Topic 8 Temperature and Ideal Gases

Guiding Questions:

- What is the difference between temperature and heat? What is internal energy?
- How can we define a simple model of a gas that is valid for real gases under certain assumptions?
- How can we relate microscopic behaviour of gas molecules to the macroscopic properties of a gas?

Content

- Thermal equilibrium
- Temperature scales
- Equation of state
- Kinetic theory of gases
- Kinetic energy of a molecule

Learning Outcomes

Candidates should be able to:

- (a) show an understanding that regions of equal temperature are in thermal equilibrium.
- (b) explain how empirical evidence leads to the gas laws and to the idea of an absolute scale of temperature (i.e. the thermodynamic scale that is independent of the property of any particular substance and has an absolute zero).
- (c) convert temperatures measured in degrees Celsius to kelvin: T/K = T/°C + 273.15.
- (d) recall and use the equation of state for an ideal gas expressed as pV = nRT, where *n* is the amount of gas in moles.
- (e) state that one mole of any substance contains 6.02×10^{23} particles and use the Avogadro number N_A = 6.02×10^{23} mol⁻¹.
- (f) state the basic assumptions of the kinetic theory of gases.
- (g) explain how molecular movement causes the pressure exerted by a gas and hence derive the relationship $pV = \frac{1}{3}Nm < c^2 >$, where *N* is the number of gas molecules (a simple model considering the one-dimensional collisions and then extending to three dimensions using $< c_x^2 >= \frac{1}{3} < c^2 >$ is sufficient).
- (h) recall and apply the relationship that the mean kinetic energy of a molecule of an ideal gas is proportional to the thermodynamic temperature (i.e. $\frac{1}{2}m < c^2 >= \frac{3}{2}kT$) to new situations or to solve related problems.

Introduction

Heat and temperature are often used interchangeably in everyday language. However, these terms have different and specific meanings in physics. Macroscopically, temperature can be defined in terms of its measurement, while heat refers to the energy transferred between two systems at different temperatures. Linking these two ideas is the zeroth law of thermodynamics, which states that when two objects at different temperature are placed in thermal contact, there will be energy exchange between them until thermal equilibrium is reached. The zeroth law allows the use of thermometers to measure temperature. When thermal equilibrium is achieved, the thermometer reflects its own temperature as well as the temperature of the other body that it is in thermal contact with.

(a) show an understanding that regions of equal temperature are in thermal equilibrium.

Temperature is a measure of degree of hotness of an object. Thermal energy moves from objects at a higher temperature to objects at a lower temperature.

<u>Note:</u> temperature does NOT measure the amount of thermal energy in an object. It only indicates in which direction thermal energy will flow unaided.

Temperature is a fundamental quantity (i.e. it cannot be defined from other physical quantities) that measures the degree of hotness of a body as indicated on a calibrated scale. When two systems have different temperatures, a net thermal energy is transferred from the system at a higher temperature to a system at a lower temperature.

Heat is energy that flows from a higher-temperature object to a lower-temperature object because of differences in temperature.

Two systems are in thermal contact with each other if there is a mechanism for the transfer of thermal energy (i.e. by conduction, convection or radiation). Thermal contact may not necessarily mean physical contact only as thermal energy can be transmitted even through a vacuum by radiation (e.g. in space!).

When two objects in thermal contact has no <u>net</u> heat transfer between them, they are said to be in **thermal equilibrium** and are at the same temperature.

The **zeroth law of thermodynamics** states that if objects A and B are separately in thermal equilibrium with object C, then object A and B are in thermal equilibrium with each other.

Object C can function as a measuring device known as a thermometer, and the temperature can be measured on a suitable temperature scale.



Watch this video about Heat & Temperature!

(b) explain how empirical evidence leads to the gas laws and to the idea of an absolute scale of temperature (i.e. the thermodynamic scale that is independent of the property of any particular substance and has an absolute zero).

Thermometers are devices that are used to measure the temperature of a system. All thermometers are based on the principle that some physical property (known as thermometric property) of a system changes as the system's temperature changes.

Empirical temperature scale is a scale of temperature that is based on the variation with temperature of a property of a substance, <u>assuming</u> that the property varies linearly with temperature.

How do we set up an empirical temperature scale?

Step 1: Choose an appropriate thermometric property.

We choose a physical property that varies with temperature. This property must be reproducible.

