx^3+\dfrac{k}{x^2}$ for $x>0$, has a gradient function of $f'(x)=3x^2-\dfrac{96}{x^3}$, where h and k are constants.

(i) Find the value of h and of k. [3]

(ii) Showing clear working, determine whether f(x) is an increasing or decreasing function for $0<x<2$. [3]

(iii) Determine the nature of the turning point of the graph. [3]

4 (i) Show that $\frac{d}{dx}\left(\cos^3 3x\right) = -9\cos^2 3x \sin 3x$. [2]

Hence evaluate each of the following.

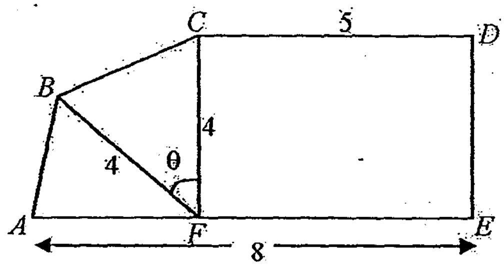
(ii) $\int_0^{\frac{\pi}{6}} \cos^2 3x \sin 3x \, dx$. [3]

(iii) $\int_0^{\frac{\pi}{6}} \sin^3 3x \, dx$. [4]

5 (i) Express $\frac{2x^2-3x+1}{9x^3-6x^2+x}$ as partial fractions. [6]

(ii) Hence find $\int \frac{2x^2-3x+1}{9x^3-6x^2+x}\,dx$. [3]

6



In the diagram, *AFE* is a straight line, $BF = CF = 4$ m, $CD = 5$ m and $AE = 8$ m. Given also that *CDEF* is a rectangle and $\angle BFC = \theta$. The area of the pentagon *ABCDE* is T m^2.

(i) Show that T can be expressed as $a + b\sin\theta + c\cos\theta$, where a, b and c are constants to be found. [3]

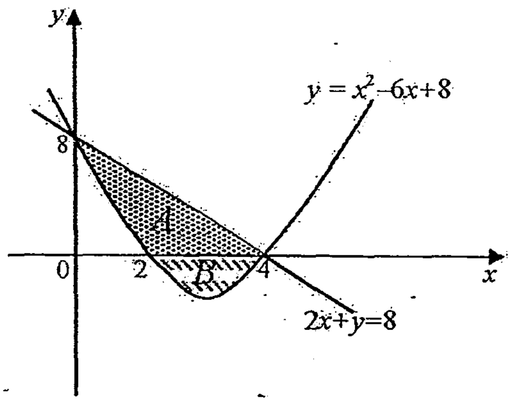
(ii) Express T in the form $a + R\cos(\theta - \alpha)$, where $R > 0$ and α is an acute angle. [3]

(iii) Find the maximum value of T and the corresponding value of θ. [2]

(iv) Explain whether T can be equal to 20 m^2. [2]

7 (a) Find the area bounded by the curve $y = \ln(x-1)$, the lines $y = -2$, $y = 2$ and the y-axis. [4]

(b) The diagram shows part of the curve $y = x^2 - 6x + 8$ and the line $2x + y = 8$. Find the ratio of shaded region A to the shaded region B. [7]



8 It is given that $\cos 2\theta = a + b$ and $\sin 2\theta = a - b$.

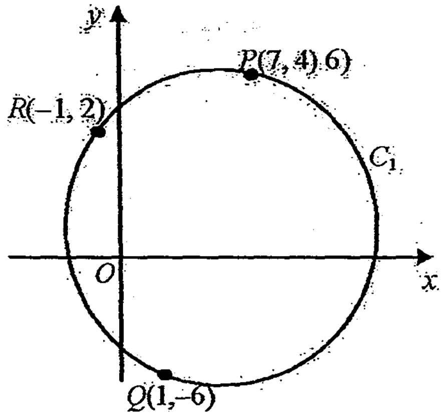
(i) Show that $2(a^2 - b^2) = \sin 4\theta$. [2]

(ii) Find the value of $a^2 + b^2$. [4]

(iii) Deduce that the value of $\tan 2\theta = -\frac{4}{3}$ if $\frac{a}{b} = -\frac{1}{7}$. [3]

(iv) Hence, without the use of a calculator, find the possible values of $\tan\theta$. [3]

9



In the diagram, which is not drawn to scale, P, Q and R are points on the circle C_1.

(i) Show that PQ is the diameter of the circle C_1 and hence find the centre of C_1. [5]

(ii) Find the equation of C_1 in the form $x^2+y^2+px+qy+r=0$, where p, q and r areintegers. [3]

(iii) Given that C_2 is a reflection of the circle C_1 in the line $x=-1$, find the centre of C_2. [2]

(iv) Determine whether Q lies inside or outside the circle C_2. [2]

[Turn over

10 **(a)** Solve the following equations.

(i) $\log_4 x - \log_4 0.5 = \log_{16}(7x-3)$. [4]

(ii) $\log_a e^{2x} + \log_a 2 = \log_a(15-e^x)$. [4]

(b) The population, P, of a certain type of bacteria, given a favourable growth medium, can be modelled by the equation $P = Ae^{kt}$, where A and k are constants and t is the time in hours.It is known that the population, P, doubles every 6.5 hours andthere were approximately 1000 bacteria at the start, how many bacteria will there be in a day and a half?Leave your answer correct to 4 significant figures? [4]

Name:	Register No:	Class:

Marking Scheme

ƎIRLS' SCHOOL
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ꞮY EXAMINATION

ADDITIONALMATHEMATICS4047/02

Paper 2

21August 2014

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[Turn over

Marking Scheme

1 The curves $y = m\sqrt[3]{x}$ and $y = \dfrac{2m}{kx^2}$ meet at the point (1, 5), where m and k are constants.

(i) Find the value of m and of k. [2]

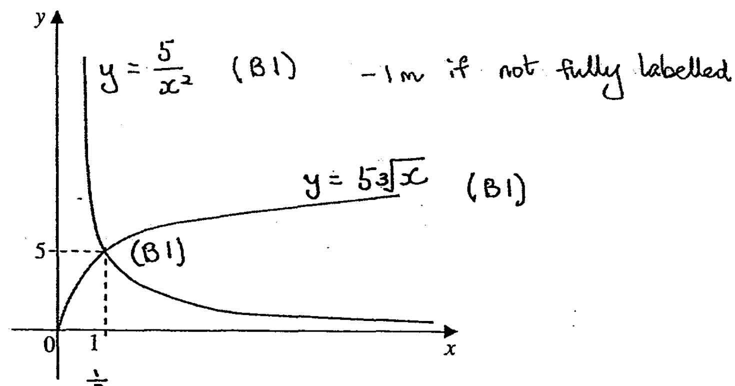
(ii) Sketch the two curves on the same axes, for $x > 0$. [3]

(iii) The normal to the curve $y = m\sqrt[3]{x}$ at (1, 5) meet the x-axis at P. Find the coordinates of P. [3]

(i) sub (1, 5) into $y = m\sqrt[3]{x}$, $m = 5$ (B1)

sub (1, 5) into $y = \dfrac{2m}{kx^2}$, $k = \dfrac{2(5)}{(5)(1)^2} = 2$ (B1)

(ii)



(iii) $y = 5x^{\frac{1}{3}}$

$\dfrac{dy}{dx} = \dfrac{5}{3}x^{-\frac{2}{3}}$ (B1)

At $x = 1$, $\dfrac{dy}{dx} = \dfrac{5}{3}$

$\therefore$ grad of normal $= -\dfrac{3}{5}$ (M1)

eqn' of normal is $y - 5 = -\dfrac{3}{5}(x - 1)$

when $y = 0$, $x = \dfrac{28}{5}$ $\therefore$ coord of $P = \left(\dfrac{28}{5}, 0\right)$ (A1)

2 (i) Given that coefficient of x^{-3} in the expansion of $\left(ax^2 - \dfrac{1}{x}\right)^{12}$ is -1760, where a is a constant, find the value of a. [4]

$T_{r+1} = \binom{12}{r}(ax^2)^{12-r}\left(-\dfrac{1}{x}\right)^r$ (M1)

$= \binom{12}{r}a^{12-r}(-1)^r x^{24-3r}$

$\therefore 24 - 3r = -3 \Rightarrow r = 9$ (A1)

$\therefore$ coeff of $x^{-3} = \binom{12}{9}a^3(-1)^9$

$a^3 = \dfrac{-1760}{-220} \Rightarrow a = 2$ (M1, A1)

(ii) Using the value of a found in part (i), find the coefficient of x^{-3} in the expansion of

$(1+5x^3)\left(ax^2-\frac{1}{x}\right)^{12}$. [4]

When $24-3r=-6 \Rightarrow r=10$ (M1)

$\therefore T_{11} = \binom{12}{10}(2)^{12-10}(-1)^{10}x^{-6}$

$= 264x^{-6}$ (A1)

hence $(1+5x^3)(\ldots -1760x^{-3}+264x^{-6}+\ldots)$

coeff of $x^{-3} = -1760+5(264)$ (M1)

$= -440$ (A1)

3 Given the graph $f(x)=hx^3+\frac{k}{x^2}$ for $x>0$, has a gradient function of $f'(x)=3x^2-\frac{96}{x^3}$, where h and k are constants.

(i) Find the value of h and of k. [3]

(ii) Showing clear working, determine whether $f(x)$ is an increasing or decreasing function for $0<x<2$. [3]

(iii) Determine the nature of the turning point of the graph. [3]

(i) $f'(x) = 3hx^2 - \frac{2k}{x^3}$ (M1)

$\therefore h=1$, $k=48$ (A1, A1)

(ii) $f'(x) = \frac{3x^5-96}{x^3}$

$= \frac{3(x^5-32)}{x^3}$ (M1)

For $0<x<2$, $x^3>0$, $x^5-32<0$ (M1)

$\therefore f'(x)<0$ ie. $f(x)$ is a decreasing fn for $0<x<2$ (A1)

(iii) For turning pt $f'(x)=0$

$3x^5-96=0$ (M1)

$x=2$

$f''(x) = 6x+\frac{288}{x^4}$ (M1)

$f''(2)>0$

$\therefore$ turning pt. is a min pt. (A1)

4 (i) Show that $\frac{d}{dx}\left(\cos^3 3x\right) = -9\cos^2 3x \sin 3x$. [2]

Hence evaluate each of the following.

(ii) $\int_0^{\frac{\pi}{6}} \cos^2 3x \sin 3x \, dx$. [3]

(iii) $\int_0^{\frac{\pi}{6}} \sin^3 3x \, dx$. [4]

(i) $\frac{d}{dx}(\cos 3x)^3 = 3(\cos 3x)^2(-3\sin 3x)$ (B1) (B1)

$= -9\cos^2 3x \sin 3x$

(ii) $-9\int_0^{\pi/6}(\cos^2 3x \sin 3x)\,dx = \left[\cos^3 3x\right]_0^{\frac{\pi}{6}}$ (M1)

$= 0^3 - 1^3$ (M1)

$\therefore \int_0^{\frac{\pi}{6}}(\cos^2 3x \sin 3x)\,dx = \frac{1}{9}$ (A1)

(iii) $\cos^2 3x \sin 3x = (1-\sin^2 3x)(\sin 3x)$ (M1)

$= \sin 3x - \sin^3 3x$

$\therefore \int_0^{\frac{\pi}{6}}(\sin 3x - \sin^3 3x)\,dx = \frac{1}{9}$

$\int_0^{\frac{\pi}{6}} \sin 3x\,dx - \int_0^{\frac{\pi}{6}} \sin^3 3x\,dx = \frac{1}{9}$ (M1)

$\therefore \int_0^{\frac{\pi}{6}} \sin^3 3x\,dx = \left[-\frac{\cos 3x}{3}\right]_0^{\frac{\pi}{6}} - \frac{1}{9}$

$= \frac{1}{3} - \frac{1}{9}$ (M1)

$= \frac{2}{9}$ (A1)

5 (i) Express $\frac{2x^2-3x+1}{9x^3-6x^2+x}$ as partial fractions. [6]

(i) $9x^3 - 6x^2 + x = x(9x^2-6x+1)$

$= x(3x-1)^2$ (B1)

$\therefore \frac{2x^2-3x+1}{x(3x-1)^2} = \frac{A}{x} + \frac{B}{(3x-1)} + \frac{C}{(3x-1)^2}$ (M1, M1)

$\therefore 2x^2 - 3x + 1 = A(3x-1)^2 + Bx(3x-1) + Cx$

when $x=0$, $A=1$ (M1, A 2, 1, 0)

when $x=\frac{1}{3}$, $C=\frac{2}{3}$

when $x=1$, $B=-\frac{7}{3}$

$\therefore \frac{2x^2-3x+1}{x(3x-1)^2} = \frac{1}{x} - \frac{7}{3(3x-1)} + \frac{2}{3(3x-1)^2}$

(−1M if final ans not shown)

(ii) Hence find $\int \frac{2x^2-3x+1}{9x^3-6x^2+x}\,dx$. [3]

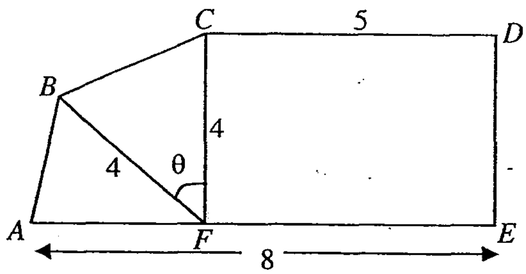
(ii) $\int \frac{2x^2-3x+1}{9x^3-6x^2+x}\,dx = \int \frac{1}{x} - \frac{7}{3(3x-1)} + \frac{2}{3(3x-1)^2}\,dx$

$= \ln x - \frac{7}{3} \times \frac{\ln(3x-1)}{3} + \frac{2}{3} \times \frac{(3x-1)^{-1}}{3(-1)} + c$

$= \ln x - \frac{7}{9}\ln(3x-1) - \frac{2}{9(3x-1)} + c$

(B1) (B1) (B1)

6



In the diagram, *AFE* is a straight line, *BF* = *CF* = 4 m, *CD* = 5 m and *AE* = 8 m. Given also that *CDEF* is a rectangle and ∠*BFC* = θ. The area of the pentagon *ABCDE* is *T* m^2.

(i) Show that *T* can be expressed as $a+b\sin\theta+c\cos\theta$, where *a*, *b* and *c* are constants to be found. [3]

(ii) Express *T* in the form $a+R\cos(\theta-\alpha)$, where $R > 0$ and α is an acute angle. [3]

(iii) Find the maximum value of *T* and the corresponding value of θ. [2]

(iv) Explain whether *T* can be equal to 20 m^2. [2]

(i) $T = \frac{1}{2} AF \cdot BF \sin(90° - \theta) + \frac{1}{2} BF \cdot CF \sin\theta + (CF \cdot CD)$ (M1)

$= \frac{1}{2}(8-5)(4)\cos\theta + \frac{1}{2}(4)(4)\sin\theta + (4 \times 5)$ (M1)

$= 6\cos\theta + 8\sin\theta + 20$ (A1)

(ii) $T = 20 + R\cos(\theta - \alpha)$

where $R = \sqrt{6^2+8^2} = 10$, $\tan\alpha = \frac{8}{6}$, $\alpha = 53.130°$ (M1, A1, A1)

$\therefore T = 20 + 10\cos(\theta - 53.1°)$ (−1 if final ans not shown)

(iii) max $T = 10 + 20 = 30$ (A1)

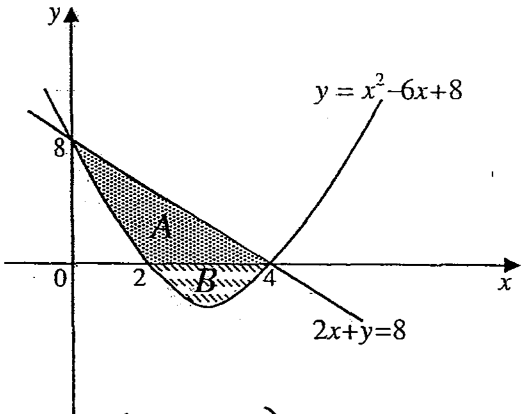
when $\cos(\theta - 53.130°) = 1 \Rightarrow \theta \approx 53.1°$ (A1)

(iv) When $T = 20$, $10\cos(\theta - 53.130°) = 0 \Rightarrow \theta = 143.130°$ (NA)

$\therefore T \neq 20$ (M1, A1)

7 **(a)** Find the area bounded by the curve $y = \ln(x-1)$, the lines $y = -2$, $y = 2$ and the y-axis. [4]

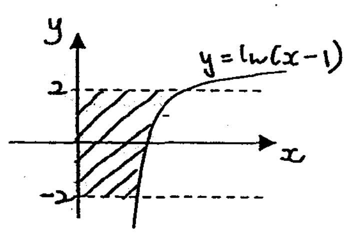
(b) The diagram shows part of the curve $y = x^2 - 6x + 8$ and the line $2x + y = 8$. Find the ratio of shaded region A to the shaded region B. [7]



7 a) $y = \ln(x-1)$

$x = e^y + 1$ (M1)

$\therefore$ Area $= \int_{-2}^{2} e^y + 1 \, dy$ (M1)



$= \left[e^y + y\right]_{-2}^{2}$ (M1)

$= (e^2 + 2) - (e^{-2} - 2)$

$= 4 + e^2 - \frac{1}{e^2}$ or 11.3 unit^2 (A1)

7b) Area of A $= \frac{1}{2}(4)(8) - \int_0^2 (x^2 - 6x + 8)\, dx$ (M1)

$= 16 - \left[\frac{x^3}{3} - 3x^2 + 8x\right]_0^2$ (M1) – integration

$= \frac{28}{3}$ unit^2 (A1)

Area of B $= \left|\int_2^4 (x^2 - 6x + 8)\, dx\right|$ (M1)

$= \left|\left[\frac{x^3}{3} - 3x^2 + 8x\right]_2^4\right|$ (M1)

$= \frac{4}{3}$ unit^2 (A1)

$\therefore$ Ratio of A to ratio of B $= 7 : 1$ (A1)

8 It is given that $\cos 2\theta = a + b$ and $\sin 2\theta = a - b$.

(i) Show that $2(a^2 - b^2) = \sin 4\theta$. [2]

(ii) Find the value of $a^2 + b^2$. [4]

(iii) Deduce that the value of $\tan 2\theta = -\frac{4}{3}$ if $\frac{a}{b} = -\frac{1}{7}$. [3]

(iv) Hence, without the use of a calculator, find the possible values of $\tan\theta$. [3]

8 i) $2(a^2 - b^2)$

$= 2(a+b)(a-b)$ (M1)

$= 2\sin 2\theta \cos 2\theta$

$= \sin 4\theta$ (B1)

ii) $(a+b)^2 = (\cos 2\theta)^2$

$a^2 + b^2 + 2ab = \cos^2 2\theta$ .... (1) (M1)

$(a-b)^2 = (\sin 2\theta)^2$

$a^2 + b^2 - 2ab = \sin^2 2\theta$ .... (2) (M1)

(1) + (2) $2(a^2 + b^2) = \sin^2 2\theta + \cos^2 2\theta$ (M1)

$\therefore a^2 + b^2 = \frac{1}{2}$ (A1)

(iii) $\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta}$

$= \frac{a-b}{a+b}$ (M1)

$= \frac{\frac{a}{b} - 1}{\frac{a}{b} + 1}$ (M1)

$= \frac{-\frac{1}{7} - 1}{-\frac{1}{7} + 1}$ (A1)

$= -\frac{4}{3}$

(iv) $\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$

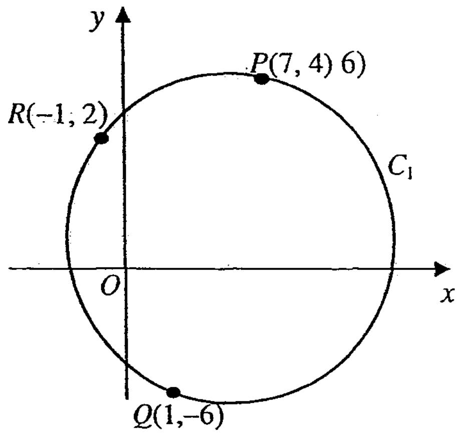
$\therefore \frac{2\tan\theta}{1 - \tan^2\theta} = -\frac{4}{3}$ (M1)

$4\tan^2\theta - 6\tan\theta - 4 = 0$

$2(2\tan\theta + 1)(\tan\theta - 2) = 0$ (M1)

$\therefore \tan\theta = -\frac{1}{2}$ or 2 (A1)

9



In the diagram, which is not drawn to scale, P, $Q$and $R$are points on the circle C_1.

(i) Show that PQ is the diameter of the circle $C_1$and hence find the centre of C_1. [5]

(ii) Find the equation of C_1 in the form $x^2+y^2+px+qy+r=0$, where p, q and r areintegers. [3]

(iii) Given that C_2 is a reflection of the circle C_1 in the line $x=-1$, find the centre of C_2. [2]

(iv) Determine whether Q lies inside or outside the circle C_2. [2]

(i) $\text{grad}_{RQ} = \frac{2-(-6)}{-1-1} = -4$ (B1)

$\text{grad}_{RP} = \frac{4-2}{7-(-1)} = \frac{1}{4}$ (B1)

Since $\text{grad}_{RQ} \times \text{grad}_{RP} = -4 \times \frac{1}{4} = -1$ (M1)

$\therefore RQ \perp RP$

$\therefore \angle QRP = 90°$ ($\angle$ in semi-circle) (M1)

$\therefore PQ$ is the diameter of C_1

$\therefore$ Centre of $C_1 = \left(\frac{1+7}{2}, \frac{-6+4}{2}\right) = (4, -1)$ (A1)

(ii) radius of $C_1 = \sqrt{(4-1)^2+(-1-(-6))^2} = \sqrt{34}$ (M1)

eqn of C_1: $(x-4)^2+(y+1)^2 = (\sqrt{34})^2$ (M1)

$x^2+y^2-8x+2y-17=0$ (A1)

(iii) Centre of $C_2 = (-6, -1)$, radius $= \sqrt{34}$ (M1)

$\therefore$ eqn of C_2: $(x+6)^2+(y+1)^2 = 34$ (A1)

(iv) dist of Q to centre of $C_2 = \sqrt{(1+6)^2+(-6+1)^2} = \sqrt{74}$ (M1)

Since $\sqrt{74} > \sqrt{34}$, Q lies outside C_2 (A1)

[Turn over

10 **(a)** Solve the following equations.

(i) $\log_4 x - \log_4 0.5 = \log_{16}(7x-3)$. [4]

(ii) $\log_a e^{2x} + \log_a 2 = \log_a(15-e^x)$. [4]

(b) The population, P,of a certain type of bacteria, given a favourable growth medium, can be modelled by the equation $P = Ae^{kt}$, where A and k are constants and t is the time in hours.It is known that the population, P,doubles every 6.5 hours andthere were approximately 1000 bacteria at the start, how many bacteria will there be in a day and a half?Leave your answer correct to 4 significant figures? [4]

10 a) i)

$\log_4\left(\frac{x}{\frac{1}{2}}\right) = \frac{\log_4(7x-3)}{\log_4 4^2}$ (M1)

$\log_4(2x) = \frac{1}{2}\log_4(7x-3)$ (M1)

$\log_4(4x^2) = \log_4(7x-3)$

$\therefore 4x^2 = 7x-3$

$4x^2 - 7x + 3 = 0$ (M1)

$(4x-3)(x-1) = 0$

$\therefore x = 1$ or $\frac{3}{4}$ (A1)

ii) $\log_a(2e^{2x}) = \log_a(15-e^x)$ (M1)

$\therefore 2e^{2x} + e^x - 15 = 0$ (M1)

$(2e^x - 5)(e^x + 3) = 0$

$\therefore e^x = \frac{5}{2}$ or $e^x = -3$ (rej) (A1)

$\therefore x = \ln\left(\frac{5}{2}\right)$ or 0.916 (A1)

10b) $P = Ae^{kt}$

When $t = 0$, $P = 1000$

$\therefore A = 1000$ (B1)

When $t = 6.5$, $P = 2A$,

$\therefore 2A = Ae^{k(6.5)}$ (M1)

$\therefore 6.5k = \ln 2$

$k = \frac{\ln 2}{6.5}$ (A1)

When $t = 36$, $P = 1000e^{\left(\frac{\ln 2}{6.5}\right)(36)} \approx 46480$ bacteria (A1)

Candidate Name:	Class:	Index No:

DUNMAN SECONDARY SCHOOL

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PRELIMINARY EXAMINATION 2014

SECONDARY 4 EXPRESS

ADDITIONAL MATHEMATICS 4047 / 1

2 H 18 SEP 2014

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions from Question 1 to Question 13.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

Begin your answer to Question 9 on a fresh sheet of writing paper.
At the end of the examination, hand in **Question 1 to Question 8** and **Question 9 to Question 13** separately.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

This question paper consists of 6 printed pages including the cover page.

MR VINCENT LEW [Turn over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} .$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n ,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1 .$$
$$\sec^2 A = 1 + \tan^2 A .$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A .$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B .$$
$$\cos(A \pm B) \cos A \cos B \mp \sin A \sin B .$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} .$$
$$\sin 2A = 2 \sin A \cos A .$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A .$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A} .$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} .$$
$$a^2 = b^2 + c^2 - 2bc \cos A .$$
$$\Delta = \frac{1}{2} bc \sin A .$$

1. Given that the line $y = mx + 2$ is a tangent to the curve $y = k - x^2$, where m and k are positive constants, express k in terms of m. [4]

2. It is given that $\int_{1}^{2} \frac{a}{x^2}\,dx = \frac{1}{3}$, where a is a constant.

   (i) Find the value of $\int_{2}^{4} \frac{a}{x^2}\,dx$ [3]

   (ii) Find $\int_{1}^{2} \frac{a}{x^2} + bx\ dx$ in terms of the constant b. [2]

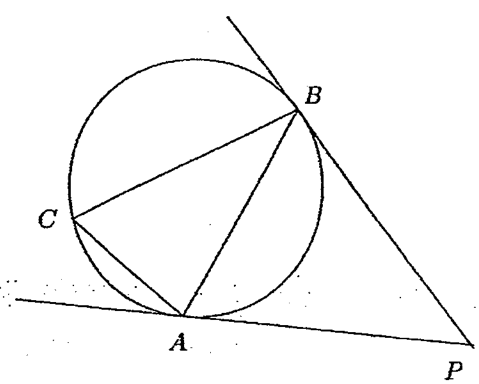
3. Find the coordinates of the stationary point of the curve $y = x^2 + \frac{1}{x^2}$ for $x > 0$, and determine the nature of the stationary point. [5]

4. Solve the pair of simultaneous equations

$$\log_8(2x + 3y) = \frac{1}{3}$$

$$\sqrt{54^x} = 3^x 6^y$$ [5]

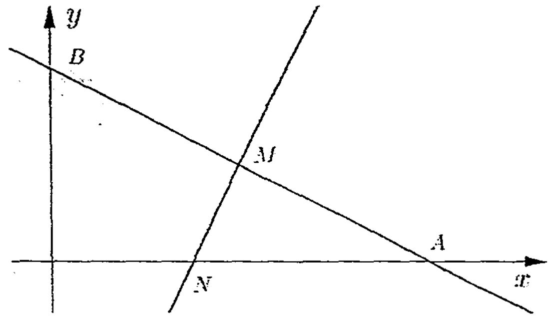
5.



The diagram shows points A, B, and C lying on a circle. The point P is such that the lines PA and PB are tangents to the circle. Given that $BA = BC$, show that

   (i) triangle BCA is similar to triangle PBA. [2]

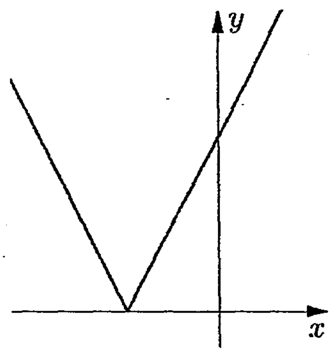
   (ii) $AC \times AP = AB^2$ [2]

6. 

In the diagram, the line $2y + x - 12 = 0$ intersects the x and y axes as points A and B respectively. The perpendicular bisector of AB passes through the line at point M and and intersects the x axis at point N. Find the coordinates of N. [6]

7. The roots of equation $2x^2 - x + 4 = 0$ are α and β.

Without evaluating the values of α and β, form an equation whose roots are α^3 and β^3. [6]

8. 

The diagram above shows part of the graph of $y = 2|x+3|$. In each of the following cases determine the number of intersections of the curve $y = a(x-h)^2 + k$ with $y = 2|x+3|$, justifying your answer.

(i) $a = 1$, $h = -3$, and $k = 0$ [2]

(ii) $a = -1$, $h = 0$, and $k > 6$ [2]

(iii) $a = -1$, $h > 3$, and $k < 9$ [2]

Begin Question 9 on a fresh sheet of writing paper

9. (i) Express the equation $3\tan^2 x = 4\sec x + 1$ as a quadratic equation in $\sec x$. [2]

(ii) Hence solve the equation $3\tan^2 x = 4\sec x + 1$ for $0 \le x \le 2\pi$ [4]

(iii) State the number of solutions of the equation $3\tan^2 x = 4\sec x + 1$ in the range $-3\pi \le x \le 3\pi$ [1]

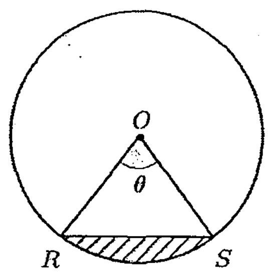
10. A particle travels in a straight line so that t seconds after passing through a fixed point O, its velocity, v ms^{-1} is given by $v = 2t^2 - 7t + 5$. Find

(i) an expression for the acceleration of the particle in terms of t. [1]

(ii) the time and velocity at which the acceleration is zero. [3]

(iii) the distance from O at which the particle first comes to rest. [4]

11.



The diagram shows a circle of radius 10 cm with a minor sector *ORS*, which subtends an angle of θ radians at O, the centre of the circle.

(i) Show that A, the area of the shaded minor segment as shown in the diagram is given by $A = 50\theta - 50\sin\theta$ [2]

(ii) Given that A is increasing at a constant rate of 5 cm^2 s^{-1}, express the rate of change of θ with respect to time, in terms of θ. [3]

(iii) Hence find the rate of change of θ when $\theta = \pi$ [2]

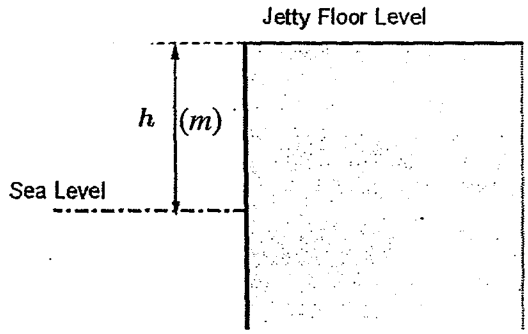
(iv) State with a reason whether the rate of change of θ of will increase or decrease for $\pi < \theta < 2\pi$ [1]

12. Variables x and y are connected by the equation $y = k\,a^x$ where a and k are constants. Using an experiment, values of x and y were obtained and a graph was drawn in which $\ln y$ was plotted on the vertical axis against x on the horizontal axis. The straight line which was obtained passed through the points (0.173, 1.73) and (4.46, 4.70). Estimate

(i) the values of a and k, to 2 significant figures [4]

(ii) the coordinates of the point on the line at which $y = e^x$ [4]

13.



The height difference, h m, between the jetty floor level and the sea level changes with time due to the tidal effects. It is given that h can be modelled mathematically as

$$h = 1.8 - 1.1 \sin kt$$

where k is a constant and t is the time in hours from midnight.

The time between two consecutive high tides is 12 hours.

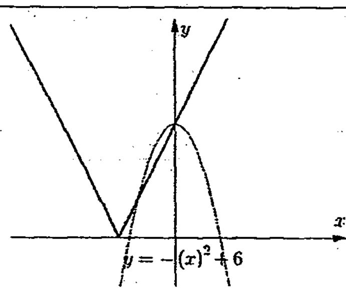
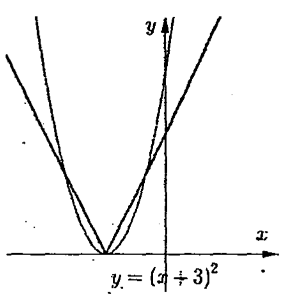
(i) Show that the value of k is $\frac{\pi}{6}$ radians per hour. [2]

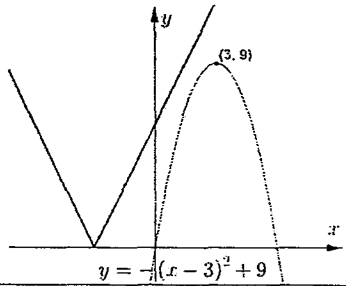
(ii) State the maximum value of the height difference h [1]

(iii) If the jetty can be used for boats to land only when the height difference, h is within range of values $0.7 \le h \le 1.5$, find the total length of time in hours in a day when the boat landings are possible. [5]

END OF PAPER

No	Marking Scheme	Marks	
1	Substitute $y=mx+2$ into $y=k-x^2$ to eliminate y $\Rightarrow mx+2=k-x^2$ $\Rightarrow x^2+mx+2-k=0$ Tangent if '$b^2-4ac=0$' ie. $m^2-4(1)(2-k)=0$ $=\quad k=\frac{1}{4}(8-m^2)$	M1 A1 M1 A1	[4]
2 i	$\int_1^2 \frac{a}{x^2}\,dx=\frac{1}{3} \Rightarrow \left[-\frac{a}{x}\right]_1^2=\frac{1}{3}$ $\Rightarrow\ -\frac{a}{2}-\left(-\frac{a}{1}\right)=\frac{1}{3} \qquad \Rightarrow a=\frac{2}{3}$ Hence $\int_2^4 \frac{a}{x^2}\,dx=\frac{2}{3}\left[-\frac{1}{x}\right]_2^4=\frac{1}{6}$	M1 A1 A1	
2 ii	$\int_1^2 \frac{a}{x^2}+bx\ dx=\int_1^2 \frac{a}{x^2}\,dx+\int_1^2 bx\ dx$ $\Rightarrow \frac{1}{3}+\left[\frac{bx^2}{2}\right]_1^2 \quad =\frac{1}{3}+\frac{4b}{2}-\frac{b}{2} \quad =\frac{1}{3}+\frac{3b}{2}$	M1, A1	[5]
3.	$\frac{dy}{dx}=2x-\frac{2}{x^3}$ Set $\frac{dy}{dx}=0 \Rightarrow 2x-\frac{2}{x^3}=0$ $\Rightarrow 2x=\frac{2}{x^3} \quad \Rightarrow \quad x^4=1 \quad \Rightarrow \quad x=1$ (as $x>0$) and $y=2$ Turning point coordinates $(1,2)$ $\frac{d^2y}{dx^2}=2+\frac{6}{x^4}$ At $x=1$, $\frac{d^2y}{dx^2}=2+\frac{6}{1^4}=8>0 \Rightarrow$ Min pt.	M1 M1 M1 M1, A1	[5]
4.	$\Rightarrow 2x+3y=8^{\frac{1}{3}}$ $\Rightarrow 2x+3y=2$ -------(1) $\Rightarrow \sqrt{(6\times 9)^x}=3^x6^y$ $\Rightarrow 6^{\frac{x}{2}}3^{\frac{2x}{2}}=3^x6^y \Rightarrow 6^{\frac{x}{2}}=6^y$ $\Rightarrow x=2y$ ---------(2) Substituting into (1) $2(2y)+3y=2 \quad \therefore y=\frac{2}{7}$ and $x=\frac{4}{7}$	M1 M1 M1 A2	[5]
5 i	$PA=PB$ (external tangents to circle) $\angle PAB=\angle PBA$ (since triangle PAB is an isoseles triangle) But also $\angle PAB=\angle ACB$ (alternate segment theorem) Since also given $BA=BC$ then $\angle ACB=\angle CAB$ (since triangle PAB is an isoseles triangle) Hence by AA similarity, triangle BCA is similar to triangle PBA	M1 A1 Deduct 1 mark if final reason not stated	

5 ii	Since triangle BCA is similar to triangle PBA $=> \frac{AC}{AB}=\frac{AB}{AP} \quad => AC \times AP = AB^2$	M1, A1	[4]
6.	Rearrange equation of line $=> y=-\frac{1}{2}x+6$ By substitutions of $y=0$ and $x=0$ we get coordinates A : (12, 0) B : (0, 6) $\therefore$ midpoint M is (6, 3) Gradient of perpendicular bisector $= -\frac{1}{-\frac{1}{2}}=2$ Coordinates of N $(k, 0)$ $=> \frac{3-0}{6-k}=2 => k=6-1.5=4.5$ $\therefore$ coordinates of N (4.5, 0)	B2 B1 A1 M1 B1	[6]
7	$=> x^2-\frac{1}{2}x+2=0$ Sum of roots $\alpha+\beta=\frac{1}{2}$ Prod of roots $\alpha\beta=2$ New sum of roots $\alpha^3+\beta^3=(\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2)$ $=(\alpha+\beta)((\alpha+\beta)^2-3\alpha\beta)$ $=\left(\frac{1}{2}\right)\left(\left(\frac{1}{2}\right)^2-3(2)\right)=-\frac{23}{8}$ New prod of roots $(\alpha^3\beta^3)=(2)^3=8$ New equation : $=> x^2+\frac{23}{8}x+8=0$ $=> or\ 8x^2+23x+64=0$	M1 M1 M1 A1 A1) A1 (either) accepted)	[6]
8.	(i) sketch shows 3 intersections  (ii) sketch shows 2 intersections  	B2 – 1 mark for correct curve shape, 1 mark for correct critical point at (-3,0) B2 – 1 mark for correct curve shape, 1 mark for correct critical point at (0, 6)	

8.	(iii) sketch shows 0 intersections 	B2 – 1 mark for correct curve shape, 1 mark for using critical point at (3, 9)	[6]
9 i	$\Rightarrow \; 3(\sec^2 x - 1) = 4\sec x + 1$	M1	
	$\Rightarrow 3\sec^2 x - 4\sec x - 4 = 0$	A1	
ii	$3\tan^2 x = 4\sec x + 1 \;\Rightarrow 3\sec^2 x - 4\sec x - 4 = 0$		
	$\Rightarrow (3\sec x + 2)(\sec x - 2) = 0$	M1	
	$\sec x = -\frac{2}{3}$ or $\sec x = 2 \;\Rightarrow \cos x = -\frac{3}{2}$ (reject!) or $\cos x = \frac{1}{2}$	M1	
	For $\cos x = \frac{1}{2} \;\Rightarrow x = \frac{\pi}{3}, \frac{5\pi}{3}$	A2	
iii	Solutions up to $3\pi = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}$ so including up to -3π : 6 solutions	B1	[7]
10i	Acceleration, $a = \frac{dv}{dt} = 4t - 7 \quad ms^{-2}$	A1	
ii.	Set $\frac{dv}{dt} = 0 \quad \Rightarrow 4t - 7 = 0 \;$ *ie.* $t = 1.75\,s$	M1,A1	
	$v = 2(1.75)^2 - 7(1.75) + 5 = -1.125\ ms^{-1} = 1.13 ms^{-1}$ to 3 s.f.	A1	
iii	Set $v = 0 \;\Rightarrow 2t^2 - 7t + 5 = 0 \Rightarrow (2t-5)(t-1) = 0 \Rightarrow t = 1s$ or $2.5\,s$	M1	
	Take $t = 1$ s for particle first coming to rest	B1	
	Displacement $s = \int v\ dt \Rightarrow \int 2t^2 - 7t + 5\ dt$		
	$\Rightarrow s = \frac{2t^3}{3} - \frac{7t^2}{2} + 5t + c.$	M1	
	At $t = 0,\; s = 0 \Rightarrow c = 0. \quad \therefore$ at $t = 1,\; s = \frac{2}{3} - \frac{7}{2} + 5 = 2\frac{1}{6}$ m (2.17 m)	A1	[8]
11i	Area of sector $ORS = \frac{1}{2}(10)^2\theta = 50\theta$	M1	
	Area of triangle ORS (using sine formula) $= \frac{1}{2}(10)^2 \sin\theta = 50\sin\theta$		
	$\therefore$ area of minor segment (shaded portion) $A = 50\theta - 50\sin\theta \quad cm^2$	A1	
ii		M1	
		M1	

iii	$\frac{dA}{d\theta}=50-50\cos\theta$ $\frac{dA}{dt}=\frac{dA}{d\theta}\times\frac{d\theta}{dt}$ $\quad\Rightarrow 5=(50-50\cos\theta)\times\frac{d\theta}{dt}$ $\Rightarrow \frac{d\theta}{dt}=\frac{5}{50-50\cos\theta}$ rad s^{-1} At $\theta=\pi$, $\frac{d\theta}{dt}=\frac{5}{(50-50\cos(\pi))}=\frac{5}{100}$ rad s^{-1} (0.005 rad s^{-1}) We note that as θ increases from π to 2π, $50-50\cos\theta$ reduces from 100 towards 0 and hence $\frac{d\theta}{dt}$ is increasing from $\frac{5}{100}$ to very large values (Allow for explanation based on $\frac{d^2\theta}{dt^2}>0$ for $\pi<\theta<2\pi$)	A1 M1, A1 B1 (reasoning on increasing value due to reducing denominator)	[8]
12 i	Taking logarithms to both sides $\ln y = x\ln a + \ln k$ Plotting $\ln y$ vs x, we have a straight line of the form $Y = mX + c$ and m, the gradient of line $\equiv \ln a$ and c, the y intercept $\equiv \ln k$ Gradient between given pts : $\frac{4.70-1.73}{4.46-0.173}=0.6928 = \ln a$ $\therefore\ a = e^{0.6928} = 2.0$ $4.70 = 0.6928(4.46) + c \Rightarrow c = 1.6101 \Rightarrow k = e^{1.6101} = 5.0$	M1 M1 A1 A1	
ii	At $y = e^x \Rightarrow \ln y = x$, then equation of the straight line becomes $x = x\ln a + \ln k \Rightarrow x = \frac{\ln k}{(1-\ln a)} = \frac{1.6101}{(1-0.6928)} = 5.241$ $\therefore \ln y = 5.241$ **the coordinates of the point is (5.24, 5.24) to 3 s.f.** Accept (5.25,5.25) where pupils substitute values of a and k found	M1 M1, A1 A1	[8]
13 i	Period = 12 h $\Rightarrow \frac{2\pi}{k}=12 \Rightarrow k=\frac{2\pi}{12}=\frac{\pi}{6}$	M1, A1	
ii	Maximum value of h = 1.8 − 1.1(−1) = 2.9 m	B1	
iii	$h=0.7$ minimum value as $1.8-1.1(1)$ $1.8 - 1.1\sin\frac{\pi}{6}t=1.5$ $\sin\frac{\pi}{6}t=0.2727 \Rightarrow \frac{\pi}{6}t=0.2727$ rad, 2.869 rad, 6.556 rad, 9. 152 rad $\Rightarrow t=0.5208$, 5.4794, 12.5210 and 17.4790 Length of time where boat landing is possible = 5.4794-0.5208 + 17.4790-12.5210 = 9.9166 h = 9.92 h	M1 M1, A1 A1 A1	[8]

END OF MARK SCHEME

Candidate Name:	Class:	Index No:

DUNMAN SECONDARY SCHOOL

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END OF YEAR EXAMINATION 2014

SECONDARY4 EXPRESS

ADDITIONAL MATHEMATICS 4047 / 02

2 H 30 MIN

17th SEPT2014

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions from Question 1 to Question 10.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, hand in **Question 1 to Question 4, Question 5 to Question 7** and **Question 8 to Question 10** separately.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

This question paper consists of 5printed pages including the cover page.

MR TEO KB

[Turn over

Mathematical Formulae

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Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

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$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots\ldots(n-r+1)}{r!}$.

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Identities

$$\sin^2 A + \cos^2 A = 1.$$

$$\sec^2 A = 1 + \tan^2 A.$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A.$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B.$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B.$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}.$$

$$\sin 2A = 2 \sin A \cos A.$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A.$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}.$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

$$a^2 = b^2 + c^2 - 2bc \cos A.$$

$$\Delta = \frac{1}{2} bc \sin A.$$

1. (a) (i) Sketch the graph of $y = 6x^{\frac{-1}{2}}$ for $x > 0$. [2]

(ii) In order to solve the equation $3x^{\frac{3}{2}} - 2x^{\frac{1}{2}} - 6 = 0$, a graph of a suitable straight line is drawn on the same set of axes in part (i). Find the equation of this line. [1]

(b) (i) Sketch the graph of $y = \ln x$ for $x>0$. [2]

(ii) Determine the equation of the straight line which would need to be drawn on the graph of $y = \ln x$ in order to obtain a graphical solution of the equation $x^2 = e^{4-x}$. Draw the straight line in your sketch in (i). [3]

2. (i) Differentiate $x \cos 2x$ with respect to x. [3]

(ii) Using your answers to part (i), find $\int x \sin 2x \, dx$ and hence find $\int_0^{\frac{\pi}{4}} x \sin 2x \, dx$. [5]

3. (i) Given the expansion of $(1 + ax)^n$ is $1 + 21x + 189x^2 + ..$, find the values of a and n. [6]

(ii) Using your answer in part (i), find the coefficient of x^2 in the expansion of [2]

$$(1 + ax)^n (2 - 12x + 42x^2).$$

4. (i) Express $\frac{2x-4}{(2x-1)(x^2-1)}$ in partial fraction. [5]

(ii) Hence find $\int \frac{x-2}{(2x-1)(x^2-1)} dx$. [3]

5. A curve has the equation $y = \text{f}(x)$, where $\text{f}(x) = \dfrac{3x-2}{2x+3}$ for $x > 0$.

(i) Obtain an expression for $\text{f}'(x)$. [2]

(ii) Find the equation of the normal to the curve at the point where the curve crosses the x-axis. [3]

(iii) Showing full working, determine whether the gradient of the curve is an increasing or decreasing function. [3]

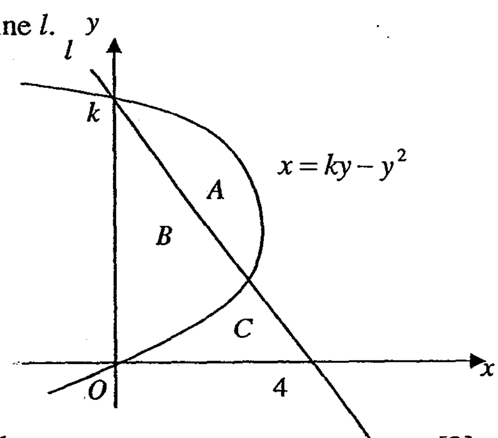
6. The line $y = 4$ is a tangent to circle C of radius 3 units. Given that the circle passes through the point A(1, –2),

(i) Show that the centre of the circle C is (1, 1). [3]

(ii) Find the equation of the circle C. [2]

(iii) The circle C cuts the x-axis at point B and D, where D lies to the right of B. Given that P is a point on the circle such that BP is a diameter of the circle C, find the exact value of the coordinates of P. [4]

7. The diagram shows a curve $x = ky - y^2$ and a straight line l.
It is given that the area bounded by the y-axis and the curve is $10\frac{2}{3}$ units^2. Find



(a) the value of k, [3]

(b) the equation of l, [2]

(c) the ratio of the areas A: B: C. [7]

8. (a) Prove the identity $\tan^4 x = \sec^2 x \tan^2 x - \sec^2 x + 1$ [3]

(b) Solve, for $0° \le x \le 360°$, the equation $\cos^2 x + 3\sin x\cos x + 1 = 0$. [5]

(c) Given that $\cos\theta = \dfrac{\sqrt{3}+1}{2\sqrt{2}}$ and $0° < \theta < 90°$,

(i) find the value of $\sin\theta$ in the form of $\dfrac{\sqrt{a-\sqrt{b}}}{a}$, [2]

(ii) hence, prove that $\tan\theta = \sqrt{7-4\sqrt{3}}$ [3]

9. (a) Solve the equation $\log_{\frac{x}{3}} x + \log_{3x} x = -2$. [4]

(b) If $\log_5 2 = 0.431$ and $\log_5 3 = 0.62$, find the value of

(i) $\log_5 \sqrt{3}$ [2]

(ii) $\log_5 13.5$ [2]

(c) In the beginning of 2010, a certain type of bacteria was found at the bottom of a seabed. It was known to grow with time, such that its population P, after t years is given by

$$P = 50\,000e^{kt}, \text{where } k \text{ is a constant.}$$

(i) Given that the population doubles in two years, show that $k = \frac{1}{2}\ln 2$. [2]

Hence, find

(ii) the year in which the population reaches 450 000, [3]

(iii) the size of the population of bacteria in 2015, giving your answer correct to the nearest 10 000. [2]

10. The diagram shows a quadrilateral$ABCDE$ in which $\angle DCE = 90^0$, AB= 12 cm, DE =7cm and $\angle BAC = \theta$ where $0° < \theta < 90°$. ΔABC is similar to ΔDEC.

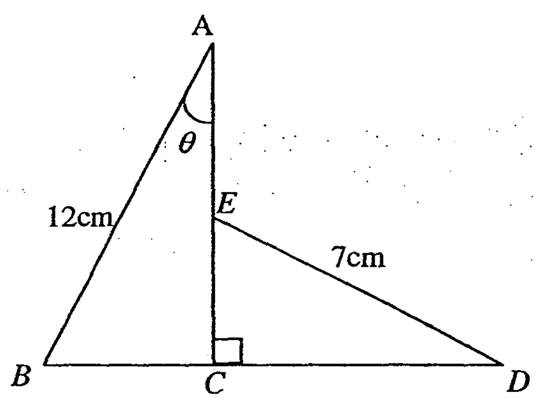
(i) Show that the perimeter, P cm, of the quadrilateral is given by [3]

$$P = 19 + 19\cos\theta + 5\sin\theta.$$

(ii) Express P in the form $19 + R\cos(\theta - \alpha)$, where $R > 0$, and α is an acute angle. [3]

(iii) Find the maximum value of the perimeter, P. [2]

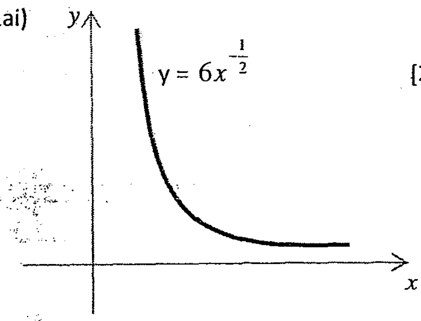
(iv) Find the value of θ for which $P = 34$. [3]



End of Paper 2

2014 EOY AMath Solutions

1ai)



[2]

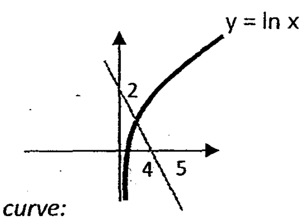
aii)

$$3x^{\frac{3}{2}} - 2x^{\frac{1}{2}} = 6$$

$$x^{\frac{1}{2}}(3x-2) = 6 \qquad [B1]$$

$$y = 3x - 2$$

1bi)



Graph of curve:

- *asymptote* *[1]*
- *Shape of graph in the correct direction* *[1]*

1bii) $\ln x^2 = \ln e^{4-x}$ [M1]

$2\ln x = 4 - x$

$y = -\dfrac{1}{2}x + 2$ [A1]

Graph of straight line: [1]

$y = -\dfrac{1}{2}x + 2$

2i) $\dfrac{d}{dx}(x\cos 2x) = x(-2\sin 2x) + \cos 2x$ [M1, M1]

$= -2x\sin 2x + \cos 2x$ [A1]

2ii) $\int -2x \sin 2x + \cos 2x \, dx = x\cos 2x + c$ [M1]

$-2\int x\sin 2x \, dx = x\cos 2x - \int \cos 2x \, dx$ [M1]

$\int x\sin 2x \, dx = -\dfrac{1}{2}x\cos 2x + \dfrac{1}{4}\sin 2x + c$ [A1]

$\displaystyle\int_0^{\frac{\pi}{4}} x\sin 2x \, dx = -\frac{1}{2}\left(\frac{\pi}{4}\right)\cos 2\left(\frac{\pi}{4}\right) + \frac{1}{4}\sin 2\left(\frac{\pi}{4}\right) - 0 = \frac{1}{4}$ [M1]/[A1]

3i) $(1 + ax)^n = 1 + nax + \dfrac{n(n-1)}{2}(a^2x^2) + ...$ [M1]

Comparing coefficients, $na = 21 \Rightarrow$ $a^2 = \dfrac{441}{n^2}$ [M1]

$\dfrac{n(n-1)}{2}(a^2) = 189 \Rightarrow \dfrac{n(n-1)}{2}\left(\dfrac{441}{n^2}\right) = 189$ [M1]

$441(n - 1) = 378n \Rightarrow$ $63n = 441 \Rightarrow$ $n = 7$ [A1]

$\therefore a = 3$ [A1]

3ii) $(1 + 3x)^7(2 - 12x^2 + 42x^2) = (1 + 21x + 189x^2 + ...)(2 - 12x + 42x^2)$ [M1]

Coefficient of $x^2 = (1 \times 42) + (21 \times -12) + (189 \times 2) = 168$ [M1]/[A1]

4i) $$\frac{2x-4}{(2x-1)(x^2-1)}=\frac{A}{2x-1}+\frac{B}{x-1}+\frac{C}{x+1}$$ [M1]

$2x-4 = A(x-1)(x+1) + B(2x-1)(x+1) + C(2x-1)(x-1)$ [M1]

When $x = 1, -2 = B(1)(2)$ $\Rightarrow$ $B = -1$

When $x = -1, -6 = C(-3)(-2)$ $\Rightarrow$ $C = -1$ [2 correct: 1m]

When $x = \frac{1}{2}, -3 = A\left(-\frac{1}{2}\right)\left(\frac{3}{2}\right)$ $\Rightarrow$ $A = 4$ [3 correct: 2m]

$$\frac{2x-4}{(2x-1)(x^2-1)}=\frac{4}{2x-1}-\frac{1}{x-1}-\frac{1}{x+1}$$ [A1]

4ii) $$\int\frac{2x-4}{(2x-1)(x^2-1)}dx = 2\ln(2x-1)-\ln(x-1)-\ln(x+1)+c$$ [M1,M1]

$$\int\frac{x-2}{(2x-1)(x^2-1)}dx = \ln(2x-1)-\frac{1}{2}\ln(x-1)-\frac{1}{2}\ln(x+1)+c$$ [A1]

5i) $$f'(x)=\frac{(2x+3)(3)-(3x-2)(2)}{(2x+3)^2}$$ [M1]

$$=\frac{6x+9-6x+4}{(2x+3)^2}=\frac{13}{(2x+3)^2}$$ [A1]

5ii) At x-axis, $y = 0$, $3x-2=0 \Rightarrow$ $x=\frac{2}{3}$ [M1]

At $x=\frac{2}{3}$, gradient $=\frac{13}{\left[2\left(\frac{2}{3}\right)+3\right]^2}=\frac{9}{13}$

Gradient of normal $=-\frac{13}{9}$ [M1]

$0=-\frac{13}{9}\left(\frac{2}{3}\right)+c$ $\Rightarrow$ $c=\frac{26}{27}$

The equation of the normal is $y=-\frac{13}{9}x+\frac{26}{27}$ [A1]

5iii) $f''(x) = 13[-2(2x+3)^{-3}(2)] = \frac{-52}{(2x+3)^3}$ [M1]

When $x > 0$, $(2x+3)^3>0 \Rightarrow$ $f''(x) < 0$ [M1]

$\therefore$ f'(x) is a decreasing function. [A1]

6i) Since the max y value is 4 and the radius is 3 unit, the y coordinate of the centre is 1. [M1]
Let the centre be (x, 1)
$3^2 = (x-1)^2 + (1+2)^2$ [M1]
$9 = (x-1)^2 + 9$
$x - 1 = 0$
$x = 1$
Hence the coordinates of the centre is (1, 1). [A1]

6ii) $(x-1)^2 + (y-1)^2 = 3^2$ [M1]
$x^2 - 2x + 1 + y^2 - 2y + 1 = 9$
$x^2 + y^2 - 2x - 2y - 7 = 0$ [A1]

6iii) At B, y = 0, $x^2 - 2x - 7 = 0$ [M1]

$$x = \frac{-(2) \pm \sqrt{4 - 4(1)(-7)}}{2(1)}$$

$$= \frac{2 \pm \sqrt{32}}{2} = \frac{2 \pm 4\sqrt{2}}{2}$$

$= 1 - \sqrt{2}$ or $1 + \sqrt{2}$ (Rejected) [M1]

Hence the coordinates of B is $(1-\sqrt{2}, 0)$
Let P be (x_1, y_1)

$$(1, 1) = \left(\frac{1 - 2\sqrt{2} + x_1}{2}, \frac{0 + y_1}{2}\right)$$ [M1]

$y_1 = 2$ $x_1 = 1 + 2\sqrt{2}$
Hence the coordinates of P is $(1+2\sqrt{2}, 2)$ [A1]

7a) $$\int_0^k ky - y^2\,dy = \left[\frac{ky^2}{2} - \frac{y^3}{3}\right]_0^k = \frac{32}{2}$$ [M1]

$$\left[\frac{k^3}{2} - \frac{k^3}{3}\right] = \frac{64}{6}$$

$$\frac{3k^3 - 2k^3}{6} = \frac{64}{6}$$ [M1]

$k^3 = 64 \Rightarrow k = 4$ [A1]

7b) gradient of $l = -\frac{4}{4} = -1$

Equation of l is $y = -x + 4$ [B1]

7c) $x = 4y - y^2$
$4 - y = 4y - y^2$
$y^2 - 5y + 4 = 0$
$(y-1)(y-4) = 0$
y = 1 or y = 4 [M1]

area of B + C = 0.5 x 4 x 4 = 8 unit^2 [M1]

Area B = $\int_1^4 4-y\ dy + \int_0^1 4y-y^2\ dy$ [M1]

$= \left[4y-\frac{y^2}{2}\right]_1^4 + \left[\frac{4y^2}{2}-\frac{y^3}{3}\right]_0^1$

$= [(16-8)-(4-\frac{1}{2})] + (2-\frac{1}{3}) = \frac{37}{6}$ [M1]

Area C = $\frac{37}{6}-8=\frac{11}{6}$ [M1]

Area A = $10\frac{2}{3}-\frac{37}{6}=\frac{9}{2}$ [M1]

A : B : C = $\frac{27}{6} : \frac{37}{6} : \frac{11}{6}$ = 27 : 37 : 11 [A1]

8a) LHS = $\sec^2 x\tan^2 x-\sec^2 x+1$
$= (\tan^2 x+1)(\tan^2 x)-(\sec^2 x-1)$ [M1, M1]
$= \tan^4 x+\tan^2 x-\tan^2 x$ [M1]
$= \tan^4 x$ = RHS

8b) $\cos^2 x+3\sin x\cos x+1=0$
$\cos^2 x+3\sin x\cos x+\sin^2 x+\cos^2 x=0$
$2\cos^2 x+3\sin x\cos x+\sin^2 x=0$ [M1]

$(2\cos x+\sin x)(\cos x+\sin x)=0$

$\tan x=-2$ or	$\tan x=-1$	[M1]
basic angle = 63.43°	basic angle = 45°	[M1]
$x\approx 116.6°, 296.6°$	$x = 135°, 315°$	[A1, A1]

8ci) $\sin\theta = \frac{\sqrt{(2\sqrt{2})^2-(\sqrt{3}+1)^2}}{2\sqrt{2}}$ [M1]

$= \frac{\sqrt{8-[3+1+2\sqrt{3}]}}{2\sqrt{2}} = \frac{\sqrt{8-4-2\sqrt{3}}}{2\sqrt{2}}$

$= \frac{\sqrt{2(2-\sqrt{3})}}{2\sqrt{2}} = \frac{\sqrt{2}(\sqrt{2-\sqrt{3}})}{2\sqrt{2}} = \frac{\sqrt{2-\sqrt{3}}}{2}$ [A1]

8cii) Opposite side of $\theta = \sqrt{(2\sqrt{2})^2-(\sqrt{3}+1)^2} = \sqrt{8-[3+2\sqrt{3}+1]} = \sqrt{4-2\sqrt{3}}$

$\tan\theta = \frac{\sqrt{4-2\sqrt{3}}}{\sqrt{3}+1}$ [M1]

$\tan^2\theta = \frac{4-2\sqrt{3}}{4+2\sqrt{3}} = \frac{2-\sqrt{3}}{2+\sqrt{3}}\times\frac{2-\sqrt{3}}{2-\sqrt{3}}$ [M1]

$= \frac{4-4\sqrt{3}+3}{4-3}$ $\therefore \tan\theta = \sqrt{7-4\sqrt{3}}$ [A1]

9a) $\dfrac{\log_x x}{\log_x \dfrac{x}{3}} + \dfrac{\log_x x}{\log_x 3x} = -2$ [M1]

$\dfrac{1}{\log_x x - \log_x 3} + \dfrac{1}{\log_x 3 + \log_x x} = -2$ [M1]

Let $y = \log_x 3$

$\dfrac{1}{1-y} + \dfrac{1}{1+y} = -2 \quad \Rightarrow \quad \dfrac{2}{1-y^2} = -2 \quad \Rightarrow \quad 1 - y^2 = -1$

$y^2 = 2 \quad \Rightarrow \quad y = \sqrt{2}$ *or* $-\sqrt{2}$

$\log_x 3 = \sqrt{2}$ *or* $\log_x 3 = -\sqrt{2}$ [M1]

$x = 3^{\frac{1}{\sqrt{2}}}$ *or* $x = 3^{-\frac{1}{\sqrt{2}}}$

$x \approx 2.17$ *or* $x \approx 0.46$ [A1, A1]

9bi) $\log_5 \sqrt{3} = \dfrac{1}{2}\log_5 3$ [M1]

$= \dfrac{1}{2}(0.62) = 0.31$ [A1]

9bii) $\log_5 13.5 = \log_5 \dfrac{27}{2} = \log_5 27 - \log_5 2$ [M1]

$= 3\log_5 3 - 0.431 = 3(0.62) - 0.431 = 1.429$ [A1]

9ci) When t = 0, P = 50000 [M1]

When t = 2, $100000 = 50000e^{2k}$

$2 = e^{2k}$ [M1]

$\Rightarrow \quad \ln 2 = 2k \Rightarrow \quad k = \dfrac{1}{2}\ln 2$

9cii) $450000 = 50000e^{\left(\frac{1}{2}t\ln 2\right)}$

$9 = e^{\frac{1}{2}t\ln 2}$ [M1]

$\ln 9 = \dfrac{1}{2}t\ln 2 \quad \Rightarrow \quad t \approx 6.34$ years [M1]

When the year in which the population will reach 450000 is 2016. [A1]

9ciii) When t = 5, $P = 50000^{5\left(\frac{1}{2}\ln 2\right)}$ [M1]

P = 282842 = 280 000 [A1]

10i) $\sin\theta = \dfrac{BC}{12} \Rightarrow$ BC = 12 sin θ

$\sin\theta = \dfrac{CE}{7} \Rightarrow$ CE = 7sinθ [M1]

$\cos\theta = \dfrac{CD}{7} \Rightarrow$ CD = 7 cos θ

$\cos\theta = \dfrac{AC}{12} \Rightarrow$ AC = 12 cos θ [M1]

P = 19 + 12 sin θ + 12 cos θ + 7 cos θ − 7sinθ [A1]

= 19 + 19 cos θ + 5 sin θ

10ii) R = $\sqrt{19^2+5^2} = \sqrt{386}$ [M1]

$\tan\alpha = \dfrac{5}{19} \Rightarrow \alpha = 14.7^\circ$ [M1]

P = 19 + $\sqrt{386}\cos\left(\theta - 14.7^\circ\right)$ [A1]

10iii) Max P occurs when $\cos\left(\theta - 14.7^\circ\right) = 1$. [M1]

∴ max P = 19 + $\sqrt{386}$ ≈ 38.6 cm [A1]

10iv) $19 + \sqrt{386}\cos\left(\theta - 14.7^\circ\right) = 34$

$\cos\left(\theta - 14.7^\circ\right) = 0.7634$ [M1]

$\theta - 14.7 = 40.23^\circ$ [M1]

$\theta = 54.93^\circ \approx 54.9^\circ$ [A1]

NAME: ______________________________ ( ) CLASS: _______

Comment [CLYA1]: Insert header on every page for students to write their name, register no. and class

FAIRFIELD METHODIST SCHOOL (SECONDARY)

Comment [CLYA2]: Use Arial 12 pt font as this is similar to the font used by UCLES. Where appropriate, follow the O-Level or N-Level format.

PRELIMINARY EXAMINATION 2014
SECONDARY 4 EXPRESS

ADDITIONAL MATHEMATICS 4047/01

Comment [CLYA3]: Insert paper no. only for Sec 4/5 papers to familiarise students with O-Level format

Paper 1

Date: 26 August 2014 **Duration: 2 hours**

Comment [SZ4]: Insert Date of Exam Paper

Comment [CLYA5]: Just indicate duration of the paper, not specific time

Additional Materials: Answer Paper
Graph Paper (1 sheet)

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use anHB pencil for any diagrams or graphs.
Do not use staples, paper clips,glue or correction fluid.

Answer **all** questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 80.

At the end of the examination, fasten all your work securely together.

Comment [SZ6]: Insert instructions by following specific Subject requirement. Font: Arial Size 12

For Examiner's Use	
Paper 1	/ 80

Setter: MdmToh SL and Miss Lee CP

This question paper consists of 7 printed pagesincluding the cover page.

