H2 Topic 18 – Alternating Current

Commemorative silver coin. *In 2018, the Serbian Mint began a coin series to honor Nikola Tesla. It features a portrait of Tesla, and his induction generator on the other side. The generator generates Alternating Current and revolutionized industrial mechanics*

Content

- Characteristics of alternating currents
- The transformer
- Rectification with a diode

Learning Outcomes

Candidates should be able to:

(a) show an understanding of and use the terms period, frequency, peak value and root-mean-square (r.m.s.) values as applied to an alternating current or voltage

(b) deduce that the mean power in a resistive load is half the maximum (peak) power for a sinusoidal alternating current

(c) represent an alternating current or an alternating voltage by an equation of the form $x = x_0 \sin \omega t$

(d) distinguish between r.m.s. and peak values and recall and solve problems using the relationship $I_{rms} = I_0 / \sqrt{2}$ for the sinusoidal case

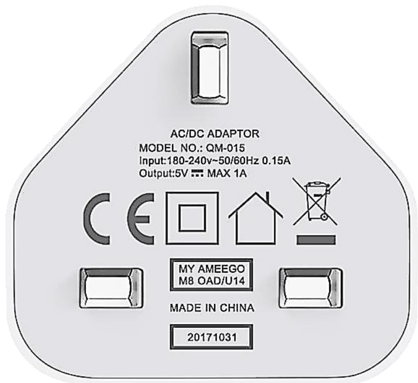
(e) show an understanding of the principle of operation of a simple iron-core transformer and recall and solve problems using $N_S / N_P = V_S / V_P = I_P / I_S$ for an ideal transformer

(f) explain the use of a single diode for the half-wave rectification of an alternating current.

18.0 Introduction

An alternating current (a.c.) is an electric ***current*** which has a flow ***direction*** that ***reverses periodically with time***. At the microscopic level, an a.c. can be considered as having the charge carriers oscillate about fixed points.

Compare that with a d.c. where the direction of flow of current is maintained in the same direction.

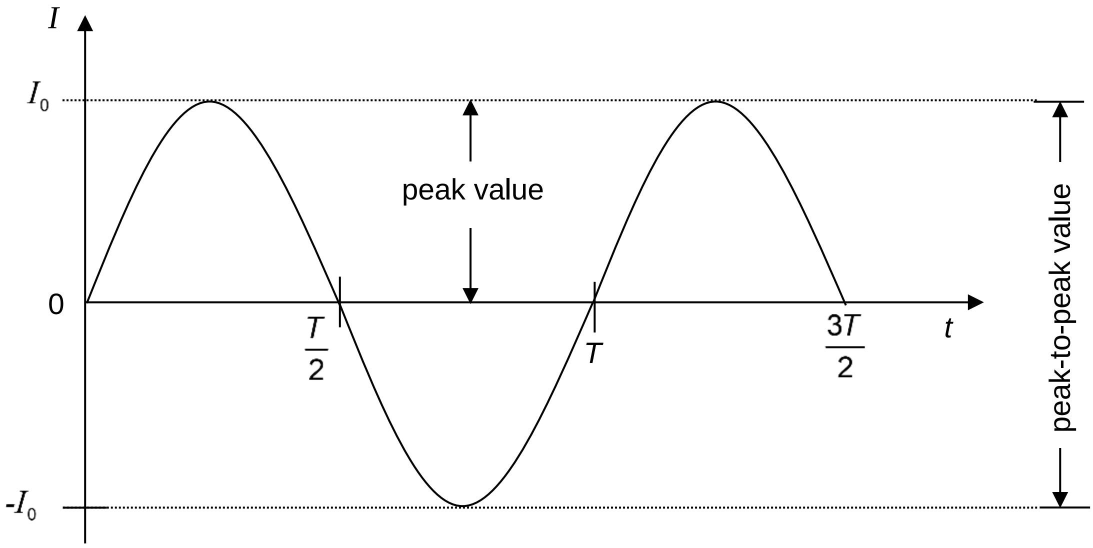


A typical AC/DC adaptor. *As seen in topic H215, this particular "handphone charger" outputs 5V D.C. at a maximum of 1 A. It accepts an input alternating voltage of 180 to 240 V_{rms}, at a frequency of 50Hz or 60 Hz, taking up an current I_{rms} of 0.15 A.*

18.1 Describing an a.c.

Quantity	*Symbol*	Description
period	T	time taken for a.c. to complete one cycle
frequency	f	number of complete cycles of the a.c. per unit time
angular frequency	ω	(for a sinusoidal a.c.), product of 2π and the frequency of the a.c., $\omega = 2\pi f$
peak value (amplitude)	I_0	maximum value of the a.c. in either direction within a cycle
peak-to-peak value		difference between the positive peak value and the negative peak value of the a.c. within a cycle
mean value	$\langle I \rangle$	average value of an a.c. over a given time interval
root-mean-square (r.m.s.) value	I_{rms}	value of a steady direct current that will dissipate thermal energy at the same average rate as the a.c. in a given resistor

A common representation of an a.c. is $I = I_0 \sin \omega t$:



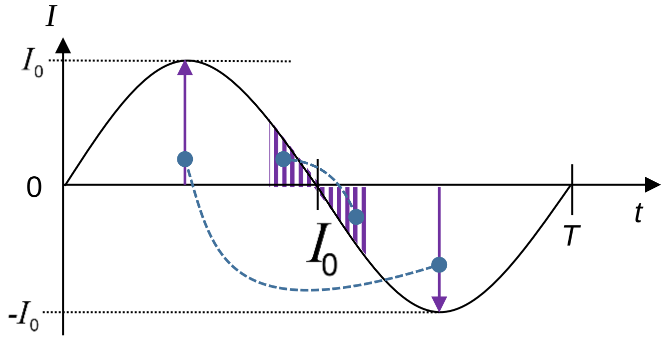
18.2 The Mean Value $\langle I \rangle$

The mean (or average) current value is **zero** for an a.c. where $I = I_0 \sin \omega t$.

$$\langle I \rangle = 0 \text{ for } I = I_0 \sin \omega t$$

Within a complete cycle, there is a positive value of current and a corresponding negative value, so mean is zero.