Examples of Thermometric Quantities are:

 Volume of a fixed mass of liquid: 	Liquid-in-glass thermometer	
Resistance of a metal:	Platinum resistance thermometer Platinum resistance thermometer Connecting Leads Mounting Thread (Sensing Lement) Rat (Sensing Lement) Rat (b) Bridge Circuit	
 Pressure of a fixed mass of gas at constant volume: 	Constant volume gas thermometer (see thermometer on p 6)	
EMF produced between junctions of dissimilar metals that are at different temperatures:	Thermocouple thermometer Dissimilar Metal 1 Junction Dissimilar Metal 2 Dissimilar Metal 2	

Step 2: Select two fixed temperature points.

We then choose an upper point and a lower point with which we will calibrate our thermometer from. For example, on the **Celsius temperature** scale, these two fixed points are the ice point of water, which is assigned to be 0 °C, and the steam point of water, which is assigned to be 100 °C.

Step 3: Calibrate the thermometer.

Lastly, we calibrate the thermometer by placing it in systems of the lower and upper fixed points. The values of the thermometric quantity at these temperatures are recorded. The value at the lower fixed point is X_0 and that of the upper fixed point is X_{100} . We then **assume a linear relationship** between these two points.

Illustration:



Using similar triangles or gradient formula, the following relationship can be obtained:

$$\theta = \frac{L_{\theta} - L_0}{L_{100} - L_0} \times 100$$

In general, for all thermometers,

$$\theta = \frac{X_{\theta} - X_{0}}{X_{100} - X_{0}} \times 100$$

 X_{θ} Value of thermometric quantity at temp θ

*X*₀ Value of thermometric quantity at ice point

 X_{100} Value of thermometric quantity at steam point

At ice-point, the mercury column of an unmarked thermometer has a height of 2.0 cm. At boiling point, the mercury column has a height of 24.0 cm. What is the temperature when the mercury column reads 18.0 cm? (Ans: 72.7 °C)

Example 2

A resistance thermometer gives a resistance of 20 Ω when the temperature is known to be -10 °C. When the temperature is 110 °C, the resistance thermometer has a resistance of 500 Ω . What is the temperature when the resistance is 360 Ω . (Ans: 75 °C)

The problem with empirical scales

Although different thermometers agree with one another at the fixed points, they may disagree at other temperatures due to their possibly non-linear variation with temperature.

Constant Volume Gas Thermometer Gas 1 capillary liquid Gas 2 h e.g. mercury fixed mark Gas 3 gas (H_e or He) -273.15 -200 -1000 100 200 T("C) flexible

Constant volume gas thermometer

It turned out that for a constant volume gas thermometer, the temperature is linearly dependent on the pressure of the gas. This relationship holds true for different types of gas, and if we extrapolate the Pressure – Temperature graph as shown above, they all meet at the same point at –273.15 °C. This point is the **absolute zero** temperature.

Absolute zero is the temperature at which all substances have minimum internal energy.

The absolute temperature scale or thermodynamic temperature scale <u>does not</u> <u>depend on the thermometric property of any particular substance</u> and has absolute zero and triple point of water as fixed points.

The **triple point of water** is the particular temperature and pressure (273.16 K, 4.58mmHg) at which the three states of water (solid, liquid and vapour states) can co-exist in equilibrium.

Properties of the Absolute scale:

- 1) The absolute or thermodynamic scale is based on the theoretical efficiency of a perfectly reversible heat engine. It is theoretical and hence is not based on any thermometric property.
- 2) The lower fixed point is the absolute zero to which a value of 0 K is allocated. No temperature at this value and lower could be attained. Hence, at 0 K, all substances have a minimum internal energy.
- 3) The upper fixed point is the Triple Point of Water, to which a value of 273.16 K is allocated.
- (c) convert temperatures measured in degrees Celsius to kelvin: T/K = T/°C + 273.15.



Watch this video on temperature conversions! Remember that Fahrenheit is not in our syllabus!

To convert temperature measured in Celsius scale to thermodynamic scale (in Kelvins)

 $T/K = \theta / °C + 273.15$

A pan of water is heated from 25°C to 80°C. Determine, in Kelvins,

- (a) the initial temperature of water
- (b) the final temperature of water
- (c) the change in temperature.





Watch this video to learn more about the three laws and the Ideal Gas Law!