Name: ______________________________ (   )   Class: __________

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

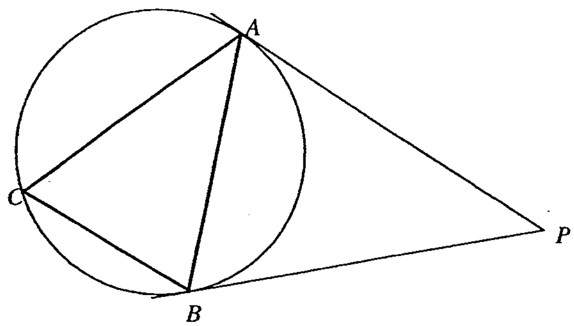
$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2}ab \sin C$$

Name: ______________________________ (   )   Class: __________

1 Given that the line $y = \frac{1}{2}x + 3$ is a tangent to the curve $y^2 = kx$, where k is a positive constant, prove that k is a multiple of 3. [4]

2



The diagram shows point A, B and C lying on a circle. The point P is such that the lines PA and PB are tangents to the circle. Given that $AB = AC$, show that $\angle APB = \angle BAC$. [4]

3 It is given that $3^{x+1} \times 2^{2x+1} = 2^{x+2}$.

(i) Find the exact value of 6^x. [3]

(ii) Hence find the value of x correct to 2 decimal places. [2]

4 Find the coordinates of the stationary point of the curve $y = 4x^2 - \frac{1}{x}, x \neq 0$ and determine the nature of this stationary point. [5]

5 It is given that $\int_1^3 px^2 dx = 7$, where p is a constant.

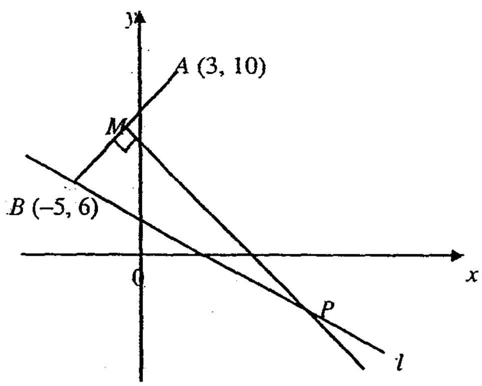
(i) Find the value of $\int_3^1 px^2 dx$. [1]

(ii) Find the value of $\int_3^9 px^2 dx$. [3]

(iii) Express $\int_1^3 (px^2 + q)\, dx$ in terms of the constant q. [2]

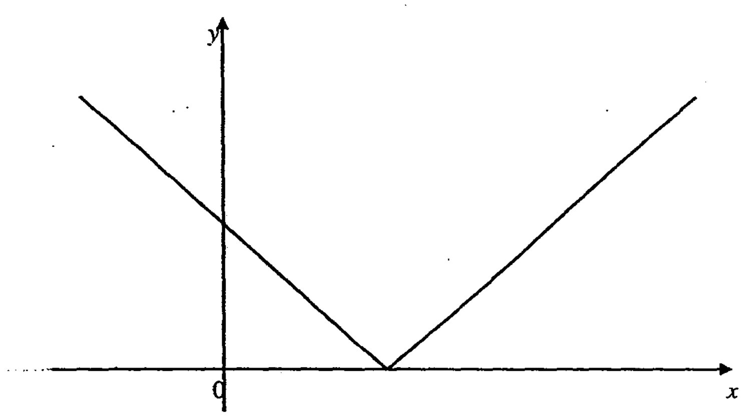
Name: ______________________________ (   )   Class: __________

6



In the diagram, M is the midpoint of the line joining the points $A(3, 10)$ and $B(-5, 6)$. The perpendicular bisector of AB intersects the line l at the point P. Given that line l is parallel to the line $6y + 7x = 0$, find the coordinates of P. [6]

7



The diagram above shows part of the graph of $y = |3 - x|$. In each of the following cases, determine the number of intersections of the line $y = mx + c$ with $y = |3 - x|$, justify your answer.

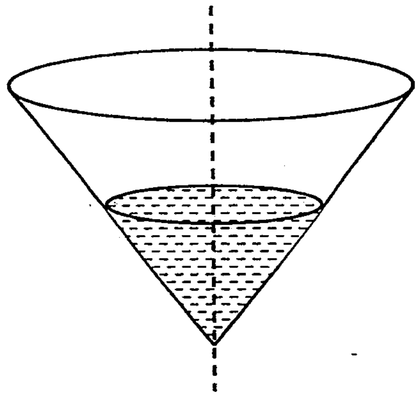
(i) $m = 1$ and $c > 1$ [2]

(ii) $m = \frac{1}{3}$ and $c = 0$ [2]

(iii) $m = -\frac{1}{3}$ and $c < 1$ [2]

Name: ______________________________ (   )   Class: _________

8



A container is in the shape of an inverted right circular cone which has a vertical axis and a base radius that is equal to its height. Water is poured into the vessel at a constant rate of 50 cm^3/s. The depth of the water is x cm.

(i) Show that the volume of water in the container is $V = \frac{1}{3}\pi\ x^3 \text{cm}^3$. [1]

Calculate, at the instant when depth of water is 20 cm, the rate of increase of

(ii) the depth of the water, in terms of π, [3]

(iii) the area of the horizontal surface of the water. [3]

9 The roots of the quadratic equation $2x^2 - 4x + 5 = 0$ are α and β.

(i) Express $\alpha^2 - \alpha\beta + \beta^2$ in terms of $(\alpha + \beta)$ and $\alpha\beta$. [1]

(ii) Find a quadratic equation whose roots are α^3 and β^3. [6]

10 (i) Express the equation $3\tan^2\theta = 5 - 2\sec\theta$ as a quadratic equation in terms of $\sec\theta$. [2]

(ii) Hence solve the equation $3\tan^2\theta = 5 - 2\sec\theta$ for $0 \le \theta \le 360°$. [4]

(iii) State the number of solutions of the equation $3\tan^2\theta = 5 - 2\sec\theta$ in the range $-540° \le \theta \le 540°$. [1]

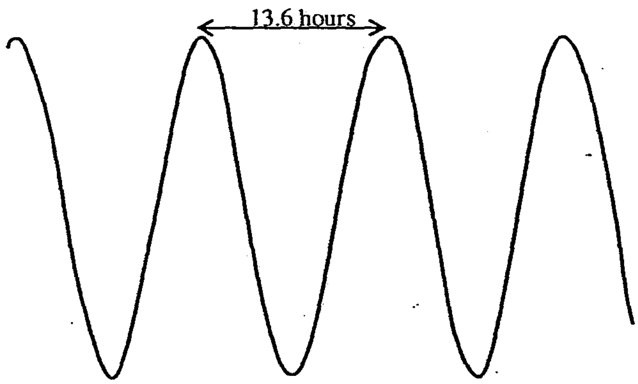
Name: ______________________________ ( ) Class: ________

11 The height of the tides in Singapore is modelled by the equation $h = 1.55 + 1.35\cos kt$, where k is a constant, and t is the time in hours after midnight. The diagram below shows the model of the height of the tides. The average time difference between high tides is 13.6 hours.

(i) Explain why this model suggest that highest tide for the day is 2.9 m. [1]

(ii) Show that the value of k is $\frac{5\pi}{34}$. [2]

(iii) Find the time for which the height of the tide first reaches 1.6 m, leaving your answer in 24-hour notation. [4]



12 A particle travels in a straight line, so that t seconds after passing through a fixed point O, its acceleration a m/s^2, is given by $a = 6t - 30$. The initial velocity of the particle is 72 m/s. Find

(i) the deceleration of the particle when $t = 3$, [1]

(ii) an expression for the velocity of the particle in terms of t, [2]

(iii) the speed of the particle when $t = 5$, [1]

(iv) an expression for the displacement of the particle in terms of t, [1]

(v) the total distance travelled by the particle for the first 6 seconds. [3]

Name: ____________________________ (   )   Class: __________

13 **(a)** The table shows experimental values of two variables x and y.

x	1	2	3	4	5
y	0.5	0.5	1.5	3.5	6

It is known that x and y are related by an equation of the form $a(x+y-b)=bx^2$, where a and b are constants.

(i) Draw a straight line graph of $(x + y)$ against x^2, using the scale of 2 cm to represent 5 units on the x^2-axis and 2 cm to represent 1 unit on the $(x + y)$ axis. [3]

(ii) Use your graph to estimate the value of a and of b. [2]

(b) In order that $y=h(1+x)^k$, where h and k are unknown constants may be represented by a straight line graph, it need to be expressed in the form $Y = mX + c$, where X and Y are functions of x and/or y, and m and c are constants.

(i) Determine an expression for Y and for X. [2]

(ii) Explain how the straight line may be used to determine the value of h. [1]

-End of Paper-

Comment [CLYA7]: Insert this at the end of the paper

FMS(S) 2014 Sec Four Express Additional Mathematics Prelim Paper 1 Answers

No.		No.	
1.	$k=0$ (NA) or $k=6$ Since 6 is multiple of 3, therefore, k is a multiple of 3 (proved).	2.	$\angle PAB = \angle BCA$ (alt segment) $\angle PBA = \angle BCA$ (alt segment) Or $\angle PBA = \angle PAB$ (PA = PB, tan fr ext. pt$P$) $\angle APB = 180° - 2\angle PAB$ ($\angle$ sum of Δ) $= 180° - 2\angle BCA$ $\angle BAC = 180° - 2\angle BCA$ ($AC = AB$, isos.Δ) Hence, $\angle APB = \angle BAC$ (shown)
3.	(i) $6^x = \frac{2}{3}$ (ii) -0.23	4.	Coordinates of the stationary point is $(-\frac{1}{2}, 3)$ The stationary point is a minimum point.
5.	(i) -7 (ii) 189 (iii) $7+2q$	6.	Coordinates of P is (7, –8).
7.	(i) Line parallel to R.H. arm; since c > 1 >–3 intersects L.H. arms; ∴ 1 intersection. Or Draw a line which is parallel to y = $x-3$ and c > 1. (ii) Through origin; since $m<$ 1 intersects both arms; ∴ 2 intersections. Or Draw a line that passes through the origin and gradient is greater than 0 but less than 1. (iii) With $m = -\frac{1}{3}$ and $c<1$; x-coordinate of intersection with x-axis is 3; ∴ 0 intersections Or Draw a line with $m= -\frac{1}{3}$; intersection with y-axis at 1 and x-axis at 3 respectively.	8.	(i) Volume of water in container, V = $\frac{1}{3}\pi(x)^2(x) = \frac{1}{3}\pi\ x^3$ (shown) (ii) $\frac{1}{8\pi}$ cm/s (ii) 400 cm^2/s
9.	(i) $(\alpha+\beta)^2 - 3\alpha\beta$ (ii)New equation: $x^2+7x+\frac{125}{8}=0$	10.	(i) $3\sec^2\theta + 2\sec\theta - 8 = 0$ (ii) $\theta = 41.40°, 318.6°$ or $\theta = 120°, 240°$. (iii) 12 solutions

<table>
<tr>
<td>11.</td>
<td>(i) Maximum height when cos kt = 1,
height = 2.9 m
(ii) $k = \frac{5\pi}{34}$ (shown)
(iii) The time is 0319.</td>
<td>12.</td>
<td>(i) Deceleration = 12 m/s^2
(ii) Velocity, $v = 3t^2 - 30t + 72$
(iii) Speed is 3 m/s
(iv) $\therefore s = t^3 - 15t^2 + 72t$
(v) Total distance travelled = 112 + (112 – 108)
= 116 m
Or integration method: $\int_0^4 v dt + \int_4^6 v dt = 116$ m</td>
</tr>
<tr>
<td>13.</td>
<td>(ai)
<table><tr><td>x^2</td><td>1</td><td>4</td><td>9</td><td>16</td><td>25</td></tr><tr><td>$x + y$</td><td>1.5</td><td>2.5</td><td>4.5</td><td>7.5</td><td>11</td></tr></table>$(x + y) = b + \frac{b}{a}x^2$
(aii) $a = 2.50 \pm 0.4$, $b = 1.0 \pm 0.2$</td>
<td>13.</td>
<td>(bi) $Y = \log y$ and $X = \log(1 + x)$

(bii) Find the vertical intercept which is log h
$h = 10^{\text{vertical intercept}}$ or $h = e^{\text{vertical intercept}}$.</td>
</tr>
</table>

Secondary 4 Express
Preliminary Examination 2014
Additional Mathematics
Paper 1
Marking Scheme

No	Working	Description	Marks allocated
1	Sub. $y=\frac{1}{2}x+3$ into $y^2=kx$ $(\frac{1}{2}x+3)^2=kx$ $\frac{1}{4}x^2+3x+9=kx$ $\frac{1}{4}x^2+x(3-k)+9=0$ Line is tangent to curve $\Rightarrow b^2-4ac=0$ $\Rightarrow(3-k)^2-36=0$ $9-6k+k^2-9=0$ $k(k-6)\ =0$ $k=0$ (NA) or $k=6$ Since 6 is multiple of 3, therefore, k is a multiple of 3.(proved).	Eliminates y completely Correct quadratic equation Use of b^2-4ac Show that $k=6$ and conclusion	M1 A1 M1 A1
2	$\angle PAB=\angle BCA$ (alt segment) $\angle PBA=\angle BCA$ (alt segment) Or $\angle PBA=\angle PAB$ (PA = PB, tan from ext. pt$P$) $\angle APB=180°-2\angle PAB$ ($\angle$ sum of Δ) $= 180°-2\angle BCA$ $\angle BAC=180°-2\angle BCA$ ($AC=AB$, isos..Δ) Hence, $\angle APB=\angle BAC$ (shown)	Angles in alternate segment Show 2 other angles equal - alternate segment or isos, PA = PB Deduce correctly.	[B1] [B1] [B1] reason [A1]
3(i)	$3^{x+1}\times 2^{2x+1}=2^{x+2}$ $3^{x+1}=2^{x+2}\div 2^{2x+1}$ $3^{x+1}=2^{-x+1}$ $3\times 3^x=\frac{2}{2^x}$ $3^x\times 2^x=\frac{2}{3}$ $6^x=\frac{2}{3}$	Apply law of indices to reduce equation to 2 terms of base 2 and 3 respectively. Break up indices	B1 B1 B1

No	Working	Description	Marks allocated
3(ii)	$x\lg 6=\lg\frac{2}{3}$ $x=\dfrac{\lg\frac{2}{3}}{\lg 6}$ $=-0.2263$ $=-0.23$		[M1] [A1]
4	$\frac{dy}{dx}=8x+\frac{1}{x^2}$ Stationary point $\Rightarrow \frac{dy}{dx}=0$ $8x+\frac{1}{x^2}=0$ $8x^3=-1$ $x=-\frac{1}{2}$ $y=3$ $\therefore$ coordinates of the stationary point is $(-\frac{1}{2},3)$ $\frac{d^2y}{dx^2}=8-\frac{2}{x^3}$ When $x=-\frac{1}{2}$, $\frac{d^2y}{dx^2}=24>0$ $\therefore$ The stationary point is a minimum point.	Differentiate $y$wrt. x When $8x+\frac{1}{x^2}=0$ Stationary point is $(-\frac{1}{2},3)$ Any valid method Conclusion is a minimum point	B1 M1 A1 M1 A1
5(i)	$\int_3^1 px^2dx=-7$		B1
5(ii)	$\int_1^3 px^2dx=7$ $\left[\frac{px^3}{3}\right]_1^3=7$ $\frac{27p-p}{3}=7$ $p=\frac{21}{26}$ $\int_3^9 px^2dx=\left[\frac{px^3}{3}\right]_3^9$ $=\frac{729p-27p}{3}$ $=\frac{702p}{3}$ $=\frac{702}{3}\times\frac{21}{26}$ $=189$	Integrate wrt. x Obtain value of p	M1 B1 A1

No	Working	Description	Marks allocated
5(iii)	Express $\int_1^3 (px^2+q)\,dx$ in terms of the constant q. $\int_1^3 (px^2+q)\,dx$ $=\left[\frac{px^3}{3}\right]_1^3+\left[qx\right]_1^3$ $or\left[\frac{px^3}{3}+qx\right]_1^3$ $=7+(3q-q)$ $=7+2q$	Integrate $\int_1^3 (px^2+q)\,dx$	M1 A1
6	$M=(-1,8)$ Gradient $AB=\frac{1}{2}$ Gradient of $\perp$ bisector of $AB=-2$ Equation BP: $y-6=-\frac{7}{6}(x+5)$ $y=-\frac{7}{6}x+\frac{1}{6}$ Equation MP: $y-8=-2(x+1)$ $y=-2x+6$ At point P, $-2x+6=-\frac{7}{6}x+\frac{1}{6}$ $x=7$ $y=-8$ Coordinates of P is $(7,-8)$		B1 M1 B1 B1 M1 A1
7(i)	Line parallel to R.H. arm; since c > 1 >–3 intersects L.H. arms; $\therefore$ 1 intersection Or Draw a line which is parallel to $y=x-3$ and $c>1$	Must write the word, line is parallel	B1 B1
7(ii)	Through origin; since $m<1$ intersects both arms; $\therefore$ 2 intersections Or Draw a line that passes through the origin and gradient is greater than 0 but less than 1.		B1 B1
7(iii)	With $m=-\frac{1}{3}$ and $c<1$; x-coordinate of intersection with x-axis is 3; $\therefore$ 0 intersections		B1 B1

No	Working	Description
8(i)	Volume of water in container, V = $\frac{1}{3}\pi(x)^2(x)$ $=\frac{1}{3}\pi\ x^3$	Show volume of water $\frac{1}{3}\times\pi\times(radius)^2\times(heig$
8 (ii)	Given $\frac{dV}{dt}=50cm^3/s$ $\frac{dV}{dx}=\pi x^2$ $\frac{dx}{dt}=\frac{dx}{dV}\times\frac{dV}{dt}$ $\frac{dV}{dx}=\pi x^2\Rightarrow\frac{dx}{dV}=\frac{1}{\pi x^2}$ $\frac{dx}{dt}=\frac{1}{\pi x^2}\times 50$ When $x=20$, $\frac{dx}{dt}=\frac{1}{\pi(20)^2}\times 50$ $=\frac{1}{8\pi}$ cm/s	Differentiate $\frac{dV}{dx}$ Uses chain rule correctly
8(iii)	Surface area of water, A = $\pi(x)^2=\pi x^2$ $\frac{dA}{dx}=2\pi x$ $\frac{dA}{dt}=\frac{dA}{dx}\times\frac{dx}{dt}$ $\frac{dA}{dx}=2\pi x\times\frac{1}{2\pi}$ When $x=20$, $\frac{dA}{dt}=2\pi(20)^2\times\frac{1}{2\pi}$ $=400$ cm²/s	Differentiate $\frac{dA}{dx}$ Uses chain rule correctly Follow through from 8(ii)
9(i)	$\alpha^2-\alpha\beta+\beta^2$ $=(\alpha+\beta)^2-2\alpha\beta-\alpha\beta$ $=(\alpha+\beta)^2-3\alpha\beta$	
9(ii)	$2x^2-4x+5=0$ $\alpha+\beta=2,\ \alpha\beta=\frac{5}{2}$ $\alpha^3+\beta^3=(\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2)$ $=(\alpha+\beta)((\alpha+\beta)^2-3\alpha\beta)$ $=2(2^2-3\times\frac{5}{2})$ $=-7$ $\alpha^3\beta^3=\left(\frac{5}{2}\right)^3=\frac{125}{8}$ New equation: $x^2+7x+\frac{125}{8}=0$ or $8x^2+56x+125=0$	Sum of roots; product of roots Correct use of factors of $\alpha^3+\beta^3$ Follow through (*ft*) on $\alpha+\beta$ ... $\alpha\beta=\frac{5}{2}$

No	Working	Description	Marks allocated
10(i)	$3\tan^2\theta = 5 - 2\sec\theta$ $3(\sec^2\theta - 1) = 5 - 2\sec\theta$ $3\sec^2\theta - 3 = 5 - 2\sec\theta$ $3\sec^2\theta + 2\sec\theta - 8 = 0$	Uses $\tan^2\theta = \sec^2\theta - 1$ (correct formula) Correct equation in any form	M1 A1
10(ii)	$3\sec^2\theta + 2\sec\theta - 8 = 0$ $(3\sec\theta - 4)(\sec\theta + 2) = 0$ $\sec\theta = \frac{4}{3}$ or $\sec\theta = -2$ $\cos\theta = \frac{3}{4}$ or $\cos\theta = -\frac{1}{2}$ $\theta = 41.40°$ or $318.6°$ or $\theta = 120°$ or $240°$	Attempt as solution (solving method – factorization or general solution) Use $\sec\theta = \frac{1}{\cos\theta}$ FT for 360 - 41.40° or 180 – 60	M1 M1 A1, A1
10(iii)	4 times/cycle × 3 cycles = 12 angles		B1
11(i)	Maximum height when cos kt = 1, height = 2.9 m		B1
11(ii)	Period between high tides = 13.6 hours $\frac{2\pi}{k} = 13.6$ $\frac{2\pi}{13.6} = k$ $k = \frac{5\pi}{34}$	Period of graph	M1 A1
11(iii)	When h = 1.6 $1.6 = 1.55 + 1.35\cos kt$ $\cos kt = \frac{1}{27}$ $\frac{5\pi}{34}t = 1.5338 \rightarrow t = 3.3198$ The time is 0319.		M1 A1 M1 A1
12(i)	$a = 6t - 30$ When t = 3, a = $6(3) - 30$ = -12 m/s² Deceleration = 12 m/s²		B1
12(ii)	Velocity, v = $\int 6t - 30dt$ $= \frac{6t^2}{2} - 30t + c$ $= 3t^2 - 30t + c$ When t = 0, v = 72, $\therefore c = 72$ Velocity, $v = 3t^2 - 30t + 72$	Integrate acceleration	M1 A1
12(iii)	When t= 5, velocity, v = $3(5)^2 - 30(5) + 72$ = -3 m/s Speed is 3 m/s	FT if velocity has a + c as constant and speed is positive value instead of negative value	B1 / FT1

<table>
<tr><th>No</th><th>Working</th><th>Description</th><th>Marks allocated</th></tr>
<tr><td>12(iv)</td><td>Displacement, s = $\int 3t^2 - 30t + 72dt$
$= t^3 - 15t^2 + 72t + c$
When t=0, s = 0, $\therefore c = 0$
$\therefore s = t^3 - 15t^2 + 72t$</td><td>Exact answer for displacement</td><td>B1</td></tr>
<tr><td>12(v)</td><td>When v = 0, $3t^2 - 30t + 72 = 0$
$t^2 - 10t + 24 = 0$
$(t-6)(t-4) = 0$
t = 4 or t = 6
When $t = 4$, s = 112
When $t = 6$, s = 108
Total distance travelled = 112 + (112 – 108)
= 116 m
Or integration method: $\int_0^4 vdt + \int_4^6 vdt = 116$ m</td><td>Find the time when v = 0

Find the distance travelled when t= 4 or $t = 6$ (does not matter if the integration in (iv)) is incorrect

Answer (follow through from (iv))</td><td>M1

M1

A1 / FT1</td></tr>
<tr><td>13(a) (i)</td><td><table><tr><td>x^2</td><td>1</td><td>4</td><td>9</td><td>16</td><td>25</td></tr><tr><td>$x+y$</td><td>1.5</td><td>2.5</td><td>4.5</td><td>7.5</td><td>11</td></tr></table>
$a(x+y-b) = bx^2$
a(x+y) – ab = bx^2
$(x+y) = b + \frac{b}{a}x^2$

Refer to attached graph</td><td>Table of values

Plot the points and best fit line</td><td>B1

P1, S1</td></tr>
<tr><td>13(a)(ii)</td><td>Vertical intercept, $b = 1.0 \pm 0.2$
Gradient of line , $\frac{b}{a} = 0.40 \pm 0.2$
$a = 2.5 \pm 0.4$</td><td>Value of b = vertical intercept

Value for a</td><td>B1

B1</td></tr>
<tr><td>13(b)(i)</td><td>$y = h(1+x)^k$
Apply either natural logarithm or common logarithm
log y = log h + log $(1 + x)^k$
log y = log h + k log (1 + x)
Y = log y and X = log(1 + x)</td><td></td><td>B1, B1</td></tr>
<tr><td>13(b)(ii)</td><td>Find the vertical intercept which is log h
$h = 10^{\text{vertical intercept}}$ or $h = e^{\text{vertical intercept}}$</td><td>Value of h</td><td>B1</td></tr>
</table>

NAME: ______________________ (   )   CLASS: ______

FAIRFIELD METHODIST SCHOOL (SECONDARY)

PRELIMARY EXAMINATION 2014
SECONDARY 4 EXPRESS

ADDITIONAL MATHEMATICS   4047/02

Paper 2

Date: 27August 2014   **Duration: 2 hours 30 minutes**

Additional Materials: Answer Paper

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

For Examiner's Use	
Total	/ 100

Setters: Miss Thio Lay Hong and Mr James Quek

This question paper consists of 7 printed pagesincluding the cover page.

 Name: ______________________________ (   )   Class: __________

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where$n$ is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\text{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2}ab \sin C$$

Name: ______________________________ (   )   Class: __________

1 Over a period of 10 years from 1 January 2001 to 1 January 2011, the population, P, of a city increased exponentially from 50000 to 250000.

Given that $P = Ae^{kt}$, where A and k are constants and t is the time in years from 1 January 2001, find

(i) the value of A and of k. [3]

If the population continues to increase at the same rate,

(ii) in which year will it first exceed 1 million? [3]

2 **(a)** Given that $\log_{13} x + \log_{13} y = \log_{13}(x - y)$, express x in terms of y. [3]

(b) Given that $u = \log_3 z$, find, in terms of u,

(i) $\log_3 \frac{3}{z}$, [1]

(ii) $\log_3 27z$, [1]

(iii) $\log_z 9$. [2]

3 **(i)** Differentiate $x\cos 2x$ with respect to x. [3]

(ii) Using your answer to part (i),find $\int x\sin 2x\, dx$ and hence show that

$$\int_{\pi}^{2\pi} x\sin 2x\, dx = -\frac{\pi}{2}.$$ [5]

4 (i) Write down the first three terms in the expansion, in ascendi
$(1-px)^{12}$, where p is a constant.

(ii) Find, in terms of p, the coefficient of x^2 in the exp
$(1+3x-5x^2)(1-px)^{12}$.

In the expansion of $(1+3x-5x^2)(1-px)^{12}$, the sum of the coefficients of
x^2 is $-4\frac{2}{3}$.

(iii) Find the values of p in its exact form.

5 (i) Sketch the graph of $y=3e^{2x+1}$.

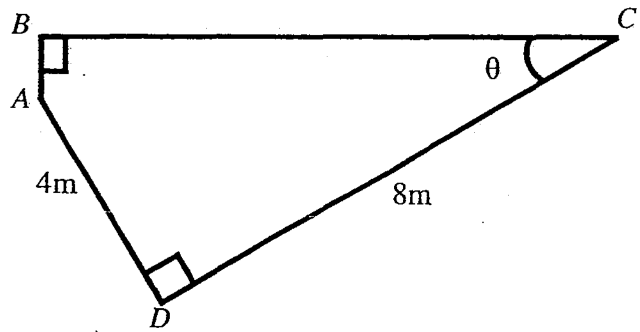
(ii) On the same diagram, sketch the graph of $y=3e^{-2x-1}$.

(iii) Calculate the coordinates of the point of intersection of your graph

(iv) Determine, with explanation, whether the tangents to the graphs
intersection are perpendicular.

Name: ______________________________ (   )   Class: __________

6 The diagram shows a model of a playground $ABCD$. The angles ABC and ADC are right angles. AD is 4m and CD is 8m. The perimeter of the playground is P m.



(i) Show that $P = 4\cos\theta + 12\sin\theta + 12$. [4]

(ii) Express P in the form $R\cos(\theta - \alpha) + 12$ where $R>0$ and α is acute. [4]

(iii) Find the value of θ for which P is a maximum. [2]

7 A curve has the equation $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = \dfrac{2x-3}{4+3x}$ for $x > 0$.

(i) Obtain an expression for $\mathrm{f}'(x)$. [2]

(ii) Find the equation of the normal to the curve at the point where the curve crosses the x-axis. [4]

(iii) Determine, with explanation, whether $\mathrm{f}(x)$ is an increasing or decreasing function. [1]

(iv) Showing full working, determine whether the gradient of the curve is an increasing or decreasing function. [3]

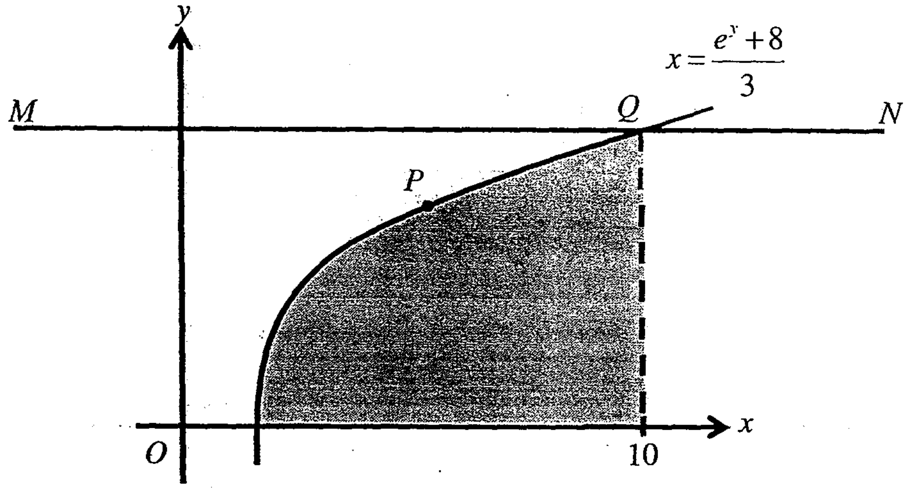
8 Given that $\dfrac{4x^3 - x^2 - 34x + 12}{x^2 - 9} = ax + b + \dfrac{2x + c}{x^2 - 9}$,

(i) find the value of each of the integers a, b and c. [4]

Hence, using partial fractions and the values of a, b and c obtained in part (i), find

(ii) $\displaystyle\int \frac{4x^3 - x^2 - 34x + 12}{x^2 - 9}\,dx$. [6]

9



The diagram shows part of the curve $x=\frac{e^y+8}{3}$ intersecting the horizontal line MN at point Q with x-coordinate = 10. The point P lies on the curve and the tangent at P is parallel to the line $5y=x+6$.

(i) Find the x-coordinate of P. [4]

(ii) Find the area of the shaded region. [6]

10 **(i)** Prove the identity $\frac{2\tan x}{1+\tan^2 x}-\sin 2x=0$. [3]

(ii) Solve the equation $\frac{8\tan x}{2+2\tan^2 x}=-1$, for $0<x<\pi$, giving your answers in terms of π. [3]

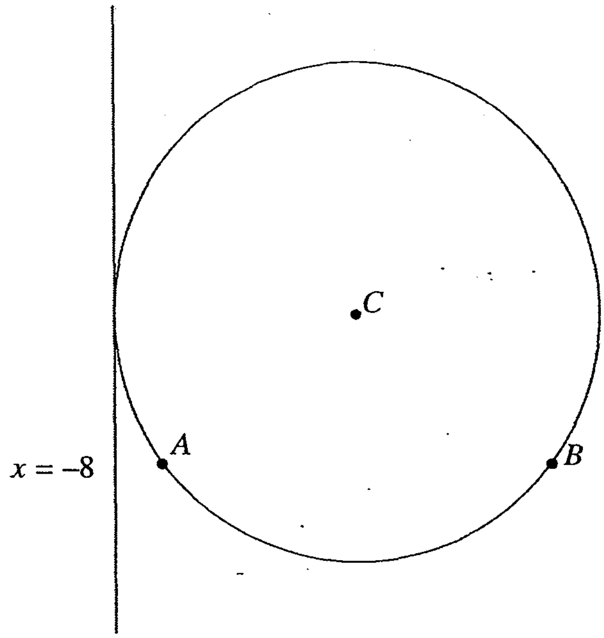
(iii) Given that $\frac{2+2\tan^2 x}{16\tan x}=-\frac{1}{3}\sec 2x$ and **without the use of calculator,**

(a) deduce that $\tan 2x=-\frac{3}{4}$, [2]

(b) find the possible values of $\tan x$. [3]

Name: ______________________________ (   )   Class: __________

11 The diagram shows two points $A(-6, 2)$ and $B(10, 2)$ on the circumference of a circle whose centre, C, lies above the x-axis. The line $x = -8$ is a tangent to the circle.



(i) Show that the radius of the circle is 10 units. [4]

(ii) Find the coordinates of C. [4]

(iii) Find the equation of the circle in the form $x^2 + y^2 + px + qy + r = 0$. [2]

(iv) Find the equations of the tangents to the circle parallel to the x-axis. [2]

~End of Paper~

FMS(S)Secondary 4 Express 2014
Additional MathematicsPreliminary ExaminationPaper 2Answers

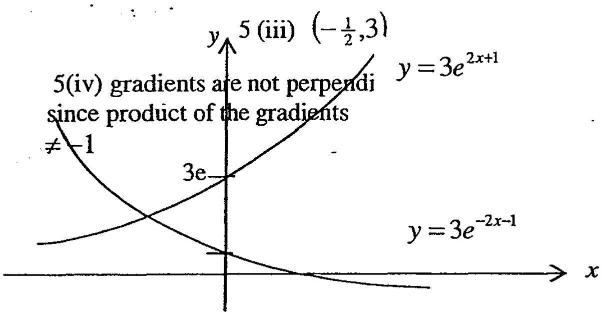
1(i) $A = 50\,000,\ k = 0.161$ 1(ii)2019

2(a) $x = \dfrac{-y}{y-1}$ or $\dfrac{y}{1-y}$ 2(b) (i) $1-u$ 2(b) (ii) $3+u$ 2(b) (iii) $\dfrac{2}{u}$

3(i) $-2x\sin 2x + \cos 2x$ 3(ii) $\frac{1}{4}\sin 2x - \frac{1}{2}x\cos 2x + C$

4(i) $1 - 12px + 66p^2x^2$ 4(ii) $66p^2 - 36p - 5$ 4(iii) $\dfrac{2}{3}$ or $\dfrac{2}{33}$

5(i)
5(ii)



6(ii) $P = 4\sqrt{10}\cos(\theta - 71.6^0) + 12$ 6(iii) $\theta = 71.6^0$

7(i) $\dfrac{17}{(4+3x)^2}$ 7(ii) $y = -\frac{17x}{4} + \frac{51}{8}$ or $8y = -34x + 102$ 7(iii) Increasing function 7(iv) decreasing function

15.7 sq units

8(i) $a = 4, b = \square 1, c = 3$

8 (ii) $\dfrac{4x^3 - x^2 - 34x + 12}{x^2 - 9} = 4x - 1 + \dfrac{1}{2(x+3)} + \dfrac{3}{2(x-3)}$

$$\frac{(4x-1)^2}{8} + \frac{1}{2}\ln(x+3) + \frac{3}{2}\ln(x-3) + C \text{ or } 2x^2 - x + \frac{1}{2}\ln(x+3) + \frac{3}{2}\ln(x-3) + C$$

9(i) $x = 7\dfrac{2}{3}$ or 7.67 9(ii) $\left(\dfrac{22}{3}\ln 22 - 7\right)\text{units}^2$ or 15.7 sq units

10(i) $LHS = \dfrac{2\tan x}{\sec^2 x} - \sin 2x = \dfrac{2\sin x}{\cos x} \div \dfrac{1}{\cos^2 x} - \sin 2x = \dfrac{2\sin x}{\cos x} \times \cos^2 x - \sin 2x$
$= 2\sin x\cos x - \sin 2x = \sin 2x - \sin 2x = 0$

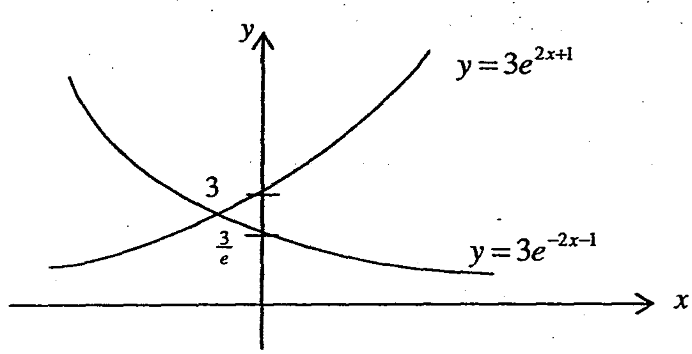
10(ii) $x = \dfrac{7\pi}{12}, \dfrac{11\pi}{12}$ 10(iii)(b) $-\dfrac{1}{3}$ or 3

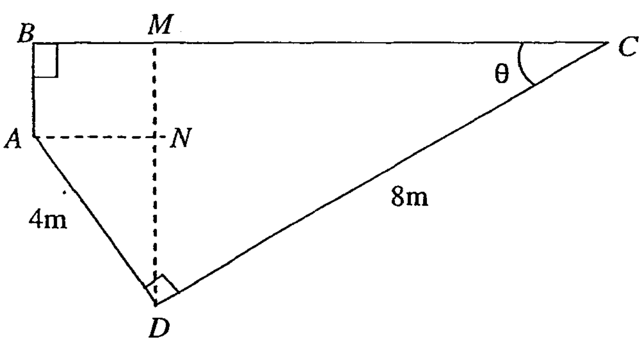
11(i) mid pt of chord formed by (-6,2) and (10,2) = (2,2) ; centre lies on $x = 2$;
distance between tangent at -8 and centre =10; hence radius = 10 units

11 (ii) are (2,8) 11(ii) $(x-2)^2 + (y-8)^2 = 100$ or $x^2 + y^2 - 4x - 16y - 32 = 0$ 11(iii) $y = 18$ or $y = -2$

Fairfield Methodist School (Secondary)
Secondary 4 Express 2014
Additional MathematicsPreliminary ExaminationPaper 2
Marking Scheme

No	Working / Description	Allocation of Marks
1i	A = 50 000 $P = Ae^{kt}$ $250000 = 50000e^{10k}$ $5 = e^{10k}$ $\ln 5 = 10k$ $k = 0.161$	B1 M1 A1
1ii	$1000000 = 50000e^{0.1609t}$ $20 = e^{0.1609t}$ $\ln 20 = 0.1609t$ $t = 18.6$ The year is 2019	M1 A1 FT1
2a	$\log_{13} x + \log_{13} y = \log_{13}(x-y)$ $\log_{13} xy = \log_{13}(x-y)$ $xy = x - y$ $xy - x = -y$ $x = \frac{-y}{y-1}$ or $\frac{y}{1-y}$	M1 for product law M1 removing log A1
2bi	$\log_3 \frac{3}{z}$ $= 1 - u$	B1
2bii	$\log_3 27z$ $= \log_3 27 + \log_3 z$ $= 3 + u$	B1
2biii	$\log_z 9$ $= 2\log_z 3$ $= \frac{2}{\log_3 z}$ $= \frac{2}{u}$	M1 Change of base A1
3(i)	$\frac{d}{dx}(\cos 2x) = -2\sin 2x$ $\frac{d}{dx}(x\cos 2x) = x(-2\sin 2x) + \cos 2x(1)$ $= -2x\sin 2x + \cos 2x$	B1 M1(product rule) FT1

No	Working / Description	Allocation of Marks
3(ii)	From (i), $\frac{d}{dx}(x\cos 2x) = -2x\sin 2x + \cos 2x$ $\int \frac{d}{dx}(x\cos 2x)dx = \int -2x\sin 2x dx + \int \cos 2x dx$ $x\cos 2x + C_1 = \int -2x\sin 2x dx + \int \cos 2x dx$ $\int 2x\sin 2x dx = \int \cos 2x dx - x\cos 2x - C_1$ $= \frac{1}{2}\sin 2x + C_2 - x\cos 2x - C_1$ $\int x\sin 2x dx = \frac{1}{4}\sin 2x - \frac{1}{2}x\cos 2x + C$ $\int_{\frac{\pi}{4}}^{\pi} x\sin 2x dx =$ $\left[\frac{1}{4}\sin 2(2\pi) - \frac{1}{2}(2\pi)\cos 2(2\pi)\right] - \left[\frac{1}{4}\sin 2(\pi) - \frac{1}{2}(\pi)\cos 2(\pi)\right]$ $= [0-\pi] - \left[0 - \frac{\pi}{2}\right] =$ (shown)	M1 B1 M1, −1m for not writing constant for final answer M1 B1
4(i)	$(1-px)^{12} = 1 - 12px + \binom{12}{2}(-px)^2 + \ldots$ $= 1 - 12px + 66p^2x^2 + \ldots$	B1 B1
4(ii)	$(1+3x-5x^2)(1-px)^{12} = (1+3x-5x^2)(1-12px+66p^2x^2+\ldots)$ Coefficient of $x^2 = 66p^2 - 36p - 5$	M1 A1
4(iii)	$(1+3x-5x^2)(1-12px+66p^2x^2+\ldots)$ Coefficient of $x = -12p+3$ $66p^2 - 36p - 5 + (-12p+3) = -4\frac{2}{3}$ $66p^2 - 48p + 2\frac{2}{3} = 0$ $198p^2 - 144p + 8 = 0$ $99p^2 - 72p + 4 = 0$ $(3p-2)(33p-2) = 0$ $p = \frac{2}{3}$ or $p = \frac{2}{33}$	B1 M1 M1 A1
5(i) &(ii)		B1 B1

No	Working / Description	Allocation of Marks
5(iii)	$y=3e^{2x+1}$ ----------(1) $y=3e^{-2x-1}$ --------(2) $3e^{2x+1}=3e^{-2x-1}$ $e^{4x+2}=1$ $4x+2=0$ $x=-\frac{1}{2}$ From (1), $y=3e^{-2(\frac{1}{2})+1}=3$ Point of intersection = $\left(-\frac{1}{2},3\right)$	 M1 A1
5(iv)	If the tangents to the graphs are perpendicular, then $m_1m_2=-1$ $y=3e^{2x+1}$ $\frac{dy}{dx}=6e^{2x+1}$ $x=-\frac{1}{2}$, $\frac{dy}{dx}=6e^{2(-0.5)+1}=6$ $y=3e^{-2x-1}$ $\frac{dy}{dx}=-6e^{-2x-1}$ $x=-\frac{1}{2}$, $\frac{dy}{dx}=-6e^{-2(-0.5)-1}=-6$ Since $6(-6)\neq-1$, the 2 tangents are **not** perpendicular. Note: Not necessary to substitute in $x=-\frac{1}{2}$	 B1 B1 M1 A1
6(i)	 $\sin\theta=\frac{DM}{8}\Rightarrow DM=8\sin\theta$ $\cos\theta=\frac{CM}{8}\Rightarrow CM=8\cos\theta$ $\sin\theta=\frac{AN}{4}\Rightarrow AN=4\sin\theta$ $\cos\theta=\frac{DN}{4}\Rightarrow DN=4\cos\theta$ $AB=DM-DN=8\sin\theta-4\cos\theta$ $P=AB+BM+MC+CD+DA$ $=8\sin\theta-4\cos\theta+4\sin\theta+8\cos\theta+8+4$ $=4\cos\theta+12\sin\theta+12$ (shown)	 B1 B1 B1 B1

No	Working / Description	Allocation of Marks
6(ii)	$4\cos\theta+12\sin\theta=R\cos(\theta-\alpha)$ $=R\cos\theta\cos\alpha+R\sin\theta\sin\alpha$ $R\cos\alpha=4$ ----------(1) $R\sin\alpha=12$ ---------(2) $(2)\div(1)$: $\tan\alpha=3\Rightarrow\alpha=\tan^{-1}3=71.57^0=71.6^0$ $(1)^2+(2)^2$: $R=\sqrt{4^2+12^2}$ $=\sqrt{160}$ $=\sqrt{16}\times\sqrt{10}$ $=4\sqrt{10}$ or 12.649 $P=4\sqrt{10}\cos(\theta-71.6^0)+12$ or $P=12.6\cos(\theta-71.6^0)+12$	M1 M1 B1 A1
6(iii)	For P to be a maximum, $\cos(\theta-71.6^0)=1$ $\theta-71.6^0=0$ $\theta=71.6^0$	M1 A1
7(i)	$f(x)=\dfrac{2x-3}{4+3x}$ $f'(x)=\dfrac{(4+3x)(2)-(2x-3)(3)}{(4+3x)^2}$ $=\dfrac{8+6x-6x+9}{(4+3x)^2}$ $=\dfrac{17}{(4+3x)^2}$	M1 A1
7(ii)	Let $y=0$ $2x-3=0$ $x=1.5$ (1.5, 0) $x=1.5$ $f'(x)=\dfrac{17}{[4+3(1.5)]^2}$ $=4/17$ Gradient of normal $=-\dfrac{17}{4}$ Equation of normal is $y-0=-\dfrac{17}{4}(x-1.5)$ $y=-\dfrac{17}{4}x+\dfrac{51}{8}$ or $8y=-34x+51$	B1 A1 M1 FT1
7(iii)	Since $(4+3x)^2>0$, $f'(x)=\dfrac{17}{(4+3x)^2}>0$ $f(x)$ is a increasing function.	B1
7(iv)	$f'(x)=\dfrac{17}{(4+3x)^2}$ $f''(x)=17(\square 2)(4+3x)^{\square 3}(3)$ $=\dfrac{-102}{(4+3x)^3}$	M1 A1

No	Working / Description	Allocation of Marks
	Since $x > 0$, $(4+3x)^3 > 0$ $f''(x) < 0$ Hence, gradient of the curve is an decreasing function.	B1
8(i)	$\dfrac{4x^3 - x^2 - 34x + 12}{x^2 - 9} = ax + b + \dfrac{2x + c}{x^2 - 9}$	
	$a = 4$	B1
	$4x - 1$	M1
	$x^2 - 9 \overline{)4x^3 - x^2 - 34x + 12}$	
	$-(4x^3 - 36x)$	
	$-x^2 + 2x + 12$	
	$-(-x^2 + 9)$	
	$2x + 3$	
	$\therefore \dfrac{4x^3 - x^2 - 34x + 12}{x^2 - 9} = 4x - 1 + \dfrac{2x + 3}{x^2 - 9}$	
	$b =$ □1	A1
	$c = 3$	A1
8(ii)	$\dfrac{2x+3}{x^2-9} = \dfrac{A}{x+3} + \dfrac{B}{x-3}$	M1
	$2x + 3 = A(x-3) + B(x+3)$	
	By comparing coefficients, $2 = A + B$ ---------(1)	M1
	$3 = -3A + 3B$ ---------(2)	
	(1) $\times 3$ +(2) : $9 = 6B \Rightarrow B = \frac{3}{2}$	A1
	(1): $A = \frac{1}{2}$	A1
	$\dfrac{2x+3}{x^2-9} = \dfrac{1}{2(x+3)} + \dfrac{3}{2(x-3)}$	
	$\dfrac{4x^3 - x^2 - 34x + 12}{x^2 - 9} = 4x - 1 + \dfrac{2x + 3}{x^2 - 9}$	
	$\dfrac{4x^3 - x^2 - 34x + 12}{x^2 - 9} = 4x - 1 + \dfrac{1}{2(x+3)} + \dfrac{3}{2(x-3)}$	
	$\int \dfrac{4x^3 - x^2 - 34x + 12}{x^2 - 9}dx = \int (4x-1)dx + \dfrac{1}{2}\int \dfrac{1}{x+3}dx + \dfrac{3}{2}\int \dfrac{1}{x-3}dx$	
	$= \dfrac{4x^2}{2} - x + \dfrac{1}{2}\ln(x+3) + \dfrac{3}{2}\ln(x-3) + C$	B2, □1 for each error
	$= 2x^2 - x + \dfrac{1}{2}\ln(x+3) + \dfrac{3}{2}\ln(x-3) + C$	
	Or $\int \dfrac{4x^3 - x^2 - 34x + 12}{x^2 - 9}dx = \int (4x-1)dx + \dfrac{1}{2}\int \dfrac{1}{x+3}dx + \dfrac{3}{2}\int \dfrac{1}{x-3}dx$	

No	Working / Description	Allocation of Marks
	$=\frac{(4x-1)^2}{2(4)}+\frac{1}{2}\ln(x+3)+\frac{3}{2}\ln(x-3)+C$ $=\frac{(4x-1)^2}{8}+\frac{1}{2}\ln(x+3)+\frac{3}{2}\ln(x-3)+C$	
9i	Gradient of tangent at P $=\frac{1}{5}$ $y=\ln(3x-8)$ $\frac{dy}{dx}=\frac{3}{3x-8}$ $\frac{3}{3x-8}=\frac{1}{5}$ $15=3x-8$ $x=\frac{23}{3}$ or $7\frac{2}{3}$ or 7.67 (3sf)	B1 M1 for differentiation M1 A1
9ii	$x=\frac{e^y+8}{3}$ $y=\ln(30-8)$ $=\ln 22$ Area bounded curve, MN, y-axis and x-axis $\int_0^{\ln 22}\frac{e^y+8}{3}dy$ $=\frac{1}{3}\left[e^y+8y\right]_0^{\ln 22}$ $=\frac{1}{3}\left[(22+8\ln 22)-(1)\right]$ $=7+\frac{8}{3}\ln 22$ Area of rectangle = 10ln22 Shaded Area $=10\ln 22-(7+\frac{8}{3}\ln 22)$ $=\frac{22}{3}\ln 22-7$ $=15.7$ sq units	B1 (y-coordinate of MN) M1 for integration A1 B1 M1 for rect-unshaded A1

No	Working / Description	Allocation of Marks
10i	$LHS=\frac{2\tan x}{1+\tan^2 x}-\sin 2x$ $=\frac{\frac{2\sin x}{\cos x}}{1+\frac{\sin^2 x}{\cos^2 x}}-\sin 2x$ $=\frac{\frac{2\sin x}{\cos x}}{\frac{\cos^2 x+\sin^2 x}{\cos^2 x}}-\sin 2x$ $=\frac{2\sin x}{\cos x}\times\frac{\cos^2 x}{\cos^2 x+\sin^2 x}-\sin 2x$ $=\frac{2\sin x}{\cos x}\times\cos^2 x-\sin 2x$ $=2\sin x\cos x-\sin 2x$ $=\sin 2x-\sin 2x$ $=0$	M1 changing tangent M1 Apply $\sin^2 x+\cos^2 x=1$ AG1
10i	$LHS=\frac{2\tan x}{1+\tan^2 x}-\sin 2x$ $=\frac{2\tan x}{\sec^2 x}-\sin 2x$ $=\frac{2\sin x}{\cos x}\div\frac{1}{\cos^2 x}-\sin 2x$ $=\frac{2\sin x}{\cos x}\times\cos^2 x-\sin 2x$ $=2\sin x\cos x-\sin 2x$ $=\sin 2x-\sin 2x$ $=0$	Alternate Method M1 changing secant form M1 changing to cosine form AG1
10ii	$\frac{8\tan x}{2+2\tan^2 x}=-1$ $2\left(\frac{2\tan x}{1+\tan^2 x}\right)=-1$ $2\sin 2x=-1$ $\sin 2x=-\frac{1}{2}$ ref angle $=\sin^{-1}\frac{1}{2}=\frac{\pi}{6}$ $2x=\frac{7\pi}{6},\frac{11\pi}{6}$ $x=\frac{7\pi}{12},\frac{11\pi}{12}$	B1 M1 find ref angle A1 for both answer

No	Working / Description	Allocation of Marks
10iiia	$\frac{2+2\tan^2 x}{16\tan x}=-\frac{1}{3}\sec 2x$ $\frac{16\tan x}{2+2\tan^2 x}=-3\cos 2x$ $4\left(\frac{2\tan x}{1+\tan^2 x}\right)=-3\cos 2x$ $4\sin 2x=-3\cos 2x$ $\frac{\sin 2x}{\cos 2x}=-\frac{3}{4}$ $\tan 2x=-\frac{3}{4}$	M1 A1 (must show sin2x/cos2x)
10iiib	$\tan 2x=-\frac{3}{4}$ $\frac{2\tan x}{1-\tan^2 x}=-\frac{3}{4}$ $8\tan x=-3+3\tan^2 x$ $3\tan^2 x-8\tan x-3=0$ $(3\tan x+1)(\tan x-3)=0$ $\tan x=-\frac{1}{3}$ or $\tan x=3$	M1 for quad equation M1 factorise or quad formula A1 for both answers correct
11i	mid pt of chord formed by (-6,2) and (10,2) = (2,2) centre lies on x= 2, dist b/w tangent at -8 and centre =10 Radius = 10 units	M1 mid pt formula, A1 M1 A1
11ii	$\sqrt{(2+6)^2+(y-2)^2}=100$ $64+(y-2)^2=100$ $(y-2)=\pm 6$ $y=8$ or -4 na coordinates are (2,8)	M1 form equation M1 solving A1 A1
11iii	$(x-2)^2+(y-8)^2=100$ $x^2+y^2-4x-16y-32=0$	M1 A1 or B2
11iv	y = 18 y = −2	B1 B1

Geylang Methodist School (Secondary)
Preliminary Examination 2014

ADDITIONAL MATHEMATICS **4047/01**

Paper 1

4 Express

Additional materials :Writing Paper **2 hours**

Setter : MrWong Han Ming / MrJohney Joseph **1Sep 2014**

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all**the questions.
Write your answers on the separate Writing Papers provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

This document consists of 6 printed pagesincluding the cover page

[Turn over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1 Without using a calculator, express $\cos 75°$ in the form

$\frac{\sqrt{a}-\sqrt{b}}{4}$, where a and b are integers. [3]

2 Find the coordinates of the point of intersection of the graphs of $y = 3x + 5$ and $y = |2-5x|+3$. [4]

3 The equation $2x^2 - 6x + 1 = 0$ has roots α and β. Without solving for α and β, find an equation that has roots $\frac{1}{\alpha^2}$ and $\frac{1}{\beta^2}$. [6]

4 In the binomial expansion of $(2+x)^n$, where n is a positive integer, the coefficient of x and the coefficient of x^2 is in the ratio 4 : 15.

Find the value of n. [6]

5 At the beginning of year 1900, the population of a country was 580 000. Due to a disease, the population decreased such that after a period of t years, the new population was given by the equation $580\,000e^{kt}$, where k is a constant.

Given that the population after 2 years was 480 000,

(i) find the value of k, [2]

(ii) calculate, to the nearest whole number, the population of the country after 5 years, [2]

(iii) find the year in which the population first fall below 100 000. [2]

[Turn over

6 (a) Given that $\lg x = a$ and $\lg y = b$, express $\lg\sqrt{\dfrac{100x^8}{y^5}}$ in terms of a and b. [3]

(b) Solve $\log_{16}[\log_2(6x-18)] = \log_9 3$. [3]

7 (i) Show that $\tan\theta + \cot\theta = 2\operatorname{cosec}2\theta$. [4]

(ii) Hence, solve $2\tan\theta + 2\cot\theta = -5$ for $0 \le \theta \le \pi$. [2]

8 It is given that $3^{x+1} \times 2^{2x+1} = 2^{x+2}$.

(i) Find the exact value of 6^x. [2]

(ii) Hence find the value of x correct to two decimal places. [2]

9 Given that $\dfrac{4x^3+7x^2-13x-2}{x^2+2x-3} = Ax + B + \dfrac{x+C}{x^2+2x-3}$,

(i) find the values of each of the integers A, B and C. [4]

Hence, using partial fractions and the values of A, B and C obtained in part **(i)**,

(ii) find $\displaystyle\int \frac{4x^3+7x^2-13x-2}{x^2+2x-3}\,dx$. [5]

10 (i) Differentiate $x^2 \ln x + x$ with respect to x. [2]

(ii) Hence evaluate $\displaystyle\int_1^3 x\ln x\,dx$, correct to one decimal place. [4]

11 Variables x and y are connected by the equation $y = ax^n$, where a and n are constants. Using experimental values of x and y, a graph was drawn in which $\lg y$ was plotted on the vertical axis against $\lg x$ on the horizontal axis. The straight line which was obtained passes through the points (2.3, 1.5) and (4.2, 5.3).

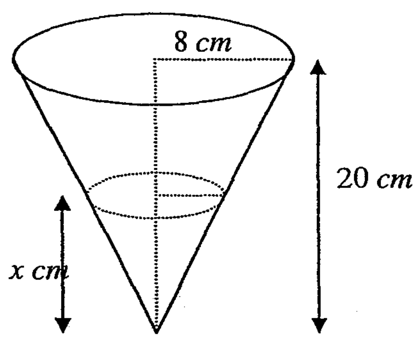
Calculate

(i) the value of n, [2]

(ii) the value of a correct to 2 significant figures, [1]

(iii) thecoordinates of the point on the line at which $y = \frac{1}{2}x$. [3]

12



[The volume of a cone of height H and radius R is given by $\frac{1}{3}\pi R^2 H$]

The diagram shows a hollow conical tank of height 20 cm and radius 8 cm. The tank is held with its circular rim horizontal. Water is then poured into the empty tank at a constant rate of 10 cm^3 s^{-1}. After t seconds, the depth of water is x cm.

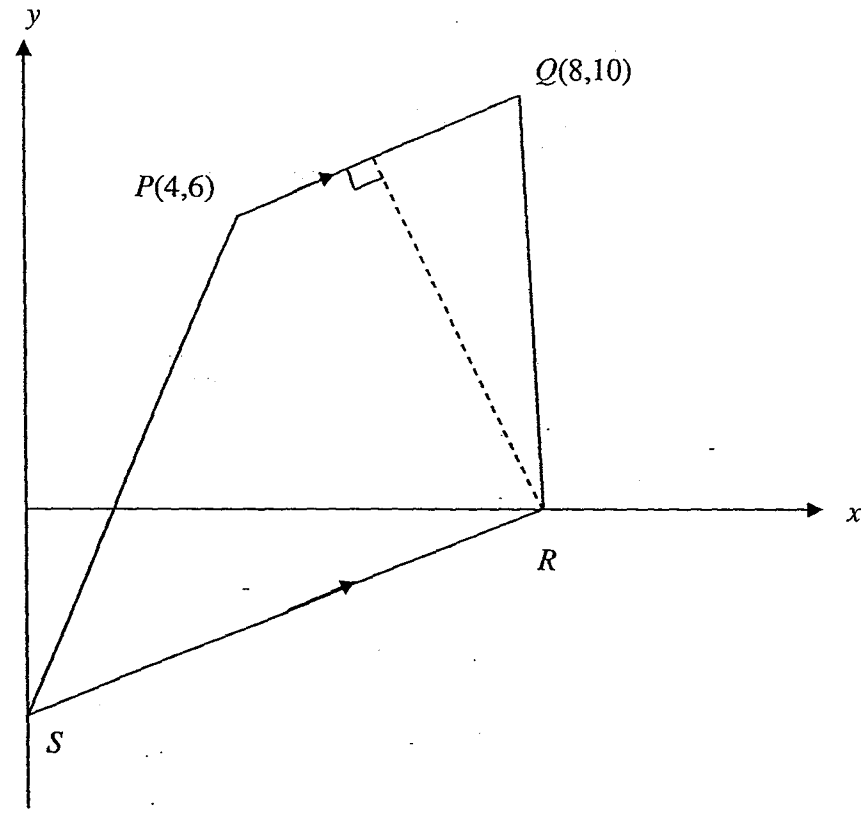
(i) Show that, the volume of the water, V cm^3, at time t is given by $V = \frac{4}{75}\pi x^3$. [2]

(ii) Find $\frac{dV}{dx}$ and hence, find the rate of change of the depth when $x = 4$. [3]

(iii) State, with reason, whether this rate will increase or decrease as t increases. [2]

[Turn over

13 **(a)**



The diagram shows a trapezium $PQRS$ in which PQ is parallel to SR. The point P is (4, 6) and the point Q is (8, 10). R and S lie on the x and y axes respectively. The point R lies on the perpendicular bisector of PQ. Find

(i) the coordinates of R and of S, [4]

(ii) the area of trapezium $PQRS$. [2]

(b) Solve the equation $3^{3x} + 3^{2x+1} - 10(3^x) - 24 = 0$. [5]

- End of Paper -

A Maths P1 Solution.

1) $\cos 75^\circ = \cos(30^\circ + 45^\circ)$

$= \cos 30^\circ \cos 45^\circ - \sin 30^\circ \sin 45^\circ$

$= \frac{\sqrt{3}}{2} \frac{\sqrt{2}}{2} - \frac{1}{2} \frac{\sqrt{2}}{2}$

$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$

$= \frac{\sqrt{6} - \sqrt{2}}{4}$

2) $y = 3x + 5$

$y = |2 - 5x| + 3$

$3x + 5 = |2 - 5x| + 3$

$3x + 2 = |2 - 5x|$

$3x + 2 = 2 - 5x$ OR $3x + 2 = -(2 - 5x)$

$8x = 0$ $\quad$ $3x + 2 = 5x - 2$

$x = 0$ $\quad$ $4 = 2x$

$x = 2$

$\therefore y = 5$ $\quad$ $y = 11$

$\therefore$ Coordinates are $(0, 5)$ & $(2, 11)$

3) $2x^2 - 6x + 1 = 0$

$\alpha + \beta = 3$

$\alpha\beta = \frac{1}{2}$

$$\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\alpha^2 + \beta^2}{(\alpha\beta)^2}$$

$$= \frac{(\alpha+\beta)^2 - 2\alpha\beta}{(\frac{1}{2})^2}$$

$$= \frac{3^2 - 2(\frac{1}{2})}{1/4}$$

$$= \frac{8}{1/4}$$

$$= 32$$

$$\frac{1}{\alpha^2} \times \frac{1}{\beta^2} = \frac{1}{(\frac{1}{2})^2}$$

$$= 4$$

$\therefore$ Eqn is $x^2 - 32x + 4 = 0$

4) $(2+x)^n = 2^n + \binom{n}{1} 2^{n-1} x$

$+ \binom{n}{2} 2^{n-2} x^2$

$+ \cdots$

coeff of $x = n 2^{n-1}$

coeff of $x^2 = \frac{n(n-1)}{2} 2^{n-2}$

$$\frac{n 2^{n-1}}{\frac{n(n-1)}{2} 2^{n-2}} = \frac{4}{15}$$

$15n 2^{n-1} = 2n(n-1) 2^{n-2}$

$15(2) = 2(n-1)$

$15 = n-1$

$n = 16$

5(i) $580\,000 e^{kt}$

when $t = 2$

$580\,000 \; e^{2k} = 480\,000$

$e^{2k} = \frac{48}{58}$

$2k = \ln \frac{48}{58}$

$k = -0.0946209982$

≈ -0.0946

5(ii) new population $= 580\,000\, e^{5k}$

$= 361377.495$

≈ 361377

(iii) $580\,000\, e^{kt} = 100\,000$

$e^{kt} = \frac{10}{58}$

$kt = \ln \frac{10}{58}$

$t = \ln \frac{10}{58} \div k$

$= 18.5778835\,7$

$\therefore$ 18 years

The year in which the population first hit below 100 000 is 1918

6a) $\lg \sqrt{\frac{100x^8}{y^5}} = \frac{1}{2} \lg \frac{100x^8}{y^5}$

$= \frac{1}{2}\left[\lg 100 + \lg x^8 - \lg y^5\right]$

$\rightarrow = \frac{1}{2}\left[10 + 8\lg x - 5\lg y\right]$ (10 circled, marked 2)

$= \frac{1}{2}\left[10 + 8a - 5b\right]$ (10 struck out, marked 1)

$= 5 + 4a - \frac{5}{2}b$

6b) $\log_{16}[\log_2(6x-18)] = \log_9 3$

$\log_{16}[\log_2(6x-18)] = \log_9 9^{\frac{1}{2}}$

$\log_{16}[\log_2(6x-18)] = \frac{1}{2}$

$\log_2(6x-18) = 16^{\frac{1}{2}}$

$\log_2(6x-18) = 4$

$6x-18 = 2^4$

$6x-18 = 16$

$6x = 34$

$x = 5\frac{2}{3}$

7(i) $\tan\theta + \cot\theta = 2\operatorname{cosec} 2\theta$

RHS $= 2\operatorname{cosec} 2\theta$

$= \frac{2}{\sin 2\theta}$

$= \frac{2}{2\sin\theta\cos\theta}$

$= \frac{1}{\sin\theta\cos\theta}$

LHS $= \tan\theta + \cot\theta$

$= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$

$= \frac{\sin^2\theta}{\sin\theta\cos\theta} + \frac{\cos^2\theta}{\sin\theta\cos\theta}$

$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$

$= \frac{1}{\sin\theta\cos\theta}$ = RHS

7(ii) $2\tan\theta + 2\cot\theta = -5$

$4\ \text{cosec}\ 2\theta = -5$

$\text{cosec}\ 2\theta = \frac{-5}{4}$

$\sin 2\theta = -\frac{4}{5}$

basic $\measuredangle = 0.927295218$

$2\theta = 4.068887872,\ 5.355890089$

$\theta = 2.03,\ 2.68$

Paper I sol.

