ANDERSON SERANGOON JUNIOR COLLEGE
JC2 Preliminary Examinations 2021
Higher 3

MATHEMATICS 9820/01

Paper 1

22 September 2021

3 hours

Additional Materials: List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

An answer booklet will be provided with this question paper. You should follow the instructions on the front cover of the booklet. If you need additional answer paper ask the invigilator for a continuation booklet.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **5** printed pages

1 For $x=5k, k\in\mathbb{Z},\ x^4+x^3+x-1\equiv 0+0+0-1\equiv -1 \pmod 5$

For $x=5k+1, k\in\mathbb{Z},\ x^4+x^3+x-1\equiv 1^4+1^3+1-1\equiv 2 \pmod 5$

For $x=5k+2, k\in\mathbb{Z},\ x^4+x^3+x-1\equiv 2^4+2^3+2-1\equiv 16+8+2-1\equiv 0 \pmod 5$

For $x=5k+3, k\in\mathbb{Z},\ x^4+x^3+x-1\equiv 3^4+3^3+3-1\equiv 81+27+3-1\equiv 0 \pmod 5$

For $x=5k+4, k\in\mathbb{Z},\ x^4+x^3+x-1\equiv 4^4+4^3+4-1\equiv 256+64+4-1\equiv 3 \pmod 5$

Therefore the solution set is $\{5k+2, 5k+3, k\in\mathbb{Z}\}$

$$x^4+x^3+x-1\equiv (x-2)(x-3)(x^2+bx+c) \pmod 5$$
$$x^4+x^3+x-1\equiv (x^2-5x+6)(x^2+bx+c) \pmod 5$$
$$x^4+x^3+x-1\equiv (x^2+1)(x^2+bx+c) \pmod 5$$
$$x^4+x^3+x-1\equiv (x^2+1)(x^2+x-1) \pmod 5$$
$$x^4+x^3+x-1\equiv (x^2+1)(x^2-4x+4) \pmod 5$$
$$x^4+x^3+x-1\equiv (x^2+1)(x-2)^2 \pmod 5$$
$$x^4+x^3+x-1\equiv (x-2)^3(x-3) \pmod 5$$

2 Let $t=\pi-x$. Then $\dfrac{\mathrm{d}x}{\mathrm{d}t}=-1$, When $x=0,\ t=\pi$. When $x=\pi,\ t=0$.

$$\int_0^\pi x\cdot \mathrm{g}(\sin x)\,\mathrm{d}x=\int_\pi^0(\pi-t)\cdot \mathrm{g}(\sin(\pi-t))(-1)\,\mathrm{d}t$$
$$=\int_0^\pi \pi \mathrm{g}(\sin t)\,\mathrm{d}t-\int_0^\pi t\cdot \mathrm{g}(\sin t)\,\mathrm{d}t$$
$$\Rightarrow 2\int_0^\pi x\cdot \mathrm{g}(\sin x)\,\mathrm{d}x=\pi\int_0^\pi \mathrm{g}(\sin x)\,\mathrm{d}x$$
$$\Rightarrow \int_0^\pi x\cdot \mathrm{g}(\sin x)\,\mathrm{d}x=\frac{\pi}{2}\int_0^\pi \mathrm{g}(\sin x)\,\mathrm{d}x \text{ (shown)}$$

(a) $$\int_0^\pi \frac{x\sin x}{\sqrt{8+\sin^2 x}}\,\mathrm{d}x=\frac{\pi}{2}\int_0^\pi \frac{\sin x}{\sqrt{8+\sin^2 x}}\,\mathrm{d}x$$
$$=-\frac{\pi}{2}\int_0^\pi \frac{-\sin x}{\sqrt{9-\cos^2 x}}\,\mathrm{d}x$$
$$=-\frac{\pi}{2}\left[\sin^{-1}\left(\frac{\cos x}{3}\right)\right]_0^\pi$$
$$=-\frac{\pi}{2}\left(\sin^{-1}\left(-\frac{1}{3}\right)-\sin^{-1}\frac{1}{3}\right)$$
$$=-\frac{\pi}{2}\left(-2\sin^{-1}\frac{1}{3}\right)=\pi\sin^{-1}\frac{1}{3}$$

(b) $\int_0^{\pi} x\sqrt{1+\sin x}\,\mathrm{d}x = \frac{\pi}{2}\int_0^{\pi} \sqrt{1+\sin x}\cdot\frac{\sqrt{1-\sin x}}{\sqrt{1-\sin x}}\,\mathrm{d}x$

$$= \frac{\pi}{2}\int_0^{\pi} \frac{\sqrt{\cos^2 x}}{\sqrt{1-\sin x}}\,\mathrm{d}x$$

$$= \frac{\pi}{2}\int_0^{\pi} \frac{|\cos x|}{\sqrt{1-\sin x}}\,\mathrm{d}x$$

$$= \frac{\pi}{2}\times 2\int_0^{\frac{\pi}{2}} \frac{\cos x}{\sqrt{1-\sin x}}\,\mathrm{d}x$$

$$= -2\pi\left[\sqrt{1-\sin x}\right]_0^{\frac{\pi}{2}}$$

$$= -2\pi\left(0-\sqrt{1}\right) = 2\pi$$

3(i) First put 1 object into each box, leaving behind $(n-k)$ objects. Then distribute the remaining $(n-k)$ objects into k boxes, with $(k-1)$ partitions.