For a gas, it is useful to know how the quantities volume V, pressure p, and temperature T are related for a fixed amount of gas. In general, the equation that relates these quantities, called the *equation of state*, is very complicated. However, when the pressure is sufficiently low and temperature sufficiently high, the equation of state can be easily found experimentally. Such a gas is known as an ideal gas, and these experimental observations are summarised below:



Putting all three together, we get the equation of state of an ideal gas.

Where *n* is the amount of gas in moles and R is the molar gas constant. (= 8.31 J mol⁻¹ K⁻¹)

An **ideal gas** is a hypothetical gas that obeys the equation of state pV = nRT perfectly at all pressure *p*, volume *V*, amount of substance *n*, and temperature *T*.

If we work with the number of molecules instead of moles, we ought to use the form

where *N* represents the number of gas atoms/molecules and *k* is known as the Boltzmann's constant (= $1.38 \times 10^{-23} \text{ J K}^{-1}$)

(e) state that one mole of any substance contains 6.02 x 10^{23} particles and use the Avogadro number N_A = 6.02 x 10^{23} mol⁻¹.

One mole is the amount of substance containing 6.02×10^{23} particles.

The mathematical relationship between *n* and *N* is

$N = nN_A$

 N_A is known as Avogadro's number, which is 6.02×10^{23} . Basically think of 1 mole as just another way to say 6.02×10^{23} .

Example 4

Find the volume occupied by one mole of an ideal gas at a temperature of 273 K and a pressure of 1.01×10^5 Pa. (Ans: 0.0225 m³)

Example 5

A gas cylinder is fitted with a safety valve which releases a gas when the pressure inside the cylinder reaches 2.0×10^6 Pa. Given that the maximum mass of this gas the cylinder can hold at 10° C is 15 kg, what would be the maximum mass at 30° C? (Ans: 14.0 kg)

An ideal gas exerts a pressure of 60Pa when its temperature is 400 K and the number of molecules present per unit volume is *N*.

Another sample of the same gas exerts a pressure of 30 Pa when its temperature is 300 K. How many molecules are present in unit volume of this second sample? (Ans: 0.667N molecules per m³)

Example 7

Two bulbs, X of volume 100 cm³ and Y of volume 50 cm³, are connected with a tube of negligible volume. A valve prevents gas to flow between the two bulbs. Initially bulb X is filled with an ideal gas at 10 °C to a pressure of 3×10^5 Pa. Bulb Y is filled with the same ideal gas at 100 °C to a pressure of 1×10^5 Pa. The valve is opened and the temperature of A and B are maintained at their initial temperatures. Determine the new equilibrium pressure of the system.





Watch this video on the kinetic theory of gases! Some symbols used might be different so just follow the notes when presenting your answer :)

Assumption (a) was not mentioned in the video but please study it as well!

(f) state the basic assumptions of the kinetic theory of gases.

So far, we were only concerned with the macroscopic *states* (pressure, volume, temperature) of the gases. However, we know from *Brownian motion* (see appendix for *more details*) that gases actually consist of moving molecules. The kinetic theory of gases is a theoretical model that seeks to explain what gave rise to the states of gases on a microscopic(molecular) level.

Before we start, we need to state the assumptions of kinetic theory of gases:

- (a) All gases consist of a very large number of atoms or molecules.
- (b) The atoms or molecules behave as if they are hard, perfectly elastic, identical spheres.
- (c) There are no forces of attraction or repulsion between atoms or molecules unless they are in collision with each other or with the walls of the containing vessel.
- (d) The molecules are in constant, random motion and obey Newton's laws of motion.
- (e) Total volume of atoms or molecules is negligible compared with the volume of the containing vessel.
- (f) Time of collisions is negligible compared with time between collisions.

(g) explain how molecular movement causes the pressure exerted by a gas and hence derive the relationship $pV = \frac{1}{3}Nm < c^2 >$, where N is the number of gas molecules (a simple model considering the one-dimensional collisions and then extending to three dimensions using $< c_x^2 >= \frac{1}{3} < c^2 >$ is sufficient).

Suppose there is a cubical box of sides *I*, containing *N* molecules of a gas (mass of each molecule = m). By considering the elastic collision of one molecule with a wall, we note that

the change in momentum of the



 $\Delta p = -2 m c_x$

molecule due to the collision is

where c_x is the x component of the molecule's velocity and direction to the right is taken as positive.

