Anglo - Chinese School (Independent)

FINAL EXAMINATIONS 2015
YEAR 3 INTEGRATED PROGRAMME
CORE MATHEMATICS
PAPER 1

FRIDAY   **2nd OCTOBER 2015**   **1 h 30 min**

Additional Material
Graph Paper (1 sheet)

INSTRUCTIONS TO CANDIDATES

- Write your index number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Answer all questions in the spaces provided.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.
- The maximum mark for this paper is 80.

For Examiner's Use

This paper consists of 14 printed pages.

[Turn over

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

*Answer **all** the questions in the spaces provided.*

1   *[Maximum mark: 7]*

(a) Simplify $\sqrt[5]{-32a^{45}b^{10}}$.

[2 marks]

(b) Simplify $\left(\dfrac{q^2}{p^4}\right)^4 \div \left(\dfrac{p^{-1}}{pq^2}\right)^3$, leaving your answer in positive indices.

[3 marks]

(c) Express $\dfrac{5}{2a-b} - \dfrac{3}{2b-4a}$ as a single fraction in its simplest form.

[2 marks]

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2   *[Maximum mark: 4]*

The height, h metres, of the water sprayed from a fountain is given as $h = -x^2 + 4x + 7$, where x metre is the horizontal distance of the water from the fountain. Find the greatest height of the water sprayed and the horizontal distance from the fountain when this occurs.

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3   *[Maximum mark:6]*

In the diagram below, ABD is a straight line. $\angle CAB = 90^o$, $CD = 20\,\text{cm}$, $BD = 7\,\text{cm}$ and the area of $\triangle BCD = 42\,\text{cm}^2$.
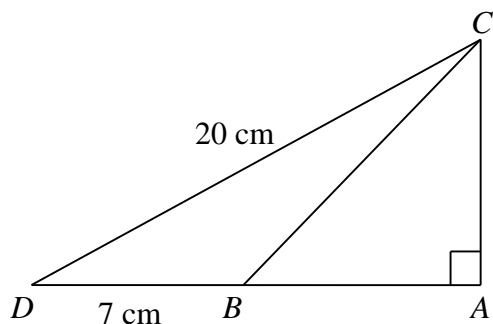
(a) Express $\sin \angle CDB$ as a fraction in its lowest terms.

[2 marks]

(b) Find the length of AB.

[2 marks]

(c) Express $\cos \angle CBD$ as a fraction in its lowest terms.

[2 marks]

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4   *[Maximum mark: 10]*

(a) Find the value of x when $\left(\dfrac{1}{3}\right)^{-2} = 27^{\frac{1}{2}} \div 9^x$.

[3 marks]

(b) Find the value of x given that $3^{12} - 3^{10} = \dfrac{8}{27^x}$.

[3 marks]

(c) Solve the equation $2^{2x^2} - 2^{x^2} - 12 = 0$, leaving your answers in surds.

[4 marks]

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5 *[Maximum mark: 5]*

(a) Factorize completely $3p(r+s) - r - s$.

[2 marks]

(b) Solve $2x = 8 - \dfrac{3}{x}$ and express your answers in the form $a \pm b\sqrt{10}$, where a and b are constants.

[3 marks]

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6   *[Maximum mark: 7]*

(a) Evaluate $\log_2 3 \times \log_3 4 + \dfrac{\ln 108 - 2\ln 2}{\ln 9}$.

[3 marks]

(b) Solve the equation $\log_{27} \dfrac{3}{\sqrt{x}} = 1 + \log_3 \sqrt{x}$.

[4 marks]

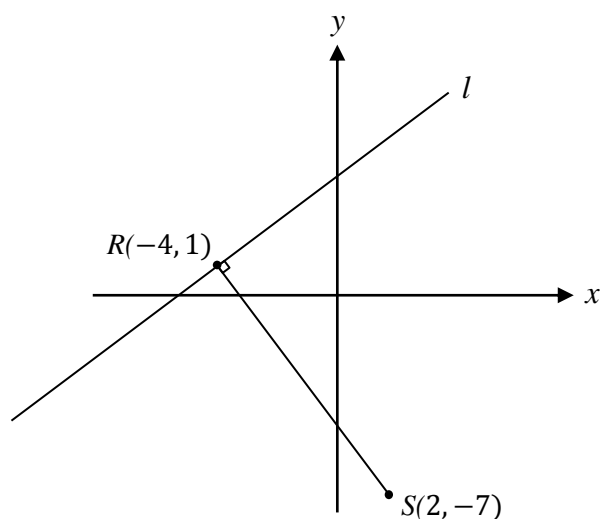...