Total number of ways $= \binom{n-k+k-1}{k-1}$

$$= \binom{n-1}{k-1}$$

(ii) First put 2 objects into each box, leaving behind $(n-2k)$ objects. Then group the remaining $(n-2k)$ objects into pairs, this gives $\frac{n-2k}{2}$ pairs. Lastly, distribute the $\frac{n-2k}{2}$ pairs into k boxes, with $(k-1)$ partitions.

Total number of ways $= \binom{\frac{n-2k}{2}+k-1}{k-1}$

$$= \binom{\frac{n}{2}-1}{k-1}$$

(iii) First put 1 object into each box, leaving behind $(n-k)$ objects. Then group the remaining $(n-k)$ objects into pairs, this gives $\frac{n-k}{2}$ pairs. Lastly, distribute the $\frac{n-k}{2}$ pairs into k boxes, with $(k-1)$ partitions.

$$\text{Total number of ways } = \binom{\frac{n-k}{2}+k-1}{k-1}$$
$$= \binom{\frac{1}{2}(n+k-2)}{k-1}$$

(iv) Let A_i be the set depicting all possible distributions of n identical objects into k distinct boxes such that every box is filled and that there are at least i box(es) with more than m objects, where $i = 1, 2, 3, \cdots, k$.

To find $|A_j|$, let's suppose that the j boxes each containing more than m objects, with the possibility that there may (or may not) be other boxes having more than m objects. First, put k objects into each of the k boxes, we are left with $(n-k)$ objects. Then we put an additional m objects into each of the j boxes, we are left with $(n-k-jm)$ objects. Now, we shall distribute these remaining $(n-k-jm)$ objects into the k boxes, with $(k-1)$ partitions. It is important to take note that there is $\binom{n}{j}$ ways to choose the j boxes to be overfilled.

So, the number of ways $= \binom{n}{j}\binom{n-k-jm+k-1}{k-1}$
$$= \binom{n}{j}\binom{n-jm-1}{k-1}$$

Using principle of inclusion and exclusion,

the total number of ways $= \sum_{j=0}^{L}(-1)^j\binom{n}{j}\binom{n-jm-1}{k-1}$, where $L = \min\left(k, \left\lfloor \frac{n-k}{m} \right\rfloor\right)$

4 **(i)** $a+b>c, \quad b+c>a, \quad c+a>b$

$$\left(\sqrt[3]{a}+\sqrt[3]{b}\right)^3 = a+b+3\left(a^{\frac{2}{3}}b^{\frac{1}{3}}+a^{\frac{1}{3}}b^{\frac{2}{3}}\right) > c+0$$

$\Rightarrow \quad \sqrt[3]{a}+\sqrt[3]{b} > \sqrt[3]{c}$. Similarly for the other two cases.

(ii) Let $p=\sqrt{a}, \quad q=\sqrt[3]{b}, \quad r=\sqrt[4]{c}$.

WLOG, suppose $p \geq q \geq r$.

Then consider: $|p-q|+|q-r| = p-q+q-r = |p-r|$.

Two of the lengths sum to the third. So these three lengths cannot form the sides of a triangle.

(iii) $\left(a^2+2bc\right)+\left(b^2+2ca\right)=(a-b)^2+2(ab+bc+ca)$

$\geq 0+2c(a+b)+2ab>2c^2+2ab>c^2+2ab$

Similarly for the other two cases. It is always possible to form such a triangle.

(iv) WLOG, let $a^2+b^2=c^2$.