Conservation of linear momentum tells us that the wall experiences a change in momentum

$$\Delta p_{wall} = 2 m c_x$$

The next time the same molecule collides with the same wall, the molecule would have t raveled a total distance of 2*I* in the x direction. Hence time between collisions

$$\Delta t = \frac{2l}{c_x}$$

Therefore, rate of change of momentum for the wall due to one molecule is

$$\frac{\Delta p_{wall}}{\Delta t} = \left(2mc_{x}\right)\left(\frac{c_{x}}{2I}\right) = \frac{mc_{x}^{2}}{I}$$

According to Newton's 2^{nd} Law, total force acting on the wall is rate of change of momentum for the wall due to all *N* molecules

$$F_{wall} = \frac{mc_{x1}^{2}}{l} + \frac{mc_{x2}^{2}}{l} + \frac{mc_{x3}^{2}}{l} + \dots$$
$$= \frac{m}{l} \underbrace{(c_{x1}^{2} + c_{x2}^{2} + c_{x3}^{2} + \dots)}_{N \text{ terms for N different molecules}}$$

If we now define mean-square-speed as $\langle c_x^2 \rangle = \frac{c_{x1}^2 + c_{x2}^2 + \ldots + c_{xN}^2}{N}$,

$$F_{wall} = \frac{Nm}{I} \langle c_x^2 \rangle$$

Pressure on the wall is force acting per unit area, hence

$$p_{wall} = \frac{F_{wall}}{I^2} = \frac{Nm}{V} \langle c_x^2 \rangle$$

Applying the Pythagoras Theorem to the 3D velocity vector for any molecule leads to

$$C^{2} = C_{x}^{2} + C_{y}^{2} + C_{z}^{2}$$

$$< C^{2} = < C_{x}^{2} + < C_{y}^{2} + < C_{z}^{2} >$$



However, since *N* is large and the molecules are moving randomly (with no preference in any direction), $\langle c_x^2 \rangle = \langle c_y^2 \rangle = \langle c_z^2 \rangle$.

Therefore,

$$\left\langle c_{x}^{2}\right\rangle = \frac{1}{3}\left\langle c^{2}\right\rangle$$

which leads us to

$$pV = \frac{1}{3}Nm\langle c^2 \rangle$$

(h) recall and apply the relationship that the mean kinetic energy of a molecule of an ideal gas is proportional to the thermodynamic temperature (i.e. $\frac{1}{2}m < c^2 >= \frac{3}{2}kT$) to new situations or to solve related problems.

If we combine this equation with the equation of state, we deduce that

$$pV = \frac{1}{3}Nm\langle c^2 \rangle = NkT$$

(multiplying both sides by $\frac{3}{2}$)

$$\frac{1}{2}m\langle c^2\rangle = \frac{3}{2}kT$$

which tells us that the average kinetic energy of an ideal gas molecule is $\frac{3}{2}kT$.

For 1 molecule,

$$\frac{1}{2}m\langle c^2\rangle=\frac{3}{2}kT$$

For N molecules,

total KE of all N gas molecules

$$=N\times\left(\frac{1}{2}m\langle c^2\rangle\right)=\frac{3}{2}NkT=\frac{3}{2}nRT$$

We usually use $\langle c^2 \rangle$ instead of the usual v^2 for square speed. Different molecules in an ideal gas actually move at different speeds. Unfortunately, the simpler 'arithmetic mean' speed does not give us the true average speed of all the gas molecules. Based on the conservation of KE, the <u>root-mean-square speed</u>, which is written as $\sqrt{\langle c^2 \rangle}$ was instead found to be the true average speed of the gas molecules.

Mean Square Speed <c²>

Consider N molecules in the gas with respective molecular speeds $c_1, c_2, c_3, c_4, \ldots, c_N$

The mean square speed $\langle c^2 \rangle$ of the molecules is the mean of the square of the speeds,

$$< c^2 > = \frac{c_1^2 + c_2^2 + c_3^2 + \dots + c_N^2}{N}$$

Root Mean Square Speed crms

The root mean square speed $c_{\rm rms}$ of the molecules is

$$C_{\rm rms} = \sqrt{\langle c^2 \rangle} = \sqrt{\frac{c_1^2 + c_2^2 + c_3^2 + \dots + c_N^2}{N}}$$

Example 8

Ten particles have the following speeds in m s⁻¹: 1.0 7.0 9.0 5.0 5.0 6.0 6.0 5.0 3.0 4.0 Calculate (a) the mean speed, and (b) the r.m.s. speed.