But a resistor heats up when an a.c. flows through it. The mean current value of an a.c. does not effectively characterise an a.c.



Example 1

The mean value of an alternating current is zero.
Explain why heating occurs when there is an alternating current in a resistor.

Heating effect is due to power dissipated in resistor:

Method 1	**Method 2**
• power dissipated is directly proportional to square of current, $P = I^2R$ • square of current is always positive	• a.c. changes direction every half cycle • but heating effect is independent of current direction

We represent the sine a.c. as

$$I = I_0 \sin \omega t\ .$$

Except at $t = n\left(\frac{T}{2}\right)$, some current is always flowing in the resistor.

Energy output for a constant power output is $E = P_{\text{constant}}t$.

If power output is changing, energy output is area under *P*-*t* graph

$$E = \int P\ dt = \int I^2R\ dt$$

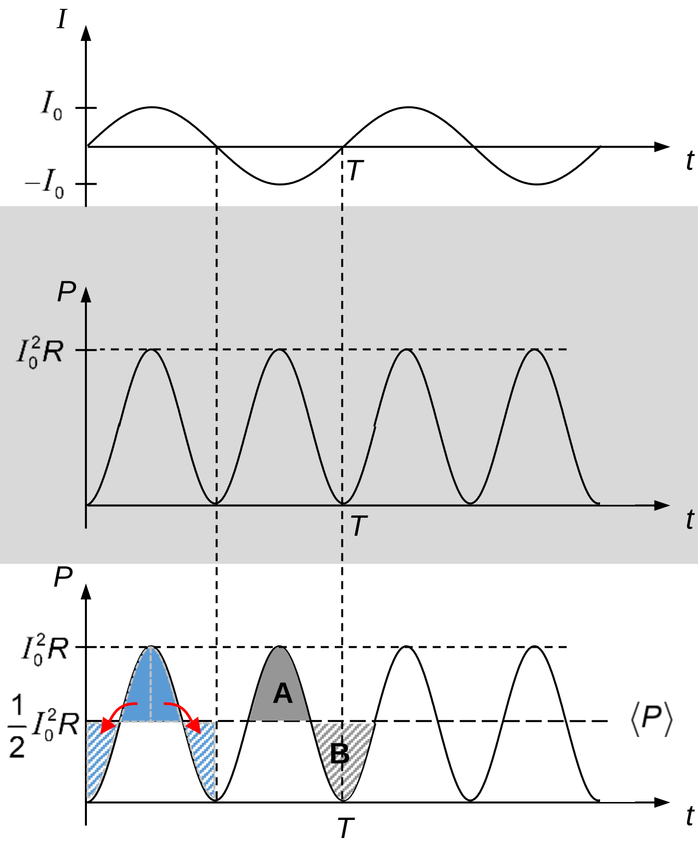
$$= R\left(\int I^2\ dt\right) = R\left(\int (I_0 \sin \omega t)^2\ dt\right)$$

$$= \left(I_0{}^2R\right)\left(\int \sin^2 \omega t\ dt\right)$$

When working with power *P*, the squaring of current makes all values positive.

average power $P_{\text{avg}} = \frac{1}{2}P_{\text{max}} = \frac{1}{2}I_0^2R$

Area A is equal to Area B.



For constant resistance *R*, observe above we **[Squared]** squared the current and **[Mean]** found the average. Comparing to an equivalent d.c. for the same heating effect:

$$P_{avg} = \frac{1}{2}I_0^2R = I_{\text{dc}}^2R$$

$$\frac{I_0^2}{2} = I_{\text{dc}}^2$$

[Root] square-root for next line

$$I_{\text{rms}} = \frac{I_0}{\sqrt{2}} = I_{dc}$$

18.3 The Root-Mean-Square Value I_{rms}

r.m.s. value of an alternating current is the value of a steady direct current that will dissipate thermal energy
at the same average rate
as the a.c. in a given resistor

for sinusoidal a.c.

$$I_{\text{rms}} = \frac{I_0}{\sqrt{2}}$$

$$V_{\text{rms}} = \frac{V_0}{\sqrt{2}}$$

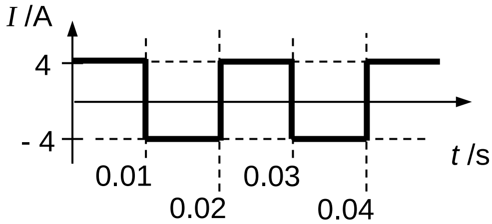
$$P_{\text{mean}} = \frac{1}{2} P_{\text{max}}$$

(includes both sin and cos)

To process r.m.s, we work backwards i.e.
- **[S]** square the function
- **[M]** find the average value
- **[R]** square-root average**d** value

Example 2
Find the (i) mean value and (ii) r.m.s. value for each of the a.c. below

(a)



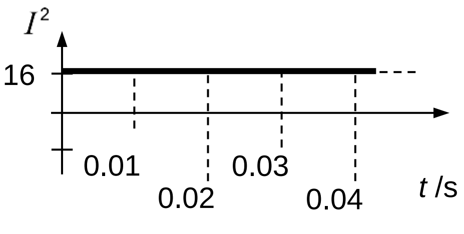
(i) mean value:

in 1 period, total area $= 4(0.01) + (-4)(0.01)$
$= 0$ As

mean current $\langle I \rangle = \dfrac{0}{T} = 0$ A

(ii) r.m.s. value:

