2024 Y5 H2 Math Holiday Assignment 1 Solutions

Qn	Suggested Solution	Comments
1	Curve $B: y^2 = 2y + 8xy - 17$ Differentiate with respect to *x*, $2y\frac{\mathrm{d}y}{\mathrm{d}x} = 2\frac{\mathrm{d}y}{\mathrm{d}x} + 8x\frac{\mathrm{d}y}{\mathrm{d}x} + 8y$ $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{4y}{y-1-4x}$ $\left.\frac{\mathrm{d}y}{\mathrm{d}x}\right\|_{x=2,y=1} = \frac{4(1)}{1-1-4(2)} = -\frac{1}{2}$ To find (a,b), let $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{4b}{b-1-4a} = -\frac{1}{2}$ $\Rightarrow b - 1 - 4a = -8b$ $\therefore a = \frac{9b-1}{4} \Rightarrow$ subst into eqn of curve $b^2 = 2b + 8b\left(\frac{9b-1}{4}\right) - 17$ $b^2 = 2b + 2b(9b-1) - 17$ $17b^2 = 17$ $b = -1$ ($\because y = 1$ is given earlier) $a = \frac{9(-1)-1}{4} = -2.5$	As there's only one "*a*" term in the curve eqn, it's easier to subst $a = \frac{9b-1}{4}$ instead of $b = \frac{4a+1}{9}$. As (2,1) is already provided in the question, students should reject $a = 2, b = 1$ & accept $a = -2.5, b = -1$ as the final answer.

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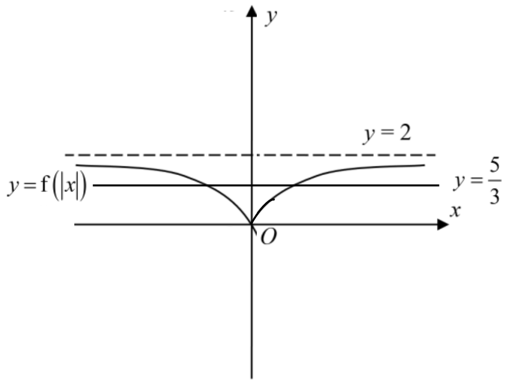
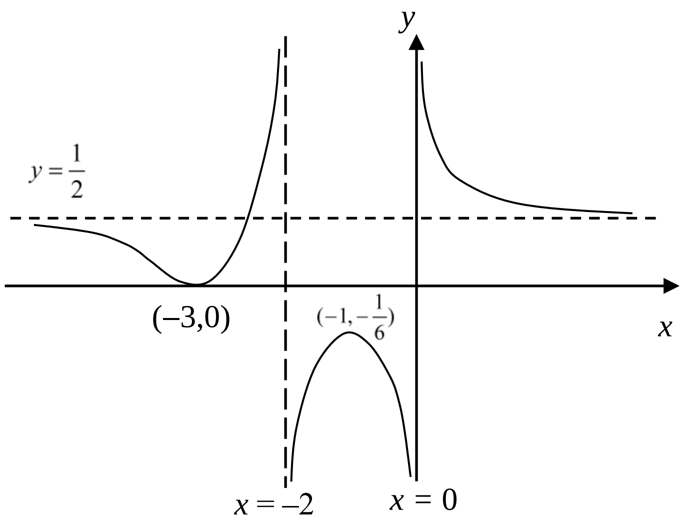
2(a)	Since $\mathrm{f}(-2) = 0$, $\mathrm{f}(1-3x) = 0$ $1 - 3x = -2$ $x = 1$ Since f(0) = 0, $\mathrm{f}(1-3x) = 0$ $1 - 3x = 0$ $x = \frac{1}{3}$ $\therefore$ The roots of the eqn are $\frac{1}{3}$ and 1.	Problem solving: Do not rush in to solve the question. The question did not ask for the sketch of $y = \mathrm{f}(1-3x)$. Thus, you may want to think of a simpler approach to solve the question. Solving this question can be done by observing the values that gives you the roots on the $y = \mathrm{f}(x)$. Concept used: If $\mathrm{f}(a) = 0$, then *a* is a root.

(b)	$3\mathrm{f}(\|x\|)-5=0$ $\mathrm{f}(\|x\|)=\dfrac{5}{3}$ Sketch the graph of $y=\mathrm{f}(\|x\|)$ and $y=\dfrac{5}{3}$.  From the graph, there are 2 distinct real roots.	Concept used: Sketch of $y=\mathrm{f}(\|x\|)$
(c)		Concepts used: 1. Sketch of $y=\dfrac{1}{\mathrm{f}(x)}$ from $y=\mathrm{f}(x)$. [see lecture notes] 2. $\mathrm{f}(x)>0 \Leftrightarrow \dfrac{1}{\mathrm{f}(x)}>0$ and vice versa. E.g. $\mathrm{f}(x)=x^3$: $2^3>0 \Leftrightarrow \dfrac{1}{2^3}>0$. [You can use this to check your sketch when you are done]