(JJ)

8 (i) $3^{x+1} \times 2^{2x+1} = 2^{x+2}$

$3^x \times 3 \times 2^{2x} \times 2 = 2^x \times 4$

$3^x \times 2^x = \frac{2}{3}$

$6^x = \frac{2}{3}$

(ii) $x \lg 6 = \lg \frac{2}{3}$

$x = \frac{\lg(\frac{2}{3})}{\lg 6}$

$= -0.23$

9 (i)

$$\begin{array}{r|l} & 4x - 1 \\ x^2+2x-3 & 4x^3 + 7x^2 - 13x - 2 \\ & 4x^3 + 8x^2 - 12x \\ \hline & -x^2 - x - 2 \\ & -x^2 - 2x + 3 \\ \hline & x - 5 \end{array}$$

$$\frac{4x^3 + 7x^2 - 13x - 2}{x^2+2x-3} = 4x - 1 + \frac{x-5}{x^2+2x-3}$$

$\therefore A = 4, B = -1, C = -5$

(ii) $x^2 + 2x - 3 = (x+3)(x-1)$

Let $\frac{x-5}{(x+3)(x-1)} = \frac{P}{x+3} + \frac{Q}{x-1}$

$x - 5 = P(x-1) + Q(x+3)$

$P = 2, \quad Q = -1$

$\therefore \frac{x-5}{(x+3)(x-1)} = \frac{2}{x+3} - \frac{1}{x-1}$

$$\int \frac{4x^3 + 7x^2 - 13x - 2}{x^2 + 2x - 3}\,dx = \int \left(4x - 1 + \frac{2}{x+3} - \frac{1}{x-1}\right)$$

$$= 2x^2 - x + 2\ln(x+3) - \ln(x-1) +$$

10 (i) $\frac{d}{dx}(x^2 \ln x + x) = x^2 \times \frac{1}{x} + (\ln x)2x + 1$

$= x + 2x\ln x + 1$

$$\int_1^3 [x + 2x\ln x + 1]\,dx = \left[x^2 \ln x + x\right]_1^3$$

$$\int_1^3 2x\ln x\,dx = \left[x^2 \ln x + x\right]_1^3 - \left[\frac{x^2}{2} + x\right]_1^3$$

$$= \left[x^2 \ln x - \frac{x^2}{2}\right]_1^3$$

$$= \left(9\ln 3 - \frac{9}{2}\right) - \left(0 - \frac{1}{2}\right)$$

$$= 9\ln 3 - 4$$

~~$= 5.89 = 5.9$ (1 d.p)~~

$$\therefore \int_1^3 x\ln x\,dx = \frac{9\ln 3 - 4}{2}$$

$= 2.9$ (1 d.p)

11. (i) $y = ax^n$

$\lg y = n\lg x + \lg a$

$n = \text{gradient}$

$= \dfrac{5.3 - 1.5}{4.2 - 2.3} = 2$

(ii) $5.3 = 2 \times 4.2 + \lg a$

$\lg a = -3.1$

$a = 10^{-3.1}$

$= 0.00079.$

(iii)

$y = \frac{1}{2}x$

$\lg y = \lg x - \lg 2$ —— (1)

$\lg y = 2\lg x - 3.1$ —— (2)

$0 = \lg x - 3.1 + \lg 2$

$\lg x = 3.1 - \lg 2$

$= 2.80$

$\lg y = 2.50$

$\therefore (2.80, 2.50).$

12(i) $\frac{x}{20} = \frac{r}{8}$

$r = \frac{2}{5}x$

$V = \frac{1}{3}\pi\left(\frac{2}{5}x\right)^2 x$

$= \frac{4\pi}{75}x^3$

(ii) $\frac{dv}{dx} = \frac{4\pi}{75}(3x^2) = \frac{4\pi}{25}x^2$

when $x = 4$, $\frac{dv}{dx} = \frac{4\pi}{25} \times 4^2 = \frac{64\pi}{25}$

$\frac{dv}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$

$10 = \frac{64\pi}{25} \cdot \frac{dx}{dt}$

$\frac{dx}{dt} = \frac{25 \times 10}{64\pi} = 1.24$ cm/s.

(iii) $\frac{dx}{dt} = \frac{dv}{dt} \div \frac{dv}{dx}$

$= 10 \div \frac{4\pi x^2}{25}$

$= \frac{125}{2\pi x^2}$

∴ The rate decreases as t increases.

13 (a) Midpoint of PQ is $(6, 8)$

Gradient of PQ $= \frac{10-6}{8-4} = 1$

Equation of the perpendicular bisector is

$y - 8 = -1(x - 6)$

$y = -x + 14$

OR use $PR = QR$

$\therefore R(14, 0)$

Equation of SR is

$y - 0 = 1(x - 14)$

$y = x - 14$

$\therefore S(0, -14)$

(b) $3^{3x} + 3^{2x+1} - 10(3^x) - 24 = 0$

Let $y = 3^x$

$y^3 + 3y^2 - 10y - 24 = 0$

Let $f(y) = y^3 + 3y^2 - 10y - 24$

$f(-2) = -8 + 12 + 20 - 24 = 0$

$\therefore y + 2$ is a factor.

Geylang Methodist School (Secondary)
Preliminary Examination 2014

ADDITIONAL MATHEMATICS **4047/02**

Paper 2

4 Express

Additional materials :Writing Paper **2 hours 30 minutes**

Setter : MrJohney Joseph **29Aug 2014**

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all**the questions.
Write your answers on the separate Writing Papers provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

This document consists of **6**printed pagesincluding the cover page

[Turn over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\text{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

1 A curve is given by the equation $y = 2 + \frac{c}{x}$ where c is a positive constant.

(i) Explain with working why the line $y = 2x - 2$ intersects the curve at two distinct points for all values of c. [3]

Given that the gradient of the curve $y = 2 + \frac{c}{x}$ at the point A $(k, 6)$ is -4.

(ii) Show that c is a multiple of k^2. [2]

(iii) Find the value of c. [2]

2 (i) Sketch the graph of $y = 9x^{\frac{1}{3}}$ for x ▯ 0. [1]

(ii) On the same diagram, sketch the graph of $y = 4x^{\frac{-1}{3}}$ for x ▯ 0. [1]

(iii) Calculate the coordinates of the point of intersection of your graphs. [2]

(iv) Determine, with explanation, whether there is a common tangent for the two curves. [4]

3 It is given that $\int_1^5 px\,dx = 10$, where p is a constant.

(i) Find the value of the constant p. [2]

(ii) Express $\int_1^5 (px + q)\,dx$ in terms of the constant q. [2]

4 The height h metres of water at a jetty on a particular day is given by $h = 1.5\,(2 + \sin \frac{\pi t}{6})$ where t is the number of hours after midnight.

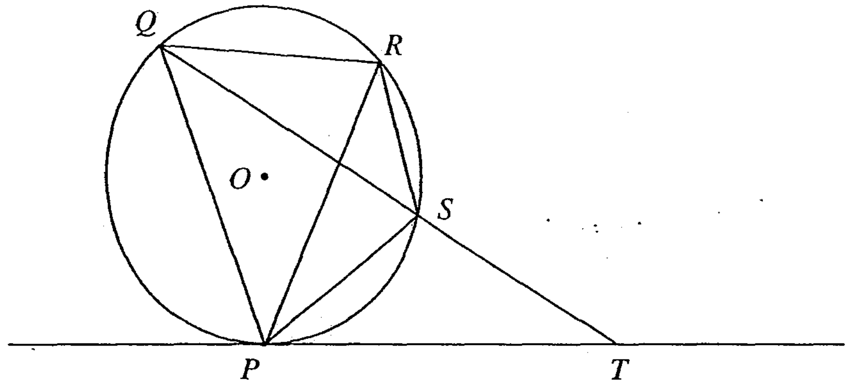
(i) Explain why the maximum height of water at the jetty for the particular day is 4.5 m. [1]

(ii) Find the height of water at 9 am. [2]

(iii) Find the first two values of t for which the height of water is 2 m. [4]

[Turn over

5 In the figure below, $RS = PS$ and the line PT is tangent to the circle. Line QT bisects angle PQR and cuts the circle at point S.



Prove that

(i) $\angle TPS = \angle SPR$, [2]

(ii) ΔSPT is similar to ΔPQT, [2]

(iii) $PT \cdot PQ = QT \cdot RS$. [4]

6 A curve has an equation $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = \dfrac{x^2+5}{x-2}$ for $x \neq 2$.

(i) Obtain an expression for $\mathrm{f}'(x)$. [2]

(ii) Find the range of values of x for which f is an increasing function. [3]

(iii) Showing full working, determine whether the gradient of the curve is an increasing or decreasing function. [3]

7 A particle travels in a straight line, so that t seconds after passing through a fixed point O, the velocity v ms^{-1}, is given by $v = \dfrac{2t-4}{t+2}$. Find

(i) the initial velocity of the particle, [1]

(ii) the acceleration of the particle when it comes to instantaneous rest, [3]

(iii) an expression for the displacement in terms of t. [4]

8 **(i)** Prove that $\sin^6 x + \cos^6 x - \sin^2 x \cos^2 x = \cos^2 2x$. [4]

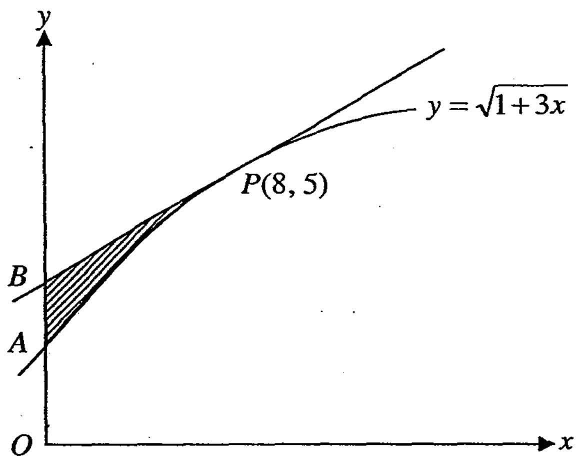
(ii) Solve the equation $\sin^6 x + \cos^6 x - \sin^2 x \cos^2 x - 1 = 0$, for $0 \le x \le \pi$, giving your answers in terms of π. [4]

(iii) Given that $\sin^6 A + \cos^6 A - \sin^2 A \cos^2 A = 3\sin^2 2A$, where $45° < A < 90°$, **without using a calculator,**

(a) deduce that $\tan 2A = -\dfrac{1}{\sqrt{3}}$, [2]

(b) find the value of $\tan A$. [3]

9



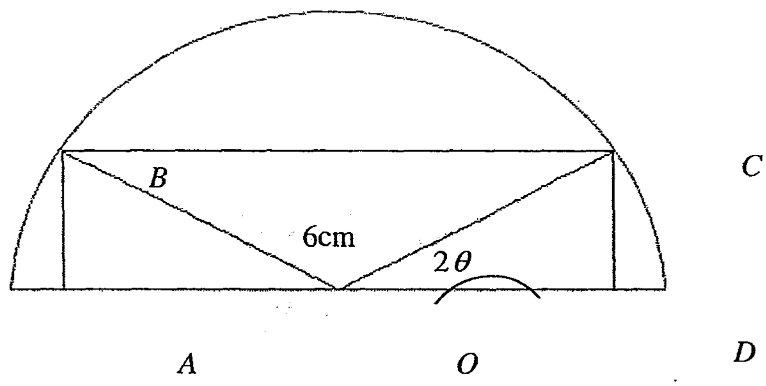
The diagram shows part of the curve $y = \sqrt{1+3x}$, intersecting the y-axis at A. The tangent to the curve at the point $P(8, 5)$ intersects the y-axis at B. Find

(i) the equation of BP, [4]

(ii) the area of the shaded region ABP. [4]

[Turn over

10 The diagram shows a rectangle $ABCD$ inscribed in a semicircle, centre O and radius6 cm, such that $\angle BOC = 2\theta$ radians.



(i) Show that the perimeter, P cm, of the rectangle is given by $P = 12\cos\theta + 24\sin\theta$. [3]

(ii) Find the values of θ for which P is 25cm. [6]

11 Find the coordinates of the stationary points on the curve $y = 15 - \frac{8}{x} - 2x$ and determine the nature of each stationary point. [8]

12 A circle with centre$C$, pass through the points (5, – 8) and (5, 0).
The line $y = 1$ is a tangent to the circle and the circle intersects the y-axis at two points. Find

(i) the radius of the circle, [4]

(ii) the coordinates of C, [4]

(iii) the equation of the circle in the form $x^2 + y^2 + ax + by + c = 0$, where a, b and c are integers, [2]

(iv) the equations of the tangents to the circle parallel to the y- axis. [2]

- End of Paper -

A Maths Paper II Solution.

(JJ)

1(i) $2 + \frac{c}{x} = 2x - 2$

$2x^2 - 4x - c = 0$

$b^2 - 4ac = (-4)^2 - 4(2)(-c)$

$= 16 + 8c$

$> 0 \quad (c > 0)$

$\therefore y = 2x - 2$ intersects $y = 2 + \frac{c}{x}$ at two distinct points.

(ii) $\frac{dy}{dx} = -\frac{c}{x^2}$

$-4 = -\frac{c}{k^2}$

$c = 4k^2$

(iii) $6 = 2 + \frac{c}{k}$

$k = \frac{c}{4}$

$c = 4\left(\frac{c}{4}\right)^2$

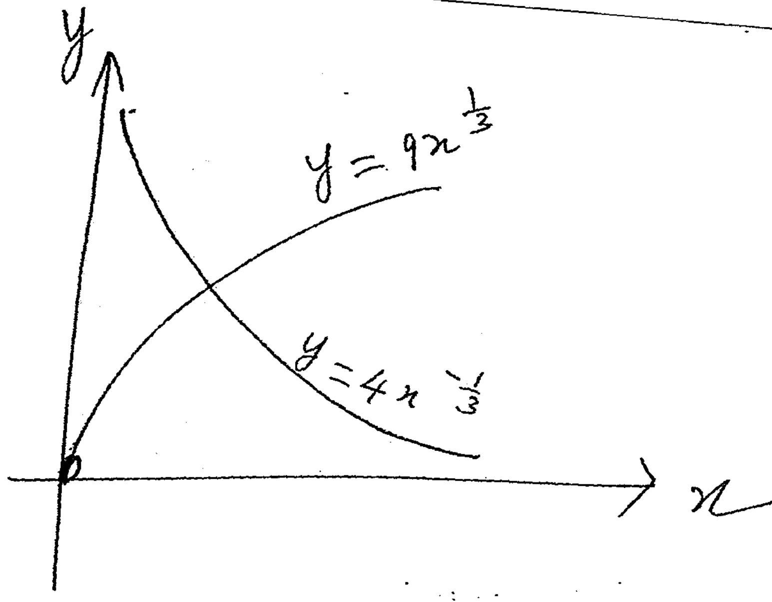
$c^2 - 4c = 0$

$c(c-4) = 0$

$c = 0$ or $c = 4$

$c = 4$

2(i), (ii)



(iii)

$$9x^{\frac{1}{3}} = 4x^{-\frac{1}{3}}$$

$$x^{\frac{2}{3}} = \frac{4}{9}$$

$$x = \left(\frac{4}{9}\right)^{\frac{3}{2}}$$

$$= \frac{8}{27}$$

$$y = 9\times\left(\frac{8}{27}\right)^{\frac{1}{3}} = 6$$

$$\therefore \left(\frac{8}{27}, 6\right)$$

(iv)

$$y = 9x^{\frac{1}{3}}$$

$$\frac{dy}{dx} = 9\times\frac{1}{3}x^{-\frac{2}{3}} = 3x^{-\frac{2}{3}}$$

$$y = 4x^{-\frac{1}{3}}$$

$$\frac{dy}{dx} = 4\times -\frac{1}{3}x^{-\frac{4}{3}} = -\frac{4}{3}x^{-\frac{4}{3}}$$

when gradients are equal,

$$3x^{-\frac{2}{3}} = -\frac{4}{3}x^{-\frac{4}{3}}$$

$x^{\frac{2}{3}} = -\frac{4}{9}$ which is not possible as x;

3 (i) $\int_1^5 px\,dx = 10$

$\left[p\frac{x^2}{2}\right]_1^5 = 10$

$p\left(\frac{25}{2} - \frac{1}{2}\right) = 10$

$12p = 10$

$p = \frac{5}{6}$

(ii) $\int_1^5 (px+q)\,dx = \int_1^5 px\,dx + \int_1^5 q\,dx$

$= 10 + q[x]_1^5$

$= 10 + q(5-1)$

$= 10 + 4q$

4 (i) $h = 1.5\left(2 + \sin\frac{\pi x}{6}\right)$

Max value of $h = 1.5(2+1)$

$= 4.5$ m $\left(\because -1 \leq \sin\frac{\pi x}{6} \leq 1\right)$

(ii) When $x = 9$

$h = 1.5\left(2 + \sin\frac{9\pi}{6}\right)$

$= 1.5(2-1)$

$= 1.5$ m.

(iii) $1.5\left(2 + \sin\frac{\pi x}{6}\right) = 2$

$3 + 1.5\sin\frac{\pi x}{6} = 2$

$\sin \frac{\pi t}{6} = -\frac{2}{3}$

Basic Angle $= 0.7297$

$\frac{\pi t}{6} = \pi + 0.7297,\ 2\pi - 0.7297$

$t = 7.39,\ 10.6$

5 (i) $\angle TPS = \angle SRP$ (Alternate Segment Theorem)

$\angle SRP = \angle SPR$ $(RS = PS)$

$\therefore \angle TPS = \angle SPR$

(ii) $\angle SPT = \angle PQT$ (Alternate Segment Theorem)

$\angle T$ is common.

$\therefore \Delta SPT$ is similar to ΔPQT

(iii) $\frac{SP}{PQ} = \frac{PT}{QT} = \frac{ST}{PT}$ (ΔSPT similar to ΔPQ

$\frac{SP}{PQ} = \frac{PT}{QT}$

$PT \cdot PQ = QT \cdot SP$

$SP = SR$ (Given)

$\therefore PT \cdot PQ = QT \cdot SR$

b(i) $f(x) = \dfrac{x^2+5}{x-2}$

$f'(x) = \dfrac{(x-2)(2x)-(x^2+5)(1)}{(x-2)^2}$

$= \dfrac{2x^2-4x-x^2-5}{(x-2)^2}$

$= \dfrac{x^2-4x-5}{(x-2)^2}$

(ii) $\dfrac{x^2-4x-5}{(x-2)^2} > 0$

$x^2-4x-5>0$

$(x-5)(x+1)>0$

$x<-1$ or $x>5$

But $x>2$.

$\therefore f(x)$ is increasing when $x>5$.

(iii) $f''(x) = \dfrac{(x-2)^2(2x-4)-(x^2-4x-5)\,2(x-2)}{(x-2)^4}$

$= \dfrac{(x-2)(2x-4)-2(x^2-4x-5)}{(x-2)^3}$

$= \dfrac{2x^2-4x-4x+8-2x^2+8x+10}{(x-2)^3}$

$= \dfrac{18}{(x-2)^3} > 0$ ... is ... increasing

7(i) $V = \frac{2t-4}{t+2}$

When $t = 0$

$V = -2$ m/s

(ii) When $V = 0$

$\frac{2t-4}{t+2} = 0$

$t = 2$

$a = \frac{(t+2)(2) - (2t-4)}{(t+2)^2}$

$= \frac{8}{(t+2)^2}$

When $t = 2$

$a = \frac{8}{(2+2)^2} = \frac{1}{2}$ m/s^2.

(iii) $S = \int v \, dt$

$= \int \frac{2t-4}{t+2} \, dt$

$$t+2 \overline{\big) 2t-4} \quad \text{quotient } 2; \quad 2t+4; \quad -8$$

$= \int \left(2 - \frac{8}{t+2}\right) dt$

$S = 2t - 8\ln(t+2) + C$

$0 = -8\ln 2 + C \Rightarrow C = 8\ln 2$

8 (i) $\sin^6 x + \cos^6 x - \sin^2 x \cos^2 x$

$= (\sin^2 x + \cos^2 x)(\sin^4 x - \sin^2 x \cos^2 x + \cos^4 x) - \sin^2 x \cos^2 x$

$= 1\,(\sin^4 x - \sin^2 x \cos^2 x + \cos^4 x) - \sin^2 x \cos^2 x$

$= \sin^4 x - 2\sin^2 x \cos^2 x + \cos^4 x$

$= (\sin^2 x - \cos^2 x)^2$

$= (-\cos 2x)^2$

$= \cos^2 2x.$

(ii) $\cos^2 2x - 1 = 0$

$\cos^2 2x = 1$

$\cos 2x = \pm 1$

$2x = 0, \pi, 2\pi$

$x = 0, \frac{\pi}{2}, \pi$
(NA) (NA)

$x = \frac{\pi}{2}$

(iii) $3\sin^2 2A = \cos^2 2A$

$\tan^2 2A = \frac{1}{3}$

$\tan 2A = -\frac{1}{\sqrt{3}}$ (~~2A~~ $90° < 2A < 180°$)

$$\frac{2\tan A}{1-\tan^2 A} = -\frac{1}{\sqrt{3}}$$

$$2\sqrt{3}\tan A = -1 + \tan^2 A$$

$$\tan^2 A - 2\sqrt{3}\tan A - 1 = 0$$

$$\tan A = \frac{2\sqrt{3} \pm \sqrt{(-2\sqrt{3})^2 - 4(1)(-1)}}{2}$$

$$= \frac{2\sqrt{3} \pm \sqrt{16}}{2}$$

$$= \frac{2\sqrt{3} \pm 4}{2}$$

$$= \sqrt{3} \pm 2$$

$$= \sqrt{3} + 2 \quad (\because \tan A > 0).$$

9(i) $y = \sqrt{1+3x}$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{1+3x}}(3)$$

When $x = 8$

$$\frac{dy}{dx} = \frac{3}{10}$$

Equation of BP is

$$y - 5 = \frac{3}{10}(x - 8)$$

$$10y - 50 = 3x - 24$$

$$10y = 3x + 26$$

(iii) $B(0, 2.6)$

Area of shaded region

$$= \frac{1}{2}(2.6+5)\times 8 - \int_0^8 \sqrt{1+3x}\,dx$$

$$= 30.4 - \left[\frac{(1+3x)^{\frac{3}{2}}}{3\times\frac{3}{2}}\right]_0^8$$

$$= 30.4 - \frac{2}{9}\left[25^{\frac{3}{2}} - 1^{\frac{3}{2}}\right]$$

$$= 30.4 - \frac{2}{9}(125-1)$$

$$= 2.84 \text{ units}^2$$

10 (i) $\sin\theta = \frac{OA}{6} \Rightarrow OA = 6\sin\theta \Rightarrow AD = 12\sin\theta$

$\cos\theta = \frac{AB}{6} \Rightarrow AB = 6\cos\theta$

Perimeter $= 2(6\cos\theta + 12\sin\theta)$

$= 12\cos\theta + 24\sin\theta$

(ii) Let $12\cos\theta + 24\sin\theta = R\cos(\theta - \alpha)$

$12\cos\theta + 24\sin\theta = R(\cos\theta\cos\alpha + \sin\theta\sin\alpha)$

$R\cos\alpha = 12$

$R\sin\alpha = 24$

$\tan\alpha = 2$

$\alpha = 1.107$

$R = \sqrt{12^2 + 24^2} = \sqrt{720} = 26.83$

$26.83\cos(\theta - 1.107) = 25$

$\cos(\theta - 1.107) = \frac{25}{26.83}$

$\theta - 1.107 = 0.3715, -0.371$

$\theta = 1.48, 0.736$

11. $y = 15 - \frac{8}{x} - 2x$

$\frac{dy}{dx} = \frac{8}{x^2} - 2$

When $\frac{dy}{dx} = 0$

$\frac{8}{x^2} - 2 = 0$

$\frac{8}{x^2} = 2$

$x^2 = 4$

$x = \pm 2$

When $x = 2$

$y = 15 - \frac{8}{2} - 2\times 2 = 7$

When $x = -2$

$y = 15 + \frac{8}{2} + 4 = 23$

∴ The stationary points are $(2, 7)$ and $(-2, 23)$

$\frac{d^2y}{dx^2} = -\frac{16}{x^3}$

When $x = 2$

$\frac{d^2y}{dx^2} = -\frac{16}{2^3} < 0$ ∴ $(2, 7)$ Max

When $x = -2$ and $(-2, 23)$ Min.

$\frac{d^2y}{dx^2} = \frac{-16}{\ldots} > 0$

12 (i) Midpoint of chord joining (5, -
is $(5, -4)$

∴ The centre lies
on the line $y = -4$ — —

$y = 1$ is tangent to the cir
∴ Radius $= 4 + 1 = 5$ uni

(ii) Let the centre be (a, -

$(a-5)^2 + (-4-0)^2 = 5^2$

$(a-5)^2 = 9$

$a - 5 = \pm 3$

$a = 2$ OR $a = 8$
(NA

∴ Centre is $(2, -4)$

(iii) $(x-2)^2 + (y+4)^2 = 5^2$

$x^2 - 4x + 4 + y^2 + 8y + 16 = 2$

$x^2 + y^2 - 4x + 8y - 5 = 0$

(iv) For tangents parall to x-axis

$x = 2 + 5$ OR $x = 2 - 5$

∴ $x = 7$ and $x = -3$ ar
tange

聖嬰中學

HOLY INNOCENTS' HIGH SCHOOL

PRELIMINARY EXAMINATION 2014
SECONDARY 4 EXPRESS

ADDITIONAL MATHEMATICS **4047 / 01**

Name	: ____________________	**Date**	: **12 August 2014**
Register No	: ____________________	**Duration**	: **2 h**
Class	: ____________________		

Additional Materials: 7 Sheets of Writing Paper

Instruction to Candidates

Write your register number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.

If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.

The use of a scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total marks for this paper is 80.

Setter: Mr Adrian Goh
Vetter: MsKohSwee Kun
MdmHayati
Mrs Nathan

2

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1. Find the set of values of the constant k for which the line $y = 3x - k$ intersects the curve $y = 2x^2 + kx - 3$ at two distinct points. [5]

2. Find the x-coordinate of the stationary point of the curve $y = \frac{1}{2}e^{2x} - e^{x} - 2x + 1$. [5]

3. The roots of the quadratic equation $4x^2 - x + 16 = 0$ are α^2 and β^2. Find the quadratic equation whose roots are α and β, where $\alpha + \beta > 0$. [6]

4. It is given that $\int_a^b \frac{5x}{2}\,dx = 5$, where a and b are constants.

   Find the possible values of k if $\int_{ka}^{kb} \frac{5x}{2}\,dx = 20$. [4]

5. (a) Express $\left(\frac{4}{2+\sqrt{5}} - 3 - 2\sqrt{5}\right)^2$ in the form $a + b\sqrt{5}$ where a and b are integers. [4]

   (b) Given that $ab - 4b + a - 4$ can be expressed as $(a-4)(b+1)$, solve
   $$6^x - 4(3^x) + 2^x - 4 = 0.$$ [4]

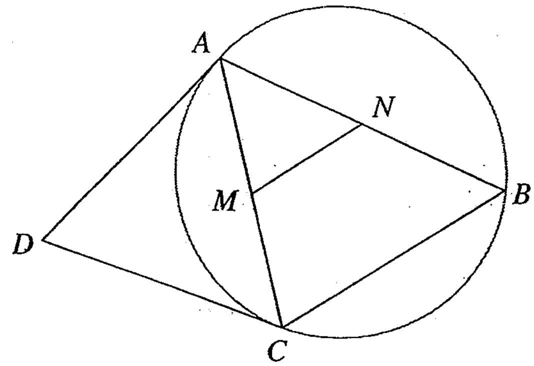
6. The variables x and y are connected by the equation $xy^2 - px - qy = 0$, where p and q are constants. Using experimental values of x and y, a graph was drawn in which y^2 was plotted on the vertical axis against $\frac{y}{x}$ on the horizontal axis. The straight line which was obtained passed through the points $(1,\ 4\frac{1}{3})$ and $(6, 6)$.

   Estimate

   (a) the value of p and of q, [4]

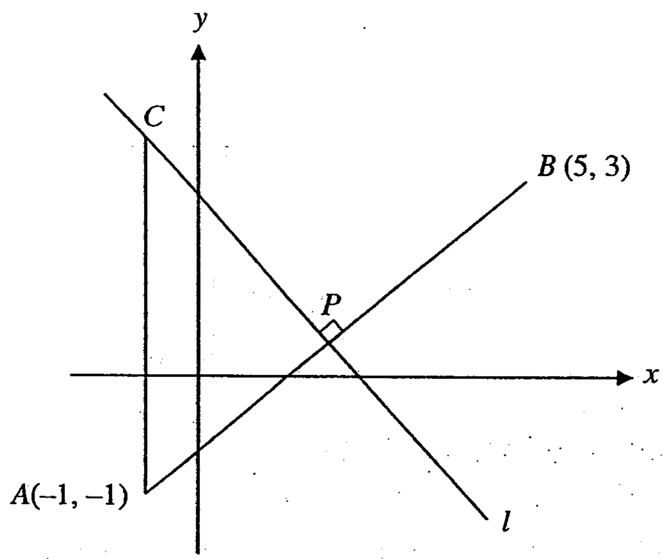
   (b) the values of x when $\frac{y}{x} = 3$. [3]

7.



The diagram shows points A, B and C lying on a circle.
The tangents to the circle at A and C intersect at D.
M and N are the mid-points of AC and AB respectively.
The line AC bisects $\angle BCD$.
Prove that ΔAMN is similar to ΔDCA. [4]

8.



In the diagram, l is the perpendicular bisector of the line joining A (–1, –1) and B (5, 3).
C is a point on l such that AC is parallel to the y-axis.

(a) Find the coordinates of C. [4]

(b) Write down the coordinates of D such that $ACPD$ is a parallelogram. [1]

(c) Find the area of the parallelogram $ACPD$. [2]

9. A particle, travelling in a straight line, passes a fixed point O on the line with a velocity of 2ms^{-1}. The acceleration, $a \text{ms}^{-2}$, of the particle, t s after passing O, is given by $a = -\frac{1}{(t+1)^2}$.

(a) Obtain an expression, in terms of t, for the velocity of the particle after passing O. [3]

(b) Show that the object will never come to rest. [1]

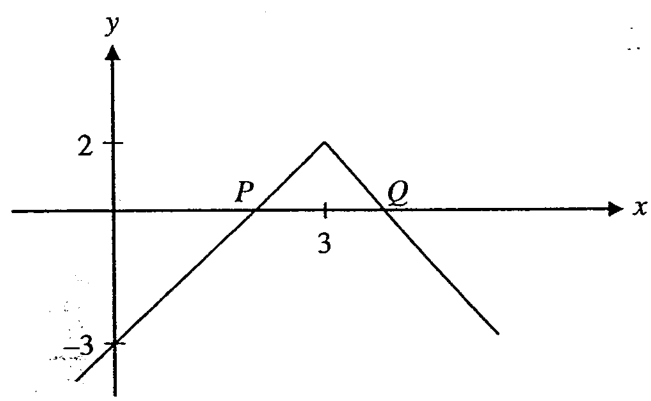
(c) Find the total distance travelled by the particle between $t = 0$ and $t = 5$. [3]

10. (a) Show that $\cot 2A$ can be expressed as $\frac{1}{2\tan A} - \frac{1}{2}\tan A$. [2]

(b) **Hence** solve $\tan A(3 - 4\cot 2A) = 3$ for $0° \le A \le 360°$. [5]

(c) Using your answers in (b), state the number of solutions of the equation $\tan\frac{1}{2}A(3 - 4\cot A) = 3$ in the range $-360° \le A \le 720°$. [1]

11.



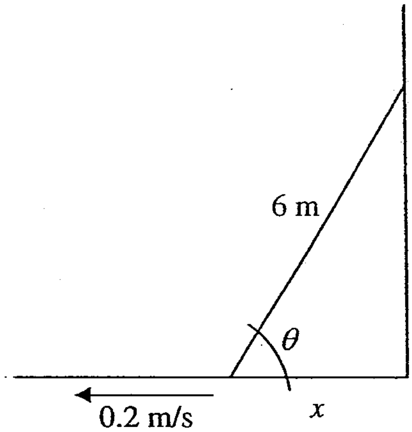
The diagram above shows part of the graph of $y = a - |bx + c|$ where a, b and c are constants.

(a) Find the values of a, b and c. [3]

(b) Find the x-coordinates of P and Q. [3]

(c) **Hence** sketch the graph of $y = |a - |bx + c||$. [2]

12.



In the diagram, a ladder, 6 m, is leaning against a vertical wall. The distance between the base of the ladder and the wall is x m. A force is exerted, pulling the base of the ladder away from the wall at a constant rate of 0.2 m/s.

(a) (i) Show that $x = 6\cos\theta$. [1]

(ii) Find $\frac{d\theta}{dt}$ at the instant when $\theta = 0.367$ radians. [3]

(b) If it took 20 seconds for the ladder to be flat on the floor, find

(i) the initial value, in radians, of θ, [2]

(ii) the time taken for $\frac{d\theta}{dt}$ to reach a value of -0.0385. [5]

End of Paper

Answers for 2014 Prelim Sec 4E A Math Paper 1

1. $k < 3$ or $k > 11$

2. $x = 0.693$

3. $x^2 - \frac{1}{2}\sqrt{17}x + 2 = 0$

4. $k = 2$ or -2

5. (a) $141 - 44\sqrt{5}$
   (b) $x = 2$

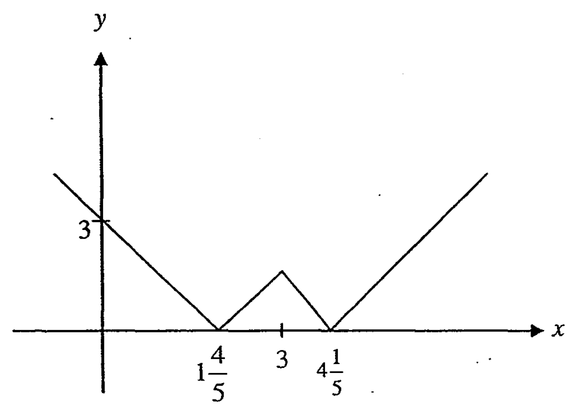
6. (a) $q = \frac{1}{3}, p = 4$
   (b) $x = 0.745, \ -0.745$

8. (a) C is $(-1, 5.5)$
   (b) D is $(2, -5\frac{1}{2})$
   (c) 19.5 sq units

9. (a) $v = \frac{1}{(t+1)} + 1$
   (b) Show $\frac{1}{(t+1)} + 1 > 0$ for all non negative values of t.
   (c) 6.79 (3sf)

10. (b) $A = 111.8°, 291.8°$
    or 45° or 225°
    (c) 6

11. (a) $a = 2$
    $c = 5$
    $b = -1\frac{2}{3}$
    (b) $x = 1\frac{4}{5}$
    or $x = 4\frac{1}{5}$
    (c)



12. (aii) $\frac{d\theta}{dt} = -0.0929$ rad / s (3sf)
    (bi) 1.23 (3sf)
    (bii) 5.01 secs

Marking Scheme for 2014 Prelim A Math Paper 1

1. $3x - k = 2x^2 + kx - 3$ [M1 for single equation]
$0 = 2x^2 + (k-3)x + k - 3$
For distinct roots, [M1 for showing knowledge that discriminant must be greater than 0]
$b^2 - 4ac > 0$
$(k-3)^2 - 4(2)(k-3) > 0$ [M1 for correct substitution]
$(k-3)(k-11) > 0$ [M1 for factorising]
$k < 3$ or $k > 11$ [A1 for answer]

2. $\frac{dy}{dx} = e^{2x} - e^x - 2$ [M1]
At stat. point,
$e^{2x} - e^x - 2 = 0$ [M1]
$(e^x - 2)(e^x + 1) = 0$ [M1]
$e^x = 2$ or -1 (rej) [A1 for rejecting]
$x = 0.693$ [A1]

3. Sum of roots: $\alpha^2 + \beta^2 = \frac{1}{4}$ [M1]
Product of roots: $\alpha^2\beta^2 = 4$ [M1]
$(\alpha+\beta)^2 = \alpha^2 + 2\alpha\beta + \beta^2$ [M1]
$= \frac{1}{4} + 2(2)$ or $\frac{1}{4} + 2(-2)$ (rej) (cannot use $\alpha\beta = -2$ because $(\alpha+\beta)^2 > 0$)

$\alpha + \beta = \sqrt{\frac{17}{4}}$ [M1]
$\alpha\beta = 2$ [M1]
The equation is
$x^2 - \frac{1}{2}\sqrt{17}x + 2 = 0$ (o.e) [A1]

4. $\left[\frac{5x^2}{4}\right]_a^b = 5$ [M1 for correct integration]
$\frac{5}{4}(b^2 - a^2) = 5$ [M1 for forming equation in a and b]
Also, $\frac{5}{4}\left(k^2b^2 - k^2a^2\right) = 20$
$\frac{5}{4}k^2\left(b^2 - a^2\right) = 20$
$k^2(5) = 20$ [M1 for removing a and b through substitution]
$k = 2$ or -2 [A1]

5. (a) $\left(\frac{8-4\sqrt{5}}{4-5}-3-2\sqrt{5}\right)^2$ [M1 for rationalising denominator]

$=\left(2\sqrt{5}-11\right)^2$ [M1 for correct simplifying]

$=20-44\sqrt{5}+121$

$=141-44\sqrt{5}$ [A1 for 141]

[A1 for $-44\sqrt{5}$]

(b) Seeing 6^x as $2^x \times 3^x$ [M1]

$a=2^x, b=3^x$ [M1]

$\left(2^x-4\right)\left(3^x+1\right)=0$

$2^x=4$ or $3^x=-1$ (reject) [M1]

$x=2$ [A1]

6. (a) $xy^2 = px+qy$

$y^2=p+q\left(\frac{y}{x}\right)$ [M1 for changing to linear form]

Gradient $=\frac{6-4\frac{1}{3}}{6-1}$ [M1]

$q=\frac{1}{3}$ [A1]

Sub (6, 6),

$6=p+\frac{1}{3}(6)$

$p=4$ [A1]

(b) When $\frac{y}{x}=3$,

$y^2=4+\frac{1}{3}(3)$ [M1]

$y=\pm\sqrt{5}$

Sub into original equation to find x:

$x(5)=4x+\frac{1}{3}\left(\sqrt{5}\right)$ or $x(5)=4x+\frac{1}{3}\left(-\sqrt{5}\right)$ [M1]

$x=0.745,\ -0.745$ [A1 for 2 answers]

7. Using Alt Seg Theorem $\angle DAC = \angle ABC$ [M1 for applying alt segthm]
Using Mid Point Theorem $MN \parallel CB$ [M1 for mid pt theorem]
Therefore $\angle DCA = \angle ACB$ ("bisects")
$= \angle AMN$ (corresponding angle)
$\angle DAC = \angle ABC$ (altseg, shown above)
$= \angle ANM$ (corresponding angle)
[M1 for showing 2 angles same]

Since 2 angles are the same, the 3rd angle must be the same, therefore ΔAMN is similar to ΔDCA. [A1 for appropriate conclusion, either through showing this statement or calculating 3rd angle]

8. (a) Coordinate of P is $(2,1)$ [M1]

Gradient of AB is $\frac{2}{3}$

Gradient of l is $-\frac{3}{2}$ [M1]

Equating gradient of PC to l OR Finding equation of l $2y = -3x + 8$

[M1]

When $x = -1$, $y = 5.5$.
C is $(-1, 5.5)$ [A1]

(b) D is $(2, -5\frac{1}{2})$ [B1]

(c) Area $= 2\left(\frac{1}{2}\times 3\times 6.5\right)$ (area of 2 triangles) [M1, or for using "shoelace" method]

$= 19.5$sq units. [A1]

9 (a) $v = \int -\frac{1}{(t+1)^2}\,dt$

$= \frac{1}{(t+1)} + c$ [M1]

When $t = 0$, $v = 2$. [M1]
$2 = 1 + c$
$c = 1$

$v = \frac{1}{(t+1)} + 1$ [A1]

(b) since $t > 0$,

So, $t + 1 > 0$

So, $\frac{1}{(t+1)} > 0$

So, $\frac{1}{(t+1)} + 1 > 0$ for all non negative values of t. [B1]

(c) $s = \int_0^5 \left(\frac{1}{t+1} + 1\right) dt$

$= [\ln(t+1) + t]_0^5$ [M1]

$= \ln 6 + 5 - 0$ [M1]

$= 6.79$ (3sf) [A1]

OR

$s = \int \left(\frac{1}{t+1} + 1\right) dt$

$= \ln(t+1) + t + c$ [M1]

$t = 0, s = 0,$

so $c = 0$

Therefore, $s = \ln(t+1) + t$ [M1]

Since never come to rest,

Dist $= \ln(5+1) + 5$

$= 6.79$ [A1]

10. (a) $\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$ [M1 for knowing to use double angle for tan 2*A*]

$\text{Cot } 2A = \frac{1 - \tan^2 A}{2\tan A}$

$= \frac{1}{2\tan A} - \frac{1}{2}\tan A$ (shown) [A1]

(b) $\tan A\left(3 - 4\left(\frac{1}{2\tan A} - \frac{1}{2}\tan A\right)\right) = 3$ [M1 for applying earlier part]

$3\tan A - 2 + 2\tan^2 A = 3$ [M1]

$(2\tan A + 5)(\tan A - 1) = 0$ [M1]

$A = 111.8°, 291.8°$ or $45°$ or $225°$ [A1 for 2-3 answers]

[A2 for 4 answers]

(c) 6 [B1]

11. (a) $a = 2$ [B1]

$c = 5$ [B1]

$b = -1\frac{2}{3}$ [B1]

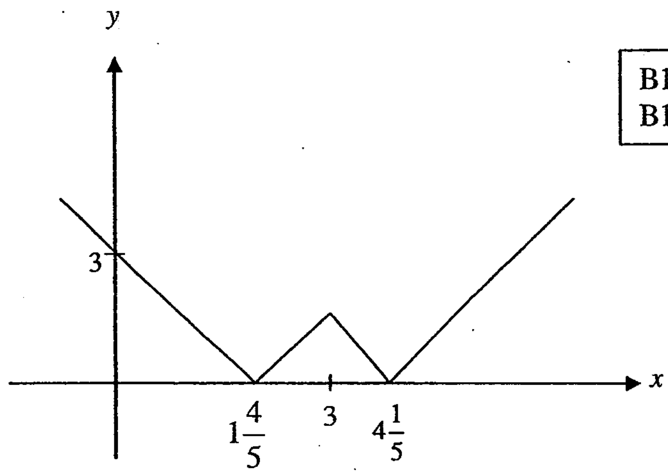
(b) When $y = 0$,

$\left|-\frac{5}{3}x + 5\right| = 2$ [M1]

$-\frac{5}{3}x + 5 = 2$ or $-\frac{5}{3}x + 5 = -2$ [M1]

$x = 1\frac{4}{5}$ or $x = 4\frac{1}{5}$ [A1]

(c)



B1 for "flipping lines"
B1 for correct y intercept value

12. (ai) $\cos\theta = \frac{x}{6}$

$x = 6\cos\ \theta$ [B1, must show previous line]

(aii) $\frac{dx}{d\theta} = -6\sin\theta$ [M1]

$\frac{dx}{dt} = \frac{dx}{d\theta} \times \frac{d\theta}{dt}$

$0.2 = -6\sin 0.367 \times \frac{d\theta}{dt}$ [M1]

$\frac{d\theta}{dt} = -0.0929$ rad / s (3sf) [A1]

(bi) distance moved in 20s = 4 m

Therefore, initial value of $x = 6 - 4 = 2$ m. [M1]

Initial value of $\theta = \cos^{-1}\frac{2}{6}$

$= 1.23$ (3sf) [A1]

(bii) When $\frac{d\theta}{dt} = -0.0385$,

$$0.2 = \frac{dx}{d\theta} \times (-0.0385) \qquad \text{[M1]}$$

$$\frac{dx}{d\theta} = \frac{0.2}{-0.0385}$$

$$-6\sin\theta = \frac{0.2}{-0.0385} \qquad \text{[M1]}$$

$$\sin\theta = \frac{0.2}{6 \times 0.0385}$$

$$\theta = 1.04675 \qquad \text{[M1]}$$

When $\theta = 1.04675$,

$$x = 6\cos 1.04675 \qquad \text{[M1]}$$

$$= 3.00233$$

Distance moved = 1.00233

Time taken = 1.00233 / 0.2

= 5.01 secs [A1]

聖嬰中學

HOLY INNOCENTS' HIGH SCHOOL

PRELIMINARY EXAMINATION 2014
SECONDARY 4 EXPRESS

ADDITIONAL MATHEMATICS
PAPER 2 **4047/02**

Name : ____________________ **Date** : **6 Aug 2014**

Register No : ____________________ **Duration :2h30 min**

Class : ____________________

Additional Materials needed
Writing Paper (8 sheets)

Instructions to Candidates

Write your register number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.

If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
Calculators should be used where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer correct to three significant figures. Give answers in degrees correct to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total marks for this paper is 100.

Setter: MsKohSwee Kun
Vetters: Mdm Noor Hayati
Mrs Nathan

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots\ldots\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

$$a^2 = b^2 + c^2 - 2bc \cos A.$$

$$\Delta = \frac{1}{2}bc \sin A.$$

Answer all the questions.

1 **(i)** Sketch the graph of $y = 5x^{\frac{1}{2}}$ for $x > 0$. [1]

(ii) On the same diagram, sketch the graph of $y = \frac{1}{5}x^{-\frac{3}{2}}$ for $x > 0$. [1]

(iii) Calculate the coordinates of the point of intersection of your graphs. [2]

(iv) Determine, with explanation, whether the tangents to the graphs at the point of intersection are perpendicular. [4]

2 **(a)** **(i)** Find the first four terms of the expansion, in ascending powers of x, of $(3+x)^5$. [2]

Henceobtain

(ii) the coefficient of x^3 in the expansion of $(3+x)^5(2x^2+x-1)$, [2]

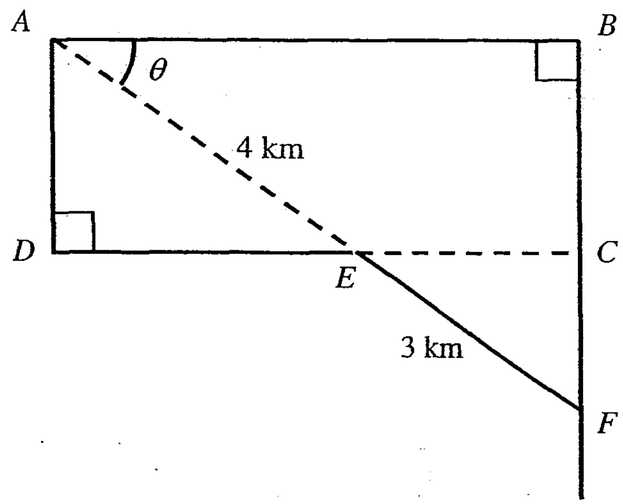
(iii) the coefficient of y^3 in the expansion of $(3-y+y^2)^5$. [2]

(b) Determine if the term independent of x exists in the expansion of $(x^2-2x)^{100}$. [2]

3 **(i)** Differentiate $3x\cos x$ with respect to x. [2]

(ii) Using your answer to part **(i)**, find $\int 2x\sin x \ dx$ and **hence**evaluate $\int_0^{\frac{\pi}{2}} 2x\sin x \ dx$. [5]

4



The diagram shows a Long Distance Race Route *FBADE*.
ABCD is a rectangular plot of land. A line through *A*, at an angle θ to *AB* intersects *DC* at *E* and *BC* produced at *F*.
During Holy Innocents' High School Long Distance Race, runners will start from point *F* and run along the straight paths *FB*, *BA*, *AD*, *DE* and return along the path *EF* to finish at *F*.
$AE = 4$ km and $EF = 3$ km.
The total length of the race is L km.

(i) Show that L can be expressed as $a\sin\theta + b\cos\theta + c$, where a, b and c are constants to be found. [3]

(ii) Express L in the form $R\cos(\theta - \alpha) + c$, where $R > 0$ and α is an acute angle. [3]

The total length of the race is found to be 18 km.

(iii) Find the value of θ. [2]

5 A curve has the equation $y = \text{f}(x)$, where $\text{f}(x) = \dfrac{x-2}{2x+1}$ for $x > 0$.

(i) Obtain an expression for $\text{f}'(x)$. [2]

(ii) Find the equation of the normal to the curve at the point where the curve crosses the x-axis. [4]

(iii) Determine, with explanation, whether f is an increasing or decreasing function. [1]

(iv) Showing full working, determine whether the gradient of the curve is an increasing or decreasing function. [3]

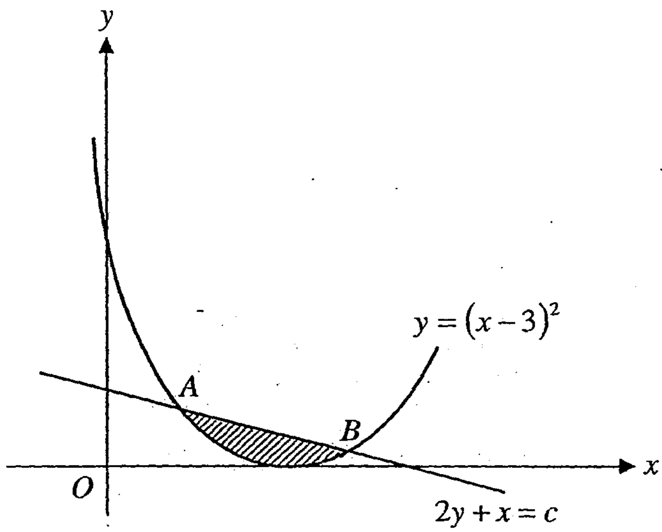
6 Given that $\dfrac{ax^3+x^2-14x-12}{x^2-9}=2x+b+\dfrac{4x+c}{x^2-9}$,

(i) find the value of each of the integers a, b and c. [4]

Hence, using partial fractions and the values of a, b and c obtained in part **(i)**, find

(ii) $\displaystyle\int \frac{ax^3+x^2-14x-12}{x^2-9}\,\mathrm{d}x$. [6]

7



The diagram shows part of the curve $y=(x-3)^2$ intersecting the line $2y+x=c$, where c is a constant, at points A and B.

The tangent at B is perpendicular to the line $2y+x=c$.

(i) Find the x-coordinate of B. [3]

(ii) **Hence** show that $c=6$. [2]

(iii) Using your answer to part **(ii)**, find the area of the shaded region. [7]

8 **(a)** **(i)** Prove the identity $\tan 2A(2\cos A - \sec A) = 2\sin A$. [4]

(ii) **Hence** solve the equation $\tan 2A(2\cos A - \sec A) = 1$, for $0 < x \leq \pi$, giving your answers in terms of π. [3]

(b) **(i)** Given that P is acute and $\tan P = \frac{4}{3}$, without using a calculator, find the value of $\sin P$ and of $\cos P$. [2]

(ii) Using the values of sin P and cos P found in part **(i)**, show that $3\cos(Q+P) + 4\sin(Q+P) = 5\cos Q$. [3]

9 A circle, centre C, passes through the points $P(1, -2)$, $Q(9, -2)$ and $R(5, 6)$.

(i) Show that the coordinates of C is (5, 1). [6]

(ii) Find the radius of the circle. [2]

(iii) Find the equation of the circle in the form $x^2 + y^2 + fx + gy + h = 0$, where f, g and h are integers. [2]

(iv) Does the point (10, 4) lie outside, inside or on the circle? Justify your answer. [2]

10 **(a)** Given that $\log_x(p^2q^3) = m$ and $\log_x(pq^2) = n$, find $\log_x \sqrt{pq}$ in terms of m and n. [3

(b) Solve the equations

(i) $3^{x+1} = 5^x$, [2

(ii) $\log_3(4x) + \log_3(x-1) = 1$. [4

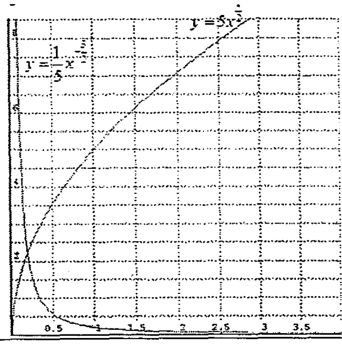
(c) The population of a village at the beginning of the year 2004 was 350. The populatio increased so that, after a period of n years, the new population is given by $350(1.08)^n$.

Find

(i) the population at the beginning of the year 2014, [

(ii) the year in which the population first reached 3680. [

End of Paper

Question	Answer		Question	Answer
1(i) & (ii)			5(iv)	Since $(2x+1)^3>0$ for $x>0$, then $f''(x)=\frac{-20}{(2x+1)^3}<0\Rightarrow$ Gradient is an decreasing function.
1(iii)	(0.2, 2.24)		6(i)	$a=2$, $b=1$ and $c=-3$
1(iv)	Since product of gradients $=5.59\times-16.77$ $\neq -1$, therefore the tangents at the point of intersection are not perpendicular.		6(ii)	$x^2+x+\frac{5}{2}\ln(x+3)$ $+\frac{3}{2}\ln(x-3)+c$
2(a)(i)	$243+405x+270x^2+90x^3$		7(i)	4
2(a)(ii)	990		7(ii)	Sub $B(4, 1)$ into the line equation $2y+x=c$ to show $c=6$.
2(a)(iii)	–630		7(iii)	$2\frac{29}{48}$ or 2.60
2(b)	The term independent of x does not exist as there should only be 101 terms in the expansion of $(x^2-2x)^{100}$ and so it is not possible to obtain any term when $r=200$ is substituted.		8(a)(i)	Refer to next page
3(i)	$3\cos x-3x\sin x$		8(a)(ii)	$A=\frac{\pi}{6},\ \frac{5\pi}{6}$
3(ii)	$\int 2x\sin x\,dx=2\sin x-2x\cos x+c$; 2		8(b)(i)	$\sin P=\frac{4}{5}$, $\cos P=\frac{3}{5}$
4(i)	L $=FB+BA+AD+DE+EF$ $=11\sin\theta+11\cos\theta+3$		8(b)(ii)	Refer to next page
4(ii)	$L=15.6\cos(\theta-45°)+3$		9(i)	Refer to next page
4(iii)	$\theta=60.4°$		9(ii)	5 units
5(i)	$\frac{5}{(2x+1)^2}$		9(iii)	$x^2-10x+y^2-2y+1$ $=0$
5(ii)	$y=10-5x$		9(iv)	Outside the circle
5(iii)	Since $(2x+1)^2>0$ for $x>0$, then $f'(x)=\frac{5}{(2x+1)^2}>0\Rightarrow f(x)$ is an increasing function.		10(a)	$\log_x\sqrt{pq}=\frac{m-n}{2}$

Question	Answer
10(b)(i)	2.15
10(b)(ii)	$x=-\frac{1}{2}$ (rej) or $1\frac{1}{2}$
10(c)(i)	756
10(c)(ii)	Year 2034

8(a)(i)

$$\tan 2A(2\cos A-\sec A)$$
$$=\tan 2A\left(2\cos A-\frac{1}{\cos A}\right)$$
$$=\tan 2A\left(\frac{2\cos^2 A-1}{\cos A}\right)$$
$$=\tan 2A\left(\frac{\cos 2A}{\cos A}\right)$$
$$=\frac{\sin 2A}{\cos 2A}\left(\frac{\cos 2A}{\cos A}\right)$$
$$=\frac{\sin 2A}{\cos A}$$
$$=\frac{2\sin A\cos A}{\cos A}$$
$$=2\sin A \text{ (proven)}$$

8(b)(ii)

$$3\cos(Q+P)+4\sin(Q+P)$$
$$=3\cos Q\cos P-3\sin Q\sin P+4\sin Q\cos P+4\cos Q\sin P$$
$$=3\bullet\frac{3}{5}\cos Q-3\bullet\frac{4}{5}\sin Q+4\bullet\frac{3}{5}\sin Q+4\bullet\frac{4}{5}\cos Q$$
$$=\frac{9}{5}\cos Q-\frac{12}{5}\sin Q+\frac{12}{5}\sin Q+\frac{16}{5}\cos Q$$
$$=\frac{25}{5}\cos Q$$
$$=5\cos Q \text{ (shown)}$$

9(i)

Method to solve:

Method 1 – Using concept of perpendicular bisectors of the sides of triangle inscribed in the circle and their intersection

Method 2 – Using concept of radius, solve simultaneous equations of $PC=QC=RC$.

聖嬰中學

HOLY INNOCENTS' HIGH SCHOOL

PRELIMINARY EXAMINATION 2014
SECONDARY 4 EXPRESS
MARKING SCHEME

ADDITIONAL MATHEMATICS **4047/02**
PAPER 2

Name : ____________________ **Date** : **06 Aug 2014**

Register No : ____________________ **Duration :2h30 min**

Class : ____________________

Additional Materials needed
Writing Paper (10 sheets)

Instructions to Candidates

Write your register number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.

If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
Calculators should be used where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer correct to three significant figures. Give answers in degrees correct to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total marks for this paper is 100.

Setter: MsKohSwee Kun
Vetter: Mdm Noor Hayati

This paper consists of 19 printed pages, inclusive of this cover page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots\ldots\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

$$a^2 = b^2 + c^2 - 2bc \cos A.$$

$$\Delta = \frac{1}{2} bc \sin A.$$

Answer all the questions.

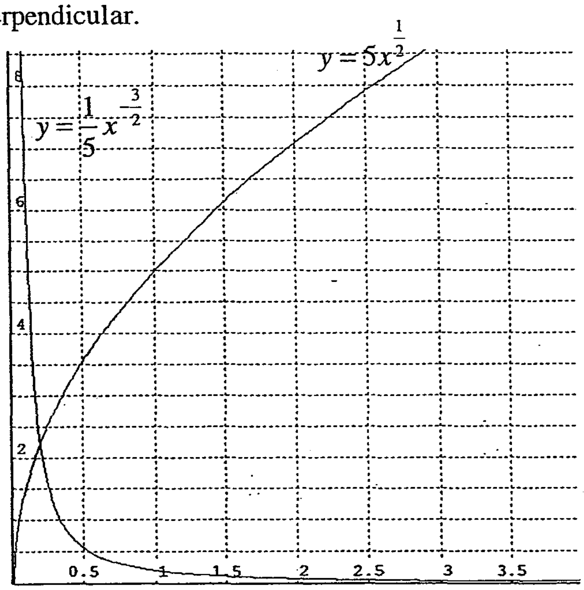
1 (i) Sketch the graph of $y = 5x^{\frac{1}{2}}$ for $x > 0$. [1]

(ii) On the same diagram, sketch the graph of $y = \frac{1}{5}x^{-\frac{3}{2}}$ for $x > 0$. [1]

(iii) Calculate the coordinates of the point of intersection of your graphs. [2]

(iv) Determine, with explanation, whether the tangents to the graphs at the point of intersection are perpendicular. [4]

Solution:
(i) & (ii)



(iii)

$5x^{\frac{1}{2}} = \frac{1}{5}x^{-\frac{3}{2}}$

$25x^2 = 1$ --- M1

$x = 0.2$

Point of intersection (0.2, 2.24) --- A1 (o.e)

(iv)

$\frac{d\left(5x^{\frac{1}{2}}\right)}{dx} = \frac{5}{2}x^{-\frac{1}{2}} = \frac{5}{2\sqrt{x}}$ --- M1 (Differentiation)

At $x = 0.2$, gradient of tangent $= \frac{5}{2\sqrt{0.2}} = 5.59$

$\frac{d\left(\frac{1}{5}x^{-\frac{3}{2}}\right)}{dx} = -\frac{3}{10}x^{-\frac{5}{2}} = -\frac{3}{10x^{\frac{5}{2}}} = -\frac{3}{10(0.2)^{\frac{5}{2}}} = -16.77$ --- M1 (Differentiation)

Since product of gradients $= 5.59 \times -16.77 \neq -1$, therefore the tangents at the point of intersection are not perpendicular. --- M1, A1 (M1 for value of gradients)

2 (a) (i) Find the first four terms of the expansion, in ascending powers of x, of $(3+x)^5$. [2]

Hence obtain

(ii) the coefficient of x^3 in the expansion of $(3+x)^5(2x^2+x-1)$, [2]

(iii) the coefficient of y^3 in the expansion of $(3-y+y^2)^5$. [2]

(b) Determine if the term independent of x exists in the expansion of $(x^2-2x)^{100}$. [2]

Solution:

(a)(i)

$(3+x)^5$

$=3^5+\binom{5}{1}(3^4)x+\binom{5}{2}(3^3)x^2+\binom{5}{3}(3^2)x^3$

$=243+405x+270x^2+90x^3$ --- B2 (B1 for any 2 correct terms)

(a)(ii)

$(3+x)^5(2x^2+x-1)$

$=(243+405x+270x^2+90x^3+...)(2x^2+x-1)$

Terms in x^3

$=405\times 2x^3+270x^3-90x^3$ --- M1 (Expanding the terms with x^3)

$=990x^3$

Coefficient of $x^3=990$ --- A1

(a)(iii)

$(3-y+y^2)^5$

$=(3+y^2-y)^5$

$=243+405(y^2-y)+270(y^2-y)^2+90(y^2-y)^3$

$=243+405y^2-405y+270(y^4-2y^3+y^2)+90(y^2-y)(y^4-2y^3+y^2)$

$=243+405y^2-405y+270y^4-540y^3+270y^2-90y^3+....$

--- M1 (Replacing x with y^2-y and expanding to find terms that include y^3)

Terms in y^3

$=-540y^3-90y^3$

$=-630y^3$

The coefficient of y^3 is -630. --- A1

(b)

General term of the expansion of $(x^2-2x)^{100}$, $T_{r+1}=\binom{100}{r}(x^2)^{100-r}(-2x)^r$.

Term independent of $x \Rightarrow$ power of $x=0$

Therefore,
$x^{200-2r+r}=x^0$
$200-r=0$
$r=200$

--- M1 (Show that $r=200$ if term independent of x exists)

The term independent of x does not exist as there should only be 101 terms in the expansion of $(x^2-2x)^{100}$ and so it is not possible to obtain any term when $r=200$ is substituted.

--- A1 (Explain that it is not possible for term independent of x to exist by quoting that there are only 101 terms in the expansion)

3 **(i)** Differentiate $3x\cos x$ with respect to x. [2]

(ii) Using your answer to part **(i)**, find $\int 2x\sin x \; dx$ and **hence** evaluate $\int_0^{\frac{\pi}{2}} 2x\sin x \; dx$. [5]

Solution:

(i)

$$\frac{d(3x\cos x)}{dx}$$
$$= 3\cos x - 3x\sin x$$

--- B1 for $3\cos x$ and B1 for $-3x\sin x$

(ii)

$$\int (3\cos x - 3x\sin x)\,dx = 3x\cos x + c_1$$ --- M1 (Integration as reverse of differentiation)

$$\int 3\cos x\,dx - \int 3x\sin x\,dx = 3x\cos x + c_1$$

$$3\int \cos x\,dx - \frac{3}{2}\int 2x\sin x\,dx = 3x\cos x + c_1$$

$$3\sin x - \frac{3}{2}\int 2x\sin x\,dx = 3x\cos x + c_2$$ --- M1 (Differentiating 3 cos x)

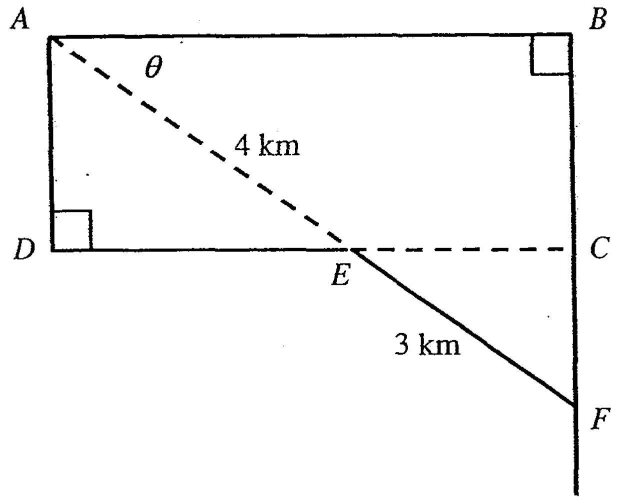
$$\frac{3}{2}\int 2x\sin x\,dx = 3\sin x - 3x\cos x + c_2$$

$$\int 2x\sin x\,dx = \frac{2(3\sin x - 3x\cos x)}{3} + c$$

$$\int 2x\sin x\,dx = 2\sin x - 2x\cos x + c$$ --- A1 (no marks if no constant c is seen)

$$\int_0^{\frac{\pi}{2}} 2x\sin x\;\mathrm{d}x$$
$$= [2\sin x - 2x\cos x + c]_0^{\frac{\pi}{2}}$$
$$= \left[2\sin\frac{\pi}{2} - 2\left(\frac{\pi}{2}\right)\cos\frac{\pi}{2} + c\right] - [2\sin 0 - 2(0)\cos 0 + c]$$ --- M1
$$= 2(1) - 0 + c - (0 + c)$$
$$= 2$$ --- A1

4



The diagram shows a Long Distance Race Route *FBADE*.
ABCD is a rectangular plot of land. A line through *A*, at an angle θ to *AB* intersects *DC* at *E* and *BC* produced at *F*.
During Holy Innocents' High School Long Distance Race, runners will start from point *F* and run along the straight paths *FB*, *BA*, *AD*, *DE* and return along the path *EF* to finish at *F*.
$AE = 4$ km and $EF = 3$ km.
The total length of the race is *L* km.

(i) Show that *L* can be expressed as $a\sin\theta + b\cos\theta + c$, where *a*, *b* and *c* are constants to be found. [3]

(ii) Express *L* in the form $R\cos(\theta - \alpha) + c$, where $R > 0$ and α is an acute angle. [3]

The total length of the race is found to be 18 km.