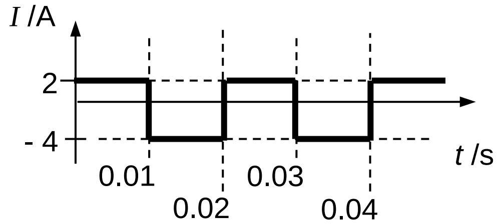
Graph of $I^2 - t$ (**[S]**quared):



[M]ean value is $I_{\text{ms}} = 16$ A^2

[R]oot $I_{\text{rms}} = \sqrt{\langle I^2 \rangle} = 4$ A

(b)



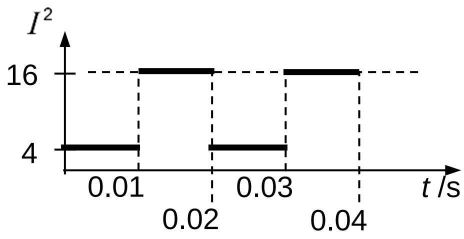
(i) mean value:

in 1 period, total area $= 2(0.01) + (-4)(0.01)$
$= -0.02$ As

mean current $\langle I \rangle = \dfrac{-0.02}{0.02} = -1$ A

(ii) r.m.s. value:

Graph of $I^2 - t$ (**[S]**quared):



in 1 period, total area $= 4(0.01) + (16)(0.01)$
$= 0.20$ A^{2}s

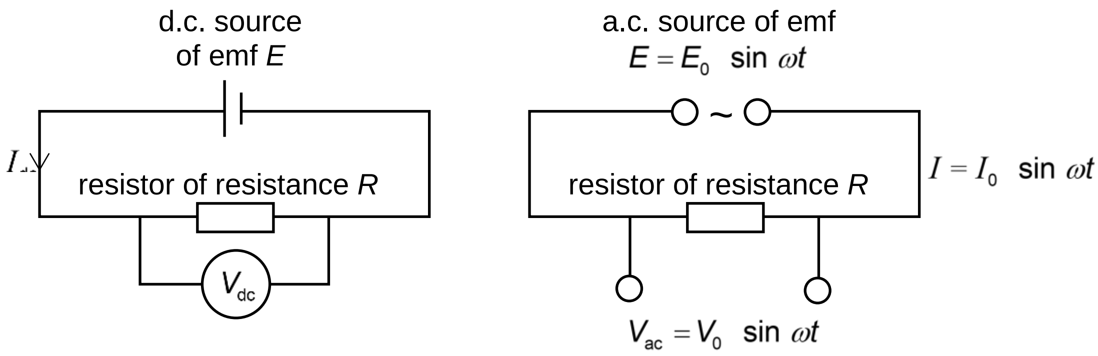
[M]ean value is $I_{\text{ms}} = \dfrac{0.20}{0.02} = 10$ A^2

[R]oot $I_{\text{rms}} = \sqrt{\langle I^2 \rangle} = 3.2$ A

Note: Determine carefully **one period** of a.c before performing r.m.s. calculations.

18.4 D.C. versus A.C. Circuits

Most methods of circuit analysis applied in d.c. can be used in a.c. circuits as well. Consider:



D.C.	Quantity	A.C. (sinusoidal)
Instantaneous, peak and mean p.d. are the same constant value V_{dc}	instantaneous voltage across resistor	$V_{ac} = V_0 \sin \omega t$
	peak voltage across resistor	V_0
	r.m.s. voltage across resistor	$V_{rms} = \dfrac{V_0}{\sqrt{2}}$
$I_{dc} = \dfrac{V_{dc}}{R}$ Instantaneous, peak and mean current are the same constant value	instantaneous current flowing through resistor	$I_{ac} = \dfrac{V_{ac}}{R} = \left(\dfrac{1}{R}\right)(V_0 \sin \omega t) = \left(\dfrac{V_0}{R}\right)(\sin \omega t) = I_0 \sin \omega t$
	peak current flowing through resistor	$I_0 = \dfrac{V_0}{R}$
	r.m.s. current flowing through resistor	$I_{rms} = \sqrt{\langle I_{ac}{}^2 \rangle} = \dfrac{I_0}{\sqrt{2}}$
$P_{dc} = I_{dc}V_{dc} = I_{dc}{}^2 R = \dfrac{V_{dc}^2}{R}$ Instantaneous, peak and mean power are the same constant value	instantaneous power dissipated in resistor	$P_{ac} = I_{ac}^2 R = (I_0 \sin \omega t)^2 R = (I_0{}^2 R) \sin^2 \omega t$
	peak power dissipated in resistor	$P_0 = I_0{}^2 R$
	mean power dissipated in resistor	$\langle P \rangle = I_{rms}V_{rms} = \left(\dfrac{I_0}{\sqrt{2}}\right)\left(\dfrac{V_0}{\sqrt{2}}\right) = \dfrac{1}{2}(I_0 V_0) = \dfrac{1}{2}P_0$

Example 3

A tourist from the U.S.A. brought along an electric water kettle designed to operate with 110 V to Singapore and plugs it into a local 240 V outlet. The heater breaks down due to overheating.

(a) If the heater typically draws 500 W, find the resistance of the heating coil.

(b) Determine the power consumption when operated using a 240 V outlet.

(c) Calculate the r.m.s. current when operated using the 240 V outlet.