The molar mass of hydrogen gas is 2.016 g. A container of 1 mole of hydrogen gas is at pressure 2.00 x 10^5 Pa and temperature 300 K. Find the density of the gas and the c_{rms} of the hydrogen gas molecules.

Example 10

Find the relationship between root-mean-square speed crms and

- (a) the temperature for a fixed mass of gas;
- (b) mass of different gases at the same temperature;
- (c) temperature and molar mass;
- (d) Hence, deduce the c_{rms} of a gas at 450.0 K given that the c_{rms} of the same gas at 50.0 K is 800 ms⁻¹.

Consider 1 mole of oxygen (molar mass = 32 g) at a temperature of 300 K in a cubical container of sides 10 cm. Calculate

- (a) the average KE of a molecule;
- (b) the total KE;
- (c) the average value of the square of the speeds;
- (d) the root-mean-square speed;
- (e) the average momentum of a molecule;
- (f) the average force exerted by the molecules on the walls of the container using your answer in (e);
- (g) the average pressure exerted by the molecules using your answer in (f). Verify this answer using the equation of state.

Annex A: (for extra reading)

Calorimetry



A calorimeter is an insulated container that enables the careful measurement of heat. It is designed to minimise heat flow from the surroundings.

A typical constant volume calorimetry, called a bomb calorimeter, consists of a hollow aluminium cylinder of known mass containing a known quantity of water; the cylinder is inside a larger aluminium cylinder with insulated wall with an evacuated space separating the 2 cylinders. An insulated lid fits over the opening of the cylinder, where there are 2 small holes in the lid, one for a thermometer and the other a stirring device to help the content reach equilibrium faster.

Suppose an object at one temperature is placed in a calorimeter with the water and aluminium cylinder at another temperature. By conservation of energy, all the heat that flows out of 1 substance (Q<0) flows into some other substance (Q>0). If no heat flows to or from the environment, the total heat into the object, water and aluminium must equal to 0.

 $Q_w + Q_a + Q_O = 0$

Centigrade and Celsius temperature scales

In 1741 Anders Celsius, professor of astronomy at the University of Uppsala, Sweden, introduced a temperature scale with 0 the temperature at which water boiled and 100 the temperature at which water froze. Shortly after his death, the scale became known as the centigrade scale, but the fixed points were reversed, making 0 degrees the freezing point of water and 100 degrees its boiling point at atmospheric pressure.

In 1887 the International Commission on Weights and Measures adopted "as the standard thermometric scale for the international services of weights and measures the centigrade scale of the hydrogen thermometer, having as fixed points the temperature of melting ice (0°) and the vapor of distilled water boiling (100°) at standard atmospheric pressure, the hydrogen being taken at an initial manometric pressure of one meter of mercury."

The Celsius scale is the centigrade scale with one change. Defined in 1954 at the 10th General Conference of Weights and Measures, temperature on the Celsius scale is the temperature on the Kelvin scale minus 273.15. This definition makes values on the Celsius and centigrade scale agree within less than 0.1 degree. For everyday purposes, the scales are identical. One reason for doing away with the word "centigrade," was that it might be confused with one-hundredth of a grade, a unit of plane angle.

Why was the centigrade scale abandoned?

The problem was that the ice point, the "temperature of melting ice...at standard atmospheric pressure," which was used to define zero degrees on the centigrade scale, cannot be measured with enough precision. Ideally one takes the temperature of a bath of pure, air-saturated water containing pure melting ice. But as ice melts it surrounds itself with a layer of insulating meltwater that is not air-saturated. The bath cannot be stirred because that would heat it.

In contrast, the Kelvin scale has a nearby set point, the triple point of water. The triple point is the temperature and pressure at which water can exist simultaneously as a solid, liquid and gas. Measurements of the temperature of the triple point are reproducible with an variation of 0.000 050 K or less. By the definition of the Kelvin scale the triple point of water is 273.16 kelvin. Replacing the hard-to-measure ice point with the triple point made possible more precise measurements. To indicate the change, the "zero is freezing, 100 is boiling" scale was given a new name.

Annex B: Evidence of Continuous Random Molecular Motion - Brownian Motion

According to the Kinetic Theory, gases are made up of molecules and that these molecules are in continuous random motion.

Brownian motion gives direct evidence for the perpetual molecular movement. It is named after biologist Robert Brown who is credited with its discovery in 1827.