(iii) Find the value of θ. [2]

Solution:
(i)
$AB = 7\cos\theta$
$DE = 4\cos\theta$
$AD = BC = 4\sin\theta$
Since *ABCD* is a rectangle, *AB* is parallel to *DC*. Therefore, angle $CEF = \theta$ (corresponding angle)
$CF = 3\sin\theta$ --- M1

L
$= FB + BA + AD + DE + EF$
$= 3\sin\theta + 4\sin\theta + 7\cos\theta + 4\sin\theta + 4\cos\theta + 3$ --- M1
$= 11\sin\theta + 11\cos\theta + 3$ --- A1

(ii)
$L = 11\sin\theta + 11\cos\theta + 3$
$L = 11\cos\theta + 11\sin\theta + 3$

$R = \sqrt{11^2 + 11^2} = \sqrt{242} = 11\sqrt{2} = 15.556$ --- M1

$\alpha = \tan^{-1}\left(\frac{11}{11}\right) = 45°$ --- M1

Therefore, $L = 15.6\cos(\theta - 45°) + 3$ --- A1

(iii)

$15.556\cos(\theta - 45°) + 3 = 18$

$\cos(\theta - 45°) = 0.96425$

$(\theta - 45°) = \cos^{-1} 0.96425$ --- M1

$\theta = 15.366 + 45$

$\theta = 60.4°$ --- A1

5 A curve has the equation $y = f(x)$, where $f(x) = \dfrac{x-2}{2x+1}$ for $x > 0$.

(i) Obtain an expression for $f'(x)$. [2]

(ii) Find the equation of the normal to the curve at the point where the curve crosses the x-axis. [4]

(iii) Determine, with explanation, whether f is an increasing or decreasing function. [1]

(iv) Showing full working, determine whether the gradient of the curve is an increasing or decreasing function. [3]

Solution:

(i)

$f'(x) = \dfrac{2x+1-2(x-2)}{(2x+1)^2}$ --- M1 (Applying Quotient Rule)

$= \dfrac{5}{(2x+1)^2}$ --- A1

(ii)

Curve crosses x-axis, y = 0.

Therefore, $\dfrac{x-2}{2x+1} = 0$

$x = 2$

At $x = 2$, gradient of tangent = $\dfrac{5}{(2(2)+1)^2} = \dfrac{1}{5}$ --- M1 (gradient of tangent)

Therefore, gradient of normal = –5 --- M1 (gradient of normal)

Equation of normal:

$y - 0 = -5(x-2)$ --- M1 (Form line equation)

$y = 10 - 5x$ --- A1 (o.e.)

(iii)

$f'(x) = \dfrac{5}{(2x+1)^2}$

Since $(2x+1)^2 > 0$ for $x > 0$, then $f'(x) = \dfrac{5}{(2x+1)^2} > 0 \Rightarrow f(x)$ is an increasing function.

--- B1

(iv)

$f''(x) = 5 \times -2 \times 2 \times (2x+1)^{-3}$ --- M1 (Applying Quotient Rule)

$= -\dfrac{20}{(2x+1)^3}$ --- M1

Since $(2x+1)^3 > 0$ for $x > 0$, then $f''(x) = \dfrac{-20}{(2x+1)^3} < 0 \Rightarrow$ Gradient is an decreasing function. --- B1

6 Given that $\dfrac{ax^3+x^2-14x-12}{x^2-9}=2x+b+\dfrac{4x+c}{x^2-9}$,

(i) find the value of each of the integers a, b and c. [4]

Hence, using partial fractions and the values of a, b and c obtained in part **(i)**, find

(ii) $\displaystyle\int\frac{ax^3+x^2-14x-12}{x^2-9}\,dx$. [6]

Solution:

(i)

$$\frac{ax^3+x^2-14x-12}{x^2-9}=2x+b+\frac{4x+c}{x^2-9}$$

$ax^3+x^2-14x-12=(2x+b)(x^2-9)+4x+c$ --- M1

By comparing coefficient of x^3,

$a=2$ --- A1

By comparing coefficient of x^2,

$b=1$ --- A1

By comparing constant term,

$-12=-9b+c$

$-12=-9(1)+c$

$c=-3$ --- A1

(ii)

$$\frac{2x^3+x^2-14x-12}{x^2-9}=2x+1+\frac{4x-3}{x^2-9}$$

$$\frac{4x-3}{x^2-9}=\frac{4x-3}{(x+3)(x-3)}$$

$\dfrac{4x-3}{(x+3)(x-3)}=\dfrac{A}{x+3}+\dfrac{B}{x-3}$ --- M1 (Break into partial fractions)

$4x-3=A(x-3)+B(x+3)$

Comparing coefficient of x,

$4=A+B$ --- (1)

Comparing constant terms,

$-3=-3A+3B$

$1=A-B$ --- (2)

(1) + (2):

$5=2A$ --- M1 (Solving simultaneous equations)

$A = \frac{5}{2}$ --- A1

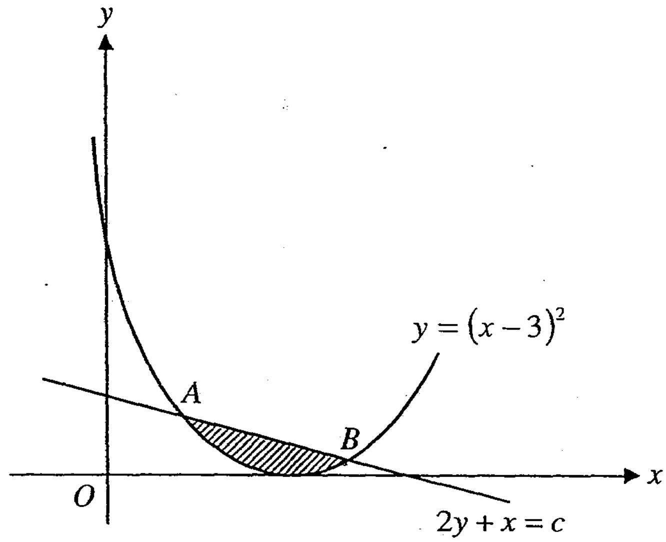
Substitute $A = \frac{5}{2}$ into (1),

$B = 4 - \frac{5}{2} = \frac{3}{2}$ --- A1

$$\int \frac{ax^3 + x^2 - 14x - 12}{x^2 - 9}\,\mathrm{d}x$$
$$= \int \frac{2x^3 + x^2 - 14x - 12}{x^2 - 9}\,\mathrm{d}x$$
$$= \int 2x + 1 + \frac{5}{2(x+3)} + \frac{3}{2(x-3)}\,\mathrm{d}x$$
$$= x^2 + x + \frac{5}{2}\ln(x+3) + \frac{3}{2}\ln(x-3) + c$$

--- B2 (minus 1 mark for each error)

7



The diagram shows part of the curve $y=(x-3)^2$ intersecting the line $2y +$

constant, at points A and B.

The tangent at B is perpendicular to the line $2y + x = c$.

(i) Find the x-coordinate of B.

(ii) **Hence** show that $c = 6$.

(iii) Using your answer to part **(ii)**, find the area of the shaded region.

(iii) Using **(ii)**, find the area of the shaded region.

Solution:

(i)

$$\frac{d(x-3)^2}{dx}=2(x-3) \qquad \text{--- M1}$$

$2y+x=c$

$$y=-\frac{1}{2}x+\frac{1}{2}c$$

Therefore, gradient of tangent at $B=2$ --- M1

$2(x-3)=2$

$x-3=1$

$x=4$

The x-coordinate of B is 4. --- A1

(ii)

Sub $x=4$ into $y=(x-3)^2$:

$y=(4-3)^2=1$ --- M1

Since the point B also lies on the line $2y+x=c$, it satisfies the line equation.

Sub $B(4, 1)$ into the line equation $2y+x=c$:

$2(1)+4=c$

$c=6$ (shown)

--- A1 (Obtain $c=6$ by substituting the coordinates of B into the line equation)

(iii)

$y=(x-3)^2$ --- (1)

$2y+x=6$ --- (2)

Sub (1) into (2):

$2(x-3)^2+x=6$ --- M1 (Solving simultaneous equations)

$2(x^2-6x+9)+x-6=0$

$2x^2-11x+12=0$

$(2x-3)(x-4)=0$

$x=1\frac{1}{2}$ or 4 --- M1

When $x=1\frac{1}{2}$,

$2y+1\frac{1}{2}=6$

$2y=4\frac{1}{2}$

$y=2\frac{1}{4}$ --- M1

Area of shaded region

= Area of trapezium – Area under the curve

$=\left[\frac{1}{2}\times\left(1+2\frac{1}{4}\right)\times\left(4-1\frac{1}{2}\right)\right]-\int_{1\frac{1}{2}}^{4}(x-3)^2\ dx$ --- M1 (Area of trapezium)

$=\frac{65}{16}-\left[\frac{(x-3)^3}{3}\right]_{1\frac{1}{2}}^{4}$ --- M1 (Integrating equation of curve)

$=\frac{65}{16}-\left[\frac{1}{3}-\left(-\frac{9}{8}\right)\right]$ --- M1 (Finding area using definite integral)

$= 2\frac{29}{48}\text{units}^2$ or 2.60 units^2 --- A1

8 **(a)** **(i)** Prove the identity $\tan 2A(2\cos A - \sec A) = 2\sin A$.

(ii) **Hence** solve the equation $\tan 2A(2\cos A - \sec A) = 1$, for $0 < x \le \pi$, answers in terms of π.

(b) **(i)** Given that P is acute and $\tan P = \frac{4}{3}$, without using a calculator, find $\sin P$ and of $\cos P$.]

(ii) Using the values of $\sin P$ and $\cos P$ found in part **(i)**, show that $3\cos(Q+P) + 4\sin(Q+P) = 5\cos Q$.]

Solution:

(a)(i)

$\tan 2A(2\cos A - \sec A)$

$= \tan 2A\left(2\cos A - \frac{1}{\cos A}\right)$ --- M1 ($\sec A = \frac{1}{\cos A}$)

$= \tan 2A\left(\frac{2\cos^2 A - 1}{\cos A}\right)$

$= \tan 2A\left(\frac{\cos 2A}{\cos A}\right)$ --- M1 ($2\cos^2 A - 1 = \cos 2A$)

$= \frac{\sin 2A}{\cos 2A}\left(\frac{\cos 2A}{\cos A}\right)$

$= \frac{\sin 2A}{\cos A}$

$= \frac{2\sin A\cos A}{\cos A}$ --- M1 ($\sin 2A = 2\sin A\cos A$)

$= 2\sin A$ (proven) --- A1

(a)(ii)

$\tan 2A(2\cos A - \sec A) = 1$

$2\sin A = 1$

$\sin A = \frac{1}{2}$ --- M1

$A = \frac{\pi}{6}, \frac{5\pi}{6}$ --- A2 (A1 for each correct answer)

(b)(i)

By Pythagoras' Theorem, hypotenuse = 5.

$\sin P = \frac{4}{5}$ --- B1

$\cos P = \frac{3}{5}$ --- B1

(b)(ii)

$3\cos(Q+P)+4\sin(Q+P)$

$= 3\cos Q\cos P - 3\sin Q\sin P + 4\sin Q\cos P + 4\cos Q\sin P$ --- M2 (Expansion)

$= 3\bullet\frac{3}{5}\cos Q - 3\bullet\frac{4}{5}\sin Q + 4\bullet\frac{3}{5}\sin Q + 4\bullet\frac{4}{5}\cos Q$

$= \frac{9}{5}\cos Q - \frac{12}{5}\sin Q + \frac{12}{5}\sin Q + \frac{16}{5}\cos Q$

$= \frac{25}{5}\cos Q$

$= 5\cos Q$ (shown) --- A1

9 A circle, centre C, passes through the points $P(1, -2)$, $Q(9, -2)$ and $R(5, 6)$.

(i) Show that the coordinates of C is $(5, 1)$. [6]

(ii) Find the radius of the circle. [2]

(iii) Find the equation of the circle in the form $x^2 + y^2 + fx + gy + h = 0$, where f, g and h are integers. [2]

(iv) Does the point $(10, 4)$ lie outside, inside or on the circle? Justify your answer. [2]

Solution:
(i)

As the circle passes through all 3 points, the intersection of the perpendicular bisectors of PR and PQ is the centre as P, Q and R to the intersection of the perpendicular bisectors have equal length which is the radius (Concept of perpendicular bisector)

Midpoint of PQ

$= \left(\frac{1+9}{2}, -2\right)$

$= (5, -2)$

Equation of the perpendicular bisector of PQ : $x = 5$

--- M1 (or able to see R is above midpoint of PQ and conclude that x-coordinate of centre is 5)

Gradient of perpendicular bisector of $PR = -\frac{1}{2}$ --- M1

Midpoint of PR

$= \left(\frac{1+5}{2}, \frac{-2+6}{2}\right)$ --- M1

$= (3, 2)$

$\frac{y-2}{x-3} = -\frac{1}{2}$ --- M1 (Forming line equation)

$2(y-2) = 3 - x$

$2y - 4 = 3 - x$

$2y = 7 - x$

Equation of the perpendicular bisector of PR : $2y = 7 - x$ ---M1

Sub $x = 5$ into $2y = 7 - x$,

$2y = 7 - 5$

$y = 1$ --- A1

Therefore, centre of the circle is $(5, 1)$

(**Alternative method:** Using concept on radius. Form equations where $PC = RC = QC$ and solve simultaneous equations)

(ii)

Radius (From P to centre of circle)

$= \sqrt{(1-5)^2 + (-2-1)^2}$ --- M1

$= 5$ units --- A1

(**Alternative method:** Use of diagram, since R is directly on top of centre, therefore $RC = 5$ units)

(iii)

Equation of circle, C:

$(x-5)^2 + (y-1)^2 = 25$ --- M1 (or for finding f, g and h)

$x^2 - 10x + y^2 - 2y + 1 = 0$ --- A1

(iv)

Length from the point (10, 4) to centre of circle

$= \sqrt{(10-5)^2 + (4-1)^2}$

--- M1 (Find length from point to centre)

$= \sqrt{25+9}$

$= \sqrt{34}$

$= 5.83$ units

Since 5.83 > radius of the circle, this implies that the point (10, 4) lies outside the circle.

--- A1

(*Do not accept sketching of diagram*)

10 **(a)** Given that $\log_x(p^2q^3)=m$ and $\log_x(pq^2)=n$, find $\log_x\sqrt{pq}$ in terms of m and n. [3]

(b) Solve the equations

(i) $3^{x+1}=5^x$, [2]

(ii) $\log_3(4x)+\log_3(x-1)=1$. [4]

(c) The population of a village at the beginning of the year 2004 was 350. The population increased so that, after a period of n years, the new population is given by $350(1.08)^n$.

Find

(i) the population at the beginning of the year 2014, [2]

(ii) the year in which the population first reached 3680. [2]

Solution:

(a)

$\log_x\sqrt{pq}=\log_x(pq)^{\frac{1}{2}}=\frac{1}{2}\log_x(pq)$

$\log_x(p^2q^3)-\log_x(pq^2)=m-n$ --- M1

$\log_x\frac{(p^2q^3)}{(pq^2)}=m-n$

$\log_x(pq)=m-n$ --- M1

$\frac{1}{2}\log_x(pq)=\frac{m-n}{2}$

$\log_x\sqrt{pq}=\frac{m-n}{2}$ --- A1

(b)(i)

$3^{x+1}=5^x$

$\lg 3^{x+1}=\lg 5^x$

$(x+1)\lg 3=x\lg 5$ --- M1

$x\lg 3+\lg 3=x\lg 5$

$x(\lg 5-\lg 3)=\lg 3$

$x=\frac{\lg 3}{\lg 5-\lg 3}=2.15$ --- A1

Alternative Method:

$3^{x+1} = 5^x$

$3^x \bullet 3 = 5^x$

$3 = \left(\frac{5}{3}\right)^x$ --- M1

$x \lg\left(\frac{5}{3}\right) = \lg 3$

$x = 2.15$ --- A1

(b)(ii)

$\log_3(4x) + \log_3(x-1) = 1$

$\log_3[4x(x-1)] = 1$ --- M1 (Combining terms using law of log)

$4x(x-1) = 3$

$4x^2 - 4x - 3 = 0$ --- M1 (Obtaining quadratic equation)

$(2x+1)(2x-3) = 0$

$x = -\frac{1}{2}$ (rej) or $1\frac{1}{2}$ --- A1 for rejecting negative value, A1 for $1\frac{1}{2}$

(c)(i)

Population at the beginning of 2014

$= 350(1.08)^{10}$ --- M1 (Sub $n = 10$)

$= 755.62$

$= 756$ --- A1

(c)(ii)

$350(1.08)^n = 3680$

$(1.08)^n = \frac{3680}{350}$

$n = \lg\left(\frac{3680}{350}\right) \div \lg 1.08$

$n = 30.6$ --- M1 (Finding n)

Therefore, the population first reached 3680 in the year 2034. --- A1

1 Given that $9^{x-1} \times 2^{2x+2} = 6^{3x}$,

(i) find the exact value of 6^x, [3]

(ii) hence solve the equation $9^{x-1} \times 2^{2x+2} = 6^{3x}$. [2]

2 The gradient function at any point (x, y) on a curve is given by $e^x - 1$. It is given that the curve passes through the point $(1, e)$.

(i) Find the equation of the curve. [3]

(ii) Show that the equation of the tangent to the curve at the point where the curve cuts the y-axis is parallel to the x-axis. [2]

3 The roots of the equation $3x^2 - 6kx - 6k + 22 = 0$ are α and β. If $\alpha^3 + \beta^3 = 24$, find the values of k. [7]

4 Show that the curve $y = x^2 + k^2 + 1 - kx$ is always positive for all real values of x. [3]

5 By expressing $\sin 3x$ as $\sin(2x + x)$, show that $\sin 3x = 3\sin x - 4\sin^3 x$. [3]

Hence

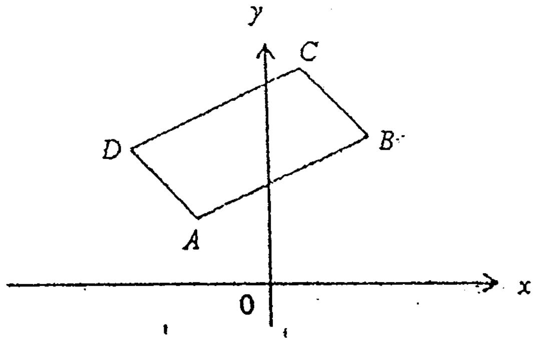
(i) solve the equation $6\sin x = 1 + 8\sin^3 x$ for $-\frac{2\pi}{3} < x < \frac{2\pi}{3}$, [4]

(ii) state the maximum value of $5 + 16\sin^3 x - 12\sin x$. [2]

6 Sketch the graph of $y = |3\sin 2x - 1|$ for $0 < x < \pi$. [2]

Hence find the range of values of c such that the equation $|3\sin 2x - 1| = c$ has 2 distinct real roots. [2]

7 The diagram shows a parallelogram $ABCD$. The coordinates of A, B and D are $(-2,1)$, $(4,3)$ and $(-5,4)$ respectively.



(i) Find the coordinates of C. [3]

(ii) Find the area of the parallelogram $ABCD$. [2]

(iii) Given further that the perpendicular bisector of DC cuts the x-axis at $(h, 0)$, find the value of h. [3]

(iv) Explain why the diagonal AC and BD are not perpendicular to each other. Justify your answer with appropriate workings. [2]

8 It is given that $f'(x) = \dfrac{8}{(2x+1)^n}$, where n is a constant.

(a) Given further that $x \neq -\dfrac{1}{2}$ and $f(1) = 0$, find an expression for $f(x)$ if

(i) $n = 1$, [3]

(ii) $n = 4$. [3]

(b) Write down the range of values of n for which $f(x)$ does not have any stationary points. Support your answer with appropriate workings. [2]

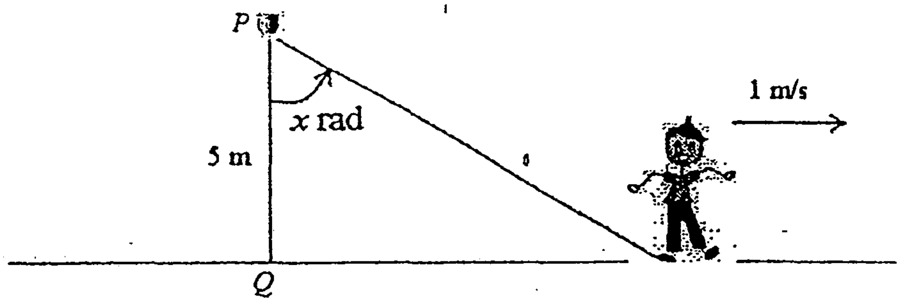
9 Two variables, x and y, are related by an equation $y = k(x-1)^h$, where k and h are constants. The table below shows their experimental values obtained.

x	2.26	3.0	4.0	4.5	5.2	7.0
y	7.94	20.0	45.8	61.3	88.2	180

(i) Express the equation $y = k(x-1)^h$ in a form suitable for drawing a straight line graph. [1]

(ii) Using graph paper, draw this straight line graph. Hence find the value of h and k. [6]

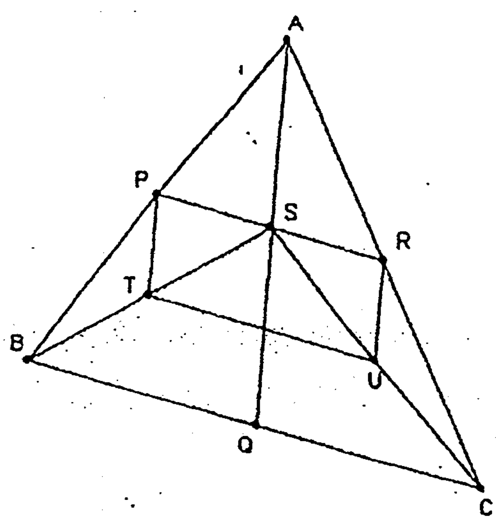
10 The diagram shows a boy walking away from the surveillance camera on a straight path. He is moving at a constant speed of 1 m/s. The surveillance camera is mounted at a point P which is 5 m vertically above a point Q along the path. The camera follows the motion of the boy rotating at P with an angle of x radian.



(i) **Express the distance, s metre, moved by the boy from Q in terms of x.** [1]

(ii) **Hence find the rate of change in the angle of rotation of the surveillance camera when the boy is 7 m from Q.** [4]

11 In triangle ABC, P, Q and R are the midpoints of AB, BC and AC respectively. The lines AQ and PR intersect at S. T and U are the midpoints of BS and SC respectively. Prove that $PRUT$ is a parallelogram. [4]



12(a) Given that $y = x(x+k)^3$ has a stationary point at $(2, 0)$,

(i) find the value of k, [1]

(ii) hence find the other stationary point and determine the nature of all the stationary points. [3]

(b) The table shows the values of x and the sign of $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for a particular function $y = f(x)$. Without finding the function $y = f(x)$, write down all the x-coordinates of the stationary points and determine the nature of each stationary point. [3]

x	0^-	0	0^+	1^-	1	1^+	2^-	2	2^+
Sign of $\frac{dy}{dx}$	+	0	–	–	–	–	–	0	+
Sign of $\frac{d^2y}{dx^2}$	–	–	–	–	0	+	+	+	+

Legend: – denotes negative and + denotes positive

13 The blood pressure P (in mmHg) of a patient can be modelled by the equation $P = 125 - 35\cos\left(\frac{\pi}{14}t\right)$, where t is the time (in minutes) at which the blood pressure is measured.

(i) Explain why this model suggests that the blood pressure, P (in mmHg) of the patient is $90 \le P \le 160$. [2]

(ii) The patient's blood pressure is considered as 'normal' if it is less than 120 mmHg. Find the length of time for which the patient's blood pressure is not 'normal'. [4]

END OF PAPER I

Answer Key

1. (i) $6^x = \frac{4}{9}$ (ii) -0.453 (3 s.f.)

2. (i) $y = e^x - x + 1$

(ii) Since the gradient at $x = 0$ is zero, the equation of tangent is parallel to the x-axis.

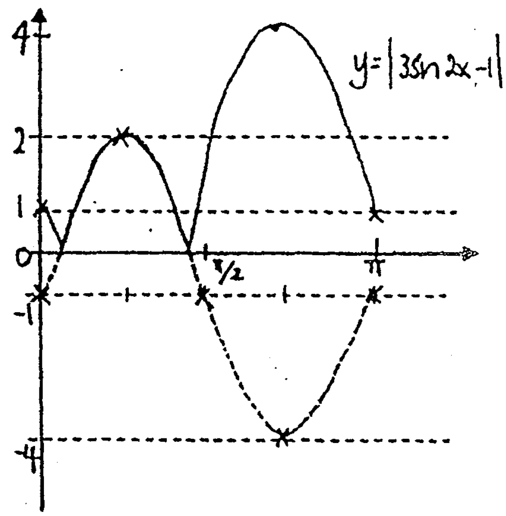
3. $\alpha + \beta = 2k$

$\alpha\beta = \frac{22-6k}{3}$ $k = -\frac{1}{2}, -3, 2$

4. Since $b^2 - 4ac < 0$ and coefficient of x^2 is positive, y does not cuts the x-axis. Hence, the curve is always positive.

5. (i) $x = \frac{\pi}{2}, \frac{5\pi}{2}, -\frac{7\pi}{2}, -\frac{11\pi}{2}$

(ii) 9

6 

$2 < c < 4$ and $c = 0$

7. (i) C = (1,6)

(ii) Area of $ABCD$ = 24 $unit^2$

(iii) $h = x = \frac{1}{3}$

(iv) Since $m_{BD} \times m_{AC} \neq -1$, hence diagonal AC and BD are not perpendicular to each other.

8. (ai) $\therefore f(x) = 4\ln(2x+1) - 4\ln 3$

or $\therefore f(x) = 4\ln\frac{(2x+1)}{3}$

(aii) $f(x) = \frac{4}{81} - \frac{4}{3(2x+1)^3}$

8. (b) Hence $n \geq 0$ for $f'(x)$ to have no stationary points.

9. (i) Plotted $\lg y = \lg k + h\lg(x-1)$

(ii) h = 2 ± 0.2

10 (i) the horizontal distance from Q. $= 5\tan x$

(ii) $\frac{dx}{dt} = 0.0676$ rad/s or $\frac{5}{74} rad/s$

12.	a(i) k= -2 a(ii) $x=2, \frac{1}{2}$ $\left(\frac{1}{2}, -1\frac{11}{16}\right)$ is a minimum point. (2, 0) is a point of inflexion.
12 (b)	$x=0$ and $x=2$ $x=0$ is a maximum point as $\frac{d^2y}{dx^2}\Big\|_{x=0}<0$ or gradient changes from + to –. $x=2$ is a minimum point as $\frac{d^2y}{dx^2}\Big\|_{x=0}>0$ or gradient changes from – to +.
13	.(i) $90 \le P \le 160$ (ii) Length of time = 21.6390 - 6.36098 = 15.3 mins

NCHS 2014 PRELIM EXAM (2) ADDITIONAL MATHEMATICS PAPER 1 (4047/01)

Qn No	Suggested Solutions
1(i)	$9^{x-1}\times 2^{2x+2}=6^{3x}$ $3^{2x-2}\times 2^{2x+2}=6^{3x}$ $3^{2x}\times\frac{1}{9}\times 2^{2x}\times 4=6^{3x}$ $6^{2x}\left(\frac{4}{9}\right)=6^{3x}$ $6^{x}=\frac{4}{9}$
(ii)	$9^{x-1}\times 2^{2x+2}=6^{3x}$ $6^{x}=\frac{4}{9}$ $x=\frac{\lg\frac{4}{9}}{\lg 6}$ $=-0.453$ (3 s.f.)
2(i)	$\frac{dy}{dx}=e^{x}-1$ $y=\int e^{x}-1\,\mathrm{dx}$ $=e^{x}-x+c$ ...... (1) Sub $(1,e)$ into (1), $e=e-1+c$ $c=1$ $y=e^{x}-x+1$ ....(2)
(ii)	When the curve cuts the $y-$axis, sub $x=0$ into (2), $y=e^{0}-0+1$ $=2$ $(0,2)$ At $(0,2)$, $\frac{dy}{dx}=e^{0}-1=0$ Since the gradient at x = 0 is zero, the equation of tangent is parallel to the *x*-axis.
3	$3x^{2}-6kx-6k+22=0$ $\alpha+\beta=2k$ $\alpha\beta=\frac{22-6k}{3}$

	$\alpha^3+\beta^3=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)$ $24=(2k)^3-3\left(\frac{22-6k}{3}\right)(2k)$ $24=8k^3-2k(22-6k)$ $24=8k^3-44k+12k^2$ $2k^3+3k^2-11k-6=0$ Let $f(k)=2k^3+3k^2-11k-6$ $f(2)=2(2)^3+3(2)^2-11(2)-6$ $=0$ $(k-2)$ is a factor. $f(-3)=2(-3)^3+3(-3)^2-11(-3)-6 \quad =0$ $(k+3)$ is a factor. $f(-\frac{1}{2})=2(-\frac{1}{2})^3+3(-\frac{1}{2})^2-11(-\frac{1}{2})-6 \quad =0$ $(2k+1)$ is a factor. $\therefore 2k^3+3k^2-11k-6=0$ $(2k+1)(k+3)(k-2)=0$ $k=-\frac{1}{2},-3,2$
4	$x^2+k^2+1-kx=0$ b^2-4ac $=(-k)^2-4(1)(k^2+1)$ $=-3k^2-4$ $=-(3k^2+4)$ since $(3k^2+4)>0$ $\because -(3k^2+4)<0$ Since since $b^2-4ac<0$ and coefficient of x^2 is positive, y does not cuts the x-axis. Hence, the curve is always positive.
5	$\sin 3x$ $=\sin(2x+x)$ $=\sin 2x\cos x+\cos 2x\sin x$ $=2\sin x\cos^2 x+\sin(1-2\sin^2 x)$ $=2\sin x(1-\sin^2 x)+\sin x-2\sin^3 x$ $=2\sin x-2\sin^3 x+\sin x-2\sin^3 x$ $=3\sin x-4\sin^3 x$ (shown)
5(i)	$6\sin x=1+8\sin^3 x$

NCHS 2014 PRELIM EXAM (2) ADDITIONAL MATHEMATICS PAPER 1 (4047/01)

$$3\sin x - 4\sin^3 x = \frac{1}{2}$$

$$\sin 3x = \frac{1}{2}$$

$$basic\angle = \frac{\pi}{6}$$

$$3x = \frac{\pi}{6}, \frac{5\pi}{6}, -\left(\pi + \frac{\pi}{6}\right), -\left(2\pi - \frac{\pi}{6}\right)$$

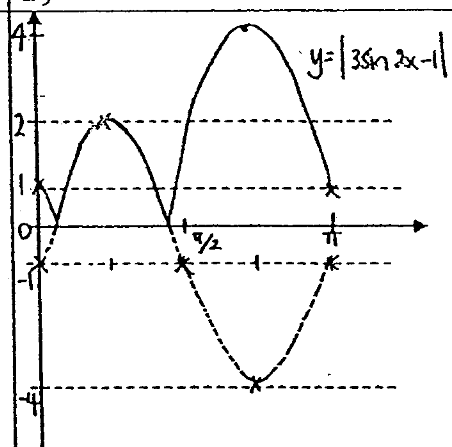
$$x = \frac{\pi}{2}, \frac{5\pi}{2}, -\frac{7\pi}{2}, -\frac{11\pi}{2}$$

5(ii)

$$5 + 16\sin^3 x - 12\sin x$$
$$= 5 - 4(3\sin x - 4\sin^3 x)$$
$$= 5 - 4\sin 3x$$

Maximum value $5 + 16\sin^3 x - 12\sin x$
$= 5 - (-4)$
$= 9$

6



$2 < c < 4$ and $c = 0$

7(i)

Midpoint BD = midpoint AC

$$\left(\frac{4-5}{2}, \frac{3+4}{2}\right) = \left(\frac{x-2}{2}, \frac{y+1}{2}\right)$$

$-1 = x - 2$ $\quad\quad$ $3 + 4 = y + 1$

$x = 1$ $\quad\quad$ $y = 6$

C = (1, 6)

(ii)

Area of $ABCD$

$$= \frac{1}{2}\begin{vmatrix} 1 & -5 & -2 & 4 & 1 \\ 6 & 4 & 1 & 3 & 6 \end{vmatrix}$$

$$= \frac{1}{2}\left[4 - 5 - 6 + 24 - (-30 - 8 + 4 + 3)\right] = 24 \text{ unit}^2$$

NCHS 2014 PRELIM EXAM (2) ADDITIONAL MATHEMATICS PAPER 1 (4047/01)

(iii)	$m_{DC}=\frac{6-4}{1-(-5)}=\frac{1}{3}$ $m_{\perp}=-3$ Midpoint of DC = $\left(\frac{-5+1}{2},\frac{4+6}{2}\right)$ $=(-2,5)$ Equation of perpendicular bisector: $y-5=-3(x+2)$ $y=-3x-1$ Sub $y=0$, $0=-3x-1$ $h=x=-\frac{1}{3}$
(iv)	$m_{DB}=\frac{4-3}{-5-4}=-\frac{1}{9}$ $m_{AC}=\frac{6-1}{1-(-2)}=\frac{5}{3}$ Since $m_{BD}\times m_{AC}\neq-1$, hence diagonal AC and BD are not perpendicular to each other.
8(ai)	*If* $n=1$ $f'(x)=\frac{8}{(2x+1)}$ $f(x)$ $=\int\frac{8}{(2x+1)}dx$ $=4\ln(2x+1)+c$ Sub $f(1)=0$, $0=4\ln3+c$ $c=-4\ln3$ $\therefore f(x)=4\ln(2x+1)-4\ln3$ or $\therefore f(x)=4\ln\frac{(2x+1)}{3}$
8(aii)	*If* $n=4$ $f'(x)=\frac{8}{(2x+1)^4}$ $f(x)$ $=\int 8(2x+1)^{-4}dx$ $=-\frac{4}{3}(2x+1)^{-3}+c$ Sub $f(1)=0$,

	$0=-\frac{4}{3}(3)^{-3}+c$ $c=\frac{4}{81}$ $\therefore f(x)=\frac{4}{81}-\frac{4}{3(2x+1)^3}$
8(b)	At stationary point, $f'(x)=0$ For $\frac{8}{(2x+1)^n}=0$ to be defined, $n<0$. Hence $n\geq 0$ for $f'(x)$ to have no stationary points.
9	(i) Plotted $\lg y=\lg k+h\lg(x-1)$ (ii) G1 – All points plotted correctly; G1 – Join all points with a best straight line and appropriate choice of scale T1- Table $k=5.01\pm 1.4$ $h=2\pm 0.2$
10(i)	Let s be the horizontal distance from Q. $\tan x=\frac{-s}{5}$ $s=5\tan x$
10(ii)	When $s=7$, $7=5\tan x$ $x=0.95055$ $\frac{ds}{dx}=5\sec^2 x$ $\frac{dx}{dt}=\frac{dx}{ds}\times\frac{ds}{dt}$ $=\frac{1}{5\sec^2 x}\times 1$ $=\frac{\cos^2 x}{5}$ Sub $x=0.95055$, $\frac{dx}{dt}=\frac{\cos^2 0.95055}{5}$ $=0.0676$ rad/s or $\frac{5}{74}rad/s$
11	In ΔABC: $AP=PB$ (given) $AR=RC$ (given) $PR=\frac{1}{2}BC$ and $PR // BC$ (Midpoint Theorem) In ΔSBC:

	$ST = BT$ (given) $SU = UC$ (given) $TU = \frac{1}{2}BC$ and $TU // BC$ (Midpoint Theorem) $\therefore PR = TU$ and $PR // TU$ Hence *PRUT* is a parallelogram. (shown)
12(ai)	Sub (2, 0) into $y = x(x+k)^3$, $0 = 2(2+k)^3$ $k = -2$
12(aii)	$y = x(x-2)^3$ $\frac{dy}{dx} = x(3)(x-2)^2 + (x-2)^3$ $= (x-2)^2(4x-2)$ At stationary point, $\frac{dy}{dx} = 0$. $(x-2)^2(4x-2) = 0$ $x-2=0$ or $(4x-2)=0$ $x = 2, \frac{1}{2}$ Sub $x = 0.5$, $y = 0.5(0.5\text{-}2)^3 = -1\frac{11}{16}$ Ans: $\left(\frac{1}{2}, -1\frac{11}{16}\right)$ x: 0.4, 0.5, 0.6; $\frac{dy}{dx}$: -, 0, + $\left(\frac{1}{2}, -1\frac{11}{16}\right)$ is a minimum point. x: 1.9, 2, 2.1; $\frac{dy}{dx}$: +, 0, + (2, 0) is a point of inflexion.
12(b)	$x = 0$ and $x = 2$ $x = 0$ is a maximum point as $\frac{d^2y}{dx^2}\Big\|_{x=0} < 0$ or gradient changes from + to –. $x = 2$ is a minimum point as $\frac{d^2y}{dx^2}\Big\|_{x=0} > 0$ or gradient changes from – to +.

x	0.4	0.5	0.6
$\frac{dy}{dx}$	-	0	+

x	1.9	2	2.1
$\frac{dy}{dx}$	+	0	+

NCHS 2014 PRELIM EXAM (2) ADDITIONAL MATHEMATICS PAPER 1 (4047/01)

13(i)	$P = 125 - 35\cos\left(\frac{\pi}{14}t\right)$ $P_{max} = 125 - 35(-1) = 160$ $P_{min} = 125 - 35(1) = 90$ Hence $90 \le P \le 160$
13(ii)	$125 - 35\cos\left(\frac{\pi}{14}t\right) = 120$ $35\cos\left(\frac{\pi}{14}t\right) = 5$ $\cos\left(\frac{\pi}{14}t\right) = \frac{1}{7}$ Basic $\angle = 1.4274$ $\left(\frac{\pi}{14}t\right) = 1.4274, 2\pi - 1.4274$ $t = 6.36098, 21.6390$ Length of time = 21.6390 - 6.36098 = 15.3 mins

Answer ALL Questions

1. The initial mass of a radioactive substance is $5\,\mu g$. Given that the mass of a radioactive substance, m μg, at time t minutes, is given by $m = Ae^{kt}$, where A and k are constants.

   (i) Find the value of A. [1]

   (ii) Hence, given further that the mass of the radioactive substance decays to one third of its initial mass when $t = 60$ min, find the value of k. [2]

   (iii) With the values found in (i) and (ii), find the rate at which m is decreasing when $t = 10$ min. [3]

2. (i) Show $\frac{d}{dx}\left(\tan^3 2x\right) = 6\left(\sec^4 2x - \sec^2 2x\right)$. [3]

   (ii) Hence find $\int_{\frac{\pi}{3}}^{\pi} \sec^4 2x\,dx$. [4]

3. (i) Prove the identity $\frac{2\cos A}{2\sin A - \operatorname{cosec} A} = -\tan 2A$. [3]

   (ii) Hence solve the equation $\frac{6\cos 2x}{2\sin 2x - \operatorname{cosec} 2x} + \sqrt{3} = 0$, for $0 < x < \pi$. Express your answers in term of π. [5]

4. (i) Given that $\log_4(2a-1) + \log_2 2b = \log_2 b$ where $b \neq 0$, find the value of a. [4]

   (ii) Given that $3^k = h$, find $\log_3\left(\frac{27}{h}\right)$ and $\log_h 9$ in terms of k. [5]

5. (i) Sketch the graph of $y = -\frac{8}{27}x^3$. [1]

   (ii) Sketch the graph of $y = -\frac{27}{8}x^{-3}$ on the same diagram as (i). [2]

   (iii) Point A and B are the two intersections of your graphs. Calculate the coordinates of points of intersection of your graphs. Hence write down two facts about OA and OB, where point O is at the origin. [6]

6. (i) Express $\frac{2x^4 + 3x^3 - x^2 + 4}{(x^2-4)(x-2)}$ as partial fraction. [7]

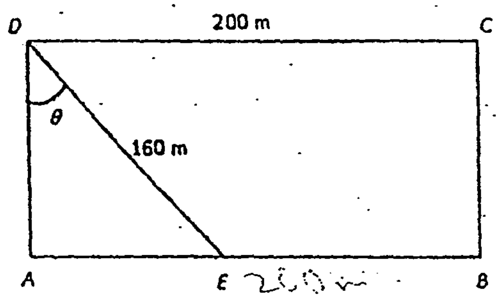
   (ii) Hence find $\int \frac{2x^4 + 3x^3 - x^2 + 4}{(x^2-4)(x-2)}\,dx$ for $x > 2$. [4]

7. Sketch the graphs of $y = x^2 - 2x + 2$ and $y = 2x + 2$ on the same diagram showing the coordinates of the intersections. Hence find the area enclosed by the two graphs. [9]

8. (i) Given that the coefficient of x^6 and x^{10} in the expansion of $\left(x^3 - \frac{2}{x}\right)^{10}$ are a and b respectively, find the value of $\frac{a}{b}$. [5]

(ii) Given the first three terms in the expansion, in ascending powers of x, of $(2+px)^n$ are $2048 + 16x + qx^2$ where n, p and q are constants. Find the value of n, p and q. [6]

9. The diagram shows a rectangular field $ABCD$ with length 200 m. Students ran along the straight tracks BC, CD, DE and EB to complete the running course. Given that $DE = 160$m and $\angle ADE = \theta$ where $0 < \theta < \frac{\pi}{2}$.



(i) Show that the total distance, P m, of the running course is given by $P = r + h\sin\theta + k\cos\theta$, where r, h and k are constants to be found. [3]

(ii) Express P in the form $r + R\cos(\theta + \alpha)$, where $R > 0$ and α is an acute angle. [3]

(iii) Find the value of θ which $P = 0.65$ km. [4]

10. Given that the y-axis is a tangent to the circle C_1 at point A which the y-coordinate is 4 and the circle passes through point $B(4, 5)$. Find

(i) the coordinates of the centre and radius of the circle, [5]

(ii) the equation of the circle in the form $x^2 + y^2 + ax + by + c = 0$, where a, b and c are constants, [2]

(iii) the equation of another circle C_2 which is the image of the circle C_1 being reflected along the x-axis. Express the equation of circle C_2 in the form $x^2 + y^2 + px + qy + r = 0$, where p, q and r are constants. [2]

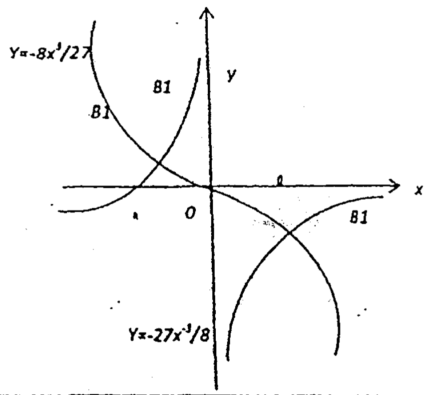
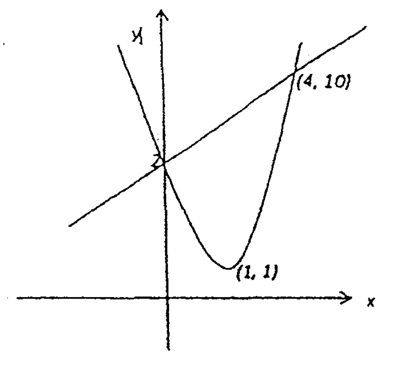
11. A curve has an equation $y = \ln\left(\frac{\sqrt{5-2x}}{3x+1}\right)$ for $1 < x < 2\frac{1}{2}$. Find

(i) $\frac{dy}{dx}$, [3]

(ii) the equation of the normal to the curve at the point A which the tangent to the curve at this point is parallel to the line $7y + 10x = 1$. Express the equation of the normal in the form of $y = mx + \ln a + b$ which m, a and b are constants. [8]

END OF PAPER 1

Answer Key

1.	(i)	$A = 5$	2.	(i)	$6(\sec^4 2x - \sec^2 2x)$
	(ii)	$k = -0.0183$		(ii)	$\sqrt{3}$
	(iii)	$rate\ decreasing = 0.0762\ \mu g/\text{min}$	4.	(i)	$a = \frac{5}{8}$
3	(ii)	$x = \frac{\pi}{24}, \frac{7\pi}{24}, \frac{13\pi}{24}, \frac{19\pi}{24}$		(ii)	$\frac{2}{k}$
5	(i) (ii)		7.		
	(iii)	A(-1.5,1), B(1.5,-1) Point A, O and B are collinear. O is the mid-point of AB.			Parabolic curve with y-intercept of 2 (B1) Min pt (B1) Str. Line with y-intercepts of 2 and +ve gradient (B1) Showing values or coordinates of two intersection (B1)
6.	(i)	$\frac{2x^4+3x^3-x^2+4}{(x^2-4)(x-2)} = 2x+7+\frac{41}{2(x-2)}+\frac{14}{(x-2)^2}+\frac{1}{2(x+2)}$			$10\frac{2}{3}$
	(ii)	$x^2+7x+\frac{41}{2}\ln(x-2)-\frac{14}{(x-2)}+\frac{1}{2}\ln(x+2)+C$			
8	(i)	$\frac{a}{b} = -\frac{5}{3}$	9	(i)	$560+160\cos\theta-160\sin\theta$
	(ii)	$p = \frac{1}{704}, q = \frac{5}{88}$		(ii)	$P = 560+226\cos(\theta+\frac{\pi}{4})$
				(iii)	$\theta = 0.376$
10.	(i)	$(2\frac{1}{8}, 4)$, $radius = 2\frac{1}{8}$	11.	(i)	$-\frac{1}{(5-2x)}-\frac{3}{3x+1}$
	(ii)	$x^2+y^2-\frac{17}{4}x+8y+16=0$		(ii)	$y = \frac{7}{10}x+\ln\frac{1}{7}-\frac{7}{5}$

Additional Mathematics – Secondary 4 Express

Nan Chiau High School

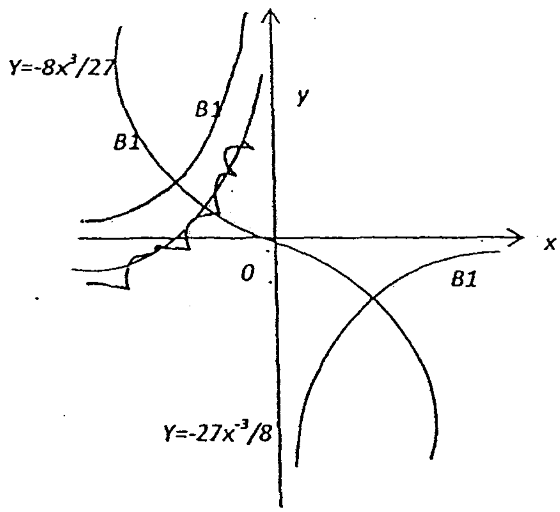
Prelim Examination (2) 2014

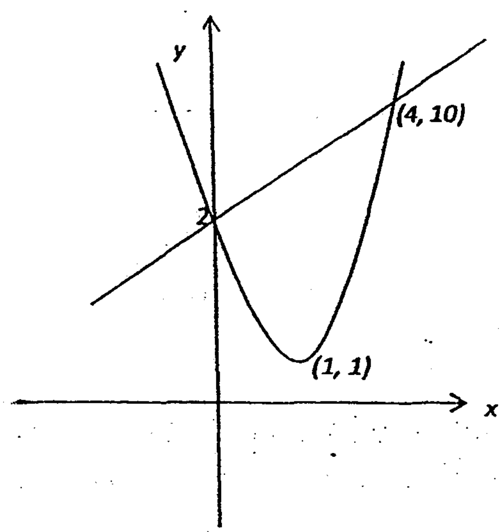
Marking Scheme

Qn No	Suggested Solutions	
1i	$m = Ae^0$	
	$A = 5$ —— B1	
1ii	$\frac{1}{3}(5) = 5e^{k(60)}$ —— M1	
	$k = -0.018310$	
	$k = -0.0183$ —— A1	
1iii	$\frac{dm}{dt} = Ake^{kt}$ —— M1	
	$= 5(-0.018310)e^{-0.018310(10)}$	
	$= -0.0762\ \mu g/\min$ —— M1	
	rate decreasing $= 0.0762\ \mu g/\min$ —— A1	
2i	$\frac{d}{dx}(\tan^3 2x) = 3\tan^2 2x(\sec^2 2x)(2)$ —— M1	
	$= 6\tan^2 2x(\sec^2 2x)$	
	$= 6(\sec^2 2x - 1)(\sec^2 2x)$ —— M1	
	$= 6(\sec^4 2x - \sec^2 2x)$ —— A1	
2ii	$\int_{\frac{\pi}{3}}^{\pi} \sec^4 2x\, dx = \frac{1}{6}\left[\tan^3 2x\right]_{\frac{\pi}{3}}^{\pi} + \int_{\frac{\pi}{3}}^{\pi} \sec^2 2x\, dx$ —— M2	
	$= \frac{1}{6}\left[\tan^3 2x\right]_{\frac{\pi}{3}}^{\pi} + \left[\frac{\tan 2x}{2}\right]_{\frac{\pi}{3}}^{\pi}$ —— M1	
	$= \sqrt{3}$ —— A1	

3i	$\dfrac{2\cos A}{2\sin A-\operatorname{cosec} A}=-\tan 2A$ $LHS=\dfrac{2\cos A}{2\sin A-\dfrac{1}{\sin A}}$ $=\dfrac{2\cos A}{\dfrac{2\sin^2 A-1}{\sin A}}$ ——— M1 $=\dfrac{2\sin A\cos A}{2\sin^2 A-1}$ $=\dfrac{\sin 2A}{-\cos 2A}$ ——— M1 $=-\tan 2A$ ——— A1 $=RHS$	
3ii	$\dfrac{6\cos 2x}{2\sin 2x-\operatorname{cosec} 2x}+\sqrt{3}=0$ $-3\tan 4x+\sqrt{3}=0$ $\tan 4x=\dfrac{\sqrt{3}}{3}$ ——— M1 $\alpha=\dfrac{\pi}{6}$ ——— M1 $4x=\dfrac{\pi}{6},\dfrac{7\pi}{6},\dfrac{13\pi}{6},\dfrac{19\pi}{6}$ ——— M2 $x=\dfrac{\pi}{24},\dfrac{7\pi}{24},\dfrac{13\pi}{24},\dfrac{19\pi}{24}$ ——— A1	
4i	$\log_4(2a-1)+\log_2 2b=\log_2 b$ $\dfrac{1}{2}\log_2(2a-1)+\log_2 2b=\log_2 b$ ——— M1 $\log_2(2a-1)4b^2=\log_2 b^2$ ——— M1 $(2a-1)4b^2=b^2$ $b^2(8a-5)=0$ ——— M1 $(8a-5)=0$ $a=\dfrac{5}{8}$ ——— A1 $3^k=h$	

——— M1

4ii	$\log_3 h = k$ $\log_3\left(\frac{27}{h}\right) = \log_3 27 - \log_3 h$ —— M1 $= 3 - k$ —— A1 $\log_h 9 = 2\log_h 3$ —— M1 $= \frac{2}{k}$ —— A1	
5i,ii		
5iii	$-\frac{8}{27}x^3 = -\frac{27}{8}x^{-3}$ —— M1 $x^6 = \frac{729}{64}$ $x = 1.5$ or -1.5 —— M1 A(-1.5,1), B(1.5,-1) —— A2 **Point A, O and B are collinear. O is the mid-point of AB.** (must prove). lie on ē same pt. —— A2	
6i	$\frac{2x^4+3x^3-x^2+4}{(x^2-4)(x-2)} = 2x+7+\frac{21x^2+12x-52}{(x^2-4)(x-2)}$ —— M2 $\frac{21x^2+12x-52}{(x-2)^2(x+2)} = \frac{A}{(x-2)} + \frac{B}{(x-2)^2} + \frac{C}{(x+2)}$ —— M1 $21x^2+12x-52 = A(x-2)(x+2)+B(x+2)+C(x-2)^2$	

	$Let\ x = 2$ $4B = 56$ $B = 14$ ——— M1 $Let\ x = -2$ $16C = 8$ $C = \frac{1}{2}$ ——— M1 $Let\ x = 0$ $-4A + 2B + 4C = -52$ $A = \frac{41}{2}$ ——— M1 $\frac{2x^4+3x^3-x^2+4}{(x^2-4)(x-2)} = 2x+7+\frac{41}{2(x-2)}+\frac{14}{(x-2)^2}+\frac{1}{2(x+2)}$ ——— A1	
6ii	$\int \frac{2x^4+3x^3-x^2+4}{(x^2-4)(x-2)}dx = \int 2x+7+\frac{41}{2(x-2)}+\frac{14}{(x-2)^2}+\frac{1}{2(x+2)}dx$ ——— M1 $= x^2+7x+\frac{41}{2}\ln(x-2)-\frac{14}{(x-2)}+\frac{1}{2}\ln(x+2)+C$ ——— A3	
7	 Parabolic curve with y-intercept of 2 (B1) Min pt (B1) Str. Line with y –intercepts of 2 and +ve gradient (B1) Showing values or coordinates of two intersection (B1)	3

	$x^2-2x+2=2x+2$	M1
	$x^2-4x=0$	
	$x=0\ or\ 4$	
	$y=2\ or\ 10$	M1
	$A=\int_0^4 2x+2-(x^2-2x+2)dx$	
	$=\int_0^4 -x^2+4x\ dx$	M1
	$=\left[-\frac{x^3}{3}+\frac{4x^2}{2}\right]_0^4$	M1
	$=-\frac{64}{3}+32$	
	$=10\frac{2}{3}$	A1
8i	$\left(x^3-\frac{2}{x}\right)^{10}$	
	${}^{10}C_4\left(x^3\right)^4\left(-\frac{2}{x}\right)^6$	M1
	$=13440x^6$	M1
	${}^{10}C_5\left(x^3\right)^5\left(-\frac{2}{x}\right)^5$	M1
	$=-8064x^{10}$	M1
	$a=13440, b=-8064$	
	$\frac{a}{b}=-\frac{5}{3}$	A1
8ii	$(2+px)^n=2048+16x+qx^2+...$	
	$=2^n+{}^nC_1(2)^{n-1}(px)+{}^nC_2(2)^{n-2}(px)^2+...$	
	$2^n=2048$	M1
	$n=11$	A1
	${}^{11}C_1(2)^{10}(px)=16x$	M1
	$p=\frac{1}{704}$	A1

	$^{11}C_2(2)^9(px)^2 = qx^2$ —— M1	
	$^{11}C_2(2)^9\left(\frac{1}{704}x\right)^2 = qx^2$	
	$q = \frac{5}{88}$ —— A1	
9i	$P = BC + CD + DE + EB$	
	$= 160\cos\theta + 200 + 160 + (200 - 160\sin\theta)$ —— M2	
	$= 560 + 160\cos\theta - 160\sin\theta$ —— A1	
9ii	$P = 560 + 160\cos\theta - 160\sin\theta$	
	$R = \sqrt{51200}$ —— M1	
	$\tan\alpha = 1$	
	$\alpha = \frac{\pi}{4}$ —— M1	
	$P = 560 + 226\cos(\theta + \frac{\pi}{4})$ —— A1	
9iii	$560 + \sqrt{51200}\cos(\theta + \frac{\pi}{4}) = 650$ —— M1	
	$\cos(\theta + \frac{\pi}{4}) = \frac{90}{\sqrt{51200}}$	
	$\alpha 1 = 1.16174$ —— M1	
	$\theta + \frac{\pi}{4} = 1.16174$ —— M1	
	$\theta = 0.376$ —— A1	

10i	$mid-pt\ of\ AB=(2,4.5)$ — M1 $m_{AB}=\frac{1}{4}$ $m_{normal}=-4$ — M1 $4.5=-4(2)+C$ $C=12.5$ $y=-4x+12.5$ — M1 $y-coordinate=4$ $4=-4x+12.5$ $x=\frac{17}{8}$ $(2\frac{1}{8},4),\quad radius=2\frac{1}{8}$ — A2 OR Let C = (x; 4) — M1 $(x-0)^2+(4-4)^2=(x-4)^2+(4-5)^2$ — M1 $x^2=x^2-8x+16+1$ $x=\frac{17}{8}$ — M1 $(2\frac{1}{8},4),\quad radius=2\frac{1}{8}$ — A2	
10ii	$\left(x-\frac{17}{8}\right)^2+(y-4)^2=\left(\frac{17}{8}\right)^2$ — M1 $x^2+y^2-\frac{17}{4}x-8y+16=0$ — A1	
10iii	$centre\ of\ C2\ (2\frac{1}{8},-4)$ — M1 $x^2+y^2-\frac{17}{4}x+8y+16=0$ — A1	
11i	$y=\ln\frac{\sqrt{5-2x}}{3x+1}$ $=\frac{1}{2}\ln(5-2x)-\ln(3x+1)$ — M1 $\frac{dy}{dx}=\frac{-2}{2(5-2x)}-\frac{3}{3x+1}$ — M1 $=-\frac{1}{(5-2x)}-\frac{3}{3x+1}$ — A1	

11iii	$7y + 10x = 1$	
	$m1 = -\frac{10}{7}$ —— M1	
	$-\frac{1}{(5-2x)} - \frac{3}{3x+1} = -\frac{10}{7}$ —— M1	
	$60x^2 - 151x + 62 = 0$ —— M1	
	$x = 2$ or $\frac{31}{60}(rejected)$ —— M1	
	$y = \ln\frac{\sqrt{5-2(2)}}{3(2)+1}$	
	$= \ln\frac{1}{7}$ —— M1	
	$m2 = \frac{7}{10}$ —— M1	
	$y = \frac{7}{10}x + C$	
	$\ln\frac{1}{7} = \frac{7}{10}(2) + c$	
	$c = \ln\frac{1}{7} - \frac{7}{5}$ —— M1	
	$y = \frac{7}{10}x + \ln\frac{1}{7} - \frac{7}{5}$ —— A1	
	*** End of Paper ***	

南华中学

NAN HUA HIGH SCHOOL

PRELIMINARY EXAMINATION 2014

Subject	:	**Additional Mathematics**
Paper	:	**4047/01**
Level	:	**Secondary Four Express**
Date	:	**15 September 2014**
Duration	:	**2 hours**

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correcting fluid / tape.

Answer **all** the questions.
Write your answers on the separate writing paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

This paper consists of 5 printed pages.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

1 Given that $\sin A = -\frac{3}{5}$ where $180° < A < 270°$ and $\sin B = \frac{5}{13}$ where $90° < B < 180°$, find $\tan(A+B)$ without using a calculator. [2]

2. A prism has a square base of side of $(2+\sqrt{3})$ m, and its volume is $(11+6\sqrt{3})$ m^3. Find, without using a calculator, the height of the prism in the form $(a-b\sqrt{3})$ m, where a and b are integers. [4]

3. Find, in ascending powers of x, the first 4 terms of the expansion of $\left(1-\frac{x}{3}\right)^{10}$.

Hence, find the coefficient of x in the expansion of $\left(2+\frac{3}{x}\right)^2\left(1-\frac{x}{3}\right)^{10}$. [4]

4 (a) Find the value of a and k for which $\{x : -4 < x < 7\}$ is the solution set of $k - x^2 > ax$. [2]

(b) If the line $y = 4x - 1$ meets the curve $ky - 8x = 2kx^2 + 1$, find the range of values of k. [4]

5. Given the roots of $9x^2 - 13x + 36 = 0$ are α^2 and β^2, find the quadratic equation(s) whose roots are α^3 and β^3. [7]

6. Prove the identity $\dfrac{\cot^2\theta - \cos^2\theta}{\text{cosec}^2\theta\tan^2\theta} = \cot^2\theta\cos^4\theta$. [3]

7 (a) Solve the equation $3\log_8 x - 4 = 4\log_x 8$ [4]

(b) Solve the simultaneous equations

$$\log_3\left(\frac{x-5y}{x-y}\right) = 2$$

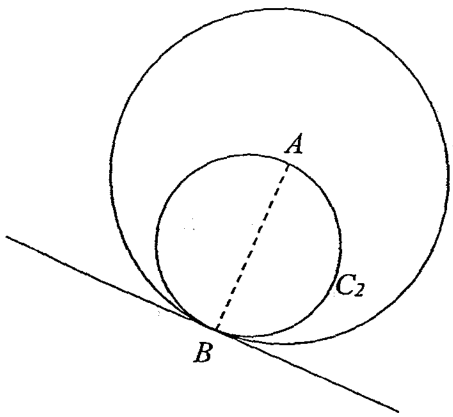
$$5e^{2x-\frac{1}{2}y} = 6 - e^{x-y}$$ [7]

8. Water is poured into an empty inverted right circular cone at a constant ...
The cone has a base radius of 6 cm and a height of 12 cm.

(i) Find the rate of change of the height of water level when the wate... 3 cm high.

(ii) State, with reason, whether this rate will increase or decrease as ...

9.



The diagram shows two circles C_1 and C_2. The equation of circle C_1 wi... $x^2+y^2-8x+2y-63=0$. AB is the diameter of circle C_2, and the tang... at point B has a gradient of $-\frac{1}{2}$. Find

(i) the centre and radius of circle C_1,

(ii) the coordinates of point B given that its y-coordinate is negative,

(iii) the equation of circle C_2.

10 (a) Without the use of a calculator, find x such that $\cos 2x = \sin 230°$ whe... $-180° \le x \le 180°$.

(b) Evaluate $\cos\left(\cos^{-1}\left(-\frac{3}{5}\right)-\cos^{-1}\left(\frac{12}{13}\right)\right)$ without using a calculator.

11. The variables x and y are related by the equation $y = 10^{-k}A^x$, where k and A are constants. Using experimental values of x and y, a graph was drawn in which $\lg y$ was plotted on the vertical axis against x on the horizontal axis. The straight line which was obtained passed through the points (20, 0.36) and (35, 1.20). Find

(i) the values of k and A, [3]

(ii) the coordinates of the point on the line at which $y = 10^{-\frac{x}{20}}$

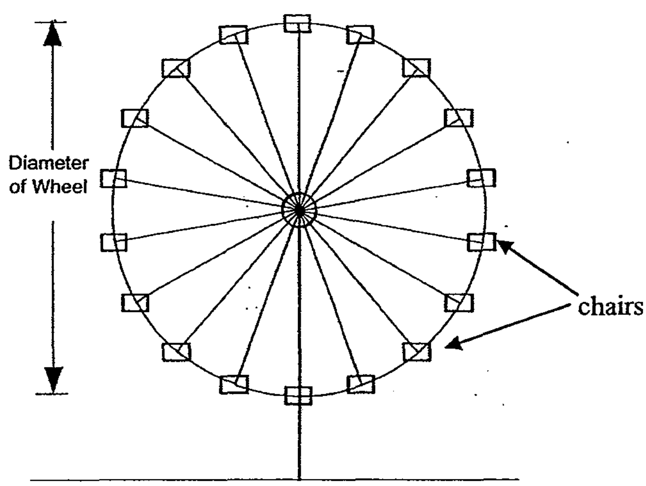
12. (a) The normal to the curve $y = \dfrac{2\ln x}{x^2}$, at the point where the curve crosses the x-axis, passes through the point $(4, p)$. Find the value of p. [4]

(b) Given that $\int_1^6 f(x)\,dx = 9$ and $\int_1^2 f(x)\,dx = 3$, find

(i) $\int_6^1 \left[\dfrac{5}{x^2} - f(x)\right] dx$. [3]

(ii) the value of the constant m for which $\int_2^6 [f(x) + mx]\,dx = -42$. [3]

13.



A Ferris wheel with a radius of 8 m is rotating at a rate of 3 revolutions per minute. The height of a chair on the Ferris wheel (measured from the ground) can be modelled by the equation, $h = a\cos kt + b$ where a, b and k are constants and t is the time in seconds after the ride starts. At the start of a ride when $t = 0$, a chair on the Ferris wheel starts at the lowest point, which is 2 m above the ground.

(i) Explain why the maximum height of any chair on the Ferris wheel is 18 m above the ground. [1]

(ii) Show that the value of k is $\dfrac{\pi}{10}$. [2]

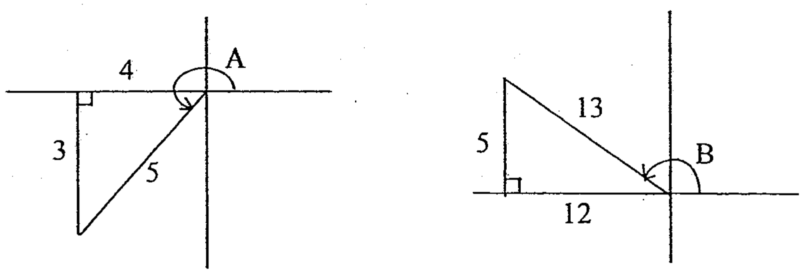
(iii) Find the values of a and b. [4]

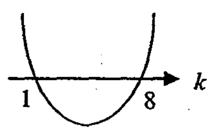
~ End of Paper ~

Answer Key

1. $\frac{16}{63}$	2. $\left(5-2\sqrt{3}\right)$
3. $1-\frac{10}{3}x+5x^2-\frac{40}{9}x^3+\ldots$; $6\frac{2}{3}$	4(a) $a=-3,\quad k=28$ (b) $k\le 1 \quad or \quad k\ge 8$
5. $27x^2\pm 35x+216=0$	7(a) $x=\frac{1}{4}$ or 64 (b) $x=-1.61,\quad y=-3.22$
8(i) 0.849 cm/s (ii) the rate will decrease	9(i) Centre: (4, -1), Radius = 8.94 units (ii) $B(0,-9)$ (iii) $(x-2)^2+(y+5)^2=20$
10(a) $-110°,\ -70°,\ 70°,\ 110°$ (b) $-\frac{16}{65}$	11(i) 0.76; 1.14 (ii) (7.17, - 0.358)
12(a) -3/2 (b)(i) $4\frac{5}{6}$ (ii) - 3	13(iii) - 8; 10

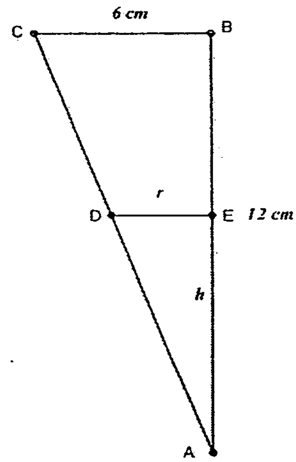
AM Prelim 2 Paper 1 Solutions

S/no	Solutions
1	 $\tan(A+B)=\dfrac{\tan A+\tan B}{1-\tan A\tan B}$ $=\dfrac{\frac{3}{4}-\frac{5}{12}}{1-\frac{3}{4}\left(-\frac{5}{12}\right)}$ $=\dfrac{4}{12}\div\left(1+\dfrac{5}{16}\right)$ $=\dfrac{1}{3}\times\dfrac{16}{21}=\dfrac{16}{63}$
2	$a-b\sqrt{3}=\dfrac{11+6\sqrt{3}}{\left(2+\sqrt{3}\right)^2}$ $=\dfrac{11+6\sqrt{3}}{7+4\sqrt{3}}$ $=\dfrac{11+6\sqrt{3}}{7+4\sqrt{3}}\times\dfrac{7-4\sqrt{3}}{7-4\sqrt{3}}$ $=\dfrac{77+42\sqrt{3}-44\sqrt{3}-72}{49-48}$ $a-b\sqrt{3}=5-2\sqrt{3}$ Hence the height of the prism is $\left(5-2\sqrt{3}\right)m$.