(a)

$$\langle P \rangle = \frac{{V_{\text{rms}}}^2}{R}$$

$$R = \frac{{V_{\text{rms}}}^2}{\langle P \rangle} = \frac{110^2}{500} = 24.2\ \Omega$$

(b)

Assume *R* remains constant as heating coil heats up

$$\langle P \rangle = \frac{{V_{\text{rms}}}^2}{R} = \frac{240^2}{24.2} = 2380\text{ W}$$

(Note that this is almost 5 times the intended power output!)

(c)

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{R} = \frac{240}{24.2} = 9.92\text{ A}$$

Note: Household electrical outlets are a.c. and are typically labelled with their r.m.s. value i.e.

(i) the a.c. supply is rated at 240 V_{rms}

(iI) the peak p.d. is about 340 V. ($V_0 = \sqrt{2}\ V_{rms} = \sqrt{2}(240) \approx 339$ V)

Example 4

A steady current of 2.0 A dissipates a certain power in a variable resistor. The variable resistor is now connected to a sinusoidal alternating current.

(a) If power dissipation remains the same, state and explain the r.m.s. value of the a.c..

(b) The resistance has to be halved in order to maintain the same power dissipation. Find the new I_{rms}.

(a) 2.0 A. The r.m.s. value of an a.c. is the value of a steady direct current that will dissipate thermal energy at the same average rate as the a.c. in a given resistor.

(b) power dissipated in resistor is $P = I^2R$
for same power dissipation

$$\frac{\langle P_{\text{ac}} \rangle}{P_{\text{dc}}} = 1$$

$$\frac{{I_{\text{rms}}}^2\left(\frac{R}{2}\right)}{{I_{\text{dc}}}^2R} = 1$$

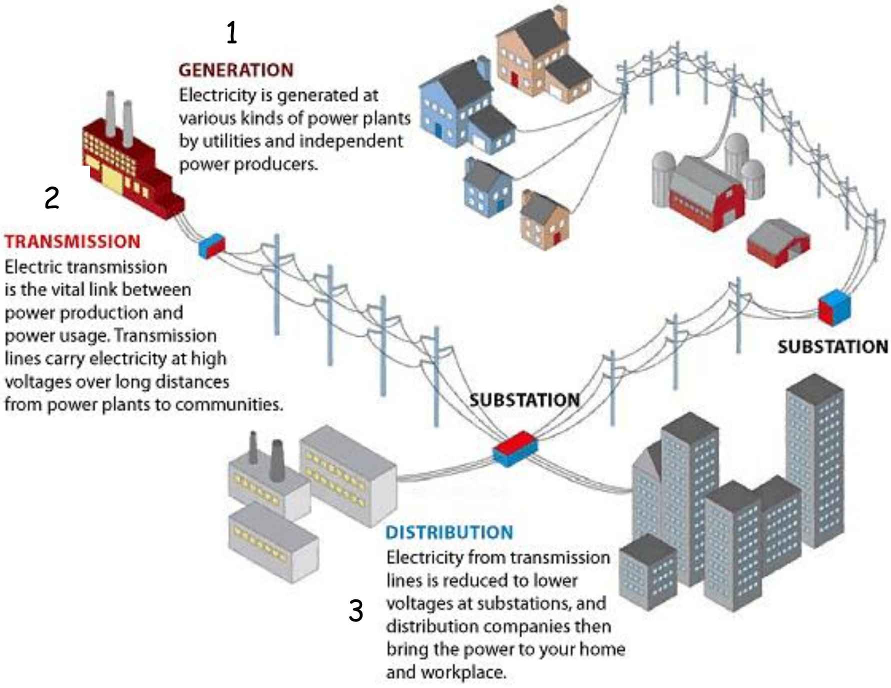
$${I_{\text{rms}}}^2 = 2{I_{\text{dc}}}^2$$

$$I_{\text{rms}} = \sqrt{2}I_{\text{dc}} = \sqrt{2}(2.0) = 2.8\text{ A}$$

Note: For exams, compute final answers according to the least no. of sf. Do not leave them in surds.

18.5 Electricity Transmission

Transmission of electrical energy is frequently done using alternating high voltages.



Example 5

For electricity transmission, suggest why

(a) high voltages are used

(b) the voltage is alternating.

(a) - for the same power transmission, a higher voltage results in a lower current
- the lower current results in less power loss as heat through $P_{\text{heat}} = I^2 R_{\text{cables}}$ in cables

(b) - generators output a.c.
- voltage can be (easily) stepped up/down
- transformers only work with a.c.
- easier to rectify than invert

Example 6

An average of 120 kW of electric power is sent to a small town from a power plant 10 km away. The transmission lines have a total resistance of 0.40 Ω. Calculate the power loss if the power is transmitted at

(a) 240 V

(b) 24 000 V.

(a) current in transmission lines

$$I_{\text{rms}} = \frac{\langle P \rangle}{V_{\text{rms}}} = \frac{120 \times 10^3}{240} = 500 \text{ A}$$

power loss

$$\langle P \rangle = I_{\text{rms}}^2 R = 500^2 (0.4) = 10^5 \text{ W}$$

small town has 20 kW power left for use

(b) current in transmission lines

$$I_{\text{rms}} = \frac{\langle P \rangle}{V_{\text{rms}}} = \frac{120 \times 10^3}{24\ 000} = 5.0 \text{ A}$$