While he was observing tiny pollen grains suspended in water under a microscope Brown noted that the tiny grains moved about in tortuous paths even though the water appeared to be perfectly still. The tiny pollen grains jostled about by the vigorous barrage of rapidly moving molecules of water.

Brownian motion can also be seen clearly in the motion of smoke particles in air and gives direct evidence of collisions between its individual molecules and the tiny smoke particles.



Figure on the left shows an arrangement for viewing smoke particles in air.

A beam of light is directed at a cell containing the smoke. Some of the light is reflected by the particles into the microscope. An observer looking down the microscope sees the particles as pinpoints of light. The particles move about in an erratic, unpredictable way, continually changing directions at random.

The cause of the motion is the continual impacts on each smoke particle by air molecules that are too small to be seen. The smoke particles are much larger than the air molecules so each smoke particle is bombarded from different sides by air molecules, but the bombardments are random and uneven. The result is that each smoke particle is pushed around haphazardly. The random nature of the bombardments is because the air molecules move about at random; the uneven nature is because the smoke particles are small enough to make even bombardment unlikely.

Essential Definition List:

Thermal Equilibrium	When two objects in thermal contact are in thermal equilibrium, there is no <u>net</u> heat transfer between them. They are at the same temperature.
Ideal Gas	An ideal gas is a hypothetical gas that obeys the equation of state ($pV = nRT$) for an ideal gas perfectly for all pressure p, volume V, amount of substance n and temperature T.
	Note: P = pressure in Pa, V = volume in m^3 , n = amount of substance measured in mol, R = molar gas constant, T = thermodynamic temperature in K.

Tutorial Questions

Temperature

- 1. Two objects, with different sizes, masses and temperatures, are placed in thermal contact. Energy travels
 - (a) from the larger object to the smaller object.
 - (b) from the object with more mass to the one with less.
 - (c) from the object at higher temperature to the object at lower temperature.
 - Which statement(s) (a), (b) or (c) is/are true?
- 2. Is it possible for two objects to be in thermal equilibrium if they are not in contact with each other? Explain.
- 3. How does the absolute (Thermodynamic) scale of temperature differ from the empirical temperature scale?
- 4. The table below gives data for two thermometers at three different temperatures (the ice-point, the steam point and room temperature).

Types of	Property	Value of property		
Thermometer	горену	ice point	steam point	room temp
Gas	Pressure in mmHg	760	1040	795
Thermistor	Current in mA	12.0	54.0	15.0

(a) Calculate the temperature of the room temperature according to each thermometer.

(b) State why the thermometers disagree in their value for room temperature.

Ideal Gases

- A partially inflated balloon contains 500 m³ of helium at 27 °C and 1.0 atm pressure. What is the volume of the helium at height where the pressure is 0.50 atm and the temperature is -3.0 °C?
 [900 m³]
- 6. A uniform capillary tube, closed at one end, contained air trapped by a thread of mercury 85 mm long. When the tube was held horizontal, the length of the air column was 50 mm; when it was held vertically with the closed end downwards, the length was 45 mm. Taking g to be 10 m.s⁻² and the density of mercury = 14 x 10³ kg.m⁻³, find the atmospheric pressure.
 [1.1 x 10⁵ Pa]
- 7. Two identical gas cylinders each contain 20 kg of compressed air at 1000 kPa pressure and 275 K. One of the cylinders is fitted with a safety valve which releases air from the cylinder into the atmosphere if the pressure in the cylinder rises above 1100 kPa. The cylinders are then moved to a room where the temperature is 310 K. Calculate

i. the pressure in the cylinder which is not fitted with a safety valve,

ii. the mass of gas lost from the cylinder fitted with the safety valve.

[1127kPa, 0.48 kg]

- Two containers of volume V and 4V are connected by a capillary tube. Initially both containers are at 280 K and contain air at a pressure of 101 kPa. The larger container is now warmed to 350 K and the smaller container cooled to 210 K.
 - i. Why may we not apply pV/T = constant to either container separately?
 - ii. Calculate the final pressure in both containers.

