JC1 H2 Mathematics (9758)
Term 4 Revision Topical Quick Check
Chapter 3 Functions

1 YIJC Promo 9758/2022/Q9 (modified)

The function f is defined by

$$\mathrm{f}: x \mapsto x^2 - 4x - 5, \text{ for } x \in \mathbb{R},\ x \le 2.$$

(a) Find $\mathrm{f}^{-1}(x)$ and state the domain of f^{-1}. [3]

(b) On the same diagram, sketch the graphs of f and f^{-1}. [3]

(c) Find the exact solution of the equation $\mathrm{f}(x) = \mathrm{f}^{-1}(x)$. [3]

The function g is defined by

$$\mathrm{g}: x \mapsto 1 - x^2, \text{ for } x \in \mathbb{R},\ x < 1.$$

(d) Explain why the composite function fg exists and find the range of fg. [3]

2 HCI Promo 9758/2022/Q6

The function h is defined as follows.

$$\mathrm{h}: x \mapsto \frac{1}{8}\left(x^3 - 6x^2 + 32\right), \qquad x \in \mathbb{R}.$$

(i) Explain why h does not have an inverse. [1]

(ii) If the domain of h is restricted to $0 \le x \le k$, state the largest value of k for which the function h^{-1} exists. [1]

Use the domain of h in part **(ii)** for the rest of this question.

(iii) Sketch the graphs of h and h^{-1} on the same diagram, showing clearly the relationship between the two graphs, and the coordinates of the end points for both graphs. [3]

(iv) Deduce the solution(s) of the equation $\mathrm{h}(x) = \mathrm{h}^{-1}(x)$. [2]

(v) The function g is defined as follows.

$$\mathrm{g}: x \mapsto \ln\left((x-3)^2 + 1\right), \qquad x \in \mathbb{R}.$$

Show that the composite function gh exists and find the exact range of gh. [2]

3 CJC Promo 9758/2022/Q10(b)

Functions g and h are defined by

$$\mathrm{g}: x \mapsto \frac{1}{1-x^2}, \qquad x \in \mathbb{R}, x > 1,$$
$$\mathrm{h}: x \mapsto 1-2x, \qquad x \in \mathbb{R}.$$

(i) Explain why the composite function gh does not exist. [2]

(ii) Find $\mathrm{hg}(x)$. [1]

(iii) Find the range of $\mathrm{hg}(x)$. [2]

(iv) By using the result in part **(ii)**, or otherwise find $(\mathrm{hg})^{-1}(4)$. [3]