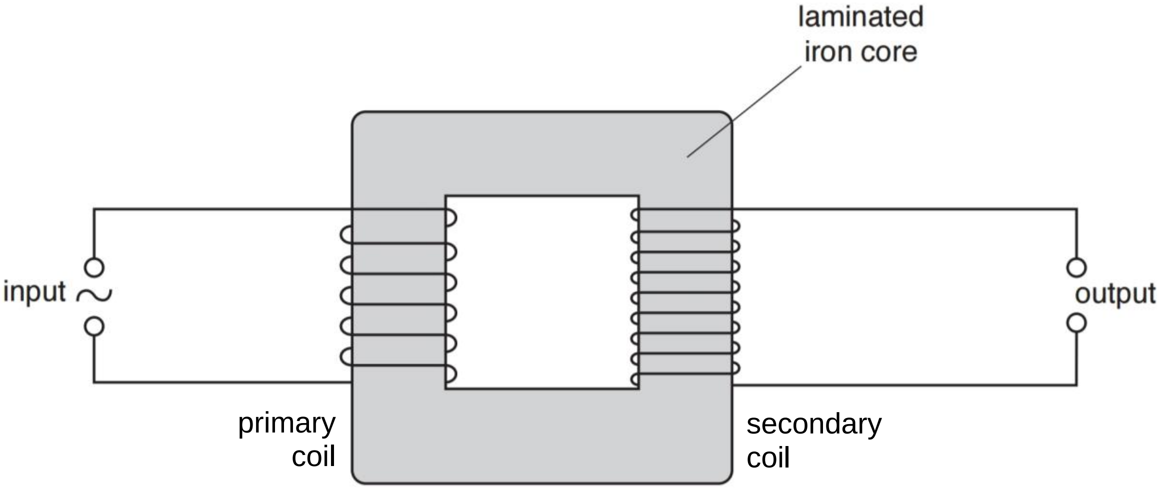
power loss

$$\langle P \rangle = I_{\text{rms}}^2 R = 5.0^2 (0.40) = 10 \text{ W}$$

small town has 119 990 W power left for use

18.6 The Transformer

A transformer allows alternating voltages to be stepped up and stepped down easily. Its circuit symbol is . The diagram below shows a step-up transformer.



in a transformer with a.c. input,

$$\frac{N_S}{N_P} = \frac{V_S}{V_P}$$

This relationship between turn-ratio and voltages applies for **both ideal and non-ideal** transformers. (*More on them in Section 18.6.2)*

Since ratio of currents is dependent on efficiency of power transformer, we strongly recommend **not** calculating the ratio of currents with turn-ratio.

18.6.1 An Application of Electromagnetic Induction

A transformer works by electromagnetic induction, and the output a.c. is not in phase with input:

[B-field generation]	An input current sets up a magnetic field in the primary coil (the magnetic field is confined and strengthened within the iron core)
[change where]	alternating current / voltage gives rise to (changing) flux in core
[flux linkage]	flux links the secondary coil
[Faraday's Law]	changing flux induces e.m.f. (in secondary coil)

A d.c. input does not work because it will give a constant magnetic field. Without a changing flux linkage with the secondary coil, there will be no induced e.m.f. across the secondary coil.

Recall that magnetic flux is the product of magnetic flux density and the area perpendicular to the flux, $\Phi = BA$. Each loop of the secondary coil experiences a magnetic flux. The magnetic flux *linkage* is then dependent on the number of turns present in secondary coil. By Faraday's Law, $E_{\text{induced}} = \frac{d\left(N_S BA\right)}{dt} = N_S \frac{d\left(BA\right)}{dt}$, the larger the number of turns in the secondary coil, the larger the induced (output). e.m.f.

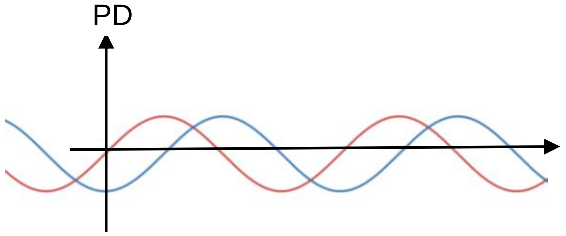
Example 7

Use Faraday's law to explain whether the output and the input potential differences of an ideal transformer are in phase.

An input current sets up a magnetic field in the primary coil. This forms a magnetic flux linkage at the secondary coil. The magnetic flux linkage at the secondary coil is in phase with the current and p.d from the primary coil.

According to Faraday's and Lenz's law, the *e.m.f* at the secondary coil is directly proportional to the ***rate of change of magnetic flux linkage***.

$$E_{\text{induced}} = -N_S \frac{d(BA)}{dt}$$



Imagine the input p.d. to be a sine function. In turn, the magnetic flux linkage will also be a sine function.

After you differentiate the sine function, you will get a negative cosine function as shown in the figure on the left

This means that the induced e.m.f is not in phase with the input potential difference.

The output potential difference is given by the induced *e.m.f*.

Hence the output potential difference **is not in phase** with the input potential difference.

18.6.2 The Ideal Transformer

> An ideal transformer is one with
> no power loss, so
> input power is equal to the output power.

Since 100% of the input power is transferred to the output,

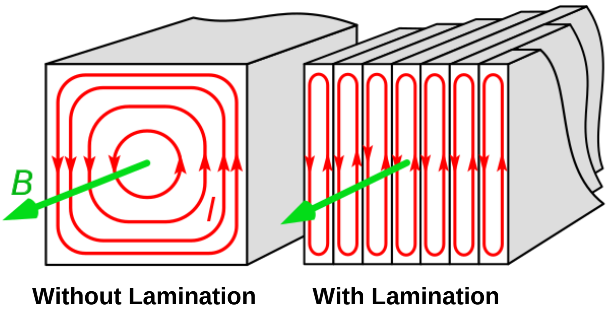
$$P_{\text{input}} = P_{\text{output}}$$
$$I_{\text{pri}}V_{\text{pri}} = I_{\text{sec}}V_{\text{sec}}$$