3	$\left(1-\frac{x}{3}\right)^{10}=1-\binom{10}{1}\left(\frac{x}{3}\right)+\binom{10}{2}\left(\frac{x}{3}\right)^2-\binom{10}{3}\left(\frac{x}{3}\right)^3+\ldots$ $=1-\frac{10}{3}x+5x^2-\frac{40}{9}x^3+\ldots$ $\left(2+\frac{3}{x}\right)^2\left(1-\frac{x}{3}\right)^{10}$ $=\left(4+\frac{12}{x}+\frac{9}{x^2}\right)\left(1-\frac{10}{3}x+5x^2-\frac{40}{9}x^3+\ldots\right)$ *Coefficient of* x: $(4)\left(-\frac{10}{3}\right)+(12)(5)+(9)\left(-\frac{40}{9}\right)$ $=-\frac{40}{3}+60-40$ $=6\frac{2}{3}$
4(a)	$k-x^2>ax$ $x^2+ax-k<0$ *if* $-4<x<7$ *is the solution, then* $(x+4)(x-7)<0$ $x^2-3x-28<0$ *By comparing,* $a=-3,\quad k=28$
4(b)	*At pt of intersection,* $k(4x-1)-8x=2kx^2+1$ $2kx^2+(8-4k)x+(1+k)=0$ *Since line meets curve, discriminant:* $(8-4k)^2-4(2k)(1+k)\geq 0$ $64-64k+16k^2-8k-8k^2\geq 0$ $8k^2-72k+64\geq 0$ $k^2-9k+8\geq 0$ $(k-8)(k-1)\geq 0$ $k\leq 1 \quad or \quad k\geq 8$ 

5	$9x^2 - 13x + 36 = 0$ $\alpha^2\beta^2 = (\alpha\beta)^2 = \frac{36}{9} = 4$ $\alpha\beta = 2 \quad or \quad -2$ $\alpha^2 + \beta^2 = \frac{13}{9}$ $(\alpha+\beta)^2 - 2\alpha\beta = \frac{13}{9}$ $(\alpha+\beta)^2 = \frac{13}{9} + 2\alpha\beta$ $When \quad \alpha\beta = -2, \quad (\alpha+\beta)^2 = \frac{13}{9} - 4 < 0 \quad (NA \sin ce\,(\alpha+\beta)^2 \geq 0)$ $When \quad \alpha\beta = 2, \quad (\alpha+\beta)^2 = \frac{13}{9} + 4 = \frac{49}{9}$ $\alpha + \beta = \pm\frac{7}{3}$ $\alpha^3 + \beta^3 = (\alpha+\beta)(\alpha^2 - \alpha\beta + \beta^2)$ $= \pm\frac{7}{3}\left(\frac{13}{9} - 2\right)$ $= \pm\frac{7}{3}\left(-\frac{5}{9}\right) = \pm\frac{35}{27}$ $\alpha^3\beta^3 = (\alpha\beta)^3 = 2^3 = 8$ Equations required: $x^2 \pm \frac{35}{27}x + 8 = 0$ $27x^2 \pm 35x + 216 = 0$
6	$\frac{\cot^2\theta - \cos^2\theta}{\mathrm{cosec}^2\theta\tan^2\theta}$ $= \dfrac{\frac{\cos^2\theta}{\sin^2\theta} - \cos^2\theta}{\frac{1}{\sin^2\theta}\left(\frac{\sin^2\theta}{\cos^2\theta}\right)}$ $= \cos^2\theta\left[\cos^2\theta\left(\frac{1}{\sin^2\theta} - 1\right)\right]$ $= \cos^4\theta(\mathrm{cosec}^2\theta - 1)$ $= \cos^4\theta\cot^2\theta$

7(a)	$3\log_8 x - 4 = \frac{4}{\log_8 x}$ $3(\log_8 x)^2 - 4\log_8 x - 4 = 0$ Let $y = \log_8 x$ $\therefore 3y^2 - 4y - 4 = 0$ $(3y+2)(y-2) = 0$ $y = -\frac{2}{3}$ or $y = 2$ $\log_8 x = -\frac{2}{3}$ or $\log_8 x = 2$ $x = 8^{-\frac{2}{3}}$ or $x = 8^2$ $x = \frac{1}{4}$ or $x = 64$
7(b)	$\log_3\left(\frac{x-5y}{x-y}\right) = 2$ $\frac{x-5y}{x-y} = 3^2 = 9$ $x - 5y = 9x - 9y$ $4y = 8x$ $y = 2x$ $5e^{2x-\frac{1}{2}y} = 6 - e^{x-y}$ $5e^{2x-x} = 6 - e^{x-2x}$ $5e^x = 6 - e^{-x}$ Multiply by e^x $5e^{2x} - 6e^x + 1 = 0$ $(5e^x - 1)(e^x - 1) = 0$ $e^x = \frac{1}{5}$ or $e^x = 1$ $x = \ln\frac{1}{5}$ or $x = 0$ $When\ x = -1.609,\quad y = -3.219$ $When\ x = 0,\ y = 0,\ \log_3\left(\frac{x-5y}{x-y}\right)$ *is undefined, so NA* $Ans: x = -1.61,\quad y = -3.22$

8(i)	*By similar* Δ, $\frac{r}{6}=\frac{h}{12}$ $r=\frac{h}{2}$ $V=\frac{1}{3}\pi r^2 h$ $=\frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h$ $=\frac{\pi}{12}h^3$ $\frac{dV}{dh}=\frac{\pi}{4}h^2, \quad \therefore \frac{dh}{dV}=\frac{4}{\pi h^2}$ $\frac{dh}{dt}=\frac{dh}{dV}\|_{h=3}\times\frac{dV}{dt}$ $=\frac{4}{\pi 3^2}\times 6$ $=\frac{8}{3\pi}$ or $0.849 cm/s$  The water level is increasing at a rate of $\frac{8}{3\pi}$ or $0.849\ cm/s$
8(ii)	$\frac{dh}{dt}=\frac{dh}{dV}\times\frac{dV}{dt}=\frac{4}{\pi h^2}\times 6=\frac{24}{\pi h^2}$ As t increases, h increases, therefore the rate will decrease.
9(i) 9(ii)	$x^2+y^2-8x+2y-63=0$ $(x-4)^2-16+(y+1)^2-1-63=0$ $(x-4)^2+(y+1)^2=80$ Centre: (4, -1), Radius = $\sqrt{80}$ = 8.94 units *eq of AB* : $\frac{y+1}{x-4}=2$ $y=2x-9$ *At B*, $(x-4)^2+(2x-8)^2=80$ $(x-4)^2+4(x-4)^2=80$ $5(x-4)^2=80$ $(x-4)^2=16$ $x-4=4$ *or* -4 $x=8$ *or* 0 $y=7$ *or* -9 $\therefore$ $B(0,-9)$ *since the* $y-$*coord is negative*

9(iii)

centre of C_2 : $\left(\dfrac{4+0}{2}, \dfrac{-1-9}{2}\right) = (2, -5)$

radius of C_2 : $\sqrt{(2-0)^2 + (-5+9)^2} = \sqrt{20}$

eq of C_2 : $(x-2)^2 + (y+5)^2 = 20$

10(a)

$$\begin{aligned} \cos 2x &= \sin 230^\circ \\ &= -\sin 50^\circ \\ &= -\cos 40^\circ \end{aligned}$$

basic $\angle = 40^\circ$

since $-180^\circ \leq x \leq 180^\circ$

$-360^\circ \leq 2x \leq 360^\circ$

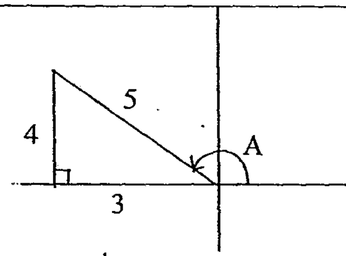
$\therefore\ 2x = 140^\circ,\ 220^\circ,\ -140^\circ,\ -220^\circ$

$x = -110^\circ,\ -70^\circ,\ 70^\circ,\ 110^\circ$

10(b)

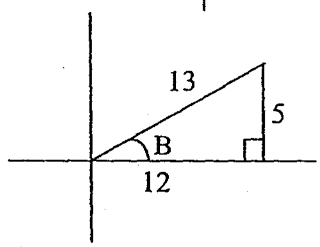
Let $A = \cos^{-1}\left(-\dfrac{3}{5}\right)$

$\cos A = -\dfrac{3}{5}$



Let $B = \cos^{-1}\left(\dfrac{12}{13}\right)$

$\cos B = \dfrac{12}{13}$



$$\begin{aligned} &\cos\left[\cos^{-1}\left(-\frac{3}{5}\right) - \cos^{-1}\left(\frac{12}{13}\right)\right] \\ &= \cos(A-B) \\ &= \cos A \cos B + \sin A \sin B \\ &= \left(-\frac{3}{5}\right)\left(\frac{12}{13}\right) + \left(\frac{4}{5}\right)\left(\frac{5}{13}\right) \\ &= \frac{-36+20}{65} \\ &= -\frac{16}{65} \end{aligned}$$

11(i)

$y = 10^{-k} A^x$

$\lg y = -k + x \lg A$

$\lg A = \dfrac{1.20 - 0.36}{35-20} = 0.056$

$A = 1.14$ (to 3 sf)

Using $(35, 1.20)$: $1.20 = -k + 35(0.056)$

$k = 0.76$

11(ii)	$y=10^{-\frac{x}{20}}$ $\lg y=-\frac{x}{20}=-k+x\lg A$ $=-0.76+0.056x$ $\left(0.056+\frac{1}{20}\right)x=0.76$ $x=7.16981=7.17$ $\lg y=-\frac{x}{20}=-0.358$ *The point is* $(7.17,-0.358)$
12(a)	when $y=0,\frac{\ln x^2}{x^2}=0$ $x=1$ $\frac{dy}{dx}=\frac{x^2\left(\frac{2}{x}\right)-2\ln x\times(2x)}{x^4}$ $=\frac{2x-4x\ln x}{x^4}=\frac{2-4\ln x}{x^3}$ gradient of tangent at this point $=\frac{2-4\ln(1)}{(1)^3}=2$ gradient of normal at this point $=-\frac{1}{2}$ Equation of normal: $y-0=-\frac{1}{2}(x-1)$ $y=-\frac{1}{2}x+\frac{1}{2}$ when $x=4$, $p=4\left(-\frac{1}{2}\right)+\frac{1}{2}=-\frac{3}{2}$
12(b)(i)	$\int_6^1\left[\frac{5}{x^2}-f(x)\right]dx$ $=\int_1^6\left[f(x)-\frac{5}{x^2}\right]dx$ $=\int_1^6 f(x)dx+\left[\frac{5}{x}\right]_1^6$ $=(9)+\frac{5}{6}-5=4\frac{5}{6}$

(ii)	$\int_2^6 [f(x)+mx]\,dx = -42$ $\int_2^6 f(x)\,dx + \left[\frac{mx^2}{2}\right]_2^6 = -42$ $(9-3)+\frac{m}{2}(36-4) = -42$ $16m = -48$ $m = -3$
13(i)	$\max h = 8+8+2 = 18\ m$ $\therefore \max$ *height of a chair is* $18\ m$ *above the ground*
13(ii)	$h = a\cos kt + b$ $3\ rev/60\ s$ $1\ rev/20\ s$ $\frac{1}{2} rev/10\ s$ $period = \frac{2\pi}{k} = 20$ $k = \frac{\pi}{10}$
13(iii)	$h = a\cos kt + b$ $When\ t = 0, \quad a\cos 0 + b = 2$ $a + b = 2 \ldots\ldots\ldots eq1$ $When\ t = 10, \quad h\ is\ \max:$ $a\cos\left(\frac{\pi}{10}\right)(10) + b = 18$ $-a + b = 18 \ldots\ldots eq2$ $eq1 + eq2: \quad 2b = 20$ $b = 10$ $a = -8$

南华中学

NAN HUA HIGH SCHOOL

PRELIMINARY EXAMINATION 2014

Subject	:	**Additional Mathematics**
Paper	:	**4047/02**
Level	:	**Secondary Four Express**
Date	:	**18 September 2014**
Duration	:	**2 hours 30 minutes**

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correcting fluid / tape.

Answer **all** the questions.
Write your answers on the separate writing paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

This paper consists of 8 printed pages.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\text{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2}ab \sin C$$

1. (a) The term containing the highest power of x in the polynomial $f(x)$ is $2x^4$ and the roots of $f(x)=0$ are 2 and -7. $f(x)$ has a remainder of -72 when divided by $(x+1)$, and a remainder of -80 when divided by $(x-1)$.
   (i) Find the expression for $f(x)$ in descending power of x. [4]
   (ii) Explain why the equation $f(x)=0$ has exactly 2 real roots. [2]

   (b) Express $\dfrac{6x^2-3x-2}{4x^3-12x^2+7x-21}$ in partial fractions. [6]

2. (a) (i) Sketch the graph of $y=5-|2x^2-x-15|$ for $-4 \le x \le 5$. [4]
   (ii) Hence find the value(s) of k such that the equation $|2x^2-x-15|=5-k$ has exactly 2 solutions. [2]

   (b) Solve the equation $|-3x+18|=6x+|x-6|$. [4]

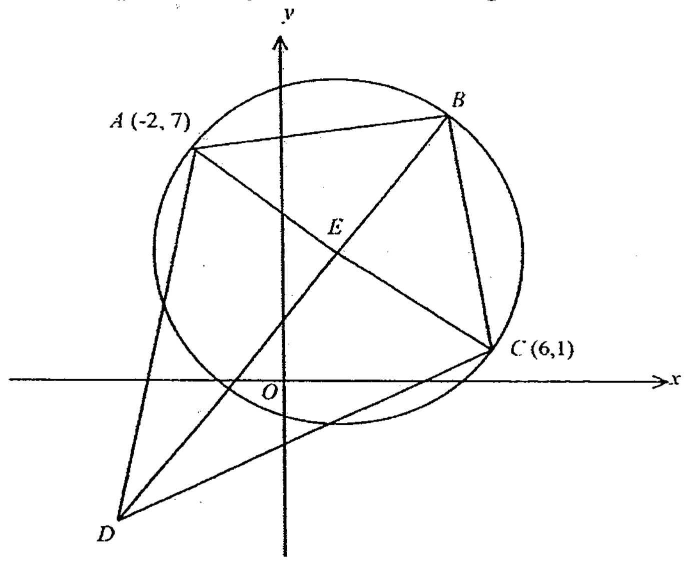
3. (a) (i) Solve the equation $6\sin^2\dfrac{x}{2}-2\cos^2 x=1$ for $0° < x < 360°$. [4]
   (ii) State the number of solutions of the equation $6\sin^2\dfrac{x}{2}-2\cos^2 x=1$ in the range $-1080° < x < 720°$. [1]

   (b) Solve the equation $\sin 2y=\cos^2 y$ for $0 \le y \le 9$. [5]

4. The function f is defined by $f(x)=a\tan\left(\dfrac{x}{2}\right)+b$, where a and b are positive integers and $-\pi \le x \le \pi$. The graph of $y=f(x)$ meets the y-axis at the point where $y=3$, and $f\left(\dfrac{\pi}{2}\right)=7$.
   (i) State the period of f. [1]
   (ii) Find the values of a and b. [2]
   (iii) Sketch the graph of $y=f(x)$. [2]

5. Differentiate $\tan^3 6\theta$ with respect to θ. Hence find [1]
   (i) $\int \tan^2 6\theta \sec^2 6\theta \, d\theta$, [1]
   (ii) $\int \sec^4 6\theta \, d\theta$. [3]

6. **Solutions to this question by accurate drawing will not be accepted.**



The diagram shows a kite *ABCD* in which *A*, *B* and C are points on a circle C_1. The coordinates of *A* and *C* are $(-2, 7)$ and $(6, 1)$ respectively.

The lines *AC* and *BD* intersect at point *E*, and point *B* lies on the curve $4y = x^2 + 7$.

(i) Find the coordinates of *B*, if the coordinates are integer values. [5]

(ii) Given that $BE : ED = 2 : 3$, find the coordinates of *D*. [3]

(iii) Show that point *E* is the centre of the circle C_1. [3]

(iv) Calculate the area of the kite *ABCD*. [2]

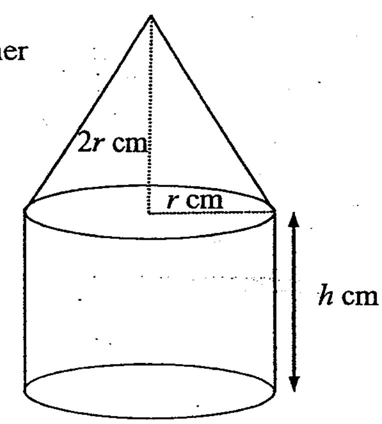
7. A container consists of a cone (of radius *r* cm) fixed to the top of a right circular cylinder of radius *r* cm and height *h* cm. Given that the height of the cone is $2r$ cm and that the volume of the container is $75\pi\,\text{cm}^3$,

(i) show that $h = \dfrac{75}{r^2} - \dfrac{2}{3}r$, [2]

(ii) determine, with explanation, whether *h* is an increasing or decreasing function, [2]

(iii) show that the total surface area, A cm², of the container is given by

$$A = \left(\sqrt{5} - \frac{1}{3}\right)\pi r^2 + \frac{150\pi}{r}.$$ [2]



If *r* varies,

(iv) showing full working, determine whether the stationary value of *A* is a minimum or maximum. [4]

8. A particle moves in a straight line so that its displacement, s metres, from a fixed point O is given by $s = 4e^{-5t} + 10t - 7$ for $0 \le t \le T$, where t is the time in seconds after passing O, and T is the time in seconds when the particle first comes to rest.

(i) Find the initial position of the particle. [1]

(ii) Find the initial velocity of the particle. [2]

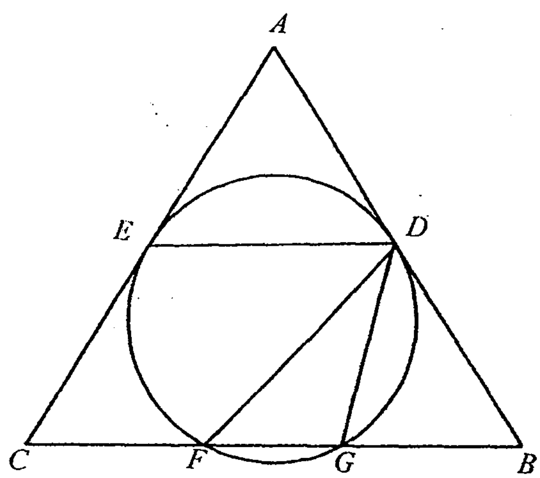
(iii) Show that $T = \dfrac{\ln 2}{5}$. [2]

The particle then travels at a constant acceleration of 10 m/s^2 for the next 2 seconds.

(iv) Sketch the velocity-time graph for the particle for $0 \le t \le (T+2)$. [2]

(v) Find the total distance travelled by the particle in the first $(T+2)$ seconds. [3]

9. In the diagram, AB and AC are tangents to the circle at point D and E respectively. BC intersects the circle at points F and G.

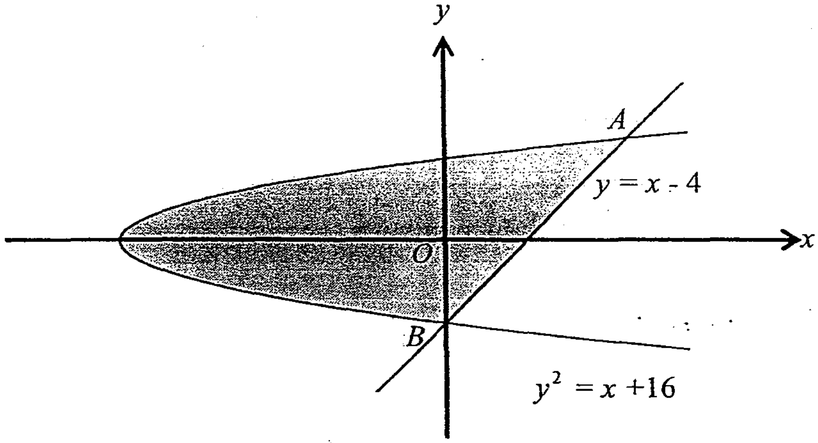


Given $AE = EC$ and $AD = DB$, prove that

(i) $AE \times BC = AC \times ED$, [3]

(ii) $BD^2 = BG \times BF$. [3]

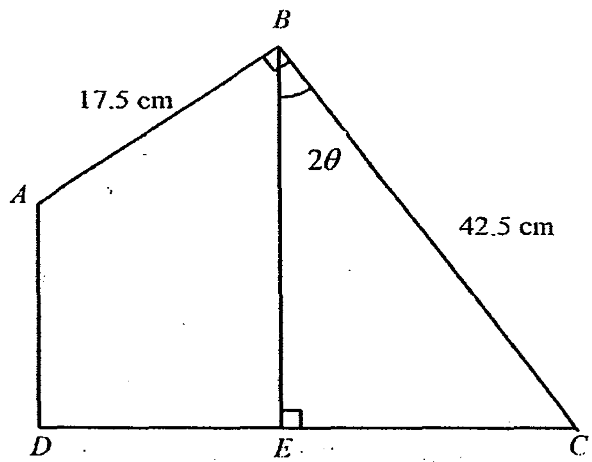
10. The diagram shows part of the curve $y^2 = x + 16$. The line $y = x - 4$ meets the curve at points A and B.



(i) Find the coordinates of A and B. [3]

(ii) Calculate the area of shaded region bounded by the curve and the straight line AB. [4]

11. The diagram shows a glass window, $ABCD$, consisting of a trapezium $ABED$ in which $AD // BE$ and a right-angled triangle BCE. It is given that $AB = 17.5$ cm, $BC = 42.5$cm, and AB and BC intersect each other perpendicularly at B, and $\angle EBC = 2\theta$, where θ varies.



(i) Show that the perimeter, P cm, of the glass window $ABCD$ is
$P = 60 + 60\cos 2\theta + 25\sin 2\theta$. [3]

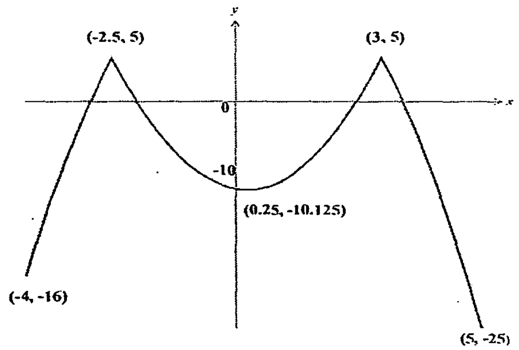
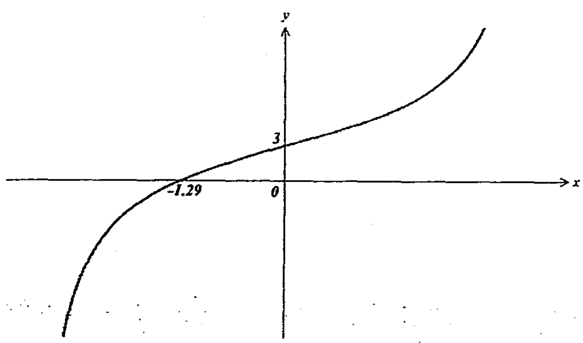
(ii) Express P in the form
$a + b\sin(2\theta + \alpha)$ where $a, b > 0$ and $0° < \alpha < 90°$. [4]

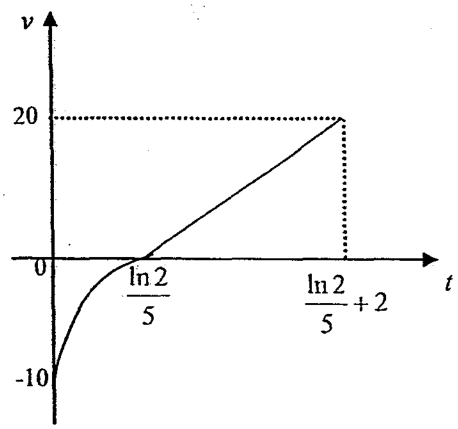
(iii) Find the value of θ for which P =100 cm. [3]

(iv) Find the maximum value of P and the corresponding value of θ. [2]

– End of Paper –

Answer Key

1.	(ai) $f(x)=2x^4+13x^3-8x^2-17x-70$ (b) $\dfrac{6x^2-3x-2}{4x^3-12x^2+7x-21}=\dfrac{1}{(x-3)}+\dfrac{2x+3}{(4x^2+7)}$
2.	(ai) $y=5-\lvert 2x^2-x-15\rvert$  (aii) Value of $k=5$ or $-16\le k<-10\frac{1}{8}$ (b) $x=1\frac{1}{2}$
3.	(ai) $x=60°,\ 300°$ (aii) 10 (b) $y=0.464,\frac{\pi}{2},3.61,\frac{3\pi}{2},6.75,\frac{5\pi}{2}$
4.	(i) 2π (ii) a=4, b=3 (iii) 
5.	(i) $18\tan^2 6\theta\sec^2 6\theta$ (ii) $\frac{1}{18}\tan^3 6\theta+C$ (iii) $\frac{1}{6}\tan 6\theta+\frac{1}{18}\tan^3 6\theta+C$
6.	(i) $B(5,8)$ (ii) $D(-2.5,-2)$ (iv) 62.5 units^2
7.	(i) $h=\frac{75}{r^2}-\frac{2}{3}r$ (iv) A is a minimum value
8.	(i) -3m from O (ii) $-10m/s$ (v) 20.6 m

	(iii) 
10.	(i) $A\ (9,5)$ and $B\ (0,-4)$ (ii) 121.5 units^2
11.	(ii) $P=60+65\sin(2\theta+67.4°)$ (iii) $37.3°$ (iv) Corresponding value of $\theta=11.3°$ Maximum value of $P=125\text{m}$

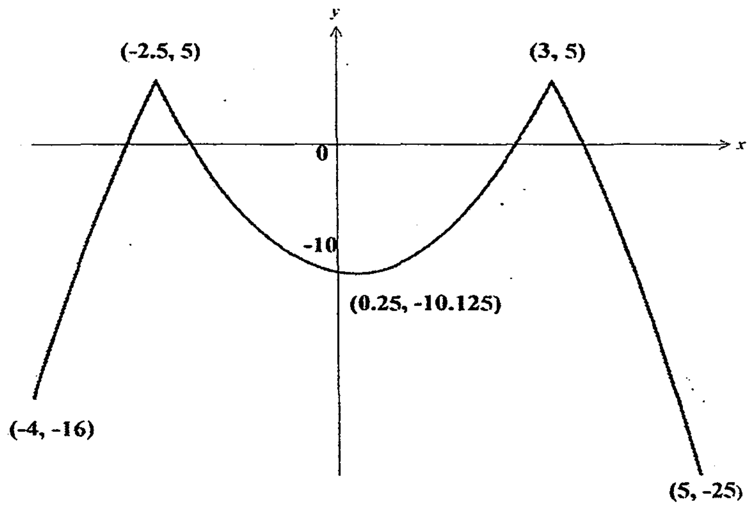
NHHS Preliminary Examination 2014- Sec 4 Additional Mathematics Paper 2 Solution

Question	Solution
1(a) (i)	$f(x)=(x-2)(x+7)(ax^2+bx+c)$ comparing coefficent of x^4 : $2x^4=ax^4$ $a=2$ $f(1)=-80$ $(1-2)(1+7)(2\times1^2+b\times1+c)=-80$ $b+c=8$-----------(1) $f(-1)=-72$ $(-1-2)(-1+7)(2\times(-1)^2+b\times-1+c)=-72$ $9b-9c=-18$--------(2) Solving Equation (1) and (2) $b=3$ $c=5$ $\therefore f(x)=(x-2)(x+7)(2x^2+3x+5)$ $=2x^4+13x^3-8x^2-17x-70$
1(a)(ii)	$f(x)=0$ $(x-2)(x+7)(2x^2+3x+5)=0$ $(x-2)=0$ or $(x+7)=0$ or $2x^2+3x+5=0$ $x=2$ or $x=-7$ The quadratic equation $2x^2+3x+5=0$ has discriminant $b^2-4ac=3^2-4\times2\times5=-31<0$ It has no real roots. Hence, $f(x)=0$ has exactly 2 real roots.
1(b)	Let $f(x)=4x^3-12x^2+7x-21$ $f(3)=4\times3^3-12\times3^2+7\times3-21=0$ $\therefore (x-3)$ is a factor of $f(x)$. By long division , $f(x)=(x-3)(4x^2+7)$ $\frac{6x^2-3x-2}{4x^3-12x^2+7x-21}=\frac{A}{(x-3)}+\frac{Bx+C}{(4x^2+7)}$ $6x^2-3x-2=A(4x^2+7)+(Bx+C)(x-3)$ Sub $x=3$, $6\times3^2-3\times3-2=A(4\times3^2+7)$ $A=1$

NHHS Preliminary Examination 2014- Sec 4 Additional Mathematics Paper 2 Solution

Question	Solution

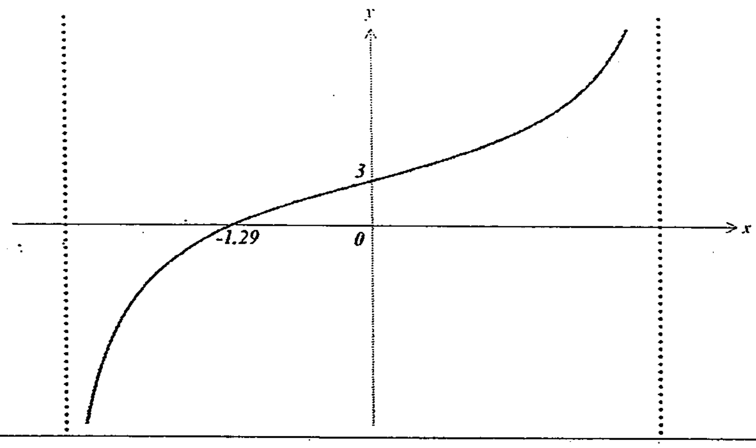
	Sub $x=0$, $6\times0^2-3\times0-2=1\times(4\times0^2+7)+C(0-3)$
	 $C=3$
	 Sub $x=1$, $6\times1^2-3\times1-2=1\times(4\times1^2+7)+(B+3)(1-3)$
	 $B=2$
	 $\dfrac{6x^2-3x-2}{4x^3-12x^2+7x-21}=\dfrac{1}{(x-3)}+\dfrac{2x+3}{(4x^2+7)}$

2(a)(i)	$y=5-\left\|2x^2-x-15\right\|$ 
2(a)(ii)	Value of $k=5$ or $-16\le k<-10\frac{1}{8}$.
2(b)	$\left\|-3x+18\right\|=6x+\left\|x-6\right\|$ $\left\|-3(x-6)\right\|=6x+\left\|x-6\right\|$ $\left\|-3\right\|\left\|x-6\right\|-\left\|x-6\right\|=6x$ $\left\|x-6\right\|=3x$ $x-6=3x$ or $x-6=-3x$ $x=-3$ or $x=1\frac{1}{2}$ Check: $x=-3$ LHS $=\left\|-3(-3)+18\right\|=27$ RHS $=6(-3)+\left\|-3-6\right\|=-9\ne LHS$ $\therefore x=-3$ is rejected. $x=1\frac{1}{2}$ LHS $=\left\|-3(1\frac{1}{2})+18\right\|=13.5$ RHS $=6(1\frac{1}{2})+\left\|1\frac{1}{2}-6\right\|=13.5=$ LHS

NHHS Preliminary Examination 2014- Sec 4 Additional Mathematics Paper 2 Solution

Question	Solution
	$\therefore\ x=1\frac{1}{2}$
3(a)(i)	$6\sin^2\frac{x}{2}-2\cos^2 x=1$ $3(1-\cos x)-2\cos^2 x-1=0$ $3-3\cos x-2\cos^2 x-1=0$ $2\cos^2 x+3\cos x-2=0$ $(2\cos x-1)(\cos x+2)=0$ $\cos x=-2\ (rej\ \text{since}\ -1\le\cos x\le 1)$ or $\cos x=\frac{1}{2}$ $x=60°,\ 300°$
3(a)(ii)	There are 5 complete revolutions in $-1080°<x<720°$. No. of solutions = 5 x 2=10
3(b)	$\sin 2y=\cos^2 y$ $2\sin y\cos y=\cos^2 y$ $\cos y(2\sin y-\cos y)=0$ $\cos y=0$ $y=\frac{\pi}{2}\ or\ \frac{3\pi}{2}\ or\ \frac{5\pi}{2}$ $or\quad \tan y=\frac{1}{2}$ Basic angle $=0.463648$ $y=0.464, 3.61, 6.75$ $\quad Ans: y=0.464, \frac{\pi}{2}, 3.61, \frac{3\pi}{2}, 6.75, \frac{5\pi}{2}$
4 (i)	The period of f is 2π.
4(ii)	$a\tan\left(\frac{0}{2}\right)+b=3$ $b=3$ $a\tan\left(\frac{\pi/2}{2}\right)+3=7$ $a\tan\left(\frac{\pi}{4}\right)=4$ $a=4$

NHHS Preliminary Examination 2014- Sec 4 Additional Mathematics Paper 2 Solution

Question	Solution
4(iii)	$y = 4\tan\left(\dfrac{x}{2}\right) + 3$ 
5	$\dfrac{d}{d\theta}\left(\tan^3 6\theta\right) = 3\tan^2 6\theta \sec^2 6\theta .\ 6$ $= 18\tan^2 6\theta \sec^2 6\theta$
5(i)	$\int 18\tan^2 6\theta \sec^2 6\theta \, d\theta = \tan^3 6\theta + C_1$ $\int \tan^2 6\theta \sec^2 6\theta \, d\theta = \dfrac{1}{18}\tan^3 6\theta + C$
5(ii)	$\int \left(\sec^2 6\theta - 1\right)\sec^2 6\theta \, d\theta = \dfrac{1}{18}\tan^3 6\theta + C$ $\int \sec^4 6\theta \, d\theta - \int \sec^2 6\theta \, d\theta = \dfrac{1}{18}\tan^3 6\theta + C$ $\int \sec^4 6\theta \, d\theta - \dfrac{1}{6}\tan 6\theta = \dfrac{1}{18}\tan^3 6\theta + C$ $\int \sec^4 6\theta \, d\theta = \dfrac{1}{6}\tan 6\theta + \dfrac{1}{18}\tan^3 6\theta + C$

NHHS Preliminary Examination 2014- Sec 4 Additional Mathematics Paper 2 Solution

Question	Solution
6(i)	The mid - point of AC: $E=\left(\frac{6-2}{2},\frac{1+7}{2}\right)$ $=(2,4)$ gradient of $AC=\frac{7-1}{-2-6}=-\frac{3}{4}$ gradient of $BD=\frac{4}{3}$ Equation of BD: $y-4=\frac{4}{3}(x-2)$ $y=\frac{4}{3}x+\frac{4}{3}$ -------------(1) substitute (1) into equation $4y=x^2+7$ $4\left(\frac{4}{3}x+\frac{4}{3}\right)=x^2+7$ $3x^2-16x+5=0$ $(3x-1)(x-5)=0$ $x=\frac{1}{3}$ (rej $\because$ x - coordinate of B should be an integer) $\therefore B(5,8)$ $\therefore x=5$ $y=8$
6(ii)	$BE=\sqrt{(5-2)^2+(8-4)^2}=5$ units $ED=5\times\frac{3}{2}=\frac{15}{2}$ units Let point D (x, y) $\sqrt{(x-2)^2+(y-4)^2}=\frac{15}{2}$ $\quad$ $(x-2)^2+\left(\frac{4}{3}\right)^2(x-2)^2=\frac{225}{4}$ $(x-2)^2+(y-4)^2=\frac{225}{4}$ -----------(2) Substitute $y=\frac{4}{3}(x+1)$ into eq (2) $(x-2)^2+(\frac{4}{3}(x+1)-4)^2=\frac{225}{4}$ $(x-2)^2=\frac{225}{4}\times\frac{9}{25}$ $x-2=4.5$ *or* -4.5 $x=6.5$ (rej since x - coordinate of D is negative) *or* -2.5

<table>
<tr><th>Question</th><th>Solution</th></tr>
<tr><td></td><td>

$y=\left(\frac{4}{3}\right)(-2.5+1)=-2$

$\therefore D(-2.5,-2)$

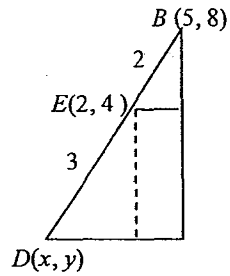
<u>**Alternative Method:**</u>

Considering similar triangles / gradients

$\frac{8-4}{4-y}=\frac{2}{3}$, $\frac{5-2}{5-x}=\frac{2}{5}$

$y=-2$, x = -2.5

D(-2.5, - 2)



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<tr><td>6(iii)</td><td>

gradient of $AB=\frac{8-7}{5+2}=\frac{1}{7}$

gradient of $BC=\frac{8-1}{5-6}=-7$

gradient of $AB\times$ gradient of $BC=-1$

$\therefore AB\perp BC, \angle ABC=90°$

Since A, B and C are points on the circle,

AC is the diameter of the circle. (rt$\angle$ in semi - circle)

E is the mid - point of diameter AC

hence it is the centre of the circle

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<tr><td>6(iv)</td><td>

Area of $ABCD$

$=\frac{1}{2}\begin{vmatrix}-2 & -2.5 & 6 & 5 & -2\\ 7 & -2 & 1 & 8 & 7\end{vmatrix}$

$=\frac{1}{2}(4-2.5+48+35+17.5+12-5+16)$

$=62.5$ units^2

Alternatively,

Area of $ABCD$ $=\frac{1}{2}AC\times BD$

$=\frac{1}{2}\times 10\times 12.5$

$=62.5$ units^2

</td></tr>
<tr><td>7(i)</td><td>

$Volume=\frac{1}{3}\pi r^2(2r)+\pi r^2h=75\pi$

$\frac{2}{3}r^3+r^2h=75$

$h=\frac{75}{r^2}-\frac{2}{3}r$

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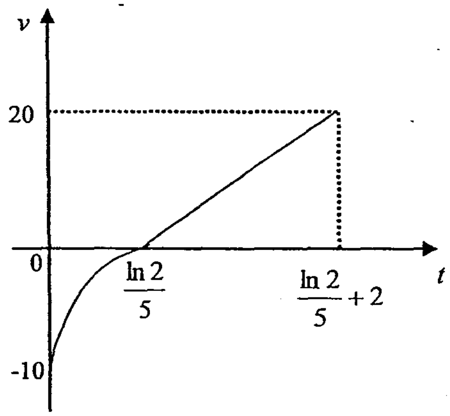
</table>

NHHS Preliminary Examination 2014- Sec 4 Additional Mathematics Paper 2 Solution

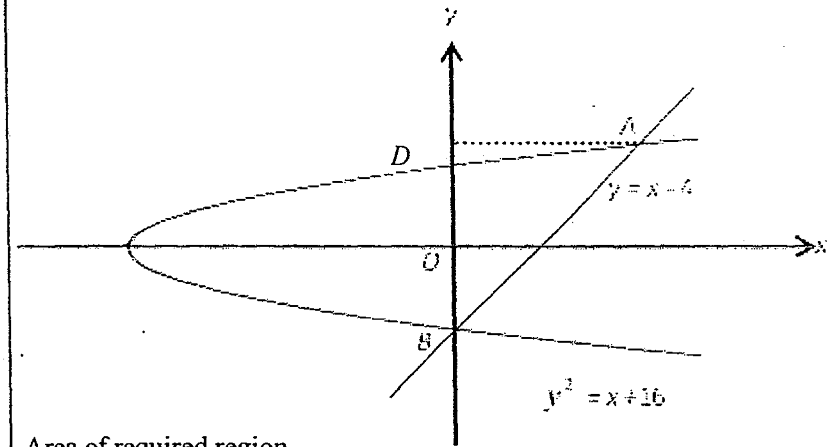
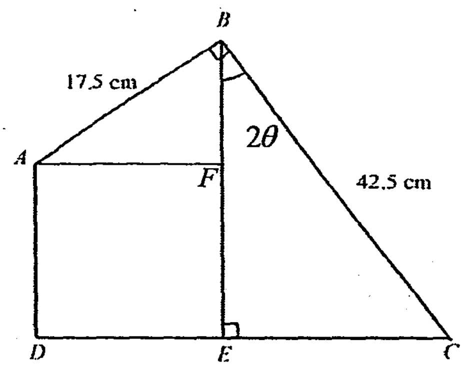
Question	Solution
7(ii)	$h=\frac{75}{r^2}-\frac{2}{3}r$ $\frac{dh}{dr}=\frac{-150}{r^3}-\frac{2}{3}$ $=-\left(\frac{150}{r^3}+\frac{2}{3}\right)<0$ *for all real values of* $r>0$ Therefore, *h is a decreasing funct4ion.*
7(iii)	$A=\pi rl+2\pi rh+\pi r^2$ $=\pi r\sqrt{4r^2+r^2}+2\pi r\left(\frac{75}{r^2}-\frac{2}{3}r\right)+\pi r^2$ $=\pi r^2\sqrt{5}+\frac{150\pi}{r}-\frac{4\pi r^2}{3}+\pi r^2$ $=\pi r^2\sqrt{5}+\frac{150\pi}{r}-\frac{\pi r^2}{3}$ $=\left(\sqrt{5}-\frac{1}{3}\right)\pi r^2+\frac{150\pi}{r}$
7(iv)	$\frac{dA}{dr}=\left(\sqrt{5}-\frac{1}{3}\right)2\pi r-\frac{150\pi}{r^2}$ $\frac{d^2A}{dr^2}=\left(\sqrt{5}-\frac{1}{3}\right)2\pi+\frac{300\pi}{r^3}$ $\frac{d^2A}{dr^2}>0$ *for all real values of* $r>0$ Therefore, the stationary value of A is a minimum.
8(i)	$s=4e^{-5t}+10t-7$ Initial position $=4e^{0(-5)}+10(0)-7$ $=4-7$ $=-3$m
8(ii)	$v=-20e^{-5t}+10$ Initial velocity $=-20e^{-5(o)}+10$ $=-10m/s$

Question	Solution
8(iii)	$v = -20e^{-5t} + 10$ At Instantaneous rest, v = 0 $-20e^{-5t} + 10 = 0$ $e^{-5t} = \frac{1}{2}$ $t = -\frac{1}{5}\ln\left(\frac{1}{2}\right)$ $\therefore T = -\frac{1}{5}(-\ln 2)$ $= \frac{\ln 2}{5} (shown)$
8(vi)	
8(v)	at t = 0, $s = -3$ m at $t = \frac{\ln 2}{5}$, $s = 4e^{-5\left(\frac{\ln 2}{5}\right)} + 10\left(\frac{\ln 2}{5}\right) - 7$ $= (2\ln 2 - 5)$m or -3.6137 m Since in the next 2 seconds, the particles travels at a constant acceleration, its velocity at $t = \frac{\ln 2}{5} + 2$ is $(0 + 2\times 10) = 20$m/s hence the distance travelled $= \frac{2\times 20}{2} = 20$m $\therefore$ the total distance travelled in the first (T + 2) seconds $= -3 - (2\ln 2 - 5) + 20$ $= 22 - 2\ln 2 = 20.6$ m

NHHS Preliminary Examination 2014- Sec 4 Additional Mathematics Paper 2 Solution

Question	Solution
9(i)	$\angle EAD = \angle CAB$ (common$\angle$) $AE = EC, \quad AD = DB$ (given) $\therefore \frac{AE}{AC} = \frac{AD}{AB} = \frac{1}{2}$ $\therefore \Delta EAD$ is similar to ΔCAB (SAS similarity) $\frac{AE}{AC} = \frac{ED}{BC}$ (corr.sides of similar Δs) $AE \times BC = AC \times ED$ (shown) **Alternatively,** $AE = EC, \quad AD = DB$ (given) $\frac{AE}{AC} = \frac{AD}{AB} = \frac{1}{2}$ $\frac{ED}{BC} = \frac{1}{2}$ (mid - point theo rem) $\frac{AE}{AC} = \frac{ED}{BC}$ $AE \times BC = AC \times ED$ (shown)
9(ii)	$\angle GBD = \angle DBF$ (common$\angle$) $\angle GDB = \angle DFB$ (tangent - chord theorem) $\therefore \Delta GBD$ is similar to ΔDBF (AA similarity) $\frac{BD}{BF} = \frac{GB}{DB}$ (corr.sides of similar Δs) $BD^2 = BG \times BF$ (shown)
10(i)	$(x-4)^2 = x + 16$ $x^2 - 8x + 16 - 16 - x = 0$ $x^2 - 9x = 0$ $x(x-9) = 0$ $x = 0 \quad or \quad 9$ $y = -4 \quad or \quad 5$ $\therefore A\ (9, 5)$ and $B\ (0, -4)$

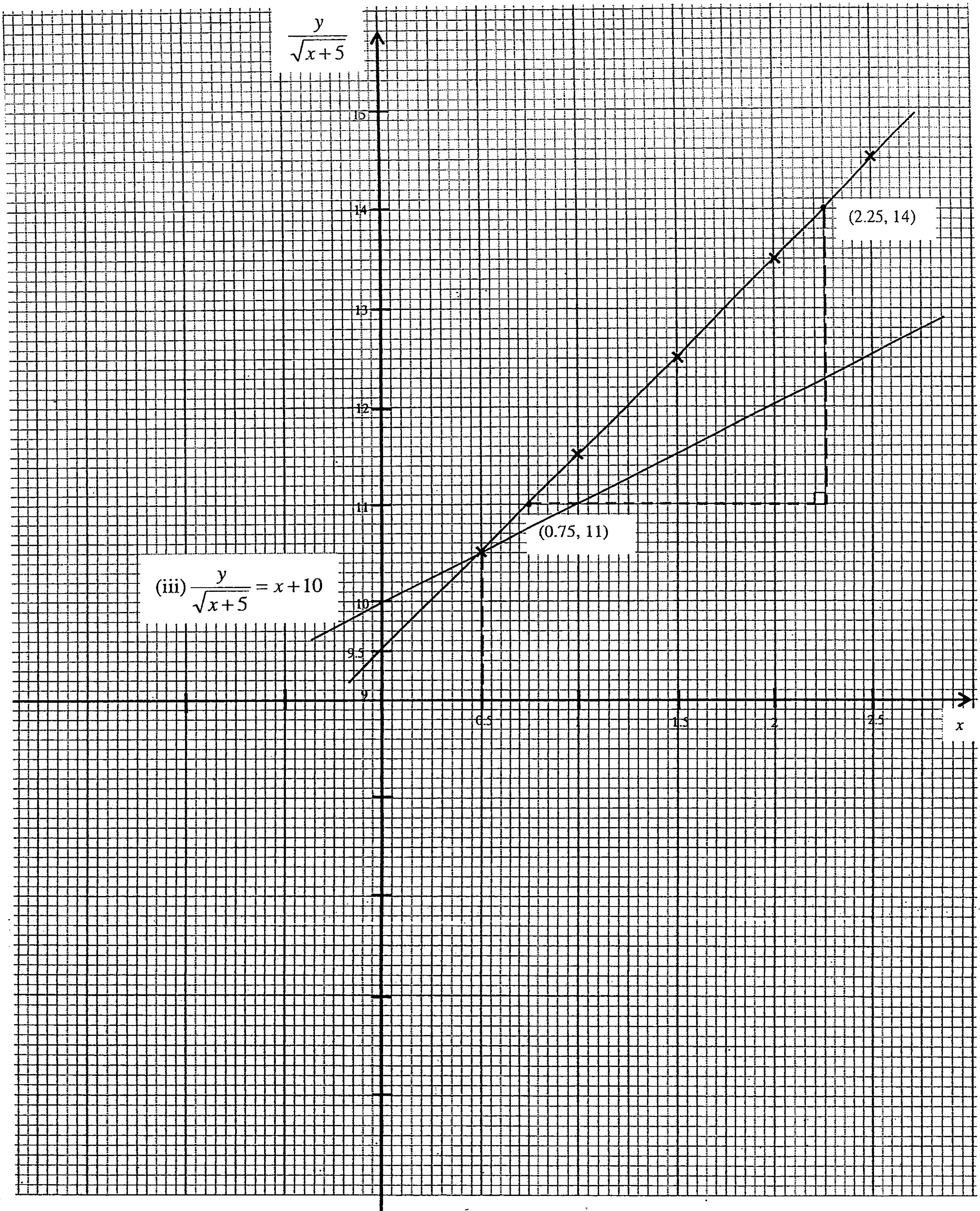
NHHS Preliminary Examination 2014- Sec 4 Additional Mathematics Paper 2 Solution

Question	Solution
10(ii)	 Area of required region $= \text{Area of } \Delta ADB - \int_4^5 (y^2-16)\,dy + 2\times\left\|\int_0^4 (y^2-16)\,dy\right\|$ $= \dfrac{9\times(5+4)}{2} - \left(\dfrac{y^3}{3}-16y\right)_4^5 + 2\left\|\left(\dfrac{y^3}{3}-16y\right)_0^4\right\|$ $= 40.5 - 4\dfrac{1}{3} + 85\dfrac{1}{3}$ $=121.5 \text{ units}^2$
11(i)	$EC = 42.5\sin(2\theta)$ $BE = 42.5\cos(2\theta)$  $\angle ABF + \angle BAF = 90°$ $\angle ABF + \angle EBC = 90°$ $\angle BAF = \angle EBC = 2\theta$ $DE = AF = 17.5\cos(2\theta)$ $BF = 17.5\sin(2\theta)$ $AD = BE - BF = 42.5\cos(2\theta) - 17.5\sin(2\theta)$ $P = AB + BC + EC + DE + AD$ $= 17.5 + 42.5 + 42.5\sin(2\theta) + 17.5\cos(2\theta) +$ $42.5\cos(2\theta) - 17.5\sin(2\theta)$ $= 60 + (42.5 - 17.5)\sin(2\theta) + (17.5 + 42.5)\cos(2\theta)$ $= 60 + 60\cos(2\theta) + 25\sin(2\theta)$ (shown)

NHHS Preliminary Examination 2014- Sec 4 Additional Mathematics Paper 2 Solution

Question	Solution
11(ii)	Let $60\cos(2\theta)+25\sin(2\theta) = b\sin(2\theta+\alpha)$ $= b\sin(2\theta)\cos\alpha + b\cos(2\theta)\sin\alpha$ $b\cos\alpha = 25 ------(1)$ $b\sin\alpha = 60 ------(2)$ $(1)^2+(2)^2:\quad b^2(\cos^2\alpha+\sin^2\alpha)=25^2+60^2$ $b^2=65^2$ $b=65 \ \ or \ \ -65 \ \ (rej \because b>0)$ $\frac{(2)}{(1)}:\quad \frac{b\sin\alpha}{b\cos\alpha}=\frac{60}{25}$ $\tan\alpha = 2.4$ $\alpha = 67.380°$ $\therefore P = 60+65\sin(2\theta+67.4°)$
11(iii)	$100 = 60+65\sin(2\theta+67.380°)$ $40 = 65\sin(2\theta+67.380°)$ $\sin(2\theta+67.380°)=\frac{40}{65}$ Basic $\angle = 37.97987°$ $0° < 2\theta < 90°$ $67.380° < 2\theta+67.380° < 157.380°$ $\therefore 2\theta+67.380° = 180°-37.97987° = 142.02°$ $\theta = 37.3°$
11(iv)	Maximum value of P occurs when $\sin(2\theta+67.380°)=1$ $(2\theta+67.380°)=90°$ $\therefore$Corresponding value of $\theta = 11.3°$ Maximum value of $P = 60+65\times 1 = 125$ cm

*Q*11



Paya Lebar Methodist Girls' School 3 ADDITI L MATHEMATICS

Paper 1

1 Given that the line $y = x + a$ is a tangent to the curve $y^2 = bx$, where a and b are positive integers, prove that $\frac{b}{a}$ is a perfect square. [4]

2 Find the coordinates of the stationary point of the curve $f(x) = 1 - \frac{2}{x^2+1}$, and determine the nature of the stationary point. [5]

3 It is given that $\int_{-3}^{2} f(x)\,dx = a$ and $\int_{-1}^{-3} f(x)\,dx = b$, where a and b are constants, and $f(x) > 0$ for all real values of x.

Find in terms of a and b,

(i) $\int_{-1}^{2} f(x)\,dx$, [2]

(ii) $\int_{-1}^{2} [2f(x) - x]\,dx$. [3]

4 It is given that $2^{3-x} = 3^{2(1-x)}$.

(i) Find the exact value of 4.5^x. [3]

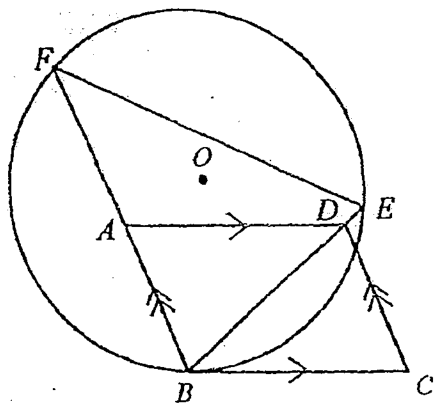
(ii) Hence find the value of x, correct to 3 significant figures. [2]

5 The roots of the equation $x^2 + px + 5 = 0$, where p is an integer, are α and β.
The roots of the equation $x^2 + 18x + q = 0$, where q is an integer, are α^3 and β^3.

(i) Express $\alpha^2 - \alpha\beta + \beta^2$ in terms of $(\alpha+\beta)$ and $\alpha\beta$. [1]

(ii) Find the value of p and of q. [6]

6



In the diagram, BC is a tangent to the circle BEF, and $ABCD$ is a parallelogram.

Prove that

(i) $\angle BAD = \angle FEB$, [3]

(ii) ΔBCD is similar to ΔFEB, [2]

(iii) $BD \times BE = BA \times BF$. [2]

7 (i) Express the equation

$$8\cot^2 2\theta = 16\operatorname{cosec} 2\theta - 15$$

as a quadratic equation in cosec 2θ. [2]

(ii) Hence solve the equation

$$8\cot^2 2\theta = 16\operatorname{cosec} 2\theta - 15$$

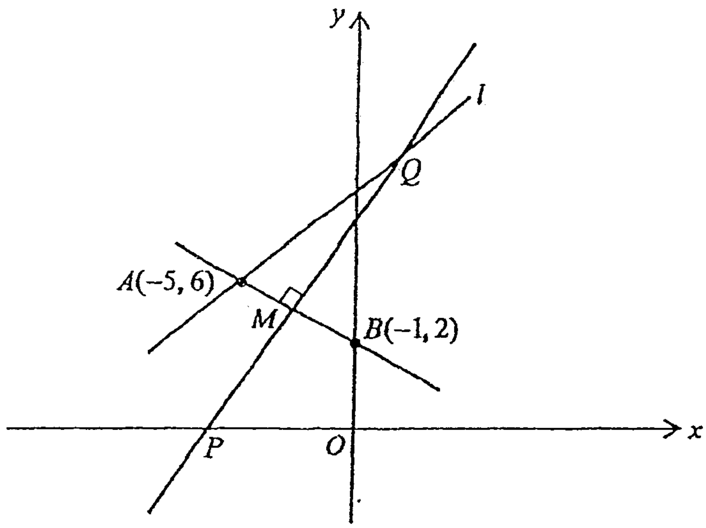
for $0° \le \theta \le 180°$. [4]

(iii) State the number of solutions of the equation

$$8\cot^2 2\theta = 16\operatorname{cosec} 2\theta - 15$$

in the range $-360° \le \theta \le 360°$. [1]

8 **Solutions to this question by accurate drawing will not be accepted.**



In the diagram, M is the midpoint of the line joining the points $A(-5, 6)$ and $B(-1, 2)$. The perpendicular bisector of AB intersects the x-axis at the point P and the line l at the point Q.

Given that the line l is parallel to the line PB, find

(i) the coordinates of Q, [4]

(ii) the area of the quadrilateral $APBQ$. [2]

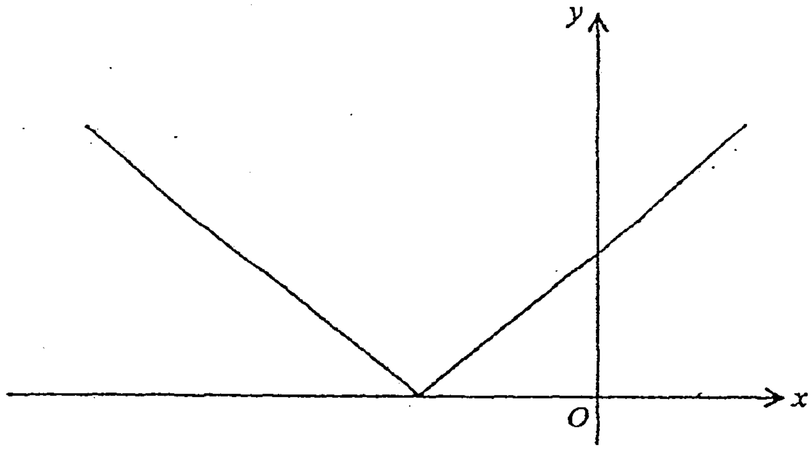
9 A particle travels in a straight line, so that t seconds after passing through a fixed point O, its velocity, v ms^{-1}, is given by $v = 3(e^{kt} + 2)$, where k is a constant.
The initial acceleration of the particle is $\frac{3}{5}$ms^{-2}.

(i) Show that the value of k is $\frac{1}{5}$. [2]

(ii) Explain why the particle would never return to O. [2]

(iii) Find the total distance travelled by the particle in the first 5 seconds. [3]

10



The diagram shows part of the graph of $y=|x+2|$. In each of the following cases, determine the number of intersections of the line $y=mx+c$ with $y=|x+2|$, justifying your answer.

(i) $m=1$ and $c>2$ [2]

(ii) $m=\frac{1}{2}$ and $c<1$ [2]

(iii) $m=-\frac{1}{2}$ and $c=0$ [2]

11 The function f is defined, for all values of x by $f(x)=2-a\sin bx$ where a and b are positive integers. The minimum value of $f(x)$ is -1. Two complete cycles of the graph of $y=f(x)$ can be sketched in the interval $-\pi \le x \le \pi$.

(i) Find the value of a and of b. [2]

Using the values of a and of b found in part (i),

(ii) find, in radians, the largest negative value of x for which $f(x)=2$, [1]

(iii) sketch the graph of $y=f(x)$ for $-\pi \le x \le \pi$, [2]

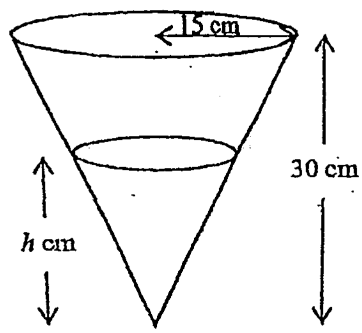
(iv) identify a possible pair of values for c and d, if the equation $a\sin bx + c\cos dx = 2$, where c and d are positive integers, has exactly 5 solutions in the interval $-\pi \le x \le \pi$. [2]

12 Using experimental values of variables x and y, a graph was drawn in which $\ln y$ was plotted on the vertical axis against x on the horizontal axis. The straight line that was obtained passed through the points $(2, 3)$ and $(10, 7)$.

(i) Show that the equation connecting x and y can be expressed in the form $y = ae^{bx}$ where a and b are constants. [4]

(ii) Find the coordinates of the point on the line at which $y = e^{2x+1}$ [3]

13



The diagram shows a hollow conical container of height 30 cm and radius 15 cm. The container that is initially completely filled with water is held fixed with its circular rim horizontal. Water is leaking from a hole at the vertex of the container at a constant rate of $10\ \text{cm}^3\text{s}^{-1}$. After t seconds, the depth of water is h cm.

(i) Show that the volume of water in the container, V cm^3, at time t, is given by

$$V = \frac{\pi h^3}{12}.$$ [2]

(ii) Find the rate of change of the depth when $h = 2$. [4]

(iii) State, with a reason, whether this rate would increase or decrease as t increases. [1]

End of Paper

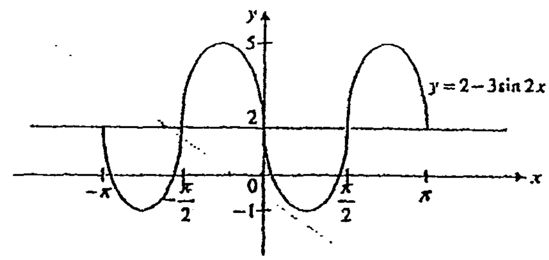
Paya Lebar Methodist Girls' School(Secondary)

Preliminary Examination 2 (2014)

Additional Mathematics (4047/1)

Answer Key

Qn	Answer	Qn	Answer
1	$\frac{b}{a} = 4$, a perfect square	7	(i) $8\text{cosec}^2 2\theta - 16\text{cosec}2\theta + 7 = 0$ (ii) $\text{cosec}2\theta = \frac{4 \pm \sqrt{2}}{4}$, $\theta = 23.8°,\ 66.2°$ (iii) 8 solutions
2	Stationary point = (0, −1) It is a minimum point	8	(i) $Q(1, 8)$ (ii) 32 square units
3	(i) $\int_{-1}^{2} f(x)\,dx = a + b$ (ii) $\int_{-1}^{2} [2f(x) - x]\,dx = 2(a+b) - \frac{3}{2}$	9	(i) $k = \frac{1}{5}$ (ii) Velocity of the particle is always positive ⇒ the particle is always moving away from O and to the right. (iii) 55.8 m (3 s.f.)
4	(i) $4.5^x = \frac{9}{8}$ (ii) $x = 0.0783$ (3 s.f.)	10	(i) For $m = 1$ and $c > 2$, the line $y = mx + c$ is above the right arm and parallel to it. ∴ The line $y = mx + c$ intersects the left arm at one point. Number of intersections = 1 (ii) For $m = \frac{1}{2}$ and $c = 1$, the line $y = mx + c$ passes through the points $(-2, 0)$ and $(0, 1)$. For $m = \frac{1}{2}$ and $c < 1$, the line $y = mx + c$ lies entirely below the graph of $y = \lvert x + 2 \rvert$. Number of intersection = 0 (iii) For $m = -\frac{1}{2}$ and $c = 0$, the line $y = mx + c$ passes through the origin, and the points $(-2, 1)$ and $(-4, 2)$. The line $y = mx + c$ intersects the graph of $y = \lvert x + 2 \rvert$ twice. Number of intersections = 2.

5	(i) $\alpha^2-\alpha\beta+\beta^2=(\alpha+\beta)^2-3\alpha\beta$ (ii) $p=-3$, $q=125$	11	(i) $a=3$, $b=2$ (ii) Largest negative value of $x=-\frac{\pi}{2}$ (iii)  (iv) one possible pair $c=d=2$.
6	(i) Show that $\angle ADB=\angle BFE$ (ii) Show that $\angle BCD=\angle FEB$, - Show that $\angle BDC=\angle FBE$ (iii) Using part (ii), $\frac{BD}{FB}=\frac{CD}{EB}$	12	(i) $y=ae^{bx}$ where $a=e^2$ $b=\frac{1}{2}$ (ii) $\left(\frac{2}{3},\frac{7}{3}\right)$
		13	(i) $V=\frac{\pi h^3}{12}$ (ii) -3.18 cms^{-1} (iii) Since $\frac{dh}{dt}=-\frac{40}{\pi h^2}$, the rate of change of the depth will increase as t increases.