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7   *[Maximum mark: 11]*

The coordinates of the points *R* and *S* are $(-4, 1)$ and $(2, -7)$ respectively.

(i)   Find the length of *RS*.

[2 marks]

(ii)   Find the equation of the line, *l*, passing through *R* and perpendicular to *RS*.

[4 marks]

(iii)   Given that $y = 1$ is the line of symmetry of ΔQRS, state the coordinates of *Q*.

[2 marks]

(iv)   Hence, find the perpendicular distance from *Q* to *SR* produced.

[3 marks]

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8   *[Maximum mark: 4]*

The diagram shows part of a straight line graph drawn to represent the equation $2 = \dfrac{p}{\lg x} + qy$. Calculate the value of p and of q.

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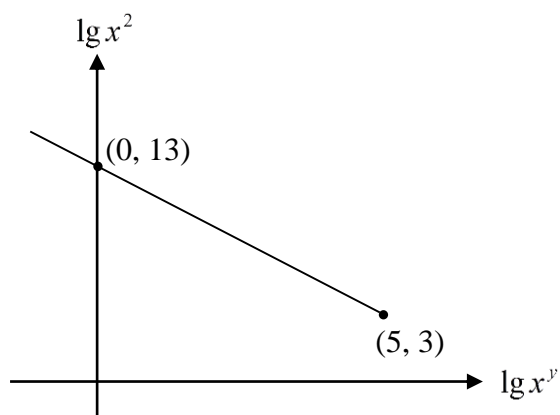
9   *[Maximum mark: 9]*

The roots of the quadratic equation $mx^2 - 4x - 3 = 0$ are α and β.

(i)   Given that $\alpha + \beta = 2$, find the value of m.

[2 marks]

(ii)   Hence, state the value of $\alpha\beta$.

[1 mark]

(iii)   Find the quadratic equation in x whose roots are $\alpha + \dfrac{1}{2\beta}$ and $\beta + \dfrac{1}{2\alpha}$.

[6 marks]

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10 *[Maximum mark: 8]*

Answer the whole of this question on a sheet of graph paper.

The variables x and y are connected by the equation $y = x + \dfrac{1}{4x} - 2$. The table below shows some values of x and the corresponding values of y, correct to 1 decimal place.

x	0.1	0.3	0.5	1.0	2.0	2.5	3.0	4.0
y	0.6	-0.9	-1.0	-0.8	0.1	m	1.1	2.1

(a) Calculate the value of m.

[1 mark]

(b) Using a scale of 4 cm to represent 1 unit on both axes, draw the graph of
   $y = x + \dfrac{1}{4x} - 2$ for $0.1 \le x \le 4$.

[4 marks]

(c) Use your graph to find the values of x in the range $0 < x \le 4$ for which
   $x + \dfrac{1}{4x} = 3$.

[3 marks]

11   *[Maximum mark: 9]*

(a) Find the range of values of p for which $\dfrac{2}{x+1} = \dfrac{5x}{p-x}$ has no real roots.

[4 marks]

(b) Given that the line $y = mx + c$ is a tangent to the curve $b^2 x^2 + y^2 = b^2$, where b, c and m are constants, show that $b^2 + m^2 = c^2$.

[5 marks]

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********** END OF PAPER 1 **********

Answers:

1a) $-2a^9b^2$

1b) $\dfrac{q^{14}}{p^{10}}$

1c) $\dfrac{13}{2(2a-b)}$

2) $-(x-2)^2 + 11$
Greatest height = 11 m
Occurs when $x = 2$ m

3a) $\dfrac{3}{5}$

3b) 9

3c) $-\dfrac{3}{5}$

4a) $-\dfrac{1}{4}$

4b) $-\dfrac{10}{3}$

4c) $\pm\sqrt{2}$

5a) $(r+s)(3p-1)$

5b) $2 \pm \dfrac{1}{2}\sqrt{10}$

6a) $3\dfrac{1}{2}$

6b) $\dfrac{1}{3}$

7i) 10

7ii) $y = \dfrac{3}{4}x + 4$

7iii) $(2, 9)$

7iv) 9.6

8) -2

9i) 2

9ii) $-\dfrac{3}{2}$

9iii) $x^2 - 1\dfrac{1}{3}x - \dfrac{2}{3} = 0$

10a) 0.6

10c) $x = 0.09$ and $x = 2.9$

11a) $p < -\dfrac{49}{40}$