and that is where we get,

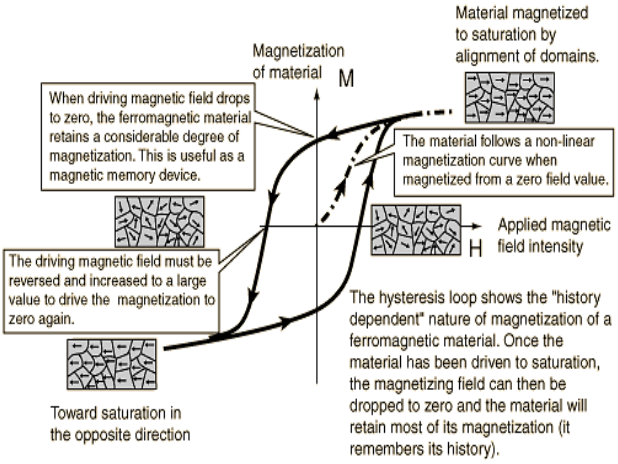
$$\boxed{\frac{N_S}{N_P} = \frac{V_S}{V_P} = \frac{I_{\text{pri}}}{I_{\text{sec}}}}$$

As opposed to ideal transformers, practical transformers have power loss that we try to minimize:

Source of power loss	**Remedy**	**Description**
joule heating of wires in primary and secondary coils	thicker wires in coils using material of lower resistivity	Power loss as heat ($P = I^2R$) is minimized when resistance R is kept small
eddy currents in core	laminated iron core	reduces size of eddy currents in core lowers heating of core
flux leakage: not all magnetic flux generated by primary coil is linked to secondary coil	use soft iron core	soft iron core improves/reduces loss of flux linkage
hysteresis loss	use of soft iron material in core	when magnetic materials undergo flux reversals, magnetic domains at the molecular level experiences friction when flipping continuously to align to the external magnetic flux, causing heating of core



Eddy currents. *Eddy current losses result from Faraday's Law. The magnetic flux is changing and forms changing flux linkage within the core. Laminations limit the size of the eddy currents and reduce power loss through heating of core.*



Example 8

10 lamps rated at 24 W, 12 V are to be connected in parallel with the mains supply of 240 V. Find

(a) the turns ratio of the transformer needed.

(b) the current drawn from the mains supply if the transformer was ideal.

(c) the current drawn from the mains supply if the transformer was 91% efficient.

(a)

Turns ratio:

$$\frac{N_S}{N_P} = \frac{V_S}{V_P} = \frac{12}{240} = \frac{1}{20} = 0.050$$

(b)

current in each lamp:

$$I_{rms} = \frac{\langle P \rangle}{V_{rms}} = \frac{24}{12} = 2.0 \text{ A}$$

10 lamps in parallel each drawing 2.0 A so total drawn current is 20 A. Ideal transformer has no power loss, so input power is equal to the output power:

$$P_{input} = P_{output}$$

$$I_{pri}V_{pri} = I_{sec}V_{sec}$$

$$I_{pri} = \frac{1}{V_{pri}} I_{sec}V_{sec} = \frac{1}{240}(20)(12) = 1.0 \text{ A}$$

(c)

$$\left(\frac{91}{100}\right) P_{input} = P_{output}$$

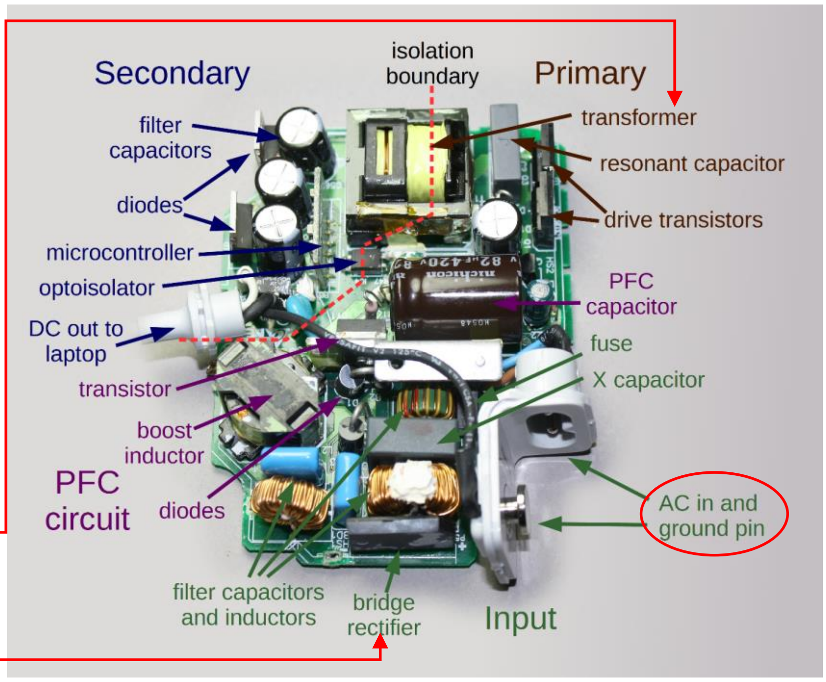
$$\left(\frac{91}{100}\right) I_{pri}V_{pri} = I_{sec}V_{sec}$$

$$I_{pri} = \left(\frac{100}{91}\right)\frac{1}{V_{pri}} I_{sec}V_{sec} = \left(\frac{100}{91}\right)\frac{1}{240}(20)(12) = 1.1 \text{ A}$$

18.7 Rectification

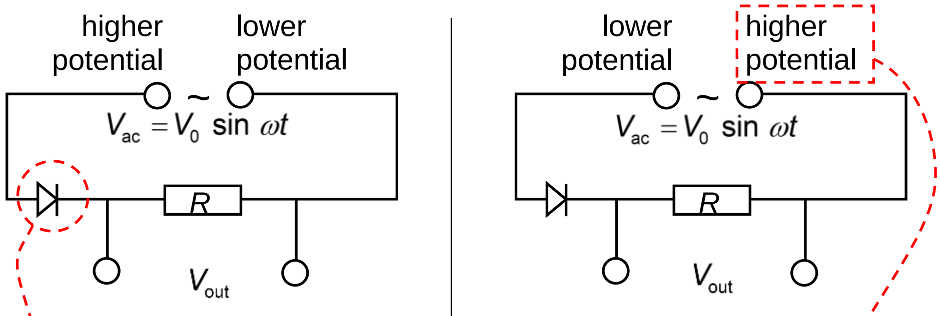
To rectify an a.c. is to convert it into a d.c. using a device which permits current to flow in one direction only. We can use a single diode to achieve half-wave rectification.