1 (i) Differentiate $x\cos\frac{x}{2}$ with respect to x. [2]

(ii) Using the answer to part (i), find $\int 2x\sin\frac{x}{2}\,dx$,

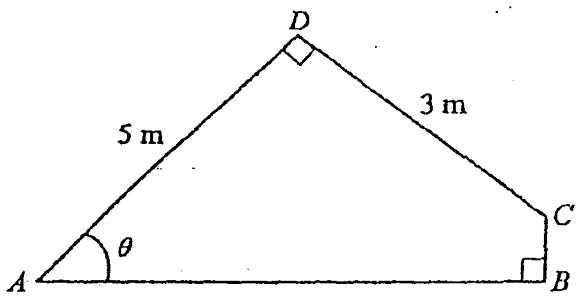
and hence show that $\int_0^{\frac{\pi}{3}} 2x\sin\frac{x}{2}\,dx = 2\left(2-\frac{\sqrt{3}\pi}{3}\right)$. [5]

2 (i) On the same diagram, sketch the graphs of $y=-\frac{1}{16}x^{\frac{4}{3}}$ and $y=-16x^{-\frac{4}{3}}$ for $x>0$. [2]

(ii) Find the coordinates of the point of intersection of the graphs. [2]

(iii) Determine, with explanation, whether the normal to the graph of $y=-\frac{1}{16}x^{\frac{4}{3}}$ and the tangent to the graph of $y=-16x^{-\frac{4}{3}}$, at the point of intersection, are perpendicular. [4]

3



The diagram shows an enclosure where $AD=5$ m, $CD=3$m and $\angle ADC=90°$.
The side AD makes an acute angle θ with the side AB.
The sides AB and BC are perpendicular to each other.
The total length of the enclosure is L m.

(i) Show that L can be expressed as $a+b\cos\theta+c\sin\theta$, where a, b and c are constants to be found. [3]

(ii) Express L in the form $a+R\cos(\theta-\alpha)$ where $R>0$ and α is an acute angle. [4]

The total length of the enclosure is found to be 11.8 m.

(iii) Find the value of θ. [2]

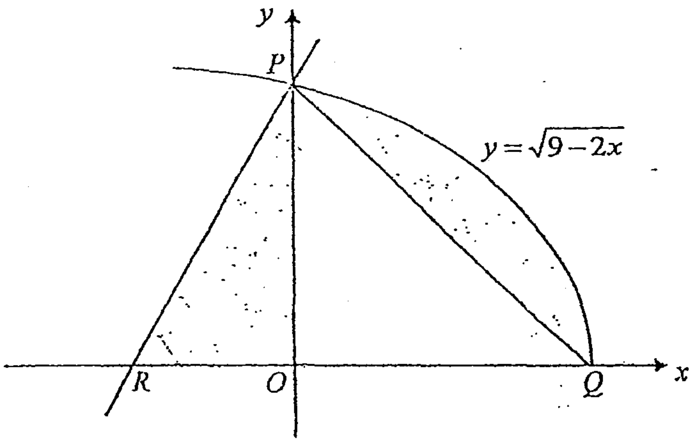
4 Given that $\dfrac{2x^3-3x^2-11x+20}{x^2-9} = 2x+a+\dfrac{bx+c}{x^2-9}$,

(i) find the value of each of the integers a, b and c. [4]

Hence using partial fractions and the values of a, b and c obtained in part (i), find

(ii) $\displaystyle\int \frac{2x^3-3x^2-11x+20}{x^2-9}\,dx.$ [6]

5



The diagram shows part of the curve $y=\sqrt{9-2x}$ that crosses the axes at the points P and Q. The lines PQ and PR are perpendicular to each other and intersect the x-axis at the points R and Q.

(i) Find the coordinates of P, Q and R. [4]

(ii) Find the total area of the shaded regions. [6]

6 A curve has the equation $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = \dfrac{\ln(1-x)}{x-1}$ for $x < 1$.

(i) Obtain an expression for $\mathrm{f}'(x)$. [2]

(ii) The tangent to the curve at the point where $x = -1$ intersects the y-axis at the point A. Find the exact coordinates of the point A. [4]

(iii) Showing all necessary working, find the range of values of x for which f is a decreasing function. [3]

(iv) Hence deduce the range of values of x for which f is an increasing function. [1]

7 (i) Write down the first three terms in the expansion, in ascending powers of x, of $\left(1-\dfrac{x}{3}\right)^n$, where n is a positive integer greater than 2. [2]

(ii) Find, in terms of n and p, the first three terms in the expansion, in ascending powers of x, of $\left(2+px+\dfrac{5}{2}x^2\right)\left(1-\dfrac{x}{3}\right)^n$, where p is a constant. [2]

In the expansion of $\left(2+px+\dfrac{5}{2}x^2\right)\left(1-\dfrac{x}{3}\right)^n$, in ascending powers of x, the first three terms are $2+\dfrac{31p}{3}x+\dfrac{25}{3}x^2$.

(iii) Find the value of n and of p. [illegible]

(iv) Hence find the coefficient of x^3 in the expansion of $\left(2+px+\dfrac{5}{2}x^2\right)\left(1-\dfrac{x}{3}\right)$ [illegible]

8 (i) Prove the identity $\cos 4A = 8\sin^4 A - 8\sin^2 A + 1$. [4]

(ii) Solve the equation $16\sin^4 A - 16\sin^2 A = -4$ for $0 < A < \pi$, giving your answers in terms of π. [3]

(iii) Given that $8\sin^4 \frac{x}{2} - 8\sin^2 \frac{x}{2} + 1 = 0$ and $\cos x \neq 0$,

(a) deduce that $\tan x = \pm 1$, [3]

(b) find the possible values of $\sec x$. [2]

9 The points $(-15, -5)$ and $(1, -5)$ are on the circumference of a circle whose centre lies below the x-axis. The line $y = -21$ is a tangent to the circle.

Find

(i) the centre and the radius of the circle, [8]

(ii) the equation of the circle in the form $x^2 + y^2 + ax + by + c = 0$, where a, b and c are integers, [2]

(iii) the equations of the tangents to the circle that are parallel to the y-axis. [2]

10 (a) Given that $u = \log_7 x$, find, in terms of u,

(i) $\log_7\left(7\sqrt{x}\right)$, [1]

(ii) $\log_x\left(\frac{1}{343}\right)$. [2]

(b) Given that $\ln\left(x^2 y\right) = p$ and $\ln\left(xy^2\right) = q$, express $\frac{x}{y}$ in terms of p and q. [3]

(c) In an experimental environment, the population of a certain bacteria can be modelled by the equation $P = 500 + Ae^{kt}$, where A and k are constants and t is the time in days. During the first 3 days of the experiment, the population of the bacteria decreased exponentially from 5500 to 5000, find

(i) the value of A and of k. [3]

If the population of the bacteria continues to decrease at the same rate,

(ii) after how many days would it first decrease to below 1000? [3]

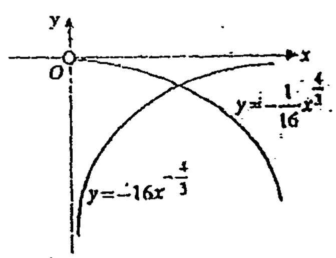
End of Paper

Paya Lebar Methodist Girls' School(Secondary)

Preliminary Examination 2 (2014)

Additional Mathematics (4047/2)

Answer Key

Qn	Answer	Qn	Answer
1	(i) $\frac{d}{dx}\left(x\cos\frac{x}{2}\right)=-\frac{x}{2}\sin\frac{x}{2}+\cos\frac{x}{2}$ (ii) $\int 2x\sin\frac{x}{2}\,dx=8\sin\frac{x}{2}-4x\cos\frac{x}{2}+C$ $\int_0^{\frac{\pi}{3}} 2x\sin\frac{x}{2}\,dx=2\left(2-\frac{\sqrt{3}\pi}{3}\right)$	7	(i) $\left(1-\frac{x}{3}\right)^n=1-\frac{n}{3}x+\frac{n(n-1)}{18}x^2+\ldots$ (ii) $1+\left(p-\frac{2n}{3}\right)x+\left[\frac{n(n-1)}{9}-\frac{np}{3}+\frac{5}{2}\right]x^2+\ldots$ (iii) $n=7$ and $p=-\frac{1}{2}$ (iv) $=-\frac{259}{27}$
2	(i)  (ii) Point of intersection = $(8,-1)$ (iii) The normal to the graph of $y=-\frac{1}{16}x^{\frac{4}{3}}$ is not perpendicular to the tangent to the graph of $y=-16x^{-\frac{4}{3}}$ as (gradient of normal)×(gradient of tangent) $=6\times\frac{1}{6}\neq-1$	8	(i) Proof (ii) $x=\frac{\pi}{4},\frac{3\pi}{4}$ (iii) (a) $\tan x=\pm1$ (b) $\sec x=\pm\sqrt{2}$

3	(i) $L=8+2\cos\theta+8\sin\theta$ where $a=8$, $b=2$ and $c=8$ (ii) $L=8+2\sqrt{17}\cos(\theta-76.0^\circ)$ (iii) $\theta=13.4^\circ$	9	(i) Centre of circle $=(-7,-11)$ Radius of circle $=10$ units (ii) $a=14$, $b=22$ and $c=70$ (iii) $x=-17$ and $x=3$
4	(i) $\dfrac{2x^3-3x^2-11x+20}{x^2-9}=2x-3+\dfrac{7x-7}{x^2-9}$ (ii) $\int\dfrac{2x^3-3x^2-11x+20}{x^2-9}\,dx$ $=\int\left[2x-3+\dfrac{14}{3(x+3)}+\dfrac{7}{3(x-3)}\right]dx$ $=x^2-3x+\dfrac{14}{3}\ln(x+3)+\dfrac{7}{3}\ln(x-3)+C$	10	(a) (i) $\log_7\left(7\sqrt{x}\right)=1+\dfrac{u}{2}$ (ii) $\log_x\left(\dfrac{1}{343}\right)=-\dfrac{3}{u}$ (b) $\dfrac{x}{y}=e^{p-q}$ (c) (i) $A=5\,000$ $k\approx-0.0351$ (to 3 s.f.) (ii) 66 days
5	(i) $P=(0,3)$, $Q=(4\frac{1}{2},0)$, $R=(-2,0)$ (ii) Total area of shaded regions $=\frac{1}{2}(2)(3)+\int_0^{\frac{9}{2}}\sqrt{9-2x}\,dx+\frac{1}{2}\left(\frac{9}{2}\right)(3)$ $=5\frac{1}{4}$ sq. units		
6	(i) $f'(x)=\dfrac{1-\ln(1-x)}{(x-1)^2}$ (ii) $A=\left(0,\dfrac{1-3\ln 2}{4}\right)$ (iii) f is a decreasing function for $x<1-e$ (iv) f is a increasing function for $1-e<x<1$		

Name: ______________________ (   )   Class: ___________

PRELIMINARY EXAMINATION
GENERAL CERTIFICATE OF EDUCATION ORDINARY LEVEL

ADDITIONAL MATHEMATICS — 4047/01

Paper 1 — **27 August 2014**

2 hours

Additional Materials: Answer Paper
Graph Paper (1 sheet)

READ THESE INSTRUCTIONS FIRST

Write your name, class, and register number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue, or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, staple all your work together with this cover sheet.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is **80**.

FOR EXAMINER'S USE

Q1		Q6		Q11	
Q2		Q7		Q12	
Q3		Q8		Q13	
Q4		Q9			
Q5		Q10			

/ 80

This document consists of 5 printed pages and 1 blank page.

圣尼各拉女校
CHIJ ST NICHOLAS GIRLS' SCHOOL
Girls of Grace · Women of Strength · Leaders with Heart

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \ldots + \binom{n}{r} a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!\,r!} = \frac{n(n-1)\ldots\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1.$$

$$\sec^2 A = 1 + \tan^2 A.$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A.$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 Express $\dfrac{\sqrt{5}}{5-2\sqrt{5}}+\sqrt{125}$ in the form $a+b\sqrt{5}$, where a and b are integers. [3]

2 Find the smallest value of the integer a for which $5x^2-ax+3$ is positive for all values of x. [3]

3 (i) Sketch the graph of $y=e^{3-x}-1$. [2]

(ii) Find the equation of the straight line which must be drawn on the graph $y=e^{3-x}-1$ to obtain the solution of the equation

$$\ln(3x+2)=\ln 2+(3-x).$$

[3]

4 The roots of the equation $x^2-3x+10=0$ are α and β. Find the quadratic equation whose roots are α^3 and β^3. [5]

5 A particle moves along the curve $y=\dfrac{e^{2x}}{x}$. Given that the y-coordinate of the particle is changing at a constant rate of 6 units per second, find the rate of change of the x-coordinate when $x=2$, giving your answer in terms of e. [5]

6 (i) In the binomial expansion of $\left(x^2+\dfrac{k}{x}\right)^6$, where k is a positive constant, the term independent of x is 240. Find the value of k. [3]

(ii) Using the same value of k found in part (i), find the coefficient of x^{12} in the expansion of $\left(1-5x^6\right)\left(x^2+\dfrac{k}{x}\right)^6$. [3]

7 Jim buys a new car. After t months its value $\$V$ is given by $V=120\,000e^{-kt}$, where k is a constant.

(i) Find the value of the car when Jim bought it. [1]

The value of the car after 12 months is expected to be \$ 100 000.

(ii) Calculate the expected value of the car after 4 years, giving your answer to 3 significant figures. [3]

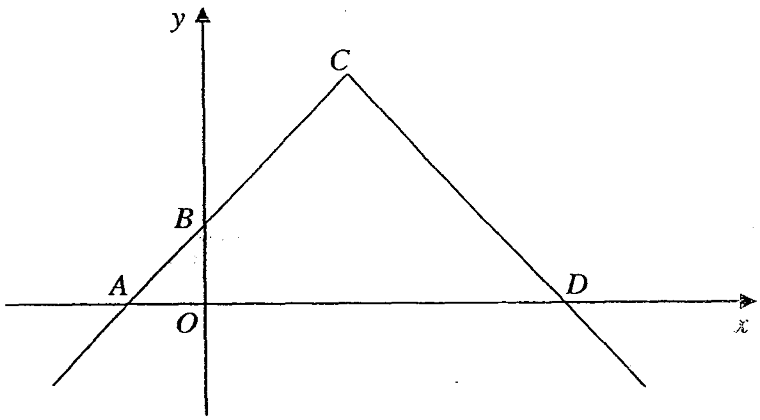
(iii) Find the age of the car, to the nearest month, when its expected value will be \$ 30 000. [2]

8 Given that $f(x) = \sin 2x - x$ for $0 \le x \le \pi$,

(i) sketch the graph represented by $y = f'(x)$,

(ii) find the range of values of x for which $f(x) = \sin 2x - x$ is an increa function of x, giving your answer in terms of π.

9



The diagram shows part of the graph of $y = 9 - |2x - 6|$.

(a) Find the coordinates of A, B, C and D.

(b) The line $y = mx$, where $m > 0$, intersects the graph of $y = 9 - |2x - 6$ one point. Find the minimum value of m.

10 A particle moves in a straight line, so that, t seconds after leaving a fixed po velocity v m/s, is given by $v = t(2 - 3t)^2$. Find

(i) an expression for the acceleration of the particle in terms of t,

(ii) the distance travelled by the particle before it comes to instantaneous re

11 **(i)** By squaring $\sin^2\theta + \cos^2\theta$, or otherwise, show that

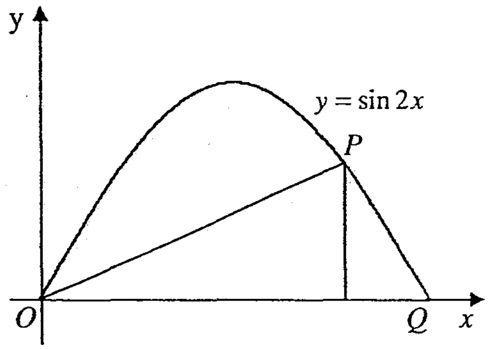
$$\sin^4\theta + \cos^4\theta = \frac{1}{4}(3 + \cos 4\theta).$$

Hence,

(ii) solve the equation $\sin^4\theta + \cos^4\theta = \frac{3}{4} + \frac{\sqrt{3}}{8}$ for $0 \le \theta \le \pi$ radians, giving your answers in terms of π.

(iii) state, for $-\pi \le \theta \le \pi$, the number of solutions of the equation $\sin^4\theta + \cos^4\theta = \frac{3}{4} + \frac{\sqrt{3}}{8}$.

12



The diagram shows part of the curve $y = \sin 2x$. P is a point on the curve and its y-coordinate is $\frac{\sqrt{3}}{2}$. The curve meets the x-axis at Q.

Find

(i) the coordinates of P and of Q in terms of π, [4]

(ii) the area of the shaded region. [5]

13 The table shows experimental values of two variables, x and y, which are connected by an equation of the form $y = \frac{a}{e^{kx}}$, where a and k are constants.

x	1	2	3	4	5	6
y	5.58	7.79	10.87	15.17	21.18	29.56

(i) Plot $\ln y$ against x for the given data and draw a straight line graph. [2]

(ii) Use your graph to estimate the value of a and of k. [4]

(iii) On the same diagram, draw the line representing $y^2 = e^{-4x+8}$ and hence find the value of x for which $ae^{2x-4} = e^{kx}$. [3]

SNGS Additional Mathematics Prelim Paper 1

1.	$2+6\sqrt{5}$	8(ii)	$0\le x<\frac{\pi}{6}$, $\frac{5\pi}{6}<x\le\pi$
2.	Smallest integer $a=-7$	9(a)	$A\left(-1\frac{1}{2},0\right)$, $B\ (0,3)$ $C=(3, 9)$, $D\left(7\frac{1}{2},0\right)$
3 (ii)	$y=\frac{3}{2}x$	(b)	minimum value of $m=2$
4.	$x^2+63x+1000=0$	10 (i)	$a=4-24t+27t^2$
5.	$\frac{8}{e^4}$ units per sec	(ii)	$\frac{4}{27}$ *or* $0.148m$
6 (i)	$k=2$	11(ii)	$\theta=\frac{\pi}{24},\frac{11\pi}{24},\frac{13\pi}{24},\frac{23\pi}{24}$
(ii)	-299	(iii)	8 solutions
7(i)	$ 120 000	12(i)	$Q\left(\frac{\pi}{2},0\right)$, $P\left(\frac{\pi}{3},\frac{\sqrt{3}}{2}\right)$
(ii)	$ 57 900 (to 3 s.f.)	(ii)	0.547 sq units
(iii)	91 months	13(ii)	$a=4.06\ (3s.f)$ $k=-0.334$
		(iii)	$x=1.15$

SNGS Add Math P1 — 2014

1) $\frac{\sqrt{5}}{5-2\sqrt{5}} + \sqrt{125}$

$= \frac{\sqrt{5}}{5-2\sqrt{5}} \cdot \frac{5+2\sqrt{5}}{5+2\sqrt{5}} + 5\sqrt{5}$

$= \frac{\sqrt{5}(5+2\sqrt{5})}{25-20} + 5\sqrt{5}$

$= \frac{5\sqrt{5}+10}{5} + \frac{25\sqrt{5}}{5}$

$= \frac{10+30\sqrt{5}}{5}$

$= 2 + 6\sqrt{5}$

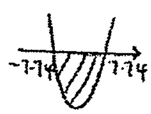
2) $5x^2 - ax + 3 > 0$ for all x,

$b^2 - 4ac < 0$

$(-a)^2 - 4(5)(3) < 0$

$a^2 - 60 < 0$

$(a+2\sqrt{15})(a-2\sqrt{15}) < 0$



$\therefore -7.74 < a < 7.74$

Smallest integer value of a is -7.

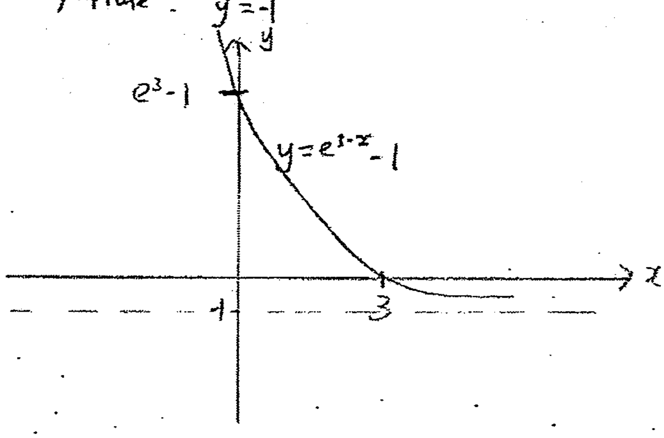
(i) $y = e^{3-x} - 1$

x-intercept : $y = 0$ $\quad e^{3-x} = 1$

$3 - x = 0 \Rightarrow x = 3$

y-intercept : $x = 0$ $\quad y = e^3 - 1 = 19.1$

Asymptote : $y = -1$



3 (ii) $\ln(3x+2) = \ln 2 + (3-x)$

$\ln(3x+2) - \ln 2 = 3 - x$

$\ln\left(\frac{3x+2}{2}\right) = 3 - x$

$\frac{3x+2}{2} = e^{3-x}$

$\frac{3x}{2} + 1 - 1 = e^{3-x} - 1$

$\therefore e^{3-x} - 1 = \frac{3}{2}x$

Equation of straight line to be drawn is $y = \frac{3}{2}x$.

4) $x^2 - 3x + 10 = 0$

Sum of roots, $\alpha + \beta = 3$

Product of roots, $\alpha\beta = 10$

For roots α^3 and β^3,

Sum of roots, $\alpha^3 + \beta^3 = (\alpha+\beta)(\alpha^2 - \alpha\beta + \beta^2)$

$= (\alpha+\beta)[(\alpha+\beta)^2 - 3\alpha\beta]$

$= 3[3^2 - 3(10)]$

$= -63$

Product of roots, $(\alpha\beta)^3 = (10)^3$

$= 1000$

$\therefore$ Quadratic equation is $x^2 + 63x + 1000 = 0$

5) $\frac{dy}{dt} = 6$ units/s

$y = \frac{e^{2x}}{x}$

$\frac{dy}{dx} = \frac{x(2e^{2x}) - e^{2x}}{x^2}$

$= \frac{e^{2x}(2x-1)}{x^2}$

When $x = 2$, $\frac{dy}{dx} = \frac{e^4(4-1)}{4} = \frac{3}{4}e^4$

$\frac{dx}{dt} = \frac{dx}{dy} \times \frac{dy}{dt}$

$= \frac{4}{3e^4} \times 6$

$= \frac{8}{e^4}$ units/s

$\therefore$ The rate of change of x is $\frac{8}{e^4}$ units/s.

(i) $6?(x^2+\frac{k}{x})^6$

$T_{r+1} = \binom{6}{r}(x^2)^{6-r}\cdot\left(\frac{k}{x}\right)^r$

$= \binom{6}{r}(x)^{12-2r}\cdot k^r\cdot x^{-r}$

$= \binom{6}{r}(k)^r\cdot x^{12-3r}$

For term independent of x, $12-3r=0$

$r=4$

$\therefore T_5 = \binom{6}{4}k^4 = 240$

$k^4 = 16$

$k = 2$ or -2 (rejected $\because k>0$)

$\therefore k=2$

ii) $(1-5x^6)(x^2+\frac{2}{x})^6$

For x^6 term in $(x^2+\frac{2}{x})^6$

$12-3r=6$

$r=2$

For x^{12} term in $(x^2+\frac{2}{x})^6$

$12-3r=12$

$r=0$

$\therefore (1-5x^6)(x^2+\frac{2}{x})^6$

$= (1-5x^6)(\cdots+\binom{6}{0}(2)^0x^{12}+\binom{6}{2}(2)^2x^6+\cdots)$

Coefficient of x^{12} is

$1(1)-5\binom{6}{2}(4)$

$= 1-300$

$= -299$

7(i) $V=120\,000e^{-kt}$

When $t=0$, $V=120\,000e^0$

The value of the car when Jim bought it is \$120 000.

(ii) $t=12$, $V=100\,000$

$100\,000 = 120\,000e^{-12k}$

$e^{-12k} = \frac{5}{6}$

$k = \frac{\ln(\frac{5}{6})}{-12}$

$= 0.015193$

$\therefore V = 120\,000e^{-0.015193t}$

4 years $= 12(4) = 48$ months

Expected value of the car after 4 years is

$V = 120\,000e^{-0.015193(48)}$

≈ 57870.37

$=$ \$57900 (to 3 s.f.)

(iii) $30\,000 = 120\,000e^{-0.015193t}$

$e^{-0.015193t} = \frac{1}{4}$

$t = \frac{\ln(\frac{1}{4})}{-0.015193}$

≈ 91.25

$=$ 91 months (to the nearest month)

The age of the car will be 91 months when its expected value is \$30 000.

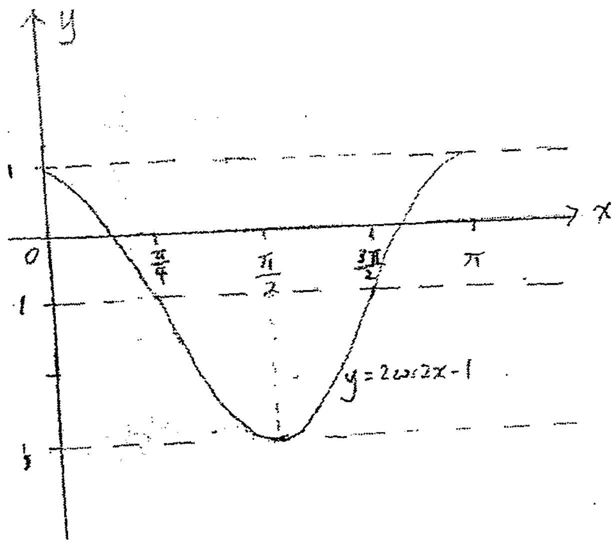
i) $f(x) = \sin 2x - x \quad 0 \le x \le \pi$

$f'(x) = 2\cos 2x - 1$

Amplitude = 2

Axis of graph is $y = -1$

Period $= \frac{2\pi}{2} = \pi$



ii) Let $f'(x) = 2\cos 2x - 1 = 0$

$\cos 2x = \frac{1}{2}$

$2x = \frac{\pi}{3}$ or $2x = 2\pi - \frac{\pi}{3}$

$x = \frac{\pi}{6}$ or $\frac{5\pi}{6}$

For $f(x) = \sin 2x - x$ to be an increasing function, $f'(x) > 0$

$\therefore 0 \le x < \frac{\pi}{6}$ or $\frac{5\pi}{6} < x \le \pi$

9(a) $y = 9 - |2x - 6|$

At x-axis, $y = 0$

$9 - |2x - 6| = 0$

$|2x - 6| = 9$

$2x - 6 = 9$ or $2x - 6 = -9$

$x = 7\frac{1}{2}$ $\quad x = -1\frac{1}{2}$

$\therefore A(-1\frac{1}{2}, 0)$

$D(7\frac{1}{2}, 0)$

At B, $x = 0$. $\therefore y = 9 - |-6|$
$= 9 - 6$
$= 3$

$\therefore B(0, 3)$

At vertex pt, $|2x - 6| = 0 \Rightarrow x = 3$

$\therefore C(3, 9)$

9(b) $y = mx$ $(m > 0)$ is an upslope straight line passing through the origin.

Minimum m = gradient of AC

$= \frac{0 - 9}{-1\frac{1}{2} - 3}$

$= 2$

(Note: The line $y = mx$ must be parallel to AC so that m is minimum and intersects the modulus graph at one point)

10) $v = t(2 - 3t)^2$

(i) Acceleration, $a = \frac{dv}{dt}$

$= t(2)(2 - 3t)(-3) + (2 - 3t)^2$

$= (2 - 3t)(-6t + 2 - 3t)$

$a = (2 - 3t)(2 - 9t)$

(ii) At instantaneous rest, $v = 0$

$t(2 - 3t)^2 = 0$

$t = 0$ or $t = \frac{2}{3}$

$v = t(4 - 12t + 9t^2)$

$= 4t - 12t^2 + 9t^3$

$s = \int v\,dt = \int 4t - 12t^2 + 9t^3\,dt$

$= 2t^2 - 4t^3 + \frac{9}{4}t^4 + c$

when $t = 0$, $s = 0$, $c = 0$

$\therefore s = 2t^2 - 4t^3 + \frac{9}{4}t^4$

Distance travelled by the particle

$= 2(\frac{2}{3})^2 - 4(\frac{2}{3})^3 + \frac{9}{4}(\frac{2}{3})^4$

$= \frac{8}{9} - \frac{32}{27} + \frac{4}{9}$

$= \frac{4}{27}$

$= 0.148$ m

Q11 i)

$$(\sin^2\theta + \cos^2\theta)^2 = \sin^4\theta + 2\sin^2\theta\cos^2\theta + \cos^4\theta$$
$$1 = \sin^4\theta + \cos^4\theta + 2\sin^2\theta\cos^2\theta$$

$$\sin^4\theta + \cos^4\theta = \frac{1}{4}(3+\cos 4\theta)$$

$$\begin{aligned}
LHS &= \sin^4\theta + \cos^4\theta \\
&= 1 - 2\sin^2\theta\cos^2\theta \\
&= 1 - 2\left(\frac{1-\cos 2\theta}{2}\right)\left(\frac{1+\cos 2\theta}{2}\right) \\
&= 1 - \frac{1}{2}(1-\cos^2 2\theta) \\
&= 1 - \frac{1}{2} + \frac{1}{2}\left(\frac{1+\cos 4\theta}{2}\right) \\
&= \frac{1}{2} + \frac{1}{2}\left(\frac{1}{2} + \frac{1}{2}\cos 4\theta\right) \\
&= \frac{1}{4}(3+\cos 4\theta) \quad \text{(shown)}
\end{aligned}$$

Note:

$\cos^2\theta = \frac{1+\cos 2\theta}{2}$

$\therefore \cos^2 2\theta = \frac{1+\cos 4\theta}{2}$ (change θ to 2θ)

ii) $\sin^4\theta + \cos^4\theta = \frac{3}{4} + \frac{\sqrt{3}}{8}$

$\frac{3}{4} + \frac{1}{4}\cos 4\theta = \frac{3}{4} + \frac{\sqrt{3}}{8}$

$\cos 4\theta = \frac{\sqrt{3}}{2}$ $\qquad 0 \le \theta \le \pi$, $0 \le 4\theta \le 4\pi$

Basic $\angle = \frac{\pi}{6}$

$4\theta = \frac{\pi}{6}$

$4\theta = 2\pi - \frac{\pi}{6}$

$4\theta = 2\pi + \frac{\pi}{6}$

$4\theta = 4\pi - \frac{\pi}{6}$

$\therefore \theta = \frac{\pi}{24}, \frac{11\pi}{24}, \frac{13\pi}{24}$ or $\frac{23\pi}{24}$

iii) For $-\pi \le \theta \le \pi$, the number of solutions of the equation $\sin^4\theta + \cos^4\theta = \frac{3}{4} + \frac{\sqrt{3}}{8}$ is $2(4) = \underline{8}$ #

12 (i)

At P, $\sin 2x = \frac{\sqrt{3}}{2}$

Basic $\angle = \frac{\pi}{3}$

$2x = \frac{\pi}{3}$ or $\pi - \frac{\pi}{3}$

$x = \frac{\pi}{6}$ (rejected) or $x = \frac{\pi}{3}$

$\therefore P\left(\frac{\pi}{3}, \frac{\sqrt{3}}{2}\right)$

Note: You can draw a horizontal line $y = \frac{\sqrt{3}}{2}$. The x-coordinate of P is the second pt of intersection between $y = \sin 2x$ and $y = \frac{\sqrt{3}}{2}$.

At Q, $\sin 2x = 0$

$2x = 0$ (rejected) or $2x = \pi$

$x = \frac{\pi}{2}$

$\therefore Q\left(\frac{\pi}{2}, 0\right)$

(ii) Area of shaded region is

$$\begin{aligned}
&\int_0^{\frac{\pi}{2}} \sin 2x \, dx - \frac{1}{2}\left(\frac{\pi}{3}\right)\left(\frac{\sqrt{3}}{2}\right) \\
&= \left[-\frac{1}{2}\cos 2x\right]_0^{\frac{\pi}{2}} - \frac{\sqrt{3}\pi}{12} \\
&= -\frac{1}{2}\cos\pi - \left(-\frac{1}{2}\right) - \frac{\sqrt{3}\pi}{12} \\
&= 1 - \frac{\sqrt{3}\pi}{12} \\
&= 0.547 \text{ sq units}
\end{aligned}$$

2014 SNG3 Prelim P1 01

13

$y = \frac{a}{e^{kx}} = ae^{-kx}$

(i) $\ln y = \ln e^{-kx} + \ln a$

$= -kx + \ln a$

Plot $\ln y$ against x

x	1	2	3	4	5	6
$\ln y$	1.72	2.05	2.39	2.72	3.05	3.39

Gradient, $-k = \frac{3.18 - 2.22}{5.4 - 2.5}$

$= 0.331$

$k = -0.331$

$Y = 0.33103X + C$

$3.18 = 0.33103(5.4) + C$

$C = 1.3924$

$\therefore \ln a = 1.3924$

$a = 4.02$

(iii) $y^2 = e^{-4x+8}$

$\ln y^2 = \ln e^{-4x+8}$

$2\ln y = -4x + 8$

$\ln y = -2x + 4$

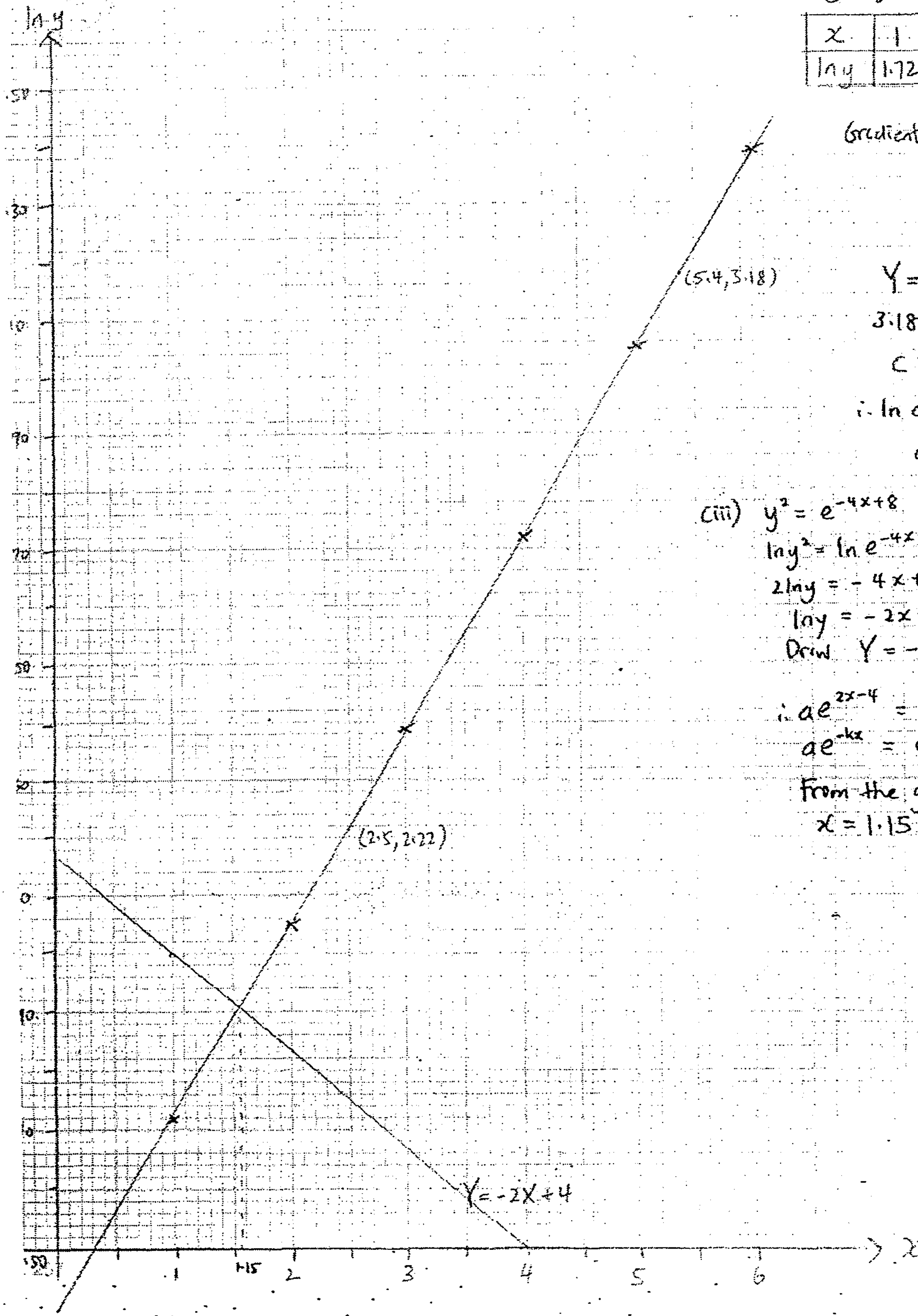
Draw $Y = -2X + 4$

$\therefore ae^{2x-4} = e^{kx}$

$ae^{-kx} = e^{-2x+4}$

From the graph,

$x = 1.15$



Name: ____________________________ (   )   Class: ______________

PRELIMINARY EXAMINATION
GENERAL CERTIFICATE OF EDUCATION ORDINARY LEVEL

ADDITIONAL MATHEMATICS 4047/02

Paper 2   28 August 2014

2 hours 30 minutes

Additional Materials: Answer Paper

READ THESE INSTRUCTIONS FIRST

Write your name, class, and register number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue, or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, staple all your work together with this cover sheet.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is **100**.

FOR EXAMINER'S USE

Q1		Q5		Q9	
Q2		Q6		Q10	
Q3		Q7		Q11	
Q4		Q8		Q12	

/100

This document consists of **6** printed pages.

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[Turn over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2+bx+c=0$,

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n=a^n+\binom{n}{1}a^{n-1}b+\binom{n}{2}a^{n-2}b^2+\ldots+\binom{n}{r}a^{n-r}b^r+\ldots+b^n,$$

where n is a positive integer and $\binom{n}{r}=\frac{n!}{(n-r)!\,r!}=\frac{n(n-1)\ldots\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A+\cos^2 A=1.$$

$$\sec^2 A=1+\tan^2 A.$$

$$\operatorname{cosec}^2 A=1+\cot^2 A.$$

$$\sin(A\pm B)=\sin A\cos B\pm\cos A\sin B$$

$$\cos(A\pm B)=\cos A\cos B\mp\sin A\sin B$$

$$\tan(A\pm B)=\frac{\tan A\pm\tan B}{1\mp\tan A\tan B}$$

$$\sin 2A=2\sin A\cos A$$

$$\cos 2A=\cos^2 A-\sin^2 A=2\cos^2 A-1=1-2\sin^2 A$$

$$\tan 2A=\frac{2\tan A}{1-\tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

$$a^2=b^2+c^2-2bc\cos A$$

$$\Delta=\frac{1}{2}bc\sin A$$

1 Using a **separate** diagram for each part, represent on the number line the solution set of

(i) $x^2 + 2x > 3$, [2]

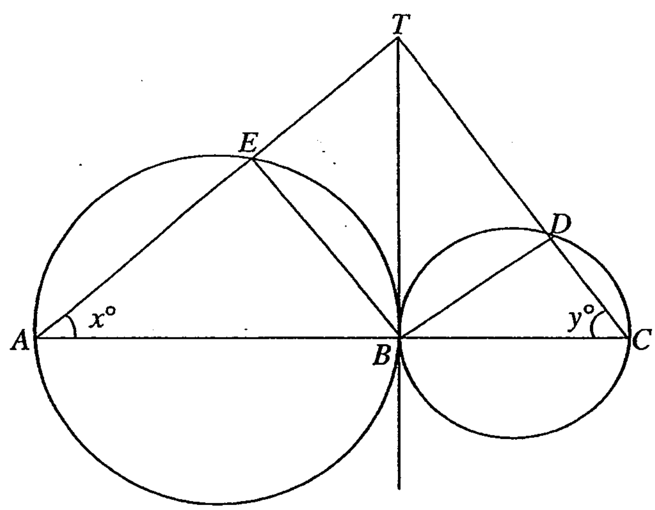
(ii) $-3 < 2(2x+1) - x^2$. [2]

State the set of values of x which satisfy both of these inequalities. [1]

2 **(i)** Differentiate $\sqrt{\cos 2x}$ with respect to x. [2]

(ii) Use your answer to part **(i)** to evaluate $\int_0^{\frac{\pi}{6}} \frac{\sin 2x}{2\sqrt{\cos 2x}}\,dx$, giving your answer in the form $a + b\sqrt{2}$, where a and b are constants. [3]

3



The diagram shows two circles and BT is a tangent to the circles at B.
AE produced and CD produced meet the tangent at T.
$\angle TAB = x^\circ$ and $\angle TCB = y^\circ$. ABC, AET and CDT are straight lines.

(i) Prove that $\angle EBD + \angle ETD = 180^\circ$. [3]

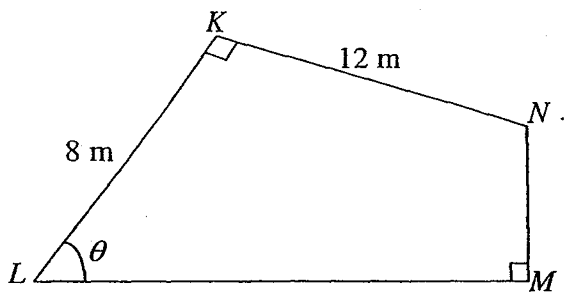
(ii) Prove that $AT \times ET = BT^2$. [3]

4 The function $f(x) = x^3 + ax^2 + bx + 4$, where a and b are constants, is exactly divisible by $(x - 2)$. Given that f(x) leaves a remainder of -3 when divided by $x + 1$,

(i) find the value of a and of b. [4]

(ii) express f(x) in the form $(x-2)(x-1-\sqrt{d})(x-1+\sqrt{d})$, where d is an integer. [3]

5



The diagram shows a pond $KLMN$ in a school. $LK = 8$ cm and $KN = 12$ m. $L\hat{K}N = L\hat{M}N = 90°$ and $K\hat{L}M = \theta$, where $0° < \theta < 90°$.

Given that the perimeter of the pond is P m,

(i) find the values of the integers a, b and of d for which

$$P = a + b\cos\theta + d\sin\theta.$$ [3]

Using the values of a, b and d found in part **(i)**,

(ii) express P in the form of $a + R\sin(\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. [4]

Hence

(iii) find the value of θ when $P = 38$. [2]

6 The function f is defined, for all values of x, by $f(x) = 2\cos\left(\frac{x}{2}\right) - 1$.

(i) State the maximum and minimum values of f (x). [2]

(ii) State the amplitude of f. [1]

(iii) State the period of f in terms of π. [1]

(iv) Find, in terms of π, the smallest positive value of x for which $f(x) = 0$. [2]

(v) Sketch the graph of $y = f(x)$ for $-2\pi \le x \le 2\pi$. [3]

7 The equation of a curve is $y = 3x^2 \ln x$. The tangent to the curve at the point $x = e^2$ meets the x-axis at A and the y-axis at B.

(i) Show that the coordinates of A are $\left(\frac{3}{5}e^2, 0\right)$. [7]

(ii) Hence find the area of triangle AOB in terms of e. [2]

8 (a) Solve the equation

(i) $\log_5\left(x^2 - 5x + 20\right) = \dfrac{1}{\log_2 5} + \dfrac{1}{\log_7 5}$, [3]

(ii) $\lg y^2 = (\lg y)^2$. [3]

(b) Solve the equation

$$8e^{4x} - 15e^{2x} - 2 = 0 .$$ [3]

9 The point $A(-8, 16)$ lies on the curve $y = kx^{\frac{5}{3}}$, where k is a constant.

(i) Show that $k = -\dfrac{1}{2}$. [1]

(ii) Sketch the graph of $y = kx^{\frac{5}{3}}$ for $x \geq 0$. [1]

(iii) On the same diagram, sketch the graph of $y = kx^{-\frac{5}{3}}$ for $x > 0$. [1]

(iv) Calculate the coordinates of the point of intersection of the two graphs. [2]

(v) Determine, with explanation, whether the tangents to the graphs at the point of intersection are perpendicular. [4]

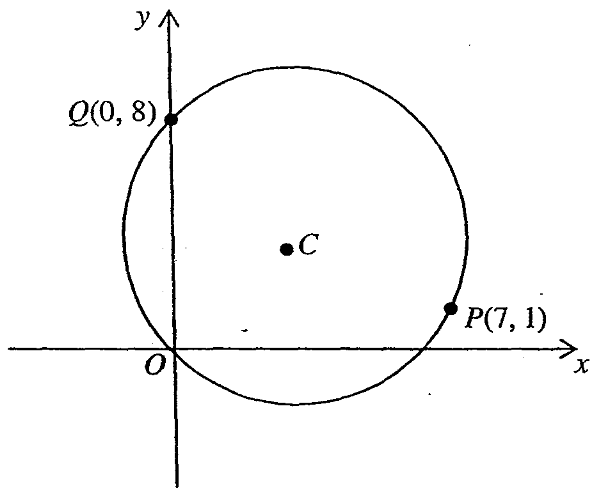
10 (i) Find $\dfrac{d}{dx}\left[\ln\left(x^2 + 9\right)\right]$. [2]

(ii) Express $\dfrac{x^2 + 45}{x^3 + 9x}$ in partial fractions. [4]

(iii) Find $\displaystyle\int \frac{x^2 + 45}{x^3 + 9x}\,dx$ and hence evaluate $\displaystyle\int_1^3 \frac{x^2 + 45}{x^3 + 9x}\,dx$. [4]

11 A curve is such that $\frac{d^2y}{dx^2} = \frac{4}{(2x-3)^2}$. The gradient of the curve at the point (2,1) is $\frac{1}{2}$.
Find the coordinates of the stationary point of the curve and determine the nature of this stationary point. [11]

12



The diagram shows a circle, centre C, that passes through the origin O.
The points $P(7, 1)$ and $Q(0, 8)$ lie on the circle.

(i) Find the equation of the perpendicular bisector of OP and of OQ. [4]

(ii) Hence show that the coordinates of C are (3, 4). [1]

(iii) Find the equation of the circle. [2]

The points R and T, which lie on the circle, are the same distance from the x-axis as the point P.

(iv) Find the equation of the tangent to the circle at R and at T. [4]

SNGS Additional Mathematics Prelim Paper 2

1.	$1 < x < 5$	7(ii)	Area $\Delta AOB = \frac{27}{10}e^6$ / $2.7e^6$ sq units
2(i)	$\frac{-\sin 2x}{\sqrt{\cos 2x}}$	8(a)(i)	$x = 2$ or $x = 3$
(ii)	$\frac{1}{2} - \frac{1}{4}\sqrt{2}$	(ii)	$x = 1$ or $x = 100$
4(i)	$a = -4, \quad b = 2$	(b)	$e^{2x} = -\frac{1}{8}$ (no solution), $x = 0.347$ (3 s.f.)
(ii)	$f(x) = (x-2)(x-1-\sqrt{3})(x-1+\sqrt{3})$	9(iv)	$\left(1, -\frac{1}{2}\right)$
5(i)	$20 - 4\cos\theta + 20\sin\theta$	10(i)	$\frac{2x}{x^2+9}$
(ii)	$P = 20 + 4\sqrt{26}\sin(\theta - 11.3°)$	(ii)	$\frac{5}{x} - \frac{4x}{x^2+9}$
(iii)	$\theta = 73.3°$ (1 *d.p.*)	(iii)	$5\ln x - 2\ln(x^2+9) + c$ $\ln 75$ *or* 4.32
6(i)	Max value = 1 Min value– 3	11.	min $(1.9, 0.973)$
(ii)	Amplitude = 2 Period $= 4\pi$	12(i)	$y = -7x + 25$
(iii)	$x = \frac{2\pi}{3}$	(iii)	$(x-3)^2 + (y-4)^2 = 25$
		(iv)	$y = -\frac{4}{3}x - \frac{1}{3}$ $y = -1$

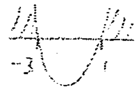
?NGS Add Math P2 — 2014

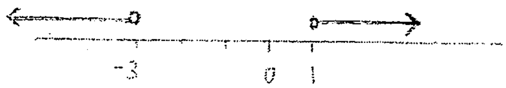
1i) $x^2 + 2x > 3$

$x^2 + 2x - 3 > 0$

$(x+3)(x-1) > 0$

$x < -3$ or $x > 1$





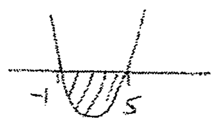
ii) $-3 < 2(2x+1) - x^2$

$0 < 4x + 2 - x^2 + 3$

$x^2 - 4x - 5 < 0$

$(x-5)(x+1) < 0$

$-1 < x < 5$



Set of values of x which satisfy both inequalities is $\{x : 1 < x < 5\}$

2i) Let $y = (\cos 2x)^{\frac{1}{2}}$

$$\frac{dy}{dx} = \frac{1}{2}(\cos 2x)^{-\frac{1}{2}} \cdot (-2)\sin 2x$$

$$= \frac{-\sin 2x}{\sqrt{\cos 2x}}$$

ii) $$\frac{d}{dx}\sqrt{\cos 2x} = \frac{-\sin 2x}{\sqrt{\cos 2x}}$$

$$-\frac{1}{2}\frac{d}{dx}\sqrt{\cos 2x} = \frac{\sin 2x}{2\sqrt{\cos 2x}}$$

$$\int_0^{\frac{\pi}{6}} \frac{\sin 2x}{2\sqrt{\cos 2x}}\,dx = -\frac{1}{2}\left[\sqrt{\cos 2x}\right]_0^{\frac{\pi}{6}}$$

$$= -\frac{1}{2}\left(\sqrt{\cos\frac{\pi}{3}} - 1\right)$$

$$= -\frac{1}{2}\left(\frac{\sqrt{2}}{2} - 1\right)$$

$$= \frac{1}{2} - \frac{1}{4}\sqrt{2}$$

3(i) $\angle EBT = x^\circ$ (alternate segment theorem)

$\angle DBT = y^\circ$ (alternate segment theorem)

Let $\angle AEB = \theta$

$\angle ETB = \theta - x^\circ$ (exterior $\angle$ of Δ)

Let $\angle BDC = \alpha$

$\angle BTD = \alpha - y^\circ$ (exterior $\angle$ of Δ)

$x^\circ + \theta = x^\circ + y^\circ + (180^\circ - \alpha - y^\circ)$ (ext. $\angle$ of Δ)

$\theta = 180^\circ - \alpha$

$\therefore \angle EBD + \angle ETD$

$= (x^\circ + y^\circ) + (\theta - x^\circ) + (\alpha - y^\circ)$

$= \theta + \alpha$

Since $\theta = 180^\circ - \alpha$

$\angle EBD + \angle ETD = 180^\circ - \alpha + \alpha$

$= 180^\circ$ (proven)

(ii) Consider ΔABT and ΔBET

$\angle BAT = \angle EBT = x^\circ$ (alternate segment theorem)

$\angle ATB = \angle BTE = \theta - x^\circ$ (common angle)

$\therefore \Delta ABT$ and ΔBET are similar.

$\frac{AT}{BT} = \frac{BT}{ET}$ (ratio of corresponding sides are equal)

$AT \times ET = BT^2$ (proven)

68

1. $f(x) = x^3 + ax^2 + bx + 4$

(i) $f(2) = 0$

$\therefore 8 + 4a + 2b + 4 = 0$

$4a + 2b = -12$

$2a + b = -6$ ---- (1)

$f(-1) = -3$

$-1 + a - b + 4 = -3$

$a - b = -6$ ---- (2)

(1) + (2) : $3a = -12$

$a = -4$

Subst $a = -4$ into (2) : $-4 - b = -6$

$b = 2$

$\therefore a = -4,\ b = 2$

(ii) $f(x) = x^3 - 4x^2 + 2x + 4 = (x-2)(x^2 + px - 2)$

Compare coeff of x : $2 = -2 - 2p$

$p = -2$

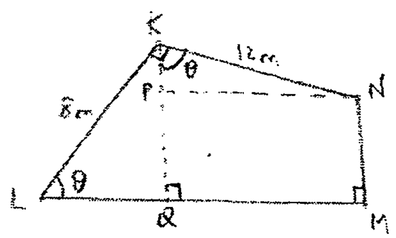
$\therefore f(x) = (x-2)(x^2 - 2x - 2)$

$= (x-2)(x^2 - 2x + (1)^2 - (1)^2 - 2)$

$= (x-2)[(x-1)^2 - 3]$

$= (x-2)(x-1+\sqrt{3})(x-1-\sqrt{3})$

i)



(i) $KQ = 8\sin\theta$

$KP = 12\cos\theta$

$NM = 8\sin\theta - 12\cos\theta$

$LQ = 8\cos\theta$

$PN = 12\sin\theta$

$LM = 8\cos\theta + 12\sin\theta$

Perimeter, $P = 8 + 12 + 8\sin\theta - 12\cos\theta + 8\cos\theta + 12\sin\theta$

$P = 20 - 4\cos\theta + 20\sin\theta$

$\therefore a = 20$

$b = -4$

$d = 20$

(ii) Let $20\sin\theta - 4\cos\theta = R\sin(\theta - \alpha)$

$R = \sqrt{20^2 + 4^2}$

$= \sqrt{416}$

$= 4\sqrt{26}$

$\alpha = \tan^{-1}\left(\frac{4}{20}\right)$

$\approx 11.310°$

$= 11.3°$ (to 1 dp)

$\therefore P = 20 + 4\sqrt{26}\sin(\theta - 11.3°)$

(iii) $P = 38$

$38 = 20 + 4\sqrt{26}\sin(\theta - 11.310°)$

$\sin(\theta - 11.310°) = \frac{18}{4\sqrt{26}}$

$= \frac{9}{2\sqrt{26}}$

Basic $\angle = 61.948°$

$\theta - 11.310° = 61.948°$

$\theta = 73.3°$ (to 1 dp)

6 (i) $f(x) = 2\cos\left(\frac{x}{2}\right) - 1$

Maximum value of $f(x) = 2(1) - 1 = 1$

Minimum value of $f(x) = 2(-1) - 1 = -3$

(ii) Amplitude $= 2$

(iii) Period $= \frac{2\pi}{\frac{1}{2}} = 4\pi$

(iv) $f(x) = 0 \Rightarrow 2\cos\left(\frac{x}{2}\right) - 1 = 0$

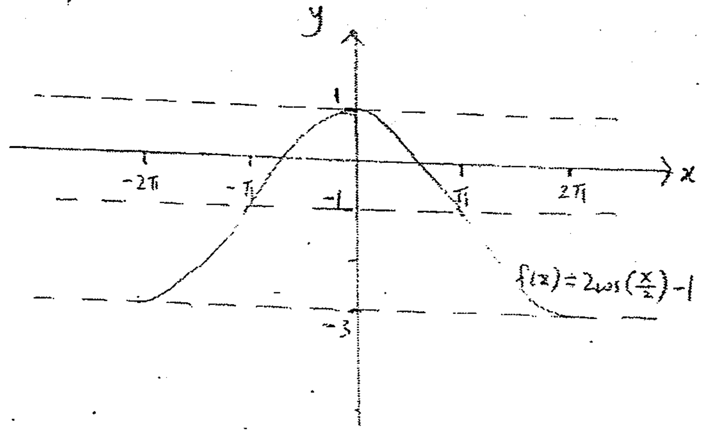
$\cos\frac{x}{2} = \frac{1}{2}$

Basic $\angle = \frac{\pi}{3}$

$\frac{x}{2} = \frac{\pi}{3}$

$x = \frac{2\pi}{3}$

(v)



1) $y = 3x^2 \ln x$

(i) $\frac{dy}{dx} = 3x^2 \cdot \frac{1}{x} + \ln x\,(6x)$

$= 3x + 6x \ln x$

At $x = e^2$

$\frac{dy}{dx} = 3e^2 + 6e^2 \ln e^2$

$= 3e^2 + 6e^2(2)$

$= 15e^2$

$y = 3(e^2)^2 \ln e^2$

$= 3e^4 \cdot (2)$

$= 6e^4$

Equation of tangent is

$y - 6e^4 = 15e^2(x - e^2)$

$= 15e^2x - 15e^4$

$y = 15e^2x - 9e^4$

At A, $y = 0$

$15e^2x = 9e^4$

$x = \frac{3}{5}e^2$

$\therefore$ Coordinates of A are $(\frac{3}{5}e^2, 0)$

(ii) At B, $x = 0$

$y = -9e^4$

Coordinates of B are $(0, -9e^4)$

Area of ΔAOB is

$\frac{1}{2}(\frac{3}{5}e^2)(9e^4)$

$= \frac{27}{10}e^6$

$= 2.7e^6$ sq units

8(a)(i)

$\log_5(x^2 - 5x + 20) = \frac{1}{\log_2 5} + \frac{1}{\log_7 5}$

$= \log_5 2 + \log_5 7$

$= \log_5 14$

$\therefore x^2 - 5x + 20 = 14$

$x^2 - 5x + 6 = 0$

$(x-3)(x-2) = 0$

$x = 3$ or 2

(ii) $\lg y^2 = (\lg y)^2$

$2\lg y = (\lg y)^2$

Let $p = \lg y$

$2p = p^2$

$p(2-p) = 0$

$p = 0$ or $p = 2$

$\lg y = 0$ $\quad$ $\lg y = 2$

$y = 10^0$ $\quad$ $y = 10^2$

$= 1$ $\quad$ $= 100$

$\therefore y = 1$ or 100

(b) $8e^{4x} - 15e^{2x} - 2 = 0$

Let $q = e^{2x}$

$\therefore 8q^2 - 15q - 2 = 0$

$(8q + 1)(q - 2) = 0$

$q = -\frac{1}{8}$ or $q = 2$

$e^{2x} = -\frac{1}{8}$ (rejected $\because e^{2x} > 0$) or $e^{2x} = 2$

$2x = \ln 2$

$x = \frac{1}{2}\ln 2$

$= 0.347$

(i) 7) $y = kx^{\frac{5}{3}}$

When $x=-8$, $y=16$

$16 = k(-8)^{\frac{5}{3}}$

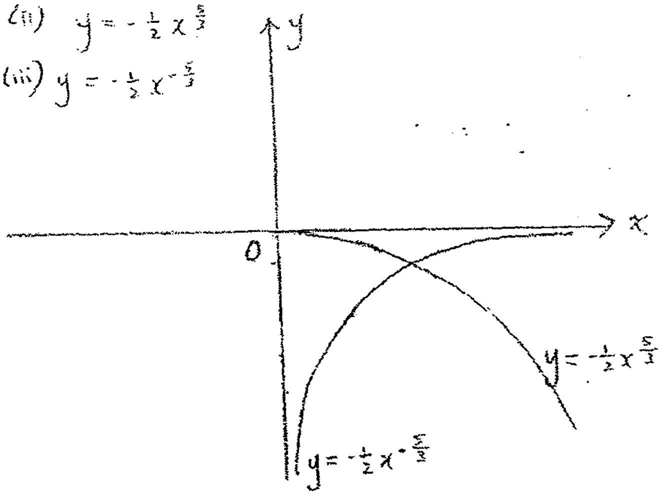
$= k(-2^3)^{\frac{5}{3}}$

$= -32k$

$k = -\frac{1}{2}$ (shown)

(ii) $y = -\frac{1}{2}x^{\frac{5}{3}}$

(iii) $y = -\frac{1}{2}x^{-\frac{5}{3}}$



Note: For $y = -\frac{1}{2}x^{-\frac{5}{3}}$

$= -\frac{1}{2x^{\frac{5}{3}}}$

Note that x cannot be zero.

∴ the y-axis is the asymptote

y cannot be zero because $x^{\frac{5}{3}} > 0$

∴ the x-axis (i.e. $y=0$) is the asymptote

iv) $-\frac{1}{2}x^{\frac{5}{3}} = -\frac{1}{2}x^{-\frac{5}{3}}$

$\frac{x^{\frac{5}{3}}}{x^{-\frac{5}{3}}} = 1$

$x^{\frac{10}{3}} = 1$

$x = 1$

When $x=1$, $y = -\frac{1}{2}(1)^{\frac{5}{3}} = -\frac{1}{2}$

Coordinates of the point of intersection are $(1, -\frac{1}{2})$.

v) $y = -\frac{1}{2}x^{\frac{5}{3}}$

$\frac{dy}{dx} = -\frac{1}{2}(\frac{5}{3})x^{\frac{2}{3}}$

$= -\frac{5}{6}x^{\frac{2}{3}}$

When $x=1$, $\frac{dy}{dx} = -\frac{5}{6}$

$y = -\frac{1}{2}x^{-\frac{5}{3}}$

$\frac{dy}{dx} = -\frac{1}{2}(-\frac{5}{3})x^{-\frac{8}{3}}$

$= \frac{5}{6}x^{-\frac{8}{3}}$

When $x=1$, $\frac{dy}{dx} = \frac{5}{6}$

Since the product of the gradients $-\frac{5}{6} \times \frac{5}{6} \neq -1$, the tangents to the graph at the point of intersection are not perpendicular.

10 i) $\frac{d}{dx}[\ln(x^2+9)]$

$= \frac{1}{x^2+9} \cdot 2x$

$= \frac{2x}{x^2+9}$

(ii) $\frac{x^2+45}{x^3+9x} = \frac{x^2+45}{x(x^2+9)} = \frac{A}{x} + \frac{Bx+C}{x^2+9}$

$x^2+45 = A(x^2+9) + x(Bx+C)$

Compare constant, $45 = 9A \Rightarrow A = 5$

Compare coeff of x^2: $1 = A+B \Rightarrow B = -4$.

Compare coeff of x: $0 = C$

$\therefore \frac{x^2+45}{x^3+9x} = \frac{5}{x} - \frac{4x}{x^2+9}$

(iii) $\int \frac{x^2+45}{x^3+9x}\,dx = \int \frac{5}{x}\,dx - 2\int \frac{2x}{x^2+9}\,dx$

$= 5\ln x - 2\ln(x^2+9) + C$

$\int_1^3 \frac{x^2+45}{x^3+9x}\,dx = \left[5\ln x - 2\ln(x^2+9)\right]_1^3$

$= 5\ln 3 - 2\ln 18 - (0 - 2\ln 10)$

$= \ln 3^5 - \ln 18^2 + \ln 10^2$

$= \ln\left(\frac{243 \times 100}{324}\right)$

$= \ln 75$ #

Note: $\int \frac{g'(x)}{g(x)}\,dx = \ln[g(x)] + C$

1) $\frac{d^2y}{dx^2} = \frac{4}{(2x-3)^2} = 4(2x-3)^{-2}$

$\frac{dy}{dx} = \int 4(2x-3)^{-2}\,dx$

$= 4 \cdot \frac{(2x-3)^{-1}}{(-1)(2)} + c$

$= -\frac{2}{2x-3} + c$

when $x = 2$, $\frac{dy}{dx} = \frac{1}{2}$

$\frac{1}{2} = -\frac{2}{4-3} + c$

$c = \frac{5}{2}$

$\therefore \frac{dy}{dx} = -\frac{2}{2x-3} + \frac{5}{2}$

At stationary point, $\frac{dy}{dx} = 0$

$-\frac{2}{2x-3} + \frac{5}{2} = 0$

$\frac{2}{2x-3} = \frac{5}{2}$

$4 = 10x - 15$

$x = 1.9$

$y = \int -\frac{2}{2x-3} + \frac{5}{2}\,dx$

$= -2\int \frac{1}{2x-3}\,dx + \frac{5}{2}\int 1\,dx$

$= -2 \cdot \frac{\ln(2x-3)}{2} + \frac{5}{2}x + c_2$

$= -\ln(2x-3) + \frac{5}{2}x + c_2$

hen $x = 2$, $y = 1$

$1 = -\ln 1 + 5 + c_2$

$c_2 = -4$

$\therefore y = -\ln(2x-3) + \frac{5}{2}x - 4$

en $x = 1.9$, $y = -\ln(0.8) + \frac{5}{2}(1.9) - 4$

$= 0.973$

$\left.\frac{d^2y}{dx^2}\right|_{x=1.9} = 4(3.8-3)^{-2} = 6.25 < 0$

$\therefore (1.9, 0.973)$ is a minimum point.

12i) Gradient of OP $= \frac{1}{7}$

Gradient of ⊥ bisector of OP $= -7$

Midpoint of OP $= \left(\frac{0+7}{2}, \frac{0+1}{2}\right) = (3.5, 0.5)$

Equation of ⊥ bisector of OP is

$y - \frac{1}{2} = -7(x - \frac{7}{2})$

$y - \frac{1}{2} = -7x + \frac{49}{2}$

$y = -7x + 25$

Midpoint of OQ $= \left(\frac{0+0}{2}, \frac{0+8}{2}\right) = (0, 4)$

Equation of ⊥ bisector of OQ is $y = 4$

(ii) $-7x + 25 = 4$

$x = 3$

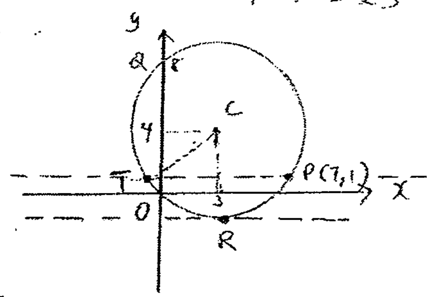
$\therefore$ Coordinates of C are $(3, 4)$

(iii) Radius $= \sqrt{(3-0)^2 + (4-8)^2} = 5$ units

Equation of circle is

$(x-3)^2 + (y-4)^2 = 25$

(iv)



Note: Red dotted lines are equidistant fr. x-axis as P.