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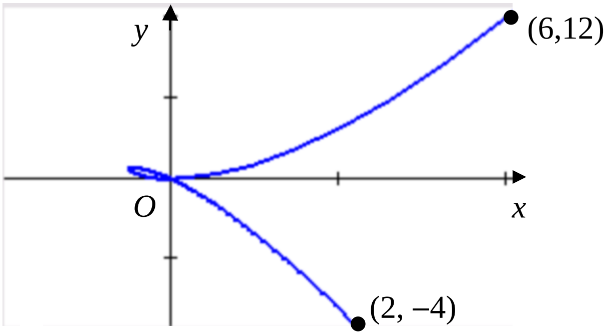
3(a)	 Since $x=t+t^2=\left(t+\frac{1}{2}\right)^2-\frac{1}{4}$ and $-2\le t\le 2$, the smallest value of x is $-\frac{1}{4}$. Thus there will be no curve when $x<-\frac{1}{4}$.	- End points should be clearly labelled and graph should pass through the origin - Explain clearly why there will be no curve when $x<-\frac{1}{4}$, by explaining why there are no t-values that give $x<-\frac{1}{4}$ (using discriminant or completing the square, etc) or why smallest value of x is $-\frac{1}{4}$ (by completing the square or drawing a graph, etc)
(b)	$\frac{dx}{dt}=1+2t,\quad \frac{dy}{dt}=2t+3t^2$ $\frac{dy}{dx}=\frac{2t+3t^2}{1+2t}$ $\left.\frac{dy}{dx}\right\|_{t=p}=\frac{2p+3p^2}{1+2p}$	- We should not sub $t = p$ from the start and differentiate w.r.t p because p is taken to be an unknown constant
(c)	At A, $x = 2$ $t^2+t=2$ $t^2+t-2=0$ $(t+2)(t-1)=0$ $t=-2$ or 1 At $t=-2, y=(-2)^3+(-2)^2\neq 2$ At $t=1, y=(1)^3+(1)^2=2$ (shown) $\therefore t=1$ at A (shown) - When the only real solution shown on the GC is $t = 1$, it does not imply that the cubic expression factorises to $(t-1)^3$ Alternatively, At A, $x = 2$ and $y = 2$ $t^2+t=2$ $t^2+t^3=2$ $t^2+t-2=0$ $t^3+t^2-2=0$ $(t+2)(t-1)=0$ $(t-1)\left(t^2+2t+2\right)=0$ $\Rightarrow t=-2$ or 1 $\Rightarrow t=1$ (no real roots for $t^2+2t+2=0$) $\therefore t=1$ at A (shown)	- There were many methods to "show" this and apart from the alternative method, when trying to solve for possible t-values, all possible solutions for t should be stated and checked/analysed – i.e. it should be clearly shown with justification why the other t-values should be rejected and why $t = 1$ will correspond to point A. - It should not be assumed that $t = 1$ is the solution without arriving at it and checking.

	Gradient of tangent at $A = \frac{2+3}{1+2} = \frac{5}{3}$ Gradient of tangent at $P = -\frac{3}{5}$ $\frac{2p+3p^2}{1+2p} = -\frac{3}{5}$ $15p^2+16p+3=0$ $p=-0.243$ or -0.824	- Final answers should be simplified as far as possible or given to 3s.f. for non-exact answers. - Students should think of using the GC for this part since non-exact answers were not required; some students tried to solve algebraically and made careless errors

Qn	Suggested Solution
4	$y=e^x-7x \xrightarrow{\text{Step 1}} y=e^{2x}-14x$ $y=e^{2x}-14x \xrightarrow{\text{Step 2}} y=14x-e^{2x}$ Replace x by $2x$ $2x>2$ $\therefore x>1$ $g(x)=14x-e^{2x}$, the domain of g is $(1,\infty)$.

Qn	Suggested Solutions	Comments
5	$\frac{1}{3}\pi r^2 h = 48\pi$ $r^2h=144 \quad --(1)$ Let S be the curved surface area of the cone cup. $S=\pi r l = \pi r\sqrt{r^2+h^2}$ Using (1), $S=\pi r\sqrt{r^2+\left(\frac{144}{r^2}\right)^2} = \pi\sqrt{r^4+\frac{144^2}{r^2}}$ Simplify the expression before differentiating. $\frac{dS}{dr}=\pi\left[\frac{1}{2}\left(r^4+\frac{144^2}{r^2}\right)^{-\frac{1}{2}}\left(4r^3-\frac{2(144^2)}{r^3}\right)\right]$ When $\frac{dS}{dr}=0$, $4r^3-\frac{2(144^2)}{r^3}=0$ $\left(r^4+\frac{144^2}{r^2}\right)^{-\frac{1}{2}}>0$, you don't have to consider this term. $4r^6=2(144^2)$ $r=\sqrt[6]{10368}$ or 4.67 (3sf)	Concepts used: 1. r, h and l are dependent on each other. As one varies, it will affect the other variables. 2. To minimise curved surface area, you will need it to be expressed in terms of 1 variable.