Laptop power adapters. *Seen here is an Apple MagSafe power adapter. Its most basic functions include using a transformer to step down the e.m.f. from main supplies, and then a rectifier to convert a.c. input into d.c. output for the laptop.*



18.7.1 Half-wave Rectification using 1 Diode

An a.c. changes direction of current flow periodically. We use conventional current and regard current to flow from higher potential end of e.m.f. source to lower potential end of e.m.f. source:



diode is in **forward-bias**

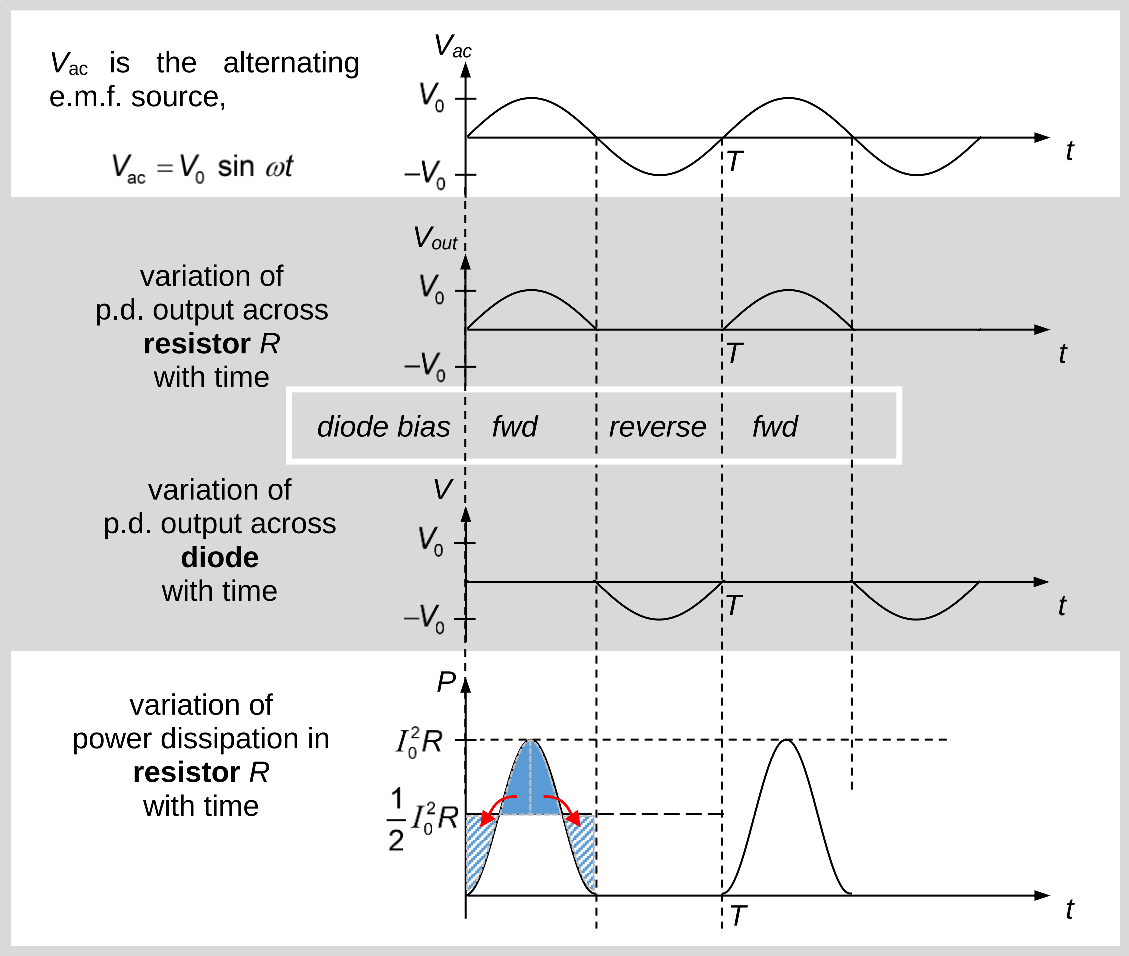
(diode has negligible resistance)

current flows counter-clockwise in circuit

diode is in **reverse-bias**

(diode has very large resistance)

negligible current in circuit


V_{ac} is the alternating e.m.f. source,
$V_{ac} = V_0 \sin \omega t$
V_{ac}
V_0
$-V_0$
T
t
variation of p.d. output across **resistor** R with time
V_{out}
V_0
$-V_0$
T
t
diode bias
fwd
reverse
fwd
variation of p.d. output across **diode** with time
V
V_0
$-V_0$
T
t
variation of power dissipation in **resistor** R with time
P
$I_0^2 R$
$\frac{1}{2} I_0^2 R$
T
t

Example 9

A sinusoidal a.c. of peak p.d. V_0 and peak current I_0 undergoes half-wave rectification. Find