Since centre of circle is $(3, 4)$ and radius is 5 units,

Equation of tangent is $y = -1$ #

When $y = 1$; $(x-3)^2 + (1-4)^2 = 25$

$(x-3)^2 = 16$

$x - 3 = 4$ or $x - 3 = -4$

$x = 7$ or -1

When $x = -1$, $y = 1$. $T(-1, 1)$

Gradient of CT $= \frac{4-1}{3+1} = \frac{3}{4}$

Gradient of tangent at T $= \frac{-1}{\frac{3}{4}} = -\frac{4}{3}$ (tan ⊥ rad)

Equation of tangent at T is $y - 1 = -\frac{4}{3}(x+1)$

$y = -\frac{4}{3}x - \frac{1}{3}$ #

1 The curve $y = (4+p)x^2 - 4x + p$ has a maximum point and cuts the line $y = -1$ at two distinct points. Find the range of values of p. [5]

2 A curve has the equation $y = 3xe^{2x+1}$.

(i) Show that the curve has only one stationary point. [4]

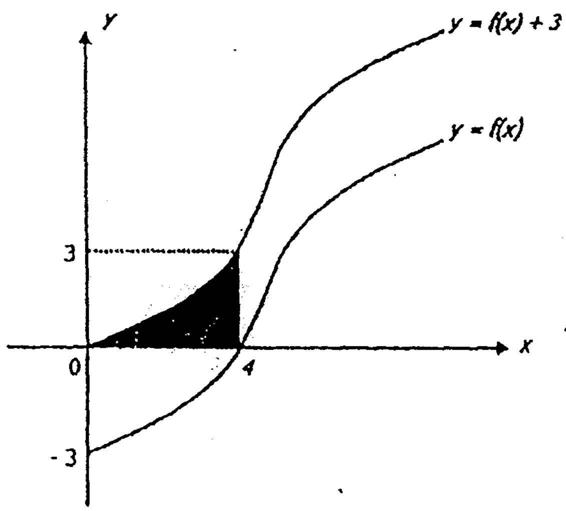
(ii) Determine the nature of this stationary point. [2]

3 The diagram below shows the graphs of $y = f(x)$ and $y = f(x) + 3$.

Given that $\int_0^8 f(x)\,dx = 16$ and $\int_0^4 f(x)\,dx = -7$, evaluate

(i) $\int_4^8 f(x)\,dx$, [1]

(ii) the area of the shaded region, [2]



4 Using a separate diagram for each part, represent on the number line the solution set of

(i) $|x| < 3$ [1]

(ii) $(2x+1)^2 > 9$ [2]

Hence, state the range of values of x which satisfy both inequalities. [2]

5 Solve the following equations in the given range:

(a) $\cos\left(2x-\frac{\pi}{6}\right)+4\sin 2x=0, \quad 0\le x\le 3$ [4]

(b) $\sec x(\tan x-2)=2\operatorname{cosec} x, \quad -180^\circ<x<180^\circ$ [4]

Begin this Section on a fresh sheet of paper.

6 Given that $\alpha^2+\beta^2=14$ and that $\alpha\beta=7$.

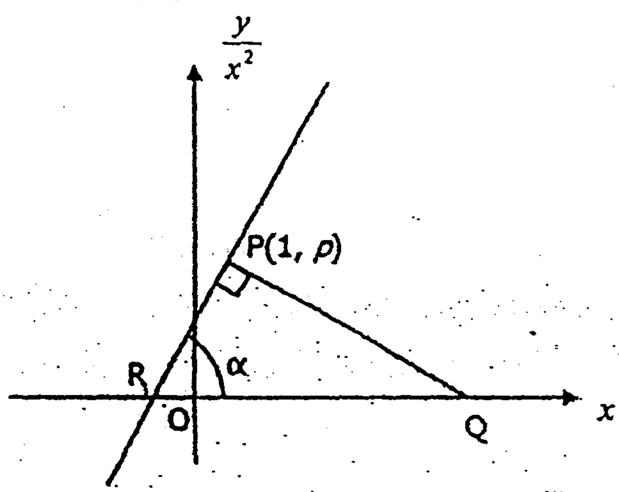
(i) find the value of $\alpha+\beta$, where $\alpha+\beta>0$, [2]

(ii) Find the quadratic equation whose roots are $\alpha-1$ and $\beta-1$. [3]

7 (i) Solve $\log_{25}[\log_2(10x+2)]=\log_9 3$. [4]

(ii) The solution of $2^{2x+3}=2^{x+1}+3$ can be expressed in the form $\log_2\frac{p}{q}$; where p and q are integers. Find the value of p and of q. [5]

8 The figure shows part of the straight line graph which is obtained by plotting $\frac{y}{x^2}$ against x for a relation between x and y given by $4y=8x^3+12x^2$. This straight line graph passes through $P\ (1, p)$ and cuts the x-axis at R. Q is a point on the horizontal axis such that $\angle QPR=90°$ and $\angle QRP=\alpha$.

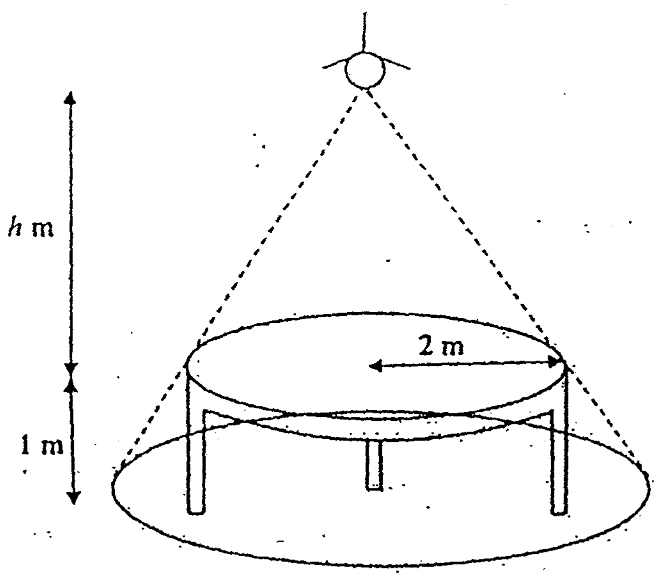


Find the value of p, of α and the x-coordinate of Q. [7]

9 A round stool has radius 2 m and height 1 m.

A small lamp O is placed h metres vertically above the centre of the stool, where $0 < h < 4$.

The lamp casts a shadow of the stool onto the ground.



(i) Show that the area of the shadow, A m^2, is given by $A = \dfrac{4\pi(h+1)^2}{h^2}$. [2]

(ii) The lamp is lowered vertically at a constant rate of $\frac{1}{8}$ m/s, find the rate of change of A when $h = 3$. [4]

Begin this Section on a fresh sheet of paper.

10 The function f is defined by $f(x) = 2 - 5x - x^2$.

(i) Express the function in the form $f(x) = a - (x+b)^2$, stating the maximum value of $f(x)$. [3]

(ii) Sketch the graph of $y = |f(x)|$ for $-6 \le x \le 1$, indicating the coordinates of the maximum point and the points of intersection with the coordinate axes. [3]

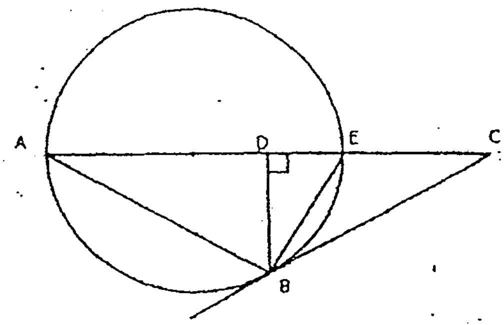
(iii) Hence, state the range of values of z in the range $-6 \le x \le 1$ for which the equation $|f(x)| = z$ has 4 solutions. [1]

11 (a) In the figure, the tangent to the circle at B meets the diameter AE produced at C. Given that BD is perpendicular to AC, prove that

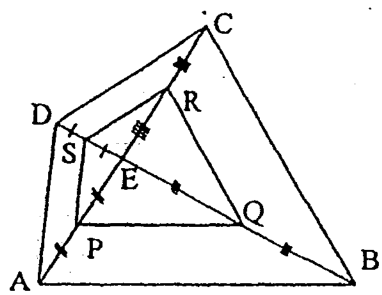
(i) EB bisects $\angle CBD$, [3]

(ii) ΔABC is similar to ΔBEC, [2]

(iii) $AB \times BC = AC \times BE$. [2]



(b) In the diagram, E is the point of intersection of the diagonals AC and BD of the quadrilateral $ABCD$. The points P, Q, R and S are the midpoints of AE, BE, CE and DE respectively.
Prove that the perimeter of $ABCD$ is twice the perimeter of $PQRS$. [3]



12 A particle moves in a straight line such that t seconds after passing through point O, its displacement, s metres, is given as $s = k\sin(\pi t)$, where k is a constant.

(i) Express the velocity v ms^{-1}, of the particle in terms of π, k and t. [1]

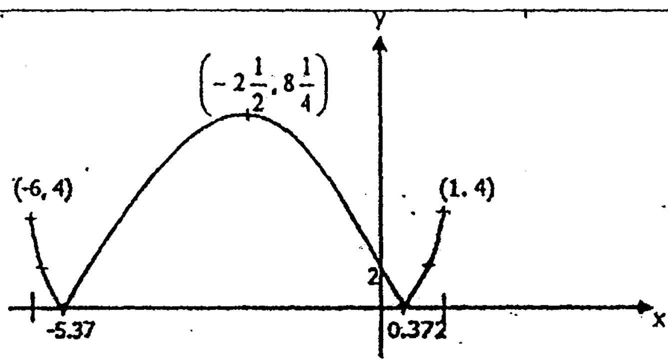
(ii) The initial velocity of the particle is 4π ms^{-1}. Find the value of k. [2]

(iii) Find the value of t at which the particle first comes to instantaneous rest. [3]

(iv) Find the total distance travelled by the particle in the first second. [3]

End of Paper

Answer key.

1.		$-5 < p < -4$	2.		The stationary point is a minimum point at $x = -\frac{1}{2}$
3.	(i)	(i) $\int_4^5 f(x)\,dx = 16-(-7) = 23$	4.	(i)	$x < 3$ *and* $-x < 3$ $x > -3$ (number line: open circles at -3 and 3, joined, axis x)
	(ii)	$5 units^2$		(ii)	Solution set: $\{x : x \in \Re,\ -3 < x < -2 \ \ or \ \ 1 < x < 3\}$
5.	(a)	$x = 1.48 rad\ (3sf)$	6	(i)	$\alpha + \beta = \sqrt{28}$ *or* $2\sqrt{7}$
	(b)	$x = -110.1°, -36.2°, 69.9°, 143.8°$		(ii)	Equation: $x^2 - (2\sqrt{7}-2)x + (8-2\sqrt{7}) = 0$
7.	(i)	$x = 3$	8.		x- coordinate of Q is 11.
	(ii)	$p = 3, q = 4$	11.	(a)	(ii) ΔABC is similar to ΔBEC. (AA similarity)
9.	(ii)	$\frac{4\pi}{27} m^2/s$			
10.	(i)	Maximum value of $f(x) = \frac{33}{4}$ or $8\frac{1}{4}$			
	(ii)				
	(iii)	$0 < z \leq 4$			
12.	(i)	$v = k\pi\cos(\pi t)$			
	(ii)	$k = 4$			
	(iii)	t = 0.5s			
	(iv)	Total distance travelled by the particle in the first second = $2(4) = 8m$			

Answer all questions.

Section A.

1. A cup of hot drink, initially at 95° C was left to stand in a room.

The temperature, $\theta° C$, at time t minutes is given by the equation $\theta = Ae^{-kt} + 28$,

where A and k are constants. At 6 minutes, the temperature dropped to 50° C.

(i) Find the value of A and of k. [3]

(ii) Find the time T minutes, where T is a whole number, after which the temperature is below 35° C. [2]

(iii) Calculate the rate at which the temperature decreases when $t = 10$. [2]

(iv) State the temperature the drink will reach when it is left to stand for a long time. [1]

2. Given that the roots of $3x^2 - 13x + 2 = 0$ are α and β,

(i) State the value of (a) $\alpha + \beta$, [1]

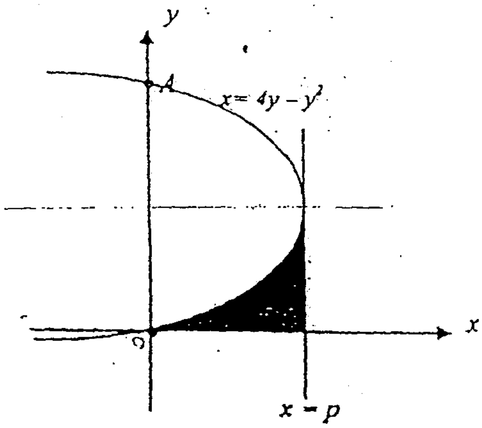
(b) $\alpha\beta$. [1]

(ii) Given further that $\alpha = \ln p$ and $\beta = \ln q$, find the value of each of the following:

(a) $\ln p^2 \times \ln q^2$, [1]

(b) $\log_q p + \log_p q$. [3]

3.



The above diagram shows a graph of the curve $x = 4y - y^2$. The curve meets the y–axis at the origin and point A. The line $x = p$ is a tangent to the curve.

(i) Find the coordinates of point A, and hence the equation of the line of symmetry. [2]

(ii) Show that $p = 4$. [2]

(iii) Calculate the area of the shaded region. [3]

4. (a) Expand $(1 + 2x)^5$ and $(1 - 2x)^5$ in ascending powers of x.

Hence write $(1 + 2x)^5 - (1 - 2x)^5$ in its simplest form. [3]

Use your expansion to find the exact value of

$(1+2\sqrt{2})^5 - (1-2\sqrt{2})^5$. [3]

(b) Given that a is positive, write down in descending powers of x, the first four terms in the expansion of $(x + \frac{a}{x})^8$, simplifying the coefficients. [3]

Given that the ratio of the coefficient of the fourth term to the coefficient of the second term is 7 : 9, find the value of a. [2]

Section B. (Start on a fresh sheet of paper.)

5. The lines $y = 8$ and $y = -2$ are tangents to a circle C_1.

The line $3y = x + 11$ is an equation of the diameter of the circle.

Find (i) the radius of the circle. [1]

(ii) the coordinates of the centre of circle C_1. [3]

Circle C_2 has the same centre as that of Circle C_1 but its area is twice that of the area of Circle C_1.

(iii) Find the equation of circle C_2 in the form $x^2 + y^2 + px + qy + r = 0$,

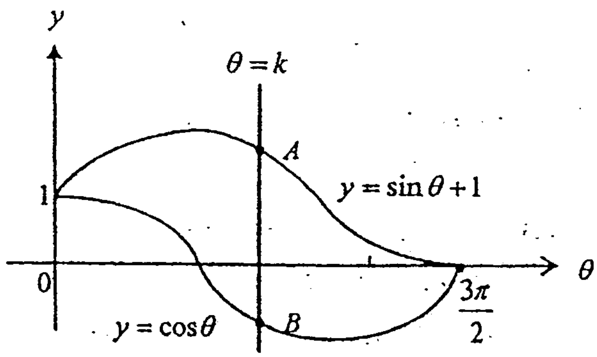
where p, q and r are integers. [3]

6. Given that $(x^4 + x^3 + 4x^2 - 3x + 1) \div (x^3 - x^2) = a + x + \dfrac{6x^2 + bx + c}{x^2(x-1)}$, find the value of each of the integer a, b, and c. [3]

Using partial fractions and the values of integer a, b, and c found,

evaluate $\displaystyle\int_3^5 \frac{x^4 + x^3 + 4x^2 - 3x + 1}{x^3 - x^2}\,dx$ [6]

7. (a)



The above are sketches of the curves $y = \sin\theta + 1$ and $y = \cos\theta$ for $0 \le \theta \le \frac{3}{2}\pi$.

The line $\theta = k$ meets the curves at points A and B respectively, where $0 \le k \le \frac{3}{2}\pi$.

Find the length of AB when $\theta = \frac{3\pi}{4}$.

Show that at this value of θ, AB is the maximum length. [4]

(b) Given $y = e^{\cos 2x}$, find $\frac{dy}{dx}$. Hence find the exact value of $\int_0^{\frac{\pi}{6}} 4e^{\cos 2x} \sin 2x\,dx$. [4]

8. (i) Show that $\frac{\sin^3\theta + \cos^3\theta}{\sin\theta + \cos\theta} = \frac{2 - \sin 2\theta}{2}$. [3]

(ii) Hence, or otherwise, given that $\frac{\sin^3\theta + \cos^3\theta}{\sin\theta + \cos\theta} = \frac{7}{10}$, show that $\sin 2\theta = \frac{3}{5}$ [2]

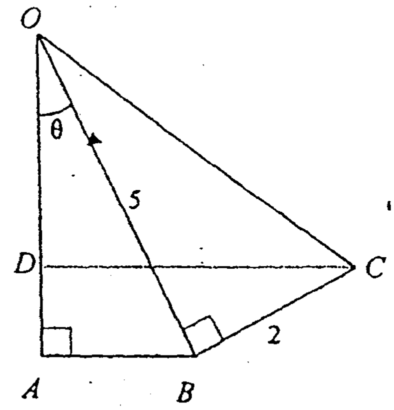
(iii) Given further that 2θ is an acute angle, without using calculator,

find the value of (a) $\cos\theta$ [2]

(b) $\tan 3\theta$. [3]

Section C (Start on a fresh sheet of paper.)

9.



The diagram shows two triangles OAB and OBC with $\angle OAB = \angle OBC = 90°$, and $\angle AOB = \theta°$.

The line CD is drawn parallel to BA.

Given that $OB = 5$ cm and $BC = 2$ cm,

(i) Show that $CD = 2\cos\theta + 5\sin\theta$. [4]

(ii) Express CD in the form $R\sin(\theta + \alpha)$. [4]

(iii) Find the maximum value of CD and the value of θ at which it occurs [3]

10. (i) Express the function $f(x) = 9x^3 + 3x^2 - 8x - 4$ as a product of three linear factors, and show that the equation $f(x) = 0$ has 2 real distinct roots. [3]

(ii) Given $y = f(x)$, find the coordinates of the turning points of the curve $y = 9x^3 + 3x^2 - 8x - 4$.

Sketch the curve $y = 9x^3 + 3x^2 - 8x - 4$.

(Proof of nature of turning point is not required.) [5]

(iii) From your sketch,

(a) state the range of values of x for which y is a decreasing function. [1]

(b) explain why the function $g(x) = 9x^3 + 3x^2 - 8x - 5$ has only one linear factor. [1]

11. The line $y = x$ intersects the curve $y = x^{\frac{1}{3}}$ at three points A, B and O, where O is the origin.

(i) Find the coordinates of A and B. [3]

(ii) Sketch the curve $y = x^{\frac{1}{3}}$ and the line $y = x$ on the same axes. [3]

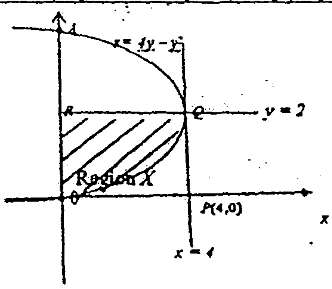
(iii) Given that the point $P\left(\frac{1}{8}, \frac{1}{2}\right)$ lies on the curve, find the perpendicular distance from the point P to the line $y = x$. [4]

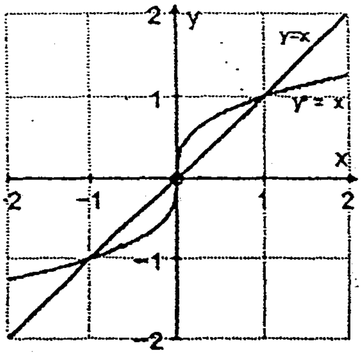
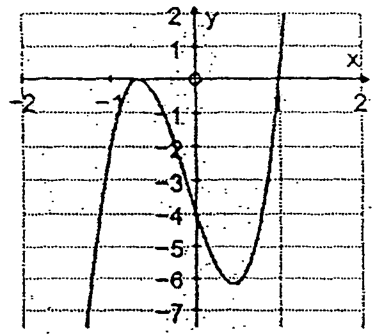
(iv) Find the gradient of the tangent to the curve at the point P.

State whether the tangent drawn at point P will meet the line $y = x$, giving a reason for your answer. [3]

End of paper

Answer Key

1.	(i)	k=0.186 (3 s.f.)	2.	(i)	(a) $\alpha+\beta=\frac{13}{3}$ (b) $\alpha\beta=\frac{2}{3}$
	(ii)	T = 13.		(ii)	(a) $2\frac{2}{3}$ (b) $26\frac{1}{6}$
	(iii)	1.94^0 C/min.	4.	(a)	$596\sqrt{2}$
	(iv)	28^0C		(b)	Since $a>0$, $a=\frac{1}{3}$
3.	(i)	A (0,4) Equation of line of symmetry is y=2	5.	(i)	5 units.
	(iii)			(ii)	Center of circle (-2,3)
				(iii)	$x^2+y^2+4x-6y-37=0$
			6	(i)	$\frac{x^4+x^3+4x^2-3x+1}{x^3-x^2}$ $=2+x+\frac{2}{x}-\frac{1}{x^2}+\frac{4}{(x-1)}$
		Required shaded area $=8-5\frac{1}{3}=2\frac{2}{3}$ units^2		(ii)	15.7 (3 sig fig.)
7	(a)	$1+\sqrt{2}$ units ∴ the value of h at $\theta=\frac{3}{4}\pi$ is maximum.	8	(iii)a.	$\cos\theta=\frac{3\sqrt{10}}{10}$
	(b)	$\int_0^{\frac{\pi}{4}} 4e^{\cos 2x}\sin 2x\,dx=2(e-\sqrt{e})$		(b)	$1\frac{4}{9}$

9	(ii)	$CD = \sqrt{29}\sin(\theta + 21.80°)$	10.	(i)	$x = 1$ or $-\frac{2}{3}$
	(iii)	$\theta = 68.20°$ $\approx 68.2°$ (to 1 dec.place.)			Stationary points are $(-\frac{2}{3}, 0)$ and $(\frac{4}{9}, -6\frac{14}{81})$
11.	(i) (ii)	A and B are points $(1, 1)$ and $(-1, -1)$ 			 Draw the line $y = 1$ on the graph of $y = f(x)$. The line meets the curve at only one point. Thus $9x^3 + 3x^2 - 8x - 5$ has only one linear factor.
	(iii)	Perpendicular distance from $(\frac{1}{8}, \frac{1}{2})$ to $(\frac{5}{16}, \frac{5}{16})$ $= \frac{3\sqrt{2}}{16}$ units			
	(iv)	**since gradient of tangent and gradient of line y=x are different, the lines will meet.**			

TANJONG KATONG SECONDARY SCHOOL
Preliminary Examination 2014
Secondary 4

CANDIDATE NAME

CLASS | | INDEX NUMBER

ADDITIONAL MATHEMATICS 4047/01

Paper 1

Wed 27August 2014

2 hours

Additional Materials: Writing Paper
Graph Paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Write your answers on the writing paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal in the case of angles in degree, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 80.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!\,r!} = \frac{n(n-1) \ldots\ldots (n-r}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 (a) Find the exact value of x if $27^x = \sqrt{3\sqrt{27}}$. [3]

(b) Find the range of values of m for which $x^2 - 2mx + 2m$ is greater than -3 for all real values of x. [4]

2 The roots of the quadratic equation $2x^2 - 4x - 1 = 0$ are α and β.

(i) Show that $\alpha^3 + \beta^3 = 11$. [3]

(ii) Find a quadratic equation whose roots are $\frac{1}{\alpha^3}$ and $\frac{1}{\beta^3}$. [3]

3 (i) Sketch the graph of $y = |3-2x| - 2$ for the domain $0 \le x \le 4$. [3]

(ii) Find the range of values of x for which $|3-2x| - 2 \le x - 2$ for the domain $0 \le x \le 4$. [4]

4 Show that $y = \ln\left(\frac{3-5x}{2+3x}\right)$ has no stationary point for $-\frac{2}{3} < x < \frac{3}{5}$. [4]

5 Given that $\int_0^4 f(x)\,dx = \int_4^7 f(x)\,dx = 5$,

(i) find $\int_7^4 2f(x)\,dx$. [2]

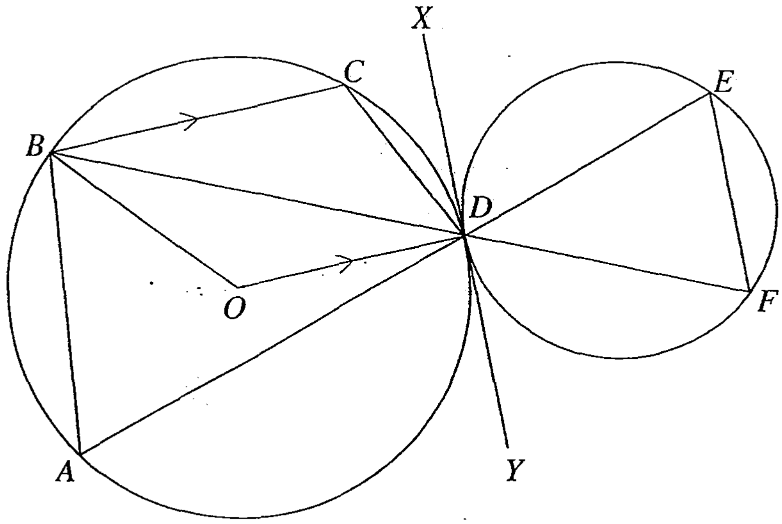
(ii) Explain the geometrical interpretation of $\int_0^7 f(x)\,dx = a$.
State the value of a. [2]

(iii) Given also that $\int_0^{12} f(x)+1\,dx = 27$, evaluate $\int_7^{12} f(x)\,dx$. [3]

6 Find all the angles between $0°$ and $360°$ which satisfy the equation

$5\sec y - 3\cos y = 5\tan y$. [4]

7 The diagram shows 2 circles touching each other at point D.
ADE and BDF are straight lines and they cut the circles at points A, B, E and F.
O is the centre of the bigger circle, BC is parallel to OD and XY is a tangent to the circles at D.



(i) Show that BD bisects $\angle OBC$. [3]

(ii) Prove that triangle ABD is similar to triangle EFD. [4]

8 A moving particle travelling in a straight line passes a fixed point O. Its velocity, $v\,\text{ms}^{-1}$ is given by $v = 8 - 5\sin 3t$, where t is the time in seconds after passing O.

Calculate

(i) the initial acceleration, [2]

(ii) the distance of the particle from O at $t = \frac{\pi}{3}$ s, [3]

(iii) the average velocity of the particle during the first $\frac{\pi}{3}$ seconds. [2]

9 Variables x and y are related by the equation $y = \frac{11x-1}{9-x}$. Given that x and y are functions of t and y increases from an initial value of 2.9 at a constant rate of 0.005 units per second. Find, after 20 seconds,

(i) the value of y, [1]

(ii) the corresponding rate of change of x. [4]

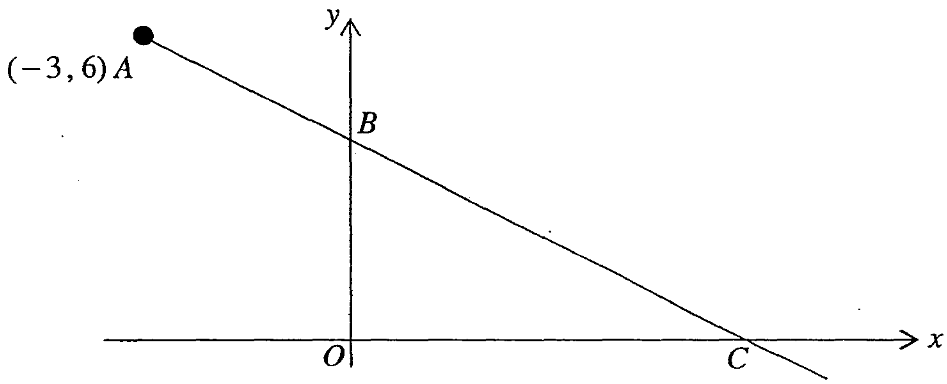
10 A line passing through $A\ (-3, 6)$ cuts the y-axis at B and x-axis at C. Given that $AB : BC = 1 : 2$, find

(i) the coordinates of B and of C, [3]

(ii) the equation of the line AC. [2]

Another line passing through M, the midpoint of AC, cuts the x-axis at N such that $\angle MNC = \angle MCN$.

(iii) Find the equation of the line MN. [3]



11 The table below shows the experimental values of x and of y which are known to be related by the equation $y = p(x+5)^{\frac{3}{2}} - q\sqrt{x+5}$, where p and q are constants.

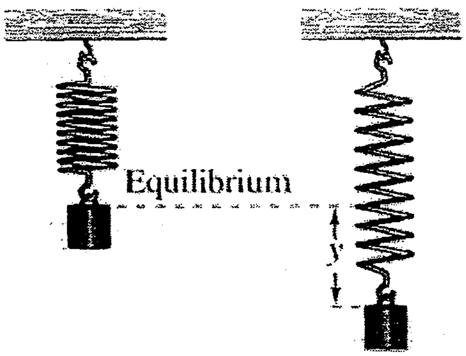
x	0.5	1	1.5	2	2.5
y	24.6	28.2	31.9	35.7	39.7

(i) On graph paper, plot $\frac{y}{\sqrt{x+5}}$ against x and draw a straight line graph. The vertical $\frac{y}{\sqrt{x+5}}$-axis should start at 9 and have a scale of 2 cm to 1. [3]

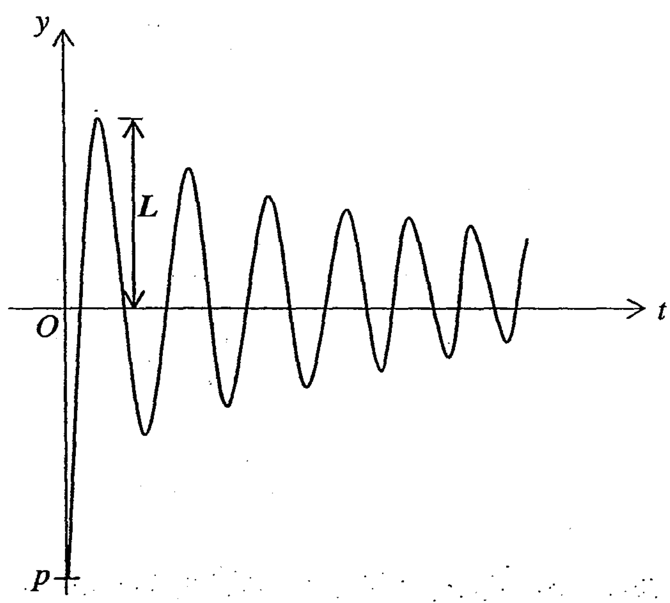
(ii) Use the graph to estimate the value of p and of q. [4]

(iii) By plotting another suitable straight line on the same axes, solve graphically the equation $p(x+5)^{\frac{3}{2}} = \sqrt{x+5}(x+10+q)$. [3]

12 The figure below shows a weight oscillating at the end of a spring. The displacement from equilibrium is given by $y = -2e^{-t}\cos 4t$, $t \geq 0$, where y is the displacement in centimetres and t is the time in seconds.



In order for the spring to start oscillating, the weight has to be first pulled down to a displacement of p cm. The motion of the spring is commonly known as Damped Harmonic Motion and the graph of the displacement from equilibrium is shown below.



(i) State the value of p. [1]

(ii) Show that $\dfrac{dy}{dt} = 2e^{-t}(4\sin 4t + \cos 4t)$. [3]

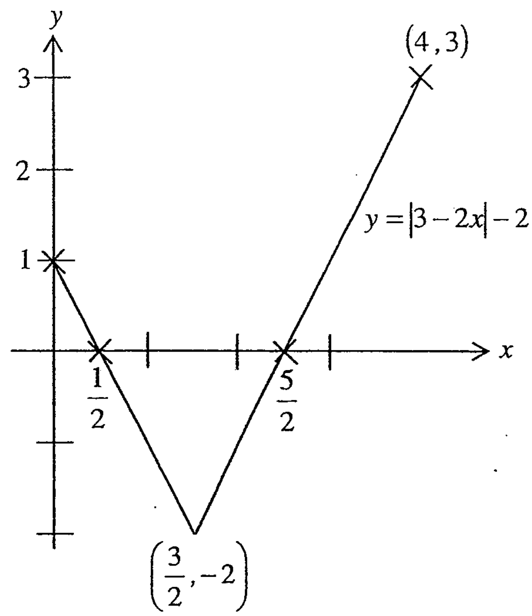
(iii) L is the displacement from equilibrium after the initial weight pull. Find L. [4]

---------- **End of Paper** ----------

Answers

1.(a) $x = \frac{5}{12}$ (b) $-1 < m < 3$ 2. (ii) $x^2 + 88x - 8 = 0$

3. (i)



(ii) $1 \le x \le 3$

4. $\frac{dy}{dx} = -\frac{19}{(3-5x)(2+3x)}$

5. (i) -10

5. (ii) The **area** bounded by $f(x)$ **and the** x**-axis** between $x = 0$ **and** $x = 7$ is a units^2. $a = 10$ (iii) 5

6. $y = 41.8°$, $138.1°$

7. (i) Let $\angle ODB = x$
$\angle OBD = x$ (base $\angle$s of isos. Δ)
$\angle DBC = x$ (alt. $\angle$s)
Since $\angle OBD = \angle DBC$,
$\Rightarrow BD$ bisects $\angle OBC$. (shown)

(ii) $\angle BDA = \angle FDE$ (vert. opp. $\angle$s)
$\angle BDX = \angle FDY$ (vert. opp. $\angle$s)
$\angle BDX = \angle BAD$ (alt. seg. thm)
$\angle FDY = \angle FED$ (alt. seg. thm)
$\Rightarrow \angle BAD = \angle FED$
ΔABD is similar to ΔEFD

8. (i) $a = -15$ m/s^2 (ii) $s = 5.04$ m (iii) 4.82 m/s

9. (i) 3 (ii) 0.0025 units/sec

10. (i) $B = (0, 4)$, $C = (6, 0)$

10 (ii) $y = -\frac{2}{3}x + 4$ (iii) $y = \frac{2}{3}x + 2$

11. (ii) $p = 2$, $q = 0.5$ (iii) $x = 0.5$

12. (i) $p = -2$ (iii) $t = 0.72415$, $L = 0.941$ cm

TANJONG KATONG SECONDARY SCHOOL
Preliminary Examination 2014
Secondary 4

CANDIDATE NAME

CLASS

INDEX NUMBER

ADDITIONAL MATHEMATICS

4047/02

Paper 2

Friday 29August 2014

2 hours 30 minutes

Additional Materials: Writing Paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Write your answers on the writing paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal in the case of angles in degree, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 100.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2}bc \sin A$$

1 **(i)** Sketch the graph of $y = x^{\frac{2}{3}}$. [1]

(ii) On a separate diagram, sketch the graph of $y = \ln(-x)$. [2]

(iii) Find the value of x where the tangents of both the graphs are parallel. [4]

2 **(i)** Express $\dfrac{-7x^2+6x-36}{(2-x)(x^2+9)}$ in partial fractions. [6]

(ii) Hence, find $\displaystyle\int \frac{-7x^2+6x-36}{(2-x)(x^2+9)}\,dx$. [2]

3 It is studied that the population, B, of a certain species of butterfly increases exponentially. Given that $B = 800(3)^{kt}$, where k is a constant and t is the time in days after the study is conducted.

(i) State the initial number of butterflies. [1]

(ii) Given that the population tripled in 18 days, show that the value of $k = \dfrac{1}{18}$. [2]

(iii) Find the number of butterflies after 30 days giving your answer to the nearest whole number. [2]

(iv) Find the number of days taken for the population to exceed 100 000. [3]

4 **(a)** Solve the equation $\log_3(2x+1) - \log_3(x-1) = 2$. [3]

(b) Solve the equation $\log_4 x - \log_x 16 = 1$. [4]

(c) Evaluate $\lg\frac{1}{2} + \lg\frac{2}{3} + \lg\frac{3}{4} + \lg\frac{4}{5} + ... + \lg\frac{99}{100}$. [2]

[Turn over

5 (i) Find $\frac{d}{dx}x\cos^2 x$. [3]

(ii) Using your answer to part (i), find $\int_0^{\frac{\pi}{4}} x\sin 2x\,dx$. [6]

6 (i) In the expansion of $(2+ax)^n$, where n is a positive integer, the coefficients of x and x^2 are in the ratio 1:7. Express a in terms of n. [4]

In the expansion of $(2+x-3x^2)(2+ax)^n$, the constant term is 512.

(ii) Find the value of n and hence, show that $a=4$. [2]

(iii) Hence find the coefficient of x in the expansion of $(2+x-3x^2)(2+ax)^n$. [3]

7 It is given that $f(x)=\frac{(x+2)^2}{p-x}, x \neq p$, where p is an integer.

(i) Obtain and simplify an expression for $f'(x)$ in terms of p. [3]

(ii) The only values of x for which $f(x)$ is an increasing function of x are those values for which $-2 < x < 10$. Show that the value of $p=4$. [4]

(iii) Hence, find the equation of the normal to the curve at the point where the curve crosses the x-axis. [3]

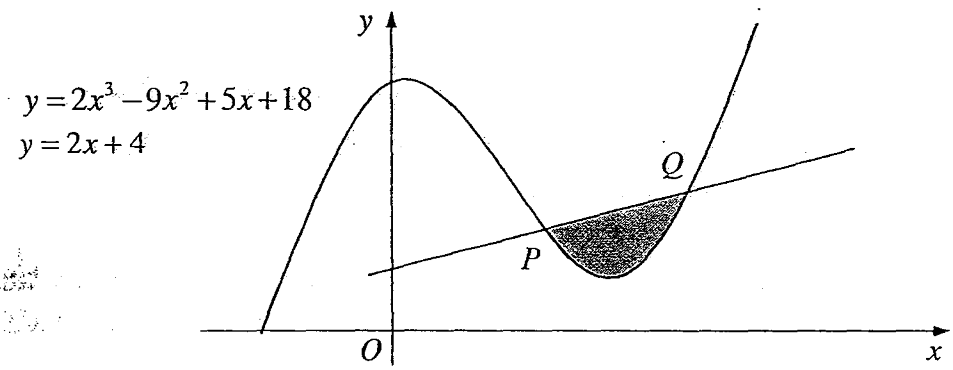
8 A circle with centre A, has a radius of 13 units. The points (–3, 7) and (–3, –17) are on the circumference of this circle. The centre of the circle lies on the right of the y-axis.

(i) Find the equation of the circle. [4]

(ii) Find the equation of the tangent to the circle at (–3, –17). [3]

(iii) Another circle is to be drawn with its centre at (24, 30), such that the two circles do not overlap. What is the maximum radius of the new circle? [3]

9



The line $y = 2x + 4$ cuts the curve $y = 2x^3 - 9x^2 + 5x + 18$ at P and Q.

(i) Find the coordinates of P and of Q. [5]

(ii) Calculate the area of the shaded region. [4]

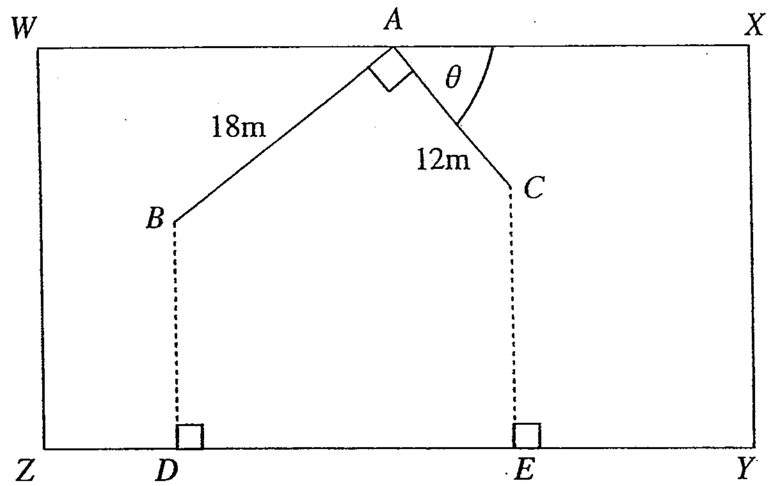
10 **(a)** **(i)** Prove the identity $\dfrac{\tan\theta}{\sec\theta - 1} + \dfrac{\tan\theta}{\sec\theta + 1} = \dfrac{2}{\sin\theta}$. [3]

(ii) Hence, solve the equation $\dfrac{\tan\theta}{\sec\theta - 1} + \dfrac{\tan\theta}{\sec\theta + 1} = 3$, for $0 < \theta < 2\pi$. [4]

(b) Given that angle A is obtuse and $\sin A = \dfrac{3}{4}$, express, without using a calculator, $\dfrac{1}{1 + \tan A}$ in the form $\dfrac{a + b\sqrt{7}}{2}$, where a and b are integers. [4]

[Turn over

11



The diagram shows a rectangle *WXYZ*. Two lines *AB* and *AC* are to be drawn on the rectangle such that $AB = 18$ m, $AC = 12$ m and angle $BAC = 90°$.

AC makes an angle of θ with *WX*, where θ can vary such that $0° < \theta < 90°$.
D and *E* are the feet of the perpendiculars from *B* and *C* to the line *YZ* respectively.

(i) Show that *DE* can be expressed as $12\cos\theta + 18\sin\theta$. [2]

(ii) Express *DE* in the form $R\cos(\theta - \alpha)$, where $R > 0$ and α is an acute angle. [4]

(iii) Find the range of values of *DE*. [2]

(iv) Given that $DE = 20$ m, find the values of θ. [2]

End of paper

7

Answers:

1(iii) -1.84

2(i) $A=-4, B=3, C=0$ 2(ii) $4\ln(2-x)+\frac{3}{2}\ln(x^2+9)+c$

3(i) 800 (ii) $\frac{1}{18}$ (iii) 4992 (iv) 80

4(a) $\frac{10}{7}$ 4(b) 16, 0.25 4(c) -2

5(i) $-x\sin 2x+\cos^2 x$ 5(ii) 0.25

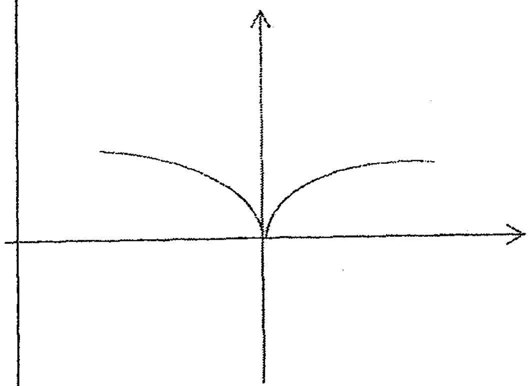
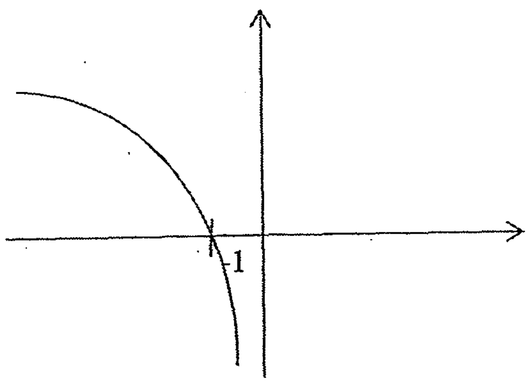
6(i) $a=\frac{28}{n-1}$ 6(ii) $n=8, a=4$ 6(iii) 8448

7(i) $\frac{-x^2+2px+4p+4}{(p-x)^2}$ 7(ii) 4 7(iii) $x=-2$

8(i) $(x-2)^2+(y+5)^2=169$ 8(ii) $y=-\frac{5}{12}x-\frac{73}{4}$ 8(iii) 28.3 9(i) $P(2,8), Q(3.5,11)$ 9(ii) 4.22

10a(ii) 0.411, 2.73 10(b) $\frac{-7-3\sqrt{7}}{2}$

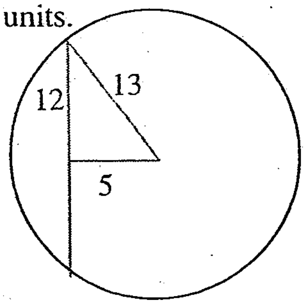
11(i) $12\cos\theta+18\sin\theta$ 11(ii) $6\sqrt{13}\cos(\theta-56.3°)$ 11(iii) $12\le DE\le 6\sqrt{13}$ 11(iv) 33.9, 78.7

No	Solution	Marks	Remarks
1(i)		B1 (graph) symmetry	
1(ii)		B1 (graph) G1 Show intercept	
1(iii)	$\frac{d}{dx}(x^{\frac{2}{3}})=\frac{2}{3}x^{-\frac{1}{3}}$ $\frac{d}{dx}\ln(-x)=\frac{1}{x}$ $\frac{2}{3}x^{-\frac{1}{3}}=\frac{1}{x}$ $x=-1.84$	B1 B1oe M1 equate A1 (reject positive)	
			7 marks
2(i)	$\frac{-7x^2+6x-36}{(2-x)(x^2+9)}=\frac{A}{2-x}+\frac{Bx+C}{x^2+9}$ $-7x^2+6x-36=A(x^2+9)+(Bx+C)(2-x)$ $x=2, A=-4$ $x=0, C=0$ $B=3$	B1 M1 multiply throughout by denominator M1 substitution or compare coefficient A1, A1, A1	
2(ii)	$\int \frac{-4}{2-x}+\frac{3x}{x^2+9}dx=4\ln(2-x)+\frac{3}{2}\ln(x^2+9)+c$	√B1, for both ln (follow through only if ln) B1 +c seen	
			8 marks

No	Solution	Marks	Remarks
3(i)	Sub $t=0$, Initial population = 800	B1	
3(ii)	$\frac{800(3)^{18k}}{800}=3$ $18k=1$ $k=\frac{1}{18}$	M1 Sub t=18 M1 (3 seen) AG	
3(iii)	$B=800(3)^{\frac{30}{18}}$ $=4992$	M1 A1 (not 3sf)	
3(iv)	$100000=800(3)^{\frac{t}{18}}$ $125=3^{\frac{t}{18}}$ $\frac{\ln 125}{\ln 3}=\frac{t}{18}$ $t=79.1$ 80 days	M1 equate or inequality M1 A1	
			8 marks
4a	$\log_3(2x+1)-\log_3(x-1)=2$ $\log_3\frac{2x+1}{x-1}=2$ $\frac{2x+1}{x-1}=9$ $2x+1=9x-9$ $x=\frac{10}{7}$	M1 quotient law in log M1 changing to index form or getting rid of log A1oe	
4b	$\log_4 x-\log_x 16=1$ $\log_4 x-\frac{2}{\log_4 x}=1$ $(\log_4 x)^2-\log_4 x-2=0$ $\log_4 x=2$ or $\log_4 x=-1$ $x=16$ or $\frac{1}{4}$	M1 change base M1 $\log_4 16=2$ (power law) M1 forming quadquad A1 both answers	
4c	$\lg\left(\frac{1}{2}\times\frac{2}{3}\times....\times\frac{99}{100}\right)$ $=\lg\frac{1}{100}$ $=-2$	M1 product law A1	
			9 marks

No	Solution	Marks	Remarks
5(i)	$\frac{d}{dx}(\cos^2 x) = -2\sin x\cos x$ $\frac{d}{dx}(x\cos^2 x)$ $= x(-2\sin x\cos x) + \cos^2 x$ (accept this) $= -x\sin 2x + \cos^2 x$	B1 diff cos$x$ = -sin$x$ B1 chain rule 2cos$x$( ) M1 product rule	
4(ii)	$\int -x\sin 2x + \cos^2 x dx = x\cos^2 x + c$ $\int \cos^2 x dx$ $= \int \frac{\cos 2x+1}{2}dx$ $= \frac{1}{4}\sin 2x + \frac{1}{2}x + c$ $-\int x\sin 2x dx = x\cos^2 x - \frac{1}{4}\sin 2x - \frac{1}{2}x + c$ $\int_0^{\frac{\pi}{4}}$ $= -\left[(\frac{\pi}{8} - \frac{1}{4} - \frac{\pi}{8}) - (0-0-0)\right]$ $= \frac{1}{4}$	M1 double angle B1 working backwards statement M1 reducing to power 1 B1 integration of cos $2x$ M1 substitution A1	
			9 marks
6(i)	$(2+ax)^n = 2^n + n2^{n-1}ax + \binom{n}{2}2^{n-2}a^2x^2 + \ldots$ Coeff of $x = n2^{n-1}a$ Coeff of $x^2 = \frac{n(n-1)}{2}2^{n-2}a^2$ $\frac{\frac{n(n-1)}{2}2^{n-2}a^2}{n2^{n-1}a} = 7$ $a = \frac{28}{n-1}$	B1 B1 M1 equate A1	
6(ii)	$(2+x-3x^2)(2^n + n2^{n-1}ax + \binom{n}{2}2^{n-2}a^2x^2 + \ldots)$ $2^{n+1} = 512$ $n = 8$ $a = 4$	A1 A1 (provided eqn is correct)	

No	Solution	Marks	Remarks
6(iii)	Coeff of x $= 2\binom{8}{1}2^7(4)^1 + 1(2)^8$ $= 8448$	B1, B1 for each term A1	
			9 marks
7(i)	$f'(x)$ $= \frac{(p-x)(2(x+2)) + (-1)(x+2)^2}{(p-x)^2}$ $= \frac{-x^2 + 2px + 4p + 4}{(p-x)^2}$	M1 quotient rule (all correct) M1 chain rule 2(x+2) A1 ISW	
7(ii)	$\frac{-x^2 + 2px + 4p + 4}{(p-x)^2} > 0$ $-x^2 + 2px + 4p + 4 > 0$ Given $-2 < x < 10$, $(x+2)(x-10) < 0$ $-x^2 + 8x + 20 > 0$ Comparing the two inequalities, $2p = 8$ $p = 4$	B1 $f'(x) > 0$ M1 M1 comparing both inequalities A1	
7(iii)	$f'(x) = \frac{-x^2 + 8x + 20}{(p-x)^2}$ At $y = 0$, $x = -2$. Gradient = 0 Equation: $x = -2$	M1 finding x coord M1 finding gradient B1 oe	
			10 marks

No	Solution	Marks	Remarks
8(i)	Midpoint of chord joining (–3, 7) and (–3, –17) is (–3, –5). Centre lies on $y = -5$. Radius = 13, length of chord 24 units. By Pythagoras,  Therefore, x-coordinate = –3+5=2 Centre (2, –5) Equation: $(x-2)^2+(y+5)^2=169$	B1 y coord SOI M1 using half the length of chord and pythagoras thm (students can use diagram to do this) Or substituting into equation of circle to find x coord A1 B1 o.e (13^2 not accepted)	
8(ii)	Gradient of centre to (–3, –17) $=\frac{12}{5}$ Gradient of tangent $=-\frac{5}{12}$ $y+17=-\frac{5}{12}(x+3)$ $y=-\frac{5}{12}x-\frac{73}{4}$	 M1 tan perp. Radius M1 forming equation B1 oe	 SOI
8(iii)	$\sqrt{(24-2)^2+(30-(-5))^2}$ $=\sqrt{1709}$ $\sqrt{1709}-13=28.3$	M1 find distance between centres M1, A1	
			10 marks
9(i)	$2x^3-9x^2+5x+18=2x+4$ $2x^3-9x^2+3x+14=0$ By trial and error, first factor : $(x-2)$ $2x^3-9x^2+3x+14=(x-2)(2x^2+bx-7)$ Comparing coefficients of x, $3=-2b-7$ $b=-5$ $(x-2)(2x^2-5x-7)=(x-2)(2x-7)(x+1)$ $x=2, x=3.5, x=-1$ $P(2,8)$ $Q(3.5,11)$	M1 equating B1 first factor M1 comparing coefficient or long division A1 coordinates of P A1 coordinates of Q	

No	Solution	Marks	Remarks
9(ii)	$\int_2^{3.5} 2x+4-(2x^3-9x^2+5x+18)dx$ $=\left[-\frac{x^4}{2}+3x^3-\frac{3x^2}{2}-14x\right]_2^{3.5}$ $=(-\frac{441}{32}-(-18))$ $=\frac{135}{32}$ $=4.21875$	M1 equation on top-equation below M2 integration A1	-1 mark for each error
			9 marks
10ai	$\frac{\tan\theta}{\sec\theta-1}+\frac{\tan\theta}{\sec\theta+1}$ $=\frac{\tan\theta(\sec\theta+1+\sec\theta-1)}{\sec^2\theta-1}$ $=\frac{\tan\theta(2\sec\theta)}{\tan^2\theta}$ $=\frac{2\sec\theta}{\tan\theta}$ $=\frac{\frac{2}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}}$ $=\frac{2}{\sin\theta}$	M1 combine fractions M1 identity M1 reciprocal functions or any other method (arrives at answer) AG	
10aii	$\frac{2\tan\theta}{\sec\theta-1}+\frac{2\tan\theta}{\sec\theta+1}=10$ $\frac{4}{\sin\theta}=10$ $\sin\theta=\frac{2}{5}$ $\theta=0.411, 2.73$	M1 using identity proven in (i) M1 A1, A1	

No	Solution	Marks	Remarks
10b	$\tan A = -\frac{3}{\sqrt{7}}$ $\frac{1}{1+\tan A}$ $= \frac{1}{1-\frac{3}{\sqrt{7}}}$ $= \frac{\sqrt{7}}{\sqrt{7}-3} \times \frac{\sqrt{7}+3}{\sqrt{7}+3}$ $= \frac{7+3\sqrt{7}}{-2}$ $= \frac{-7-3\sqrt{7}}{2}$	B1 ratio for tangent S.O.I M1 attempt to simplify fraction within fraction M1 rationalising by mutliplying conjugate surd A1	
			11 marks
11(i)	$AD' = 18\sin\theta$ $AC' = 12\cos\theta$ $DE = 12\cos\theta + 18\sin\theta$	B1 B1 Must have clear presentation	
11(ii)	$R = \sqrt{18^2 + 12^2}$ $= 6\sqrt{13}$ or $\sqrt{468}$ $\alpha = \tan^{-1}(\frac{18}{12})$ $= 56.3099°$ $R = 6\sqrt{13}\cos(\theta - 56.3°)$	M1 M1 A1 B1	
11(iii)	$12 \le DE \le 6\sqrt{13}$ or $\sqrt{468}$ or 21.63	B1 upper limit B1 lower limit	
11(iv)	$6\sqrt{13}\cos(\theta - 56.3°) = 20$ $\theta = 78.72°$	M1 A1	
			10 marks

	Class	Register Number
Name ______________________		

4047/01

14/S4PR1/AM/1

ADDITIONAL MATHEMATICS — PAPER 1

Friday 9 May 2014 2 hours

VICTORIA SCHOOL

PRELIMINARY EXAMINATION ONE
SECONDARY FOUR

Additional Material: Answer Paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen.
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The total number of marks for this paper is 80.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

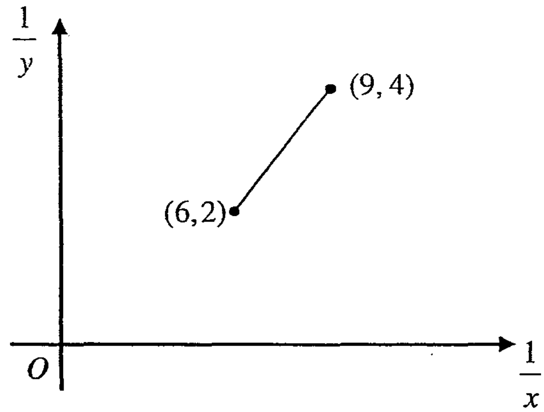
$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2\sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$\Delta = \frac{1}{2}ab\sin C$$

1 Given that $p = 3 - 2\sqrt{3}$, express $(p+1)^2 - \dfrac{6}{p}$ in the form of $a + b\sqrt{3}$, where a and b are integers. [4]

2



The diagram shows part of a straight line graph $\dfrac{py+qx}{xy} = 1$ where $\dfrac{1}{y}$ is plotted against $\dfrac{1}{x}$. The line passes through the points $(6, 2)$ and $(9, 4)$. Find the values of p and of q. [4]

3 Solve the simultaneous equations

$$25^x = \frac{1}{125^y},$$

$$\frac{27^y}{\sqrt{3}} \div \left(\sqrt{3}\right)^{x-1} = 27.$$ [5]

4 Differentiate and simplify each of the following with respect to x.

(i) $\cos^2(2-3x)$ [2]

(ii) $\ln\left(\sqrt{(2x+1)(3x-4)}\right)$ [3]

5 Given that $\int_1^3 f(x)\,dx = \int_3^5 f(x)\,dx = 55$, find the value of

(i) $\int_1^5 3f(x)\,dx + \int_5^3 f(x)\,dx$, [3]

(ii) the constant k, for which $\int_3^5 \left[f(x) - kx^2\right] dx = 6$. [2]

6 Given that the gradient function of a curve, $\frac{dy}{dx} = k\left(\cos x - \frac{3}{4}\sin 2x\right)$, where k is a constant, and the curve passes through the points $(0,4)$ and $\left(\frac{\pi}{2},0\right)$, find the equation of the curve. [5]

7 Given that $\sin A = q$ where A is an obtuse angle, obtain an expression in terms of q, for

(i) $\cos 2A$, [1]

(ii) $\tan^2 A$, [2]

(iii) $\sin 3A$. [3]

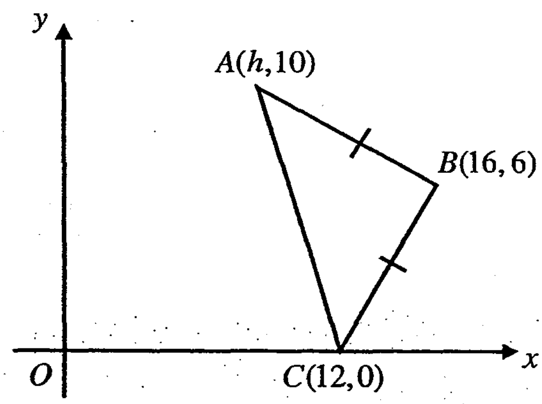
8 **(i)** Solve the equation $|2x^2 - 15| = -x$. [3]

(ii) Sketch on the same diagram, the graphs of $y = |2x^2 - 15|$ and $y = -x$. [3]

(iii) Hence, solve $|2x^2 - 15| < -x$. [1]

9 **Solutions to this question by accurate drawing will not be accepted.**

The diagram shows a triangle ABC with vertices $A(h,10)$, $B(16,6)$ and $C(12,0)$.



Given that $AB = BC$,

(i) find the value of h. [3]

A line is drawn from point B to meet the y-axis at D such that $AD = CD$. Find the

(ii) equation of BD and the coordinates of D, [4]

(iii) area of quadrilateral $ABCD$. [2]

10 Solve the following equations.

(i) $e^{x+1}-8e+13e^{1-x}=0$ [4]

(ii) $\log_2(3-x)-\log_4(x^2+15)=-1$ [5]

11 The function $\mathrm{f}(x)=2-3\sin 3x$ is defined for $0\le x\le \pi$.

(i) State the period and amplitude of f. [2]

(ii) Solve $\mathrm{f}(x)=0$ for $0\le x\le \pi$. [3]

(iii) Sketch the graph of $y=\mathrm{f}(x)$ for $0\le x\le \pi$. [3]

(iv) On the diagram drawn in part **(iii)**, draw the graph of $y=-\frac{2x}{\pi}+1$. [1]

(v) State the number of solutions, for $0\le x\le \pi$, of the equation $1+\frac{2x}{\pi}-3\sin 3x=0$. [1]

12 The expression $2x^3+mx^2+nx+3$, where m and n are constants, has a factor of $x-1$ and leaves a remainder of -9 when divided by $x+2$.