[111 kPa]

- 9. A car tire of volume 1.60×10⁻² m³ contains air at a temperature of 30.0 °C and a pressure of 2.60×10⁵ Pa. A foot pump is used to pump air into the tire to increase the pressure to 3.10×10⁵ Pa. Each stroke of the pump pushes 3.0×10⁻⁴ m³ of air at 1.0×10⁵ Pa into the tire. The temperature of the air stays constant at 30.0 °C throughout the whole process. You may assume that air behaves as an ideal gas.
 - i. Calculate the initial number of moles of air that the tire contains.
 - ii. Calculate the number of strokes needed to raise the pressure of the car tire to 3.10×10^5 Pa.
 - iii. Explain whether the answer calculate in the previous section is a minimum or a maximum number of strokes required.
 - [1.65 mol, 27, minimum]
- 10. Helium gas occupies a volume of 0.040 m³ at a pressure of 2.0x10⁵ Pa and temperature of 300 K. Calculate
 - (a) The mass of helium
 - (b) The rms speed of the molecules
 - (c) The rms speed at 432 K when the gas is heated at constant pressure to this temperature
 - (d) The rms speed of hydrogen molecules at 432 K

Take the relative molecular mass of hydrogen and helium to be 2 and 4 respectively.

[12.8 g, 1367 m.s⁻¹, 1641 m.s⁻¹, 2321 m.s⁻¹]

- 11. The atomic mass of Helium is 4.0 g. In a sample of Helium at 30 °C, find
 - (a) the root mean square speed of an atom of Helium.
 - (b) the mean average translational kinetic energy of a Helium atom in this gas.
 - (c) the total internal energy of the atoms in this gas sample, given that the mass of gas involved is 50 g. Assume the gas behaves as an ideal gas.

[1374 m.s⁻¹, 6.27×10⁻²¹ J, 4.72×10⁴ J]

- 12. The molecules of two ideal gases A and B have an average KE of 6.2×10⁻²¹ J at 27 °C. Molecular masses of gas A and B are 1.7×10⁻²⁷ kg and 3.0×10⁻²⁶ kg respectively.
 - (a) Determine the root mean square speeds of the molecules of A and B.
 - (b) The escape velocity from Earth is about 11 km s⁻¹ and from Jupiter is 60 km s⁻¹.

Which of the gases, A or B is likely to be found in the Earth's atmosphere and which in Jupiter's? Why? $[643 \text{ m.s}^{-1}]$

Data-based Question

13. An object that is at a higher temperature than its surroundings loses thermal energy by emitting electromagnetic radiation. For loss of thermal energy as electromagnetic radiation, the intensity I_{λ} of the emitted radiation of wavelength λ varies with wavelength as shown in Fig. A.



Fig. A shows the variation of I_{λ} with λ for the body when it is at 1000 K. The distribution of intensity is different at different temperatures. This is shown in Fig. B below.



- (a) (i) On the horizontal axis of Fig. B, indicate with the letter V, a wavelength that is in the visible region of the electromagnetic spectrum.
 - (ii) Hence suggest why, at a temperature of 1100 K, the object would glow with a red colour.

(b) At any temperature *T*, the graph of Fig. B shows a peak corresponding to a wavelength λ_{max} and an intensity I_{max} . Data for *T* and λ_{max} are shown in the following table.

<i>T/</i> K	λ_{\max} / nm
600	4830
700	4140
800	3610
900	3210
1000	2900
1100	2630

(i) Without drawing a graph, show that $T \ge \lambda_{max}$ = constant and determine the constant. (2.90 x 10⁻³ K m)

- (ii) Hence determine the wavelength for maximum intensity at a temperature *T* of 1200 K. (2420 nm)
- (c) The total intensity of emitted radiation from a particular body at temperature T is I_{tot} . Fig. C shows the values of lg (T/K) plotted against the corresponding values of lg ($I_{tot}/W m^{-2}$).



It is known that I_{tot} varies with T according to the relation $I_{tot} = cT^n$ where c and n are constants.

(i) Use Fig. C to determine a value for *n*.

- (4.0)
- (ii) For this body at T = 900 K, I_{tot} is found to be 71 W m⁻². Use these data and your answer in (c) (i) to determine I_{tot} for the body at a temperature of 1200 K. (224 W m⁻²)
- (d) Use your answer to (b) (ii), sketch on Fig. B, the variation with wavelength λ of intensity I_{λ} for a temperature of 1200 K.
- (e) The radiation emitted by a hot body may be used as a means of determining the temperature of the body.
 - (i) Suggest and explain a property of the radiation that could be used for this purpose.
 - (ii) Suggest one advantage and one disadvantage of this method for measuring temperature.