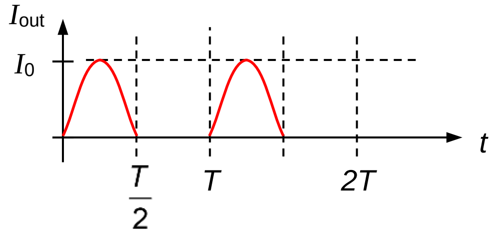
(a) output r.m.s. current I_{rms}

(b) output r.m.s. p.d. V_{rms}

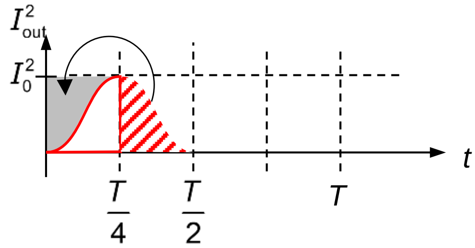
(c) output mean power $\langle P \rangle$

Solution:

(a) graph of rectified output current:



Graph of I^2 against t (**[S]**quared):



in 1 period, total area $= \frac{T}{4}\left(I_0^2\right)$

[M]ean value is $I_{\text{ms}} = \dfrac{\frac{T}{4}\left(I_0^2\right)}{T} = \dfrac{I_0^2}{4}$

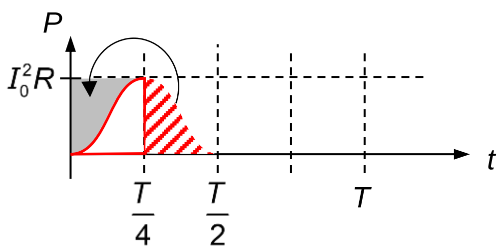
[R]oot $I_{\text{rms}} = \sqrt{\langle I^2 \rangle} = \frac{1}{2} I_0$

(b) By a similar method, $V_{\text{rms}} = \frac{1}{2} V_0$

(c) Mathematically:

$$\langle P \rangle = I_{\text{rms}} V_{\text{rms}}$$
$$= \left(\frac{I_0}{2}\right)\left(\frac{V_0}{2}\right)$$
$$= \frac{1}{4} P_0$$

Graphically:



$$\langle P \rangle = \frac{\text{total area}}{\text{one period}} = \frac{\left(\frac{T}{4}\right)\left(I_0^2 R\right)}{T} = \frac{1}{4} P_0$$

Comparing a full sinusoidal a.c. before and after half-rectification:

	sinusoidal a.c.	**half-wave rectified sinusoidal a.c.**
peak voltage peak current maximum power	$\left.\begin{matrix} V_0 \\ I_0 \\ P_0 \end{matrix}\right\}$ remains same after rectification	
root-mean-square current I_{rms}	$\dfrac{I_0}{\sqrt{2}}$	$\dfrac{I_0}{2}$
root-mean-square voltage V_{rms}	$\dfrac{V_0}{\sqrt{2}}$	$\dfrac{V_0}{2}$
mean power $\langle P \rangle$	$\dfrac{P_0}{2}$	$\dfrac{P_0}{4}$

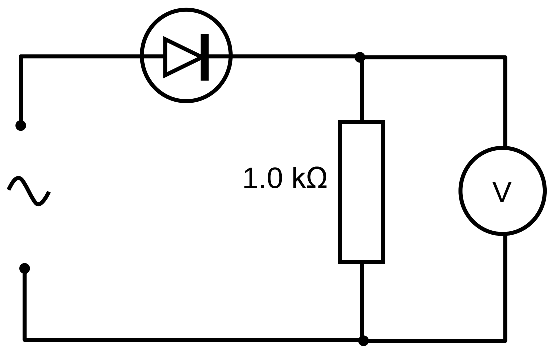
Example 10

A voltmeter reads 65 V when measuring the p.d. across a 1.0 kΩ resistor connected to a sinusoidal power source with angular frequency ω .

(a) Find average power dissipated by the resistor.

(b) State an expression for the variation of instantaneous power with time at the source.

(c) Find the peak potential difference across the 1.0 kΩ resistor.



(a) average power:

$$\langle P \rangle = \frac{{V_{rms}}^2}{R}$$
$$= \frac{65^2}{10^3} = 4.23 \text{ W}$$

(b)

$$P = \frac{V^2}{R}$$
$$= \frac{(V_0 \sin\omega t)^2}{R} = \left(\frac{V_0^2}{R}\right) \sin^2 \omega t$$

where $\left(\dfrac{V_0^2}{R}\right) = P_0$ (maximum power)

for half-wave rectified a.c., average power is $\dfrac{1}{4}P_0$

$$P_0 = 4 \times 4.23$$
$$= 16.9 \text{ W}$$
$$P = 16.9 \sin^2\omega t$$

(c) For a half-wave rectified circuit, $V_{rms} = V_0 / 2$

Peak voltage, $V_0 = 2 \times 65 = 130$ V

18.8 Ending Notes

A.C. is a relatively smaller topic and is a direct application of the previous topics on electromagnetic induction as well as D.C. circuit analysis.

Check that you are proficient with the following:

- ☐ able to describe an alternating source of electricity with the correct terms
- ☐ able to represent the alternating source of electricity using sin or cos functions with time
- ☐ able to work out that the mean power of an a.c. source is half its peak power
- ☐ able to work out r.m.s. calculations using graphical mean
- ☐ able to identify sinusoidally-changing electrical sources and therefore use $I_{\text{rms}} = \dfrac{I_0}{\sqrt{2}}$
- ☐ able to describe how a transformer works and how power loss is minimised
- ☐ calculations involving transformers and step up and step down scenarios
- ☐ explain and calculate quantities associated with half-wave rectification.