(i) Show that $m=-1$ and $n=-4$. [3]

(ii) Hence solve the equation $2x^3-x^2-4x+3=0$. [3]

(iii) Express $\frac{3x^2+4}{2x^3-x^2-4x+3}$ in partial fractions. [5]

End of Paper

Answer Key

1 $34-12\sqrt{3}$

2 $q=-\frac{1}{2}$ and $p=\frac{1}{3}$

3 $x=-1\frac{1}{5}$, $y=\frac{4}{5}$

4 i) $6\sin(2-3x)\cos(2-3x)$

ii) $\frac{12x-5}{2(2x+1)(3x-4)}$

5 i) 275

ii) $k=1\frac{1}{2}$

6 $y=-16\sin x-6\cos 2x+10$

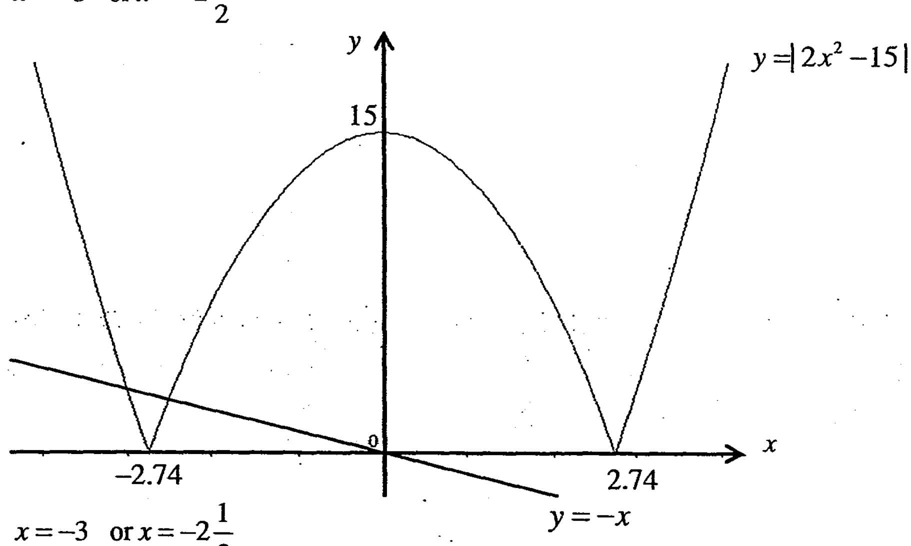
7 i) $1-2q^2$

ii) $\frac{q^2}{(1+q)(1-q)}$

iii) $q(3-4q^2)$

8 i) $x=-3$ or $x=-2\frac{1}{2}$

ii)



iii) $x=-3$ or $x=-2\frac{1}{2}$

9 i) $h=10$

ii)

$\therefore$ Equation of BD is $y=\frac{1}{5}x+2\frac{4}{5}$

$\therefore$ Coordinate of D is $\left(0, 2\frac{4}{5}\right)$.

iii) 83.2 sq units

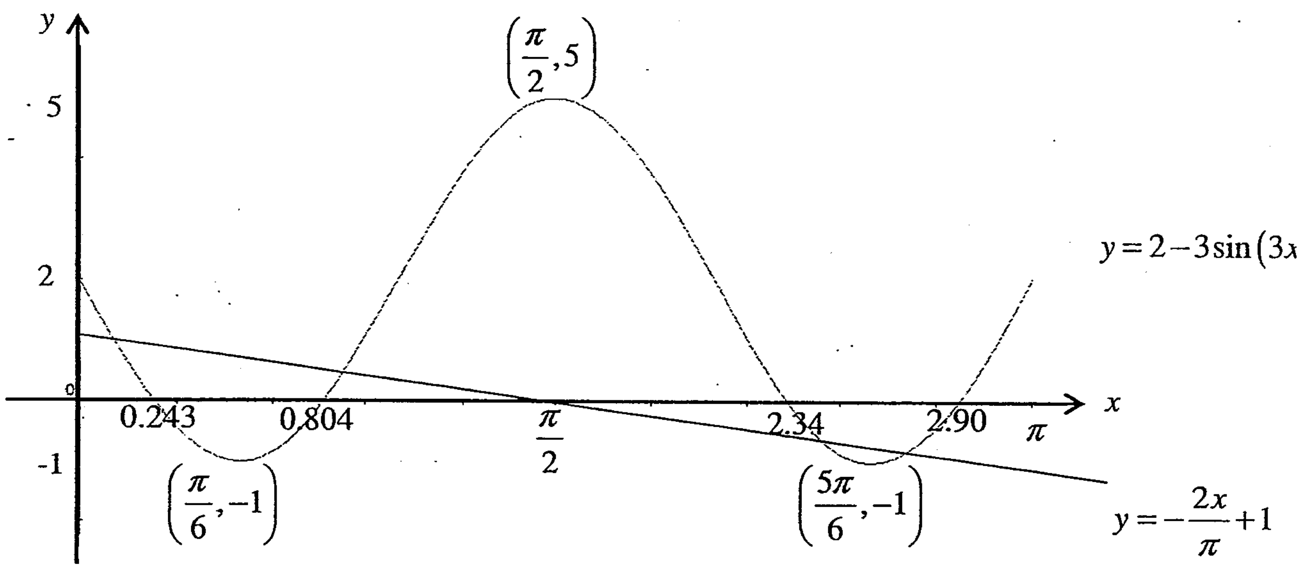
10 i) $x \approx 0.819$ or $x \approx 1.75$

ii) $x=1$

11 i) Period $=\frac{2\pi}{3}$, amplitude $=3$

ii) $x \approx 0.243,\ 0.804,\ 2.34,\ 2.90$

iii)



iv) 4 solutions

12 ii) $x=1$ or $x=-1\frac{1}{2}$

iii) $\frac{3x^2+4}{2x^3-x^2-4x+3}=\frac{16}{25(x-1)}+\frac{7}{5(x-1)^2}+\frac{43}{25(2x+3)}$

	Class	Register Number
Name MARK SCHEME		

4047/01

14/S4PR1/AM/1

ADDITIONAL MATHEMATICS

PAPER 1

Friday 9 May 2014 2 hours

VICTORIA SCHOOL

PRELIMINARY EXAMINATION ONE
SECONDARY FOUR

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$$\Delta = \frac{1}{2}ab \sin C$$

1 Given that $p = 3 - 2\sqrt{3}$, express $(p+1)^2 - \frac{6}{p}$ in the form of $a + b\sqrt{3}$, where a and b are integers. [4]

Solution:

$$(p+1)^2 - \frac{6}{p}$$

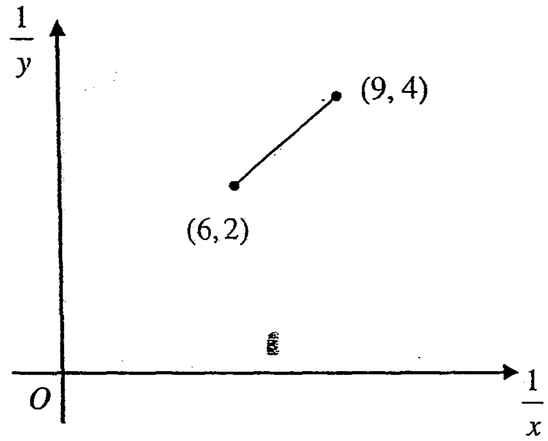
$$= \left(3 - 2\sqrt{3} + 1\right)^2 - \frac{6}{3 - 2\sqrt{3}}\left(\frac{3 + 2\sqrt{3}}{3 + 2\sqrt{3}}\right)$$ B1 × conjugate surd

$$= \left(4 - 2\sqrt{3}\right)^2 - \frac{6\left(3 + 2\sqrt{3}\right)}{9 - 12}$$

$$= 16 - 16\sqrt{3} + 12 + 2\left(3 + 2\sqrt{3}\right)$$ B2 $16 - 16\sqrt{3} + 12$, $+2\left(3 + 2\sqrt{3}\right)$ oe

$$= 34 - 12\sqrt{3}$$ A1

2



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Solution:

$$\frac{py+qx}{xy}=1$$

$$\frac{p}{x}+\frac{q}{y}=1$$

$$\frac{q}{y}=-p\left(\frac{1}{x}\right)+1$$

$$\frac{1}{y}=-\frac{p}{q}\left(\frac{1}{x}\right)+\frac{1}{q}$$ M1

$$Y=mX+c$$

Sub gradient $=\frac{4-2}{9-6}=\frac{2}{3}$ and $(6,2)$ into

$$Y=mX+c$$

$$2=\frac{2}{3}(6)+c$$

$$c=-2$$

$\therefore \frac{1}{q}=-2$ and $-\frac{p}{q}=\frac{2}{3}$ M2

$q=-\frac{1}{2}$ and $p=\frac{1}{3}$ A1

3 Solve the simultaneous equations

$$25^x = \frac{1}{125^y} \quad , \text{----------------------(1)}$$

$$\frac{27^y}{\sqrt{3}} \div \left(\sqrt{3}\right)^{x-1} = 27 \text{ ----------------------(2)}$$

[5]

Solution:

From (1)

$$25^x = \frac{1}{125^y}$$

$$5^{2x} = \frac{1}{5^{3y}}$$

$$5^{2x} = 5^{-3y}$$

$$2x = -3y \text{ ----------(3)}$$

A1 oe

From (2)

$$\frac{27^y}{\sqrt{3}} \div \left(\sqrt{3}\right)^{x-1} = 27$$

$$\frac{3^{3y}}{3^{\frac{1}{2}}} \div 3^{\frac{1}{2}x-\frac{1}{2}} = 3^3$$

$$3^{3y-\frac{1}{2}-(\frac{1}{2}x-\frac{1}{2})} = 3^3$$

$$3y - \frac{1}{2}x = 3 \text{ ----------(4)}$$

A1 oe

Sub $2x = -3y$ into (4) M1

$$3y - \frac{1}{2}x = 3$$

$$-2x - \frac{1}{2}x = 3$$

$$-2\frac{1}{2}x = 3$$

$$x = -1\frac{1}{5}$$

Sub $x = 2$ into (3) : $y = \frac{4}{5}$

$\therefore x = -1\frac{1}{5},\ y = \frac{4}{5}$ A2

4 Differentiate and simplify each of the following with respect to x.

(i) $\cos^2(2-3x)$ [2]

(ii) $\ln\left(\sqrt{(2x+1)(3x-4)}\right)$ [3]

Solution:

(i)

$$\frac{d}{dx}\cos^2(2-3x)$$

$= -2\cos(2-3x)\sin(2-3x).(-3)$ B1

$= 6\sin(2-3x)\cos(2-3x)$ A1

$= 3\sin[2(2-3x)]$

$= 3\sin(4-6x)$

ii)

$$\frac{d}{dx}\ln\left(\sqrt{(2x+1)(3x-4)}\right)$$

$= \frac{d}{dx}\left[\frac{1}{2}\ln(2x+1)+\frac{1}{2}\ln(3x-4)\right]$ B1 power law, product rule

$= \frac{1}{2x+1}+\frac{3}{2(3x-4)}$ A2 differentiation

$= \frac{6x-8+6x+3}{2(2x+1)(3x-4)}$

$= \frac{12x-5}{2(2x+1)(3x-4)}$

Alternative

(ii)

$$\frac{d}{dx}\ln\left(\sqrt{(2x+1)(3x-4)}\right)$$

$= \frac{d}{dx}\left[\frac{1}{2}\ln(6x^2-5x-4)\right]$ B1 power law & expansion

$= \frac{1}{2}\left(\frac{12x-5}{6x^2-5x-4}\right)$ M1 differentiation

$= \frac{12x-5}{2(2x+1)(3x-4)}$ A1

5 Given that $\int_1^3 f(x)\,dx = \int_3^5 f(x)\,dx = 55$, find the value of

(i) $\int_1^5 3f(x)\,dx + \int_5^3 f(x)\,dx$ [3]

(ii) the constant k, for which $\int_3^5 \left[f(x) - kx^2\right] dx = 6$. [2]

Solution:

(i)

$$\int_1^5 3f(x)\,dx + \int_5^3 f(x)\,dx$$

$$= 3\left[\int_1^3 f(x)\,dx + \int_3^5 f(x)\,dx\right] - \int_3^5 f(x)\,dx \quad \text{B1}$$

$$= 3(110) - 55 \quad \text{B1}$$

$$= 275 \quad \text{A1}$$

(ii)

$$\int_3^5 \left[f(x) - kx^2\right] dx = 6$$

$$\int_3^5 f(x)\,dx - k\int_3^5 x^2\,dx = 6$$

$$k\int_3^5 x^2\,dx = \int_3^5 f(x)\,dx - 6$$

$$k\left[\frac{x^3}{3}\right]_3^5 = 55 - 6$$

B1 integration of x^2 and sub 55

$$k\left(\frac{5^3}{3} - \frac{3^3}{3}\right) = 49$$

$$k = 1\frac{1}{2}$$

A1

6 Given that the gradient function of a curve, $\frac{dy}{dx}=k\left(\cos x-\frac{3}{4}\sin 2x\right)$, where k is a constant and the curve passes through the points $(0,4)$ and $\left(\frac{\pi}{2},0\right)$, find the equation of the curve. [5]

Solution:

Let the curve be y. It passes through $(0,4)$ and $\left(\frac{\pi}{2},0\right)$

$$\frac{dy}{dx}=k\left(\cos x-\frac{3}{4}\sin 2x\right)$$

$$=k\cos x-\frac{3k}{4}\sin 2x$$

$$y=\int\left(k\cos x-\frac{3k}{4}\sin 2x\right)\ dx$$

$$y=k\sin x+\frac{3k}{8}\cos 2x+c\text{ ------------(1)}$$ A1

At $(0,4)$,

$$4=k\sin 0+\frac{3k}{8}\cos 0+c$$ M1

$$4=\frac{3k}{8}+c$$

$$32=3k+8c\text{ --------------(2)}$$

At $\left(\frac{\pi}{2},0\right)$,

$$0=k\sin\frac{\pi}{2}+\frac{3k}{8}\cos\pi+c$$ M1

$$0=k+\frac{3k}{8}(-1)+c$$

$$0=8k-3k+8c$$

$$-5k=8c\text{ -----------------(3)}$$

Sub (3) into (2): $32=3k-5k$ M1

$$k=-16$$

Sub $k=-16$ into (3):

$$c=\frac{-5(-16)}{8}=10$$

$\therefore$ Equation of curve is $y=-16\sin x-6\cos 2x+10$ A1 oe

7 Given that $\sin A = q$ where A is an obtuse angle, obtain an expression in terms of q, for

(i) $\cos 2A$, [1]

(ii) $\tan^2 A$, [2]

(iii) $\sin 3A$. [3]

Solution:

$\sin A = q$, $\cos A = -\sqrt{1-q^2}$, $\tan A = -\dfrac{q}{\sqrt{1-q^2}}$

(i) $\cos 2A = 1 - 2\sin^2 A$

$= 1 - 2q^2$ A1

(ii) $\tan^2 A = \left(-\dfrac{q}{\sqrt{1-q^2}}\right)^2$ B1

$= \dfrac{q^2}{(1+q)(1-q)}$ A1

(iii) $\sin 3A = \sin(2A + A)$

$= \sin 2A \cos A + \cos 2A \sin A$

$= 2\sin A \cos^2 A + \cos 2A \sin A$ B1 use of formula

$= 2q\left(-\sqrt{1-q^2}\right)^2 + \left(1-2q^2\right)q$ M1 substitution

$= 2q\left(1-q^2\right) + q(1-2q^2)$

$= 2q - 2q^3 + q - 2q^3$

$= 3q - 4q^3$

$= q(3-4q^2)$ A1

> Alternative
>
> (ii) $\tan^2 A = \dfrac{\sin^2 A}{\cos^2 A}$
>
> $= \dfrac{q^2}{\left(-\sqrt{1-q^2}\right)^2}$ B1
>
> $= \dfrac{q^2}{(1+q)(1-q)}$ or $\dfrac{q^2}{1-q^2}$ A1

8 **(i)** Solve the equation $|2x^2-15|=-x$. [3]

(ii) Sketch on the same diagram, the graphs of $y=|2x^2-15|$ and $y=-x$. [3]

(iii) Hence, find the solution for $|2x^2-15|<-x$. [1]

Solution:

(i) $|2x^2-15|=-x$

$2x^2-15=-x$ or $2x^2-15=x$ M1

$2x^2+x-15=0$ $\quad 2x^2-x-15=0$

$(2x-5)(x+3)=0$ $\quad (2x+5)(x-3)=0$ M1

$x=2\frac{1}{2}$ or $x=-3$ $\quad x=-2\frac{1}{2}$ or $x=3$

(NA) (NA)

$\therefore x=-3$ or $x=-2\frac{1}{2}$ A1

(ii) B1 shape of $y=|2x^2-15|$

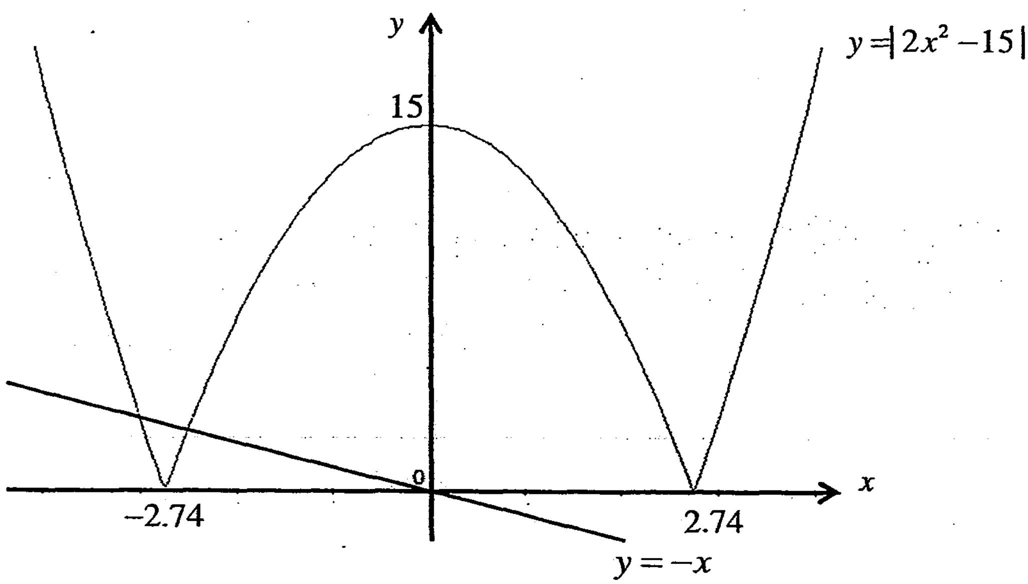
B1 line $y=-x$

B1 x and y intercepts and label

When $|2x^2-15|=0$

$2x^2-15=0$

$x=\pm\sqrt{7.5}$



(iii) $-3<x<-2\frac{1}{2}$ A1

9 **Solutions to this question by accurate drawing will not be accepted.**

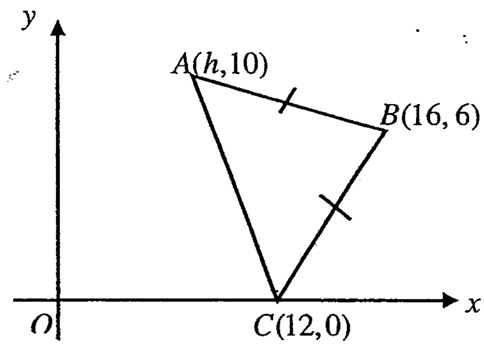
The diagram shows a triangle ABC with vertices $A(h,10)$, $B(16,6)$ and $C(12,0)$.
Given that $AB = BC$,

(i) find the value of h. [3]

A line is drawn from point B to meet the y-axis at D such that $AD = CD$. Find

(ii) the equation of BD and the coordinates of D. [4]
(iii) the area of quadrilateral $ABCD$. [2]

Solution:



(i) $AB = BC$

$\sqrt{(h-16)^2+(10-6)^2} = \sqrt{(16-12)^2+(6-0)^2}$ M1

$h^2 - 32h + 256 + 16 = 16 + 36$

$h^2 - 32h + 220 = 0$

$(h-10)(h-22) = 0$ M1

$h = 10$ or $h = 22$ (NA) A1

Alternative

(i) $4^2 + (h-16)^2 = 4^2 + 36$ M1

$(h-16)^2 = 36$

$h - 16 = -6$ or $h - 16 = 6$ M1

$h = 10$ or $h = 22$ (NA) A1

(ii) Since $AD = CD$,

BD is the perpendicular bisector of AC

Midpoint $AC = \left(\dfrac{10+12}{2}, \dfrac{10+0}{2}\right)$

$= (11, 5)$

Gradient $BM = \dfrac{6-5}{16-11} = \dfrac{1}{5}$ M1

Sub gradient and (16,6) into $y = mx + c$

$6 = \dfrac{1}{5}(16) + c$ M1

$c = 2\dfrac{4}{5}$

$\therefore$ Equation of BD is $y = \dfrac{1}{5}x + 2\dfrac{4}{5}$ A1

$\therefore$ Coordinate of D is $\left(0, 2\dfrac{4}{5}\right)$. A1

Alternative

(ii) Gradient $AC = \dfrac{0-10}{12-10} = -5$

Gradient $BD = \dfrac{1}{5}$ M1

Sub gradient and (16,6) into $y = mx + c$

$6 = \dfrac{1}{5}(16) + c$ M1

$c = 2\dfrac{4}{5}$

$\therefore$ Equation of BD is $y = \dfrac{1}{5}x + 2\dfrac{4}{5}$ A1

(iii) Area $= \dfrac{1}{2}\begin{vmatrix} 10 & 0 & 12 & 16 & 10 \\ 10 & 2.8 & 0 & 6 & 10 \end{vmatrix}$

$= \dfrac{1}{2}\left[(28+0+72+160)-(0+33.6+0+60)\right]$ M1

$= \dfrac{1}{2}(260 - 93.6)$

$= 83.2$ sq units A1

10 Solve the following equations.

(i) $e^{x+1}-8e+13e^{1-x}=0,$ [4]

(ii) $\log_2(3-x)-\log_4(x^2+15)=-1$ [5]

Solution:

(i) $e^{x+1}-8e+13e^{1-x}=0$

$e^x.e-8e+\frac{13e}{e^x}=0$

$e.e^{2x}-8e.e^x+13e=0$ — M1 multiply by e^x

$e^{2x}-8e^x+13=0$

$e^x=\frac{-(-8)\pm\sqrt{(-8)^2-4(1)(13)}}{2}$ — M1

$e^x\approx 2.268$ or $e^x\approx 5.732$

$x\approx 0.819$ or $x\approx 1.75$ — A2

(ii) $\log_2(3-x)-\log_4(x^2+15)=-1$

$\log_2(3-x)-\frac{\log_2(x^2+15)}{\log_2 4}=-1$ — M1 change of base law

$\log_2(3-x)-\frac{1}{2}\log_2(x^2+15)=-1$

$\log_2\left(\frac{3-x}{\sqrt{x^2+15}}\right)=\log_2 2^{-1}$ — M1 quotient law & power law

$\therefore \frac{3-x}{\sqrt{x^2+15}}=\frac{1}{2}$ — M1 anti log

$\sqrt{x^2+15}=6-2x$

$x^2+15=36-24x+4x^2$

$3x^2-24x+21=0$

$x^2-8x+7=0$

$(x-7)(x-1)=0$ — M1

$x=7$ or $x=1$ — A1

NA

Alternative

(ii) $\log_2(3-x)-\log_4(x^2+15)=-1$

$\log_2(3-x)-\dfrac{\log_2(x^2+15)}{\log_2 4}=-1$ M1 change of base law

$\log_2(3-x)-\dfrac{1}{2}\log_2(x^2+15)=-1$

$2\log_2(3-x)-\log_2(x^2+15)=-2$

$\log_2\dfrac{(3-x)^2}{x^2+15}=-2$ M1 quotient law & power law

$\therefore \dfrac{9-6x+x^2}{x^2+15}=2^{-2}$ M1 anti log

$x^2+15=36-24x+4x^2$

$3x^2-24x+21=0$

$x^2-8x+7=0$

$(x-7)(x-1)=0$ M1

$x=7$ or $x=1$ A1

NA

11 The function $f(x) = 2 - 3\sin 3x$ is defined for $0 \le x \le \pi$.

(i) State the period and amplitude of f. [2]

(ii) Solve $f(x) = 0$ for $0 \le x \le \pi$. [3]

(iii) Sketch the graph of $y = f(x)$ for $0 \le x \le \pi$. [3]

(iv) On the diagram drawn in part **(iii)**, draw the graph of $y = -\dfrac{2x}{\pi} + 1$. [1]

(v) State the number of solutions, for $0 \le x \le \pi$, of the equation $1 + \dfrac{2x}{\pi} - 3\sin 3x = 0$. [1]

Solution:

(i) Period $= \dfrac{2\pi}{3}$, amplitude $= 3$ B2

(ii) $f(x) = 0$ $\quad 0 \le x \le \pi$

$2 - 3\sin 3x = 0$ $\quad 0 \le 3x \le 3\pi$

$\sin 3x = \dfrac{2}{3}$

$3x$ in 1st/2nd quad

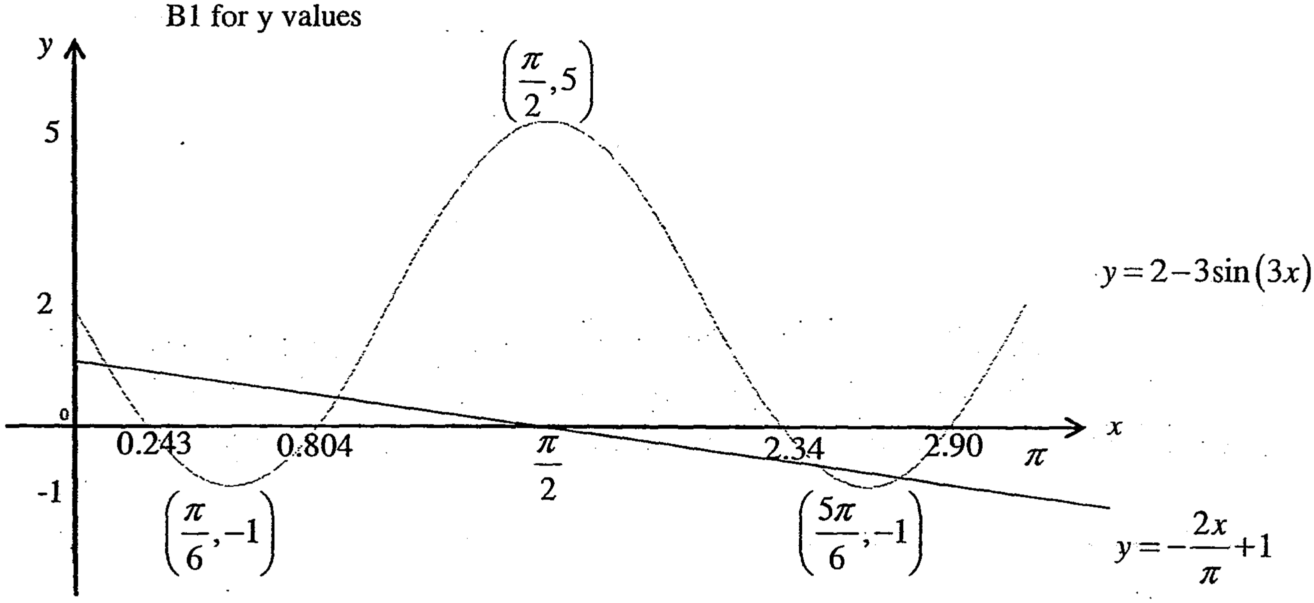
Basic □ ≈ 0.7297 A1

$3x = 0.7297, \quad \pi - 0.7297, \quad 2\pi + 0.7297, \quad 3\pi - 0.7297$

$x \approx 0.243, \quad 0.804, \quad 2.34, \quad 2.90$ A2

(iii), B1 for shape and y values
B1 for x intercepts & values
B1 for y values

(iv) B1 for straight line



(v) $1 + \dfrac{2x}{\pi} - 3\sin 3x = 0$

$2 - 3\sin 3x = -\dfrac{2x}{\pi} + 1$

$\therefore$ There are 4 solutions. A1

12 The expression $2x^3+mx^2+nx+3$, where m and n are constants, has a factor of $x-1$ and leaves a remainder of -9 when divided by $x+2$.

(i) Show that $m=-1$ and $n=-4$. [3]

(ii) Hence solve the equation $2x^3-x^2-4x+3=0$ [3]

(iii) Express $\dfrac{3x^2+4}{2x^3-x^2-4x+3}$ in partial fractions. [5]

Solution:

(i) Let $f(x)=2x^3+mx^2+nx+3$

By factor theorem,

$f(1)=0$

$2+m+n+3=0$ M1

$m=-n-5$ ------------(1)

By remainder theorem,

$f(-2)=-9$

$2(-2)^3+m(-2)^2+n(-2)+3=-9$ M1

$-16+4m-2n+3=-9$

$4m-2n=4$ ------------(2)

Sub (1) into (2):

$4(-n-5)-2n=4$

$-4n-20-2n=4$

$-6n=24$

$n=-4$

Sub $n=-4$ into (1),

$\therefore\ m=-1$ (shown) A1 sub and follow through

(ii) Let $2x^3-x^2-4x+3=(x-1)(ax^2+bx+c)=0$

Comparing coefficient of

x^3: $2=a$

x^0: $3=-c$ $\therefore c=-3$

x^2: $-1=b-a$ $\therefore b=1$

$\therefore 2x^3-x^2-4x+3=0$

$(x-1)(2x^2+x-3)=0$ A1 for $(2x^2+x-3)$

$(x-1)(x-1)(2x+3)=0$ M1

$\therefore x=1$ or $x=-1\frac{1}{2}$ A1

Alternative

(ii)

$$\begin{array}{r} 2x^2+x-3 \\ x-1\,\big)\,\overline{2x^3-x^2-4x+3} \\ \underline{2x^3-2x^2} \\ x^2-4x \\ \underline{x^2-x} \\ -3x+3 \\ \underline{-3x+3} \\ 0 \end{array}$$

(iii) Let $\dfrac{3x^2+4}{2x^3-x^2-4x+3}=\dfrac{3x^2+4}{(x-1)^2(2x+3)}=\dfrac{A}{x-1}+\dfrac{B}{(x-1)^2}+\dfrac{C}{2x+3}$ M1

$\therefore\ 3x^2+4=A(x-1)(2x+3)+B(2x+3)+C(x-1)^2$

When $x=1$,

$7=5B \qquad \therefore\ B=\dfrac{7}{5}$ A1

When $x=-1\dfrac{1}{2}$,

$10\dfrac{3}{4}=6\dfrac{1}{4}C \qquad \therefore\ C=\dfrac{43}{25}$ A1

Comparing coefficient of x^2:

$3=2A+C \qquad \therefore\ A=\dfrac{16}{25}$ A1

$\therefore\dfrac{3x^2+4}{2x^3-x^2-4x+3}=\dfrac{16}{25(x-1)}+\dfrac{7}{5(x-1)^2}+\dfrac{43}{25(2x+3)}$ A1

End of Paper

Name ____________ Class ____ Register Number ____

4047/02 **14/S4PR1/AM/2**

ADDITIONAL MATHEMATICS PAPER 2

Monday 12May 2014 2 hours 30 minutes

VICTORIA SCHOOL

PRELIMINARY EXAMINATION ONE
SECONDARY FOUR

Additional Materials: Answer Paper
Graph Paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
You are expected to use a scientific calculator to evaluate explicit numerical expressions.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \dots + \binom{n}{r} a^{n-r} b^r + \dots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\dots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1 (i) Prove that $\dfrac{\sin(A+B)+\sin(B-A)}{\cos(A+B)-\cos(B-A)} = -\cot A$. [3]

(ii) Hence, solve $\dfrac{\sin(A+B)+\sin(B-A)}{\cos(A+B)-\cos(B-A)} = 4\cos^2 A$ for $0 \le A \le \pi$, giving your answers in terms of π. [4]

2 (a) Write down and simplify the first three terms in the expansion of $\left(x+\dfrac{1}{200x^2}\right)^{10}$ in descending powers of x. Hence evaluate $(1.005)^{10}$ correct to five significant figures. [4]

(b) Find the coefficient of the third term in the expansion of $\left(a-\dfrac{1}{x}\right)^4$ and the coefficient of the second term in the expansion of $\left(2+\dfrac{x}{a}\right)^6$ in terms of a.

If the coefficient of the second term of $\left(2+\dfrac{x}{a}\right)^6$ is four times the coefficient of the third term of $\left(a-\dfrac{1}{x}\right)^4$, find the value of a. [4]

3 The roots of the quadratic equation $2x^2+x-5=0$ are α and β.

(i) Express $\alpha^2-\alpha\beta+\beta^2$ in terms of $(\alpha+\beta)$ and $\alpha\beta$. [1]

(ii) Hence, find the exact value of $\alpha^2+\beta^2$. [3]

(iii) Find a quadratic equation whose roots are α^3 and β^3. [4]

4 (i) Differentiate $\dfrac{2x}{\sin x}$ with respect to x. [3]

(ii) Using your answer to part (i), find $\displaystyle\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \left(\frac{1}{\sin x}-\frac{x\cos x}{\sin^2 x}\right) dx$. [5]

5 A curve has the equation $y = f(x)$, where $f(x) = (2-x)e^{-3x}$.

(i) Obtain an expression for $f'(x)$. [2]

(ii) Find the equation of the normal to the curve at the point where the curve crosses the x-axis. Leave your answer in terms of e. [5]

(iii) Determine, with explanation, whether f is an increasing or decreasing function for $x < 2\frac{1}{3}$. [2]

6 A circle with centre C passes through points $A(-1,7)$ and $B(0,8)$.

(i) Explain why the perpendicular bisector of AB will pass through C. [1]

(ii) Given further that the line $y = 2x-2$ passes through the centre of the circle, show that the coordinates of C is $(3,4)$. [4]

(iii) Hence, find the equation of the circle in the general form. [3]

7 (a) Find the exact values of k for which the line $y-3x=k$ is a tangent to the curve $y^2+4x+5=x^2$. [5]

(b) Calculate the smallest negative integer k for which the equation $y = 4x^2+(12-4k)x+15-7k$ is always positive for all real values of x. [3]

(c) Solve the inequality $4x^2-1 > (2x+1)(x-3)$. [2]

8 **Answer the whole of this question on a sheet of graph paper.**

The population P, in millions, of a country was recorded in certain years and the results are shown in the table below:

Year	1978	1984	1988	1993
P	12.40	14.48	16.76	21.22

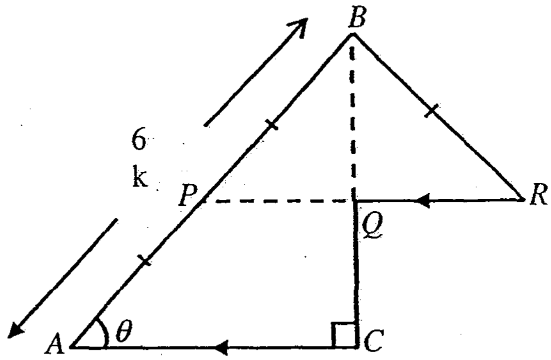
It is given that $P = 10 + ab^x$, where x is the time measured in years from January 1975 and a and b are constants. By plotting $\lg(P-10)$ against x, draw a line graph for the given data for $0 \le x \le 20$. Use a scale of 2 cm to represent 0.1 units on the vertical axis and 4 cm to represent 5 units on the horizontal axis. [4]

Use your graph to estimate,

(i) the value of a and of b. [3]

(ii) the year in which the population will reach 23.8 millions. [3]

9



In the diagram, *ABRQC* represents a cycling course. From point*A*, a cyclist travels along straight tracks *AB*, *BR*, *RQ* and *QC*, returning along the track *CA* to finish at *A*. The total length of the course is *L* km.

It is given that $AB = 6$km and *P* is the mid-point of *AB*. The length of each of the tracks *AP*, *PB* and *BR* are equal and track *QC* is also perpendicular to track *AC*. Track*QR*is parallel to the track*AC*and angle*BAC*is $\theta°$.

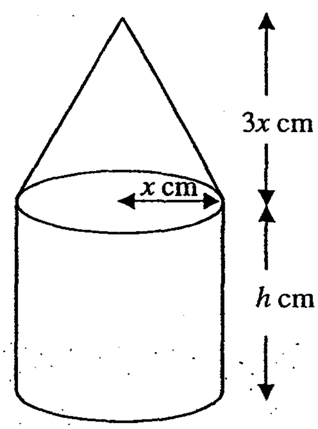
(i) Show that *L* can be expressed as $p + q\sin\theta + r\cos\theta$, where *p*, *q* and *r* are constants to be found. [3]

(ii) Express *L* in the form $p + R\cos(\theta - \alpha)$ where $R > 0$, and α is acute. [5]

The total length of the course is found to be 16.5 km.

(iii) Find the value of θ. [2]

10

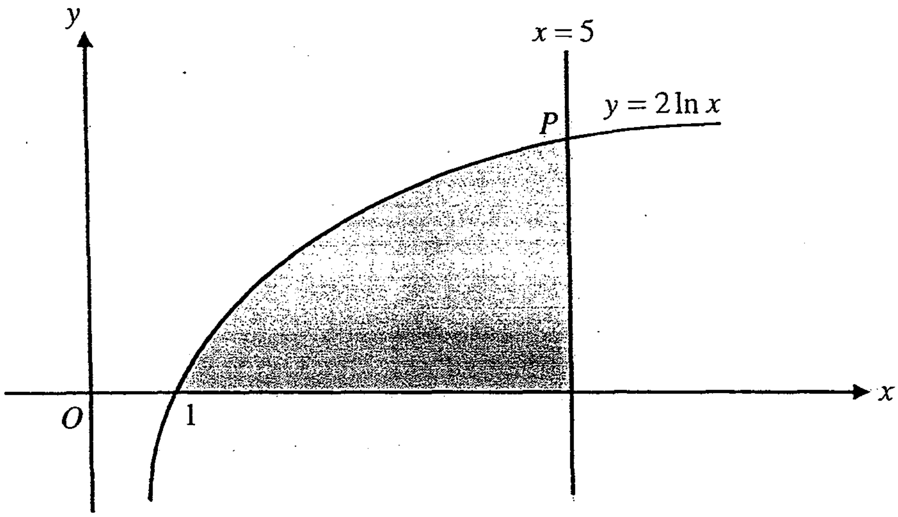


The diagram shows a solid comprising of a right-circular cone of height 3*x* cm sitting ona cylinder of base radius *x* cm and height *h* cm. The volume of the solid is 250 cm^3.

(i) Express *h* in terms of *x* and hence show that the total surface area, S cm^2, of the solid is given by $S = \pi(\sqrt{10} - 1)x^2 + \dfrac{500}{x}$. [6]

(ii) Given that *x* can vary, find, the value of *x* for which *S* has a stationary value. Find this value of *S* and determine whether it is a maximum or a minimum value. [4]

11 **(a)** The diagram below shows part of the curve $y = 2\ln x$ and the line $x = 5$.



Find the

(i) coordinates of P, [2]

(ii) area of the shaded region. [3]

(b) The concentration of caffeine, C %, in Peter's blood when he drinks a cup of coffee in the morning is represented by $C = 0.2xe^{-1.8x}$, where x is the number of hours after drinking a cup of coffee.

(i) Calculate the value of C when $x = 2$. [1]

(ii) Find the rate of change of caffeine concentration in Peter's blood, 1.5 hours after drinking a cup of coffee. [3]

(iii) What is the maximum level of caffeine concentration in Peter's blood in the morning? [3]

End of Paper

2014 A MATH PAPER 2 ANSWER KEY

1 (i) Proof (ii) $A=\frac{\pi}{2},\frac{7\pi}{12},\frac{11\pi}{12}$

2 (a) $x^{10}+\frac{1}{20}x^7+\frac{9}{8000}x^4+....$, 1.0511 **(b)** $T_3=6a^2$, $T_2=\frac{192}{a}$, $a=2$

3 (i) $\alpha^2-\alpha\beta+\beta^2=(\alpha+\beta)^2-3\alpha\beta$ **(ii)** $\alpha^2+\beta^2=5\frac{1}{4}$ **(iii)** $8x^2+31x-125=0$

4 (i) $\frac{d}{dx}\left(\frac{2x}{\sin x}\right)=\frac{2(\sin x-2x\cos x)}{\sin^2 x}$ **(ii)** -6.28

5 (i) $\therefore \mathrm{f}'(x)=e^{-3x}(3x-7)$ **(ii)** $y=e^6x-2e^6$ **(iii)** decreasing function

6 (i)By the property of circle, the perpendicular bisector of a chord passes through the centre of a circle.

(ii) Proof (iii) $x^2+y^2-6x-8y+9=0$

7 (a) $k=-6+6\sqrt{2}$ or $k=-6-6\sqrt{2}$ **(b)** -2 **(c)** $x<-2$ or $x>-\frac{1}{2}$

8 (i) $a=1.78,\ b=1.12$ **(ii)** 1995

9 (i) $L=9+9\cos\theta+3\sin\theta$ **(ii)** $L=9+9.49\cos(\theta-18.4°)$ **(iii)** $\theta=56.2°$

10 (i) $h=\frac{250-\pi x^3}{\pi x^2}$, $S=\pi\left(\sqrt{10}-1\right)x^2+\frac{500}{x}$ **(ii)** $x=3.33$ cm , Minimum $S=225$

11 (a) (i) $P(5,2\ln 5)$ **(ii)** 8.09 units^2

(b) (i) 0.0109 **(ii)** -0.0228 % per hour **(iii)** 0.0409 %

Class | Register Number

Name MARK SCHEME

4047/02

14/S4PR1/AM/2

ADDITIONAL MATHEMATICS

PAPER 2

Monday 12May 2014 2 hours 30 minutes

VICTORIA SCHOOL

PRELIMINARY EXAMINATION ONE
SECONDARY FOUR

Additional Materials: Answer Paper
Graph Paper

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$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\Delta = \frac{1}{2}ab \sin C$$

1 (i) Prove that $\dfrac{\sin(A+B)+\sin(B-A)}{\cos(A+B)-\cos(B-A)} = -\cot A$. [3]

$$\frac{\sin(A+B)+\sin(B-A)}{\cos(A+B)-\cos(B-A)} = \frac{\sin A\cos B + CosA\sin B + \sin B\cos A - \cos B\sin A}{\cos A\cos B - \sin A\sin B - [\cos B\cos A + \sin B\sin A]} \text{ ------- [B1]}$$

$$= \frac{2\sin B\cos A}{-2\sin B\sin A} \text{ ------- [B1]}$$

$$= -\frac{\cos A}{\sin A}$$

$$= -\cot A \quad \text{(shown)} \quad \text{-------- [A1]}$$

(ii) Hence, solve $\dfrac{\sin(A+B)+\sin(B-A)}{\cos(A+B)-\cos(B-A)} = 4\cos^2 A$ for $0 \le A \le \pi$, giving your answers in terms of π. [4]

$$\frac{\sin(A+B)+\sin(B-A)}{\cos(A+B)-\cos(B-A)} = 4\cos^2 A$$

$\therefore -\cot A = 4\cos^2 A$ --------- [B1]

$-\cos A = 4\cos^2 A\sin A$

$4\cos^2 A\sin A + \cos A = 0$

$\cos A(4\sin A\cos A + 1) = 0$ --------- [M1]

$\Rightarrow \cos A = 0$ or $4\sin A\cos A + 1 = 0$

$A = \dfrac{\pi}{2}$ --------- [A1]

$2\sin 2A = -1$

$\sin 2A = -\dfrac{1}{2}$

$\therefore$ basic $\angle, \alpha = \dfrac{\pi}{6}$

$2A = \dfrac{7\pi}{6}, \dfrac{11\pi}{6}$

$A = \dfrac{7\pi}{12}, \dfrac{11\pi}{12}$ --------- [A1]

2 **(a)** Write down and simplify the first three terms in the expansion of $\left(x+\frac{1}{200x^2}\right)^{10}$ in descending powers of x. Hence evaluate $(1.005)^{10}$ correct to five significant figures. [4]

$$\left(x+\frac{1}{200x^2}\right)^{10}=x^{10}+\binom{10}{1}x^9\left(\frac{1}{200x^2}\right)+\binom{10}{2}x^8\left(\frac{1}{200x^2}\right)^2+\ldots \text{ ------ [B1]}$$

$$=x^{10}+\frac{1}{20}x^7+\frac{9}{8000}x^4+\ldots \text{ ------ [A1]}$$

$$(1.005)^{10}=\left(x+\frac{1}{200}\right)^{10}, \quad \text{where } x=1.$$

$$=(1)^{10}+\frac{1}{20}(1)^7+\frac{9}{8000}(1)^4+\ldots \text{ ------ [M1]}$$

$$\approx 1.0511 \text{ ------ [A1]}$$

(b) Find the coefficient of the third term in the expansion of $\left(a-\frac{1}{x}\right)^4$ and the coefficient of the second term in the expansion of $\left(2+\frac{x}{a}\right)^6$ in terms of a.

If the coefficient of the second term of $\left(2+\frac{x}{a}\right)^6$ is four times the coefficient of the third term of $\left(a-\frac{1}{x}\right)^4$, find the value of a. [4]

For, $\left(a-\frac{1}{x}\right)^4$,

$$T_3=\binom{4}{2}a^2\left(-\frac{1}{x}\right)^2=\frac{6a^2}{x^2}$$

Coeff. of $T_3=6a^2$ --------- [A1]

For, $\left(2+\frac{x}{a}\right)^6$,

$$T_2=\binom{6}{1}2^5\left(\frac{x}{a}\right)=\frac{192}{a}x$$

Coeff. of $T_2=\frac{192}{a}$ --------- [A1]

Since, $\frac{192}{a}=4\left(6a^2\right)$ --------- [M1]

$$\therefore 48=6a^3$$

$$a^3=8$$

$$a=2 \text{ --------- [A1]}$$

3 The roots of the quadratic equation $2x^2 + x - 5 = 0$ are α and β.

(i) Express $\alpha^2 - \alpha\beta + \beta^2$ in terms of $(\alpha+\beta)$ and $\alpha\beta$. [1]

$\alpha^2 - \alpha\beta + \beta^2 = (\alpha+\beta)^2 - 3\alpha\beta$ -------- [B1]

(ii) Hence, find the exact value of $\alpha^2 + \beta^2$. [3]

$\alpha+\beta = -\frac{1}{2}$, $\alpha\beta = -\frac{5}{2}$ ---------- [B1 - for both]

$\therefore \alpha^2 - \alpha\beta + \beta^2 = (\alpha+\beta)^2 - 3\alpha\beta$ -------- [M1]

$\Rightarrow \alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= \left(-\frac{1}{2}\right)^2 - 2\left(-\frac{5}{2}\right)$

$= 5\frac{1}{4}$ -------- [A1]

(iii) Find a quadratic equation whose roots are α^3 and β^3. [4]

$\alpha^3 + \beta^3 = (\alpha+\beta)(\alpha^2 - \alpha\beta + \beta^2)$ ------- [B1]

$= (\alpha+\beta)\left[(\alpha+\beta)^2 - 3\alpha\beta\right]$

$= \left(-\frac{1}{2}\right)\left[\left(-\frac{1}{2}\right)^2 - 3\left(-\frac{5}{2}\right)\right]$ ------- [M1]

$= -\frac{31}{8}$

$\alpha^3\beta^3 = (\alpha\beta)^3$

$= \left(-\frac{5}{2}\right)^3$

$= -\frac{125}{8}$ ------- [A1]

$\therefore$ Equation with new roots;

$x^2 - \left(-\frac{31}{8}\right)x + \left(-\frac{125}{8}\right) = 0$

$x^2 + \frac{31}{8}x - 15\frac{5}{8} = 0$ ------- [A1 o.e]

$8x^2 + 31x - 125 = 0$

4 **(i)** Differentiate $\dfrac{2x}{\sin x}$ with respect to x. [3]

$$\frac{d}{dx}\left(\frac{2x}{\sin x}\right)=\frac{2\sin x-2x\cos x}{\sin^2 x} \text{ ------- [B2]}$$

$$=\frac{2(\sin x-x\cos x)}{\sin^2 x} \text{ ------- [A1]}$$

(ii) Using your answer to part **(i)**, find $\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}}\left(\frac{1}{\sin x}-\frac{x\cos x}{\sin^2 x}\right)\mathrm{d}x$. [5]

$$\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}}\left(\frac{1}{\sin x}-\frac{x\cos x}{\sin^2 x}\right)\mathrm{d}x=\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}}\left(\frac{\sin x-x\cos x}{\sin^2 x}\right)\mathrm{d}x \text{ ------- [B1]}$$

$$=\frac{1}{2}\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}}\left[\frac{2(\sin x-x\cos x)}{\sin^2 x}\right]\mathrm{d}x \text{ ------- [M1]}$$

$$=\frac{1}{2}\left[\frac{2x}{\sin x}\right]_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \text{ ------- [A1]}$$

$$=\frac{1}{2}\left[\frac{3\pi}{-1}-\frac{\pi}{1}\right] \text{ ------- [M1]}$$

$$=-2\pi$$

$$=-6.28 \text{ (3 SF) ------- [A1]}$$

5 A curve has the equation $y = f(x)$, where $f(x) = (2-x)e^{-3x}$.

(i) Obtain an expression for $f'(x)$. [2]

$\therefore f'(x) = -3(2-x)e^{-3x} + (-1)e^{-3x}$ --------- [B1]

$= e^{-3x}(-6+3x-1)$

$= e^{-3x}(3x-7)$ --------- [A1 o.e]

(ii) Find the equation of the normal to the curve at the point where the curve crosses the x-axis. Leave your answer in terms of e. [5]

At $f(x) = 0$,

$(2-x)e^{-3x} = 0$ --------- [B1]

$\therefore 2 - x = 0$ or $e^{-3x} = 0$ (NA)

$\Rightarrow x = 2$ --------- [B1]

The curve passes thru' the x – axis at $(2,0)$.

At $x = 2$,

$\therefore f'(2) = e^{-6}(6-7)$

$= -e^{-6}$

$\Rightarrow$ Gradient of normal at $x = 2$ is e^{6} --------- [M1]

$\therefore$ Equation of normal: $y = e^{6}(x-2)$ --------- [M1]

$y = e^{6}x - 2e^{6}$ --------- [A1 o.e]

(iii) Determine, with explanation, whether f is an increasing or decreasing function for $x < 2\frac{1}{3}$. [2]

$f'(x) = e^{-3x}(3x-7)$

$\Rightarrow$ For $x < 2\frac{1}{3}$, $e^{-3x} > 0$ and $(3x-7) < 0$ --------- [B1]

$\therefore e^{-3x}(3x-7) < 0$

$\Rightarrow f'(x) < 0$

$\therefore$ For $x < 2\frac{1}{3}$, f is a decreasing function. --------- [A1]

6 A circle with centre C passes through points $A(-1,7)$ and $B(0,8)$.

(i) Explain why the perpendicular bisector of AB will pass through C. [1]

By the property of circle, the perpendicular bisector of a chord passes through the centre of a circle. ------ [B1]

(ii) Given further that the line $y = 2x - 2$ passes through the centre of the circle, show that the coordinates of C is $(3,4)$. [4]

Mid-point of $AB = \left(\frac{-1+0}{2}, \frac{7+8}{2}\right)$

$= \left(-\frac{1}{2}, \frac{15}{2}\right)$ -------- [B1]

Gradient of $AB = \frac{8-7}{0-(-1)} = 1$

$\therefore$ Gradient of $\perp$ bisector $AB = -1$ -------- [M1]

$\therefore$ Equation of Gradient of $\perp$ bisector AB,

$y - \frac{15}{2} = -1\left[x - \left(-\frac{1}{2}\right)\right]$

$y = -x + 7$ ------- (1)

Given: $y = 2x - 2$ ------- (2)

Solving (1) & (2),

$2x - 2 = -x + 7$ -------- [M1]

$3x = 9$

$x = 3$

Subst. $x = 3$ into (1), $\Rightarrow y = 4$.

$\therefore$ Coordinates of C are $(3,4)$ (shown). -------- [A1]

(iii) Hence, find the equation of the circle in the general form. [3]

Coordinates of Centre is $(3,4)$.

Radius $= \sqrt{(3-0)^2 + ((8-4))^2}$ ------- [M1]

$= 5$ units

Eqn.: $(x-3)^2 + (y-4)^2 = 5^2$ -------- [M1]

In general form;

$x^2 - 6x + 9 + y^2 - 8y + 16 = 25$

$x^2 + y^2 - 6x - 8y + 9 = 0$ -------- [A1]

7 (a) Find the exact values of k for which the line $y-3x=k$ is a tangent to the curve $y^2+4x+5=x^2$. [5]

$y-3x=k \Rightarrow y=3x+k$ ------- (1) & $y^2+4x+5=x^2$ ------- (2)

Subst. (1) into (2);

$(3x+k)^2+4x+5=x^2$ ------- [M1]

$9x^2+6kx+k^2-x^2+4x+5=0$

$8x^2+(6k+4)x+(k^2+5)=0$

Since $y-3x=k$ is a tangent to the curve,

$\therefore (6k+4)^2-4(8)(k^2+5)=0$ ------- [M1 B1]

$(3k+2)^2-8(k^2+5)=0$

$9k^2+12k+4-8k^2-40=0$

$k^2+12k-36=0$

$k=\dfrac{-12\pm\sqrt{(-12)^2-4(1)(-36)}}{2(1)}$ ------- [M1]

$\therefore k=-6+6\sqrt{2}$ or $k=-6-6\sqrt{2}$ ------- [A1 o.e]

(b) Calculate the smallest negative integer k for which the equation $y=4x^2+(12-4k)x+15-7k$ is always positive for all real values of x. [3]

Since $y>0, \Rightarrow 4x^2+(12-4k)x+15-7k>0$.

$\therefore b^2-4ac<0$

$(12-4k)^2-4(4)(15-7k)<0$ --------- [B1 B1]

$(3-k)^2-(15-7k)<0$

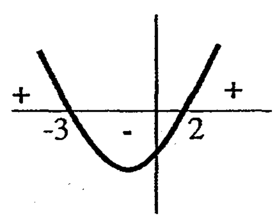
$9-6k+k^2-15+7k<0$

$k^2-k-6<0$

$(k+3)(k-2)<0$

$\Rightarrow -3<k<2$

$\therefore$ Smallest negative integer of k is -2. ------ [A1]



(c) Solve the inequality $4x^2-1>(2x+1)(x-3)$. [2]

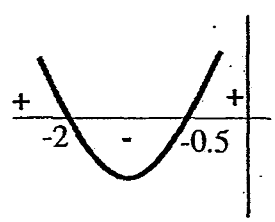
$4x^2-1>(2x+1)(x-3)$

$4x^2-1>2x^2-5x-3$

$2x^2+5x+2>0$

$(2x+1)(x+2)>0$ ------ [M1]

$\therefore x<-2$ or $x>-\dfrac{1}{2}$ ------ [A1]



8 **Answer the whole of this question on a sheet of graph paper.**

The population P, in millions, of a country was recorded in certain years and the results are shown in the table below:

Year	1978	1984	1988	1993
P	12.40	14.48	16.76	21.22

It is given that $P = 10 + ab^x$, where x is the time measured in years from January 1975 and a and b are constants. By plotting $\lg(P-10)$ against x, draw a line graph for the given data for $0 \le x \le 20$. Use a scale of 2 cm to represent 0.1 units on the vertical axis and 4 cm to represent 5 units on the horizontal axis. [4]

x	3	9	13	18
$\lg(P-10)$	0.380	0.651	0.830	1.05

.... [B1 with at least 3 SF]

[M1 --- Plot Points correctly]
[M1 --- Best-Fit Line]
[M1 --- Label axes, origin

Use your graph to estimate,

(i) the value of a and of b. [3]

$P = 10 + ab^x$

$P - 10 = ab^x$

$\lg(P-10) = (\lg b)x + \lg a$ ------ [A1]

$\lg a = 0.25$

$\Rightarrow a = 1.78$ ------ [A1]

Gradient $= \lg b = \dfrac{1.03 - 0.55}{17.5 - 7.8}$ ------ [M1] [with gradient Δ or coordinates]

$= 0.049485$

$\Rightarrow b = 1.12$

(ii) the year in which the population will reach 23.8 millions. [3]

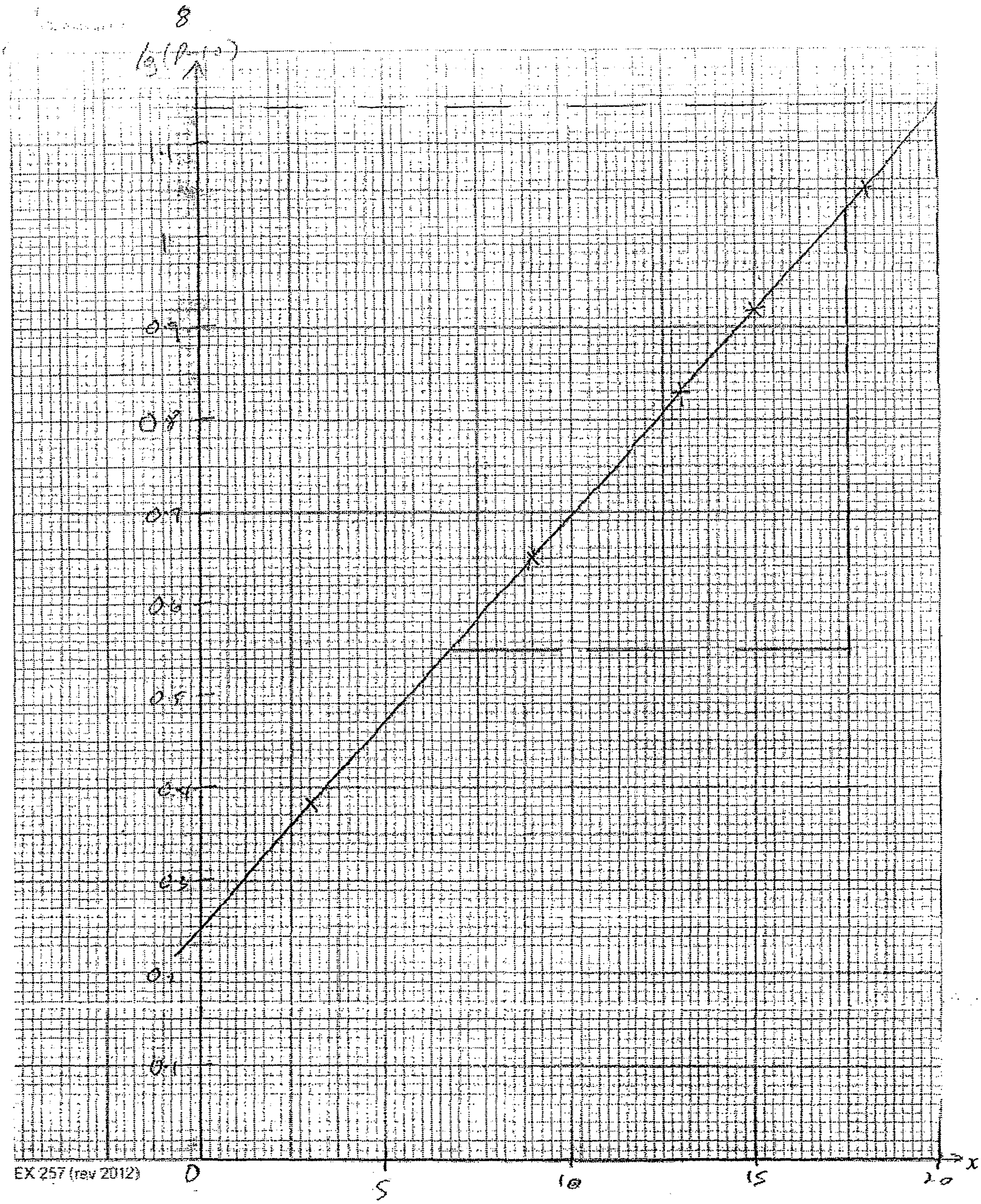
At $p = 23.8$,

$\therefore \lg(P-10) = \lg 13.8$

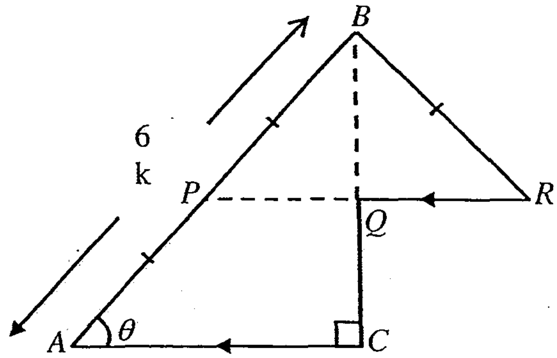
$= 1.14$ -------- [B1]

From the graph, corresponding value of x is 20. -------- [A1]

$\therefore$ The year will be in 1995. -------- [A1]


8
lg(p-10)
1.1
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
5
10
15
20
x
EX 257 (rev 2012)

9 In the diagram, *ABRQC* represents a cycling course. From point *A*, a cyclist travels along straight tracks *AB*, *BR*, *RQ* and *QC*, returning along the track *CA* to finish at *A*.
The total Length of the course is *L* km.



It is given that $AB = 6$ km and *P* is the mid-point of *AB*. The length of each of the tracks *AP*, *PB* and *BR* are equal and track *QC* is also perpendicular to track *AC*. Track *QR* is parallel to the track *AC* and angle *BAC* is $\theta°$.

(i) Show that *L* can be expressed as $p + q\sin\theta + r\cos\theta$, where *p*, *q* and *r* are constants to be found. [3]

$AP = PB = BR = 3$
$\angle BRQ = \angle BPR = \theta$
$\therefore QR = 3\cos\theta, AC = 6\cos\theta \ \& \ BC = 6\sin\theta$
$\therefore QC = \frac{1}{2}BC = 3\sin\theta$
$\Rightarrow L = AB + BR + QC + AC + RQ$
$= 6 + 3 + 3\sin\theta + 6\cos\theta + 3\cos\theta$ -------- [M2]
$L = 9 + 3\sin\theta + 9\cos\theta$ -------- [A1]

(ii) Express *L* in the form $p + R\cos(\theta - \alpha)$ where $R > 0$, and α is acute. [5]

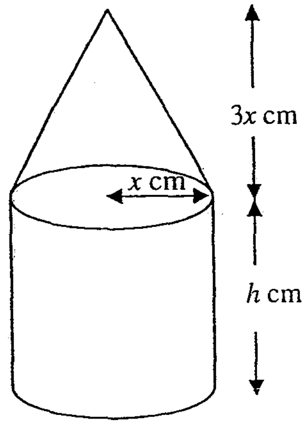
$L = 9 + 9\cos\theta + 3\sin\theta = 9 + R\cos(\theta - \alpha)$
$= 9 + R\cos\theta\cos\alpha + R\sin\theta\sin\alpha$ -------- [B1]
Comparing both sides,
$\Rightarrow R\cos\alpha = 9$ ----- (1), $R\sin\alpha = 3$ ----- (2)
$(1)^2 + (2)^2 : R^2(\cos^2\alpha + \sin^2\alpha) = 9^2 + 3^2$
$\Rightarrow R = \sqrt{90} = 9.49$ (3 SF)
$(2) \div (1)$: $\tan\alpha = \frac{1}{3}$ $\therefore \alpha = 18.43°$ --------- [M3]
$\therefore L = 9 + 9.49\cos(\theta - 18.4°)$ -------- [A1]

The total length of the course is found to be 16.5 km.

(iii) Find the value of θ. [2]

$9 + \sqrt{90}\cos(\theta - 18.43°) = 16.5$ -------- [M1]
$\sqrt{90}\cos(\theta - 18.43°) = 7.5$
$\theta - 18.43° = 37.76°$
$\theta = 56.2°$ (1 DP) -------- [A1]

10 The diagram shows a solid comprising of a right-circular cone of height $3x$ cm sitting ona cylinder of base radius x cm and height h cm. The volume of the solid is 250 cm^3.



(i) Express h in terms of x and hence show that the total surface area, S cm^2, of the solid is given by

$$S = \pi(\sqrt{10} - 1)x^2 + \frac{500}{x}.$$ [6]

Vol. of cone $= \frac{1}{3}\pi x^2(3x) = \pi x^3$

Vol. of cylinder $= \pi x^2(h) = \pi x^2 h$

$\therefore \pi x^3 + \pi x^2 h = 250$ --------- [B1]

$\pi x^2 h = 250 - \pi x^3$

$h = \dfrac{250 - \pi x^3}{\pi x^2}$ --------- [A1]

Slant height of cone, $l = \sqrt{(3x)^2 + (x)^2}$

$= \sqrt{10x^2}$

$l = \sqrt{10}x$

Curved surface area of cone $= \pi(x)(\sqrt{10}x)$ --------- [M1]

$= \pi\sqrt{10}x^2$

Surface area of cylinder $= 2\pi x h + \pi x^2$

$= 2\pi x\left(\dfrac{250 - \pi x^3}{\pi x^2}\right) + \pi x^2$ --------- [M1]

$= \dfrac{500 - 2\pi x^3}{x} + \pi x^2$

$= \dfrac{500}{x} - 2\pi x^2 + \pi x^2$

$= \dfrac{500}{x} - \pi x^2$

$\therefore$ Total Surface Area of solid, $S = \pi\sqrt{10}x^2 + \dfrac{500}{x} - \pi x^2$ --------- [M1]

$\Rightarrow S = \pi\left(\sqrt{10} - 1\right)x^2 + \dfrac{500}{x}$ --------- [A1]

(ii) Given that x can vary, find, the value of x for which S has a stationary value. Find this value of S and determine whether it is a maximum or a minimum value. [4]

$$S = \pi\left(\sqrt{10}-1\right)x^2 + \frac{500}{x}$$

$$\frac{dS}{dx} = 2\pi x\left(\sqrt{10}-1\right) - \frac{500}{x^2}$$

At stationary point, $2\pi x\left(\sqrt{10}-1\right) - \frac{500}{x^2} = 0$ --------- [M1]

$$x^3 = \frac{500}{2\pi\left(\sqrt{10}-1\right)}$$

$x = 3.326 \Rightarrow x = 3.33 \text{ cm } (3 \text{ SF})$ --------- [A1]

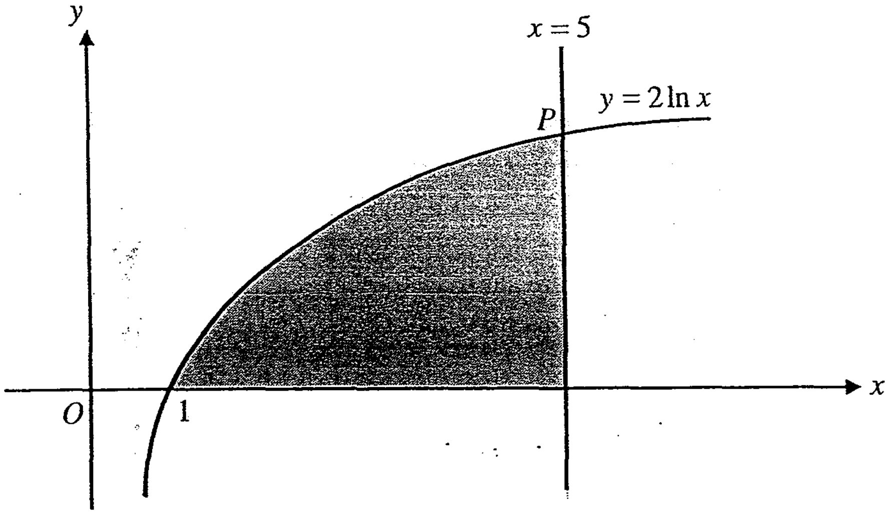
$$\frac{d^2S}{dx^2} = 2\pi\left(\sqrt{10}-1\right) + \frac{1000}{x^3}$$

At $x = 3.326$, $\frac{d^2S}{dx^2} > 0 \Rightarrow$ Minimum --------- [A1]

$\therefore$ Minimum $S = \pi\left(\sqrt{10}-1\right)(3.326)^2 + \frac{500}{3.326}$

$S = 225 \ (3 \text{ SF})$ -------- [A1]

11 **(a)** The diagram below shows part of the curve $y = 2\ln x$ and the line $x = 5$.



Find the

(i) coordinates of P, [2]

For $y = 2\ln x$
At $x = 5$, $y = 2\ln 5$ -----[M1]
$\therefore P(5, 2\ln 5)$ -----[A1]

(ii) area of the shaded region. [3]

Area of rect. $= 5 \times 2\ln 5 = 10\ln 5$
Since $y = 2\ln x$
$\ln x = \frac{1}{2}y \Rightarrow x = e^{\frac{1}{2}y}$
Area enclosed by $x = e^{\frac{1}{2}y}$ and y – axis
$= \int_0^{2\ln 5} e^{\frac{1}{2}y}\,dy$ -----[M1]
$= \left[2e^{\frac{1}{2}y}\right]_0^{2\ln 5}$
$= 2\left[e^{\ln 5} - 1\right]$
$= 2[5 - 1]$
$= 8$
$\therefore$ Shaded area $= 10\ln 5 - 8$ -----[M1]
$= 8.09$ units^2 -----[A1]

Alternative solution
Since $y = 2\ln x$
$\ln x = \frac{1}{2}y \Rightarrow x = e^{\frac{1}{2}y}$
Area enclosed by $x = e^{\frac{1}{2}y}$ and y – axis
$= \int_0^{2\ln 5} \left(5 - e^{\frac{1}{2}y}\right)dy$ -----[M2]
$= \left[5y - 2e^{\frac{1}{2}y}\right]_0^{2\ln 5}$
$= \left[10\ln 5 - 2e^{\ln 5} + 2\right]$
$= [10\ln 5 - 10 + 2]$
$= 8.09$ units^2 -----[A1]

(b) The concentration of caffeine, C %, in Peter's blood when he drinks a cup of coffee in the morning is represented by $C = 0.2xe^{-1.8x}$, where x is the number of hours after drinking a cup of coffee.

(i) Calculate the value of C when $x = 2$. [1]

$C = 0.2xe^{-1.8x}$

At $x = 2$,

$C = 0.2(2)e^{-3.6}$

$= 0.0109$ (3 SF) ------ [B1]

(ii) Find the rate of change of caffeine concentration in Peter's blood, 1.5 hours after drinking a cup of coffee. [3]

$C = 0.2xe^{-1.8x}$

$\frac{dC}{dx} = 0.2e^{-1.8x} + 0.2x\left(-1.8e^{-1.8x}\right)$ ------[B1]

$= 0.2e^{-1.8x}(1 - 1.8x)$

At $x = 1.5$,

$\frac{dC}{dx} = 0.2e^{-1.8(1.5)}\left[1 - 1.8(1.5)\right]$ ------[M1]

$= -0.0228$ % per hour. (3 SF) ------ [A1]

(iii) What is the maximum level of caffeine concentration in Peter's blood in the morning? [3]

At stationary point, $0.2e^{-1.8x}(1 - 1.8x) = 0$ ------[B1]

Since $e^{-1.8x} = 0$ (NA),

$\therefore 1 - 1.8x = 0 \Rightarrow x = 0.55556$

$\therefore \frac{d^2C}{dx^2} = -0.36e^{-1.8x}(2 - 1.8x)$

At $x = 0.55556$, $\frac{d^2C}{dx^2} < 0 \Rightarrow$ Maximum ------ [A1]

$\therefore$ Maximum $C = 0.2(0.55556)e^{-1.8(0.55556)}$

$= 0.0409$ % (3 SF) ------ [A1]

End of Paper