We prove by contradiction. Suppose the triangle in **(i)** is right-angled, then

$a^{\frac{2}{3}}+b^{\frac{2}{3}}=c^{\frac{2}{3}}$ or $b^{\frac{2}{3}}+c^{\frac{2}{3}}=a^{\frac{2}{3}}$ or $c^{\frac{2}{3}}+a^{\frac{2}{3}}=b^{\frac{2}{3}}$.

$$a^{\frac{2}{3}}+b^{\frac{2}{3}}=c^{\frac{2}{3}} \Rightarrow a^2+b^2+3\left(a^{\frac{4}{3}}b^{\frac{2}{3}}+a^{\frac{2}{3}}b^{\frac{4}{3}}\right)=c^2$$

$\Rightarrow a^{\frac{4}{3}}b^{\frac{2}{3}}+a^{\frac{2}{3}}b^{\frac{4}{3}}=0$, a contradiction, since $a,b>0$.

$$b^{\frac{2}{3}}+c^{\frac{2}{3}}=a^{\frac{2}{3}} \Rightarrow b^2+c^2+3\left(b^{\frac{4}{3}}c^{\frac{2}{3}}+b^{\frac{2}{3}}c^{\frac{4}{3}}\right)=a^2$$

$$\Rightarrow b^2+\left(a^2+b^2\right)+3\left(b^{\frac{4}{3}}c^{\frac{2}{3}}+b^{\frac{2}{3}}c^{\frac{4}{3}}\right)=a^2$$

$\Rightarrow 2b^2+3\left(b^{\frac{4}{3}}c^{\frac{2}{3}}+b^{\frac{2}{3}}c^{\frac{4}{3}}\right)=0,$ which also leads to a contradiction.

Similarly for the last case. Therefore, it is not possible for triangle in **(i)** to be right-angled.

5 (i) $\left|f(x)-1\right| \leq \left|c_1\right|\left|x\right|+\left|c_2\right|\left|x\right|^2+\ldots+\left|x\right|^n$

(ii) $\left|f(r)-1\right| \leq \dfrac{k|r|}{1-|r|}$

$k|r| \geq 1-|r| \quad (\text{since } f(r)=0)$

$|r| \geq \dfrac{1}{k+1}$

Coupled with the fact that $|r| \leq 1 \leq k+1$, we obtain the required inequality.

(iii) If r is a root, $1+c_1r+c_2r^2+\ldots+c_{n-1}r^{n-1}+r^n=0$

$$\Rightarrow r^n\left(\frac{1}{r^n}+\frac{c_1}{r^{n-1}}+\frac{c_2}{r^{n-2}}+\ldots+\frac{c_{n-1}}{r}+1\right)=0$$

$$1+c_{n-1}\left(\frac{1}{r}\right)+c_{n-2}\left(\frac{1}{r}\right)^2+\ldots+c_1\left(\frac{1}{r}\right)^{n-1}+\left(\frac{1}{r}\right)^n=0$$

Since $r>1,$ then $\left|\dfrac{1}{r}\right|<1$. Also, since the coefficients still satisfy $\left|c_r\right| \leq k$, the inequality in part **(ii)** still holds.

When $r=1,$ it is clear that $\dfrac{1}{k+1} \leq r \leq k+1$ since k is positive.

(iv) $x^5-\frac{2020}{2021+2n}x^4-\frac{2019}{2021+2n}x^3-\frac{2018}{2021+2n}x^2-\frac{2017}{2021+2n}x+1=0$

Notice that all coefficients ≤ 1. Set k=1.

Then all possible integer roots in the interval $\left[\frac{1}{2}, 2\right]$ are $\pm 2, \pm 1$.

$-2,\ 2$ are not possible, since numerator on LHS will be odd.

When $x=-1$, $\text{LHS}=-2\neq\text{RHS}$ for all n. So -1 is not a root of the equation.

When $x=1$, $4n-4032=0\Rightarrow n=1008$.

The only possible value of n is 1008, and the corresponding root is 1.

6 Let $a=2s+1$ where $s\in\mathbb{Z}^+\cup\{0\}$

Let P_n be the statement " $a^{2^n}\equiv 1\left(\text{mod } 2^{n+2}\right)$ " for $n\in\mathbb{Z}^+$.

For n = 1.

For $s=0$, $LHS=1^{2^1}=1\equiv 1\left(\text{mod}\, 2^3\right)$

For $s\geq 1$,

$LHS=(2s+1)^2=4s^2+4s+1=4s(s+1)+1$

Since there is a factor 2 in s(s+1) ,

therefore $LHS=8q+1\equiv 1\left(\text{mod}\, 2^3\right)$ where $q\in\mathbb{Z}$

$\therefore P_1$ is true.

Assume that P_k is true for some $k\in\mathbb{Z}^+$

ie $a^{2^k}\equiv 1\left(\text{mod } 2^{k+2}\right)\Rightarrow a^{2^k}-1=2^{k+2}t$ where $t\in\mathbb{Z}$

For $n=k+1$,

$$\begin{aligned}LHS&=a^{2^{k+1}}\\&=a^{2^k.2}\\&=\left(a^{2^k}\right)^2\\&=\left(2^{k+2}t+1\right)^2\\&=2^{2k+4}t^2+2^{k+3}t+1\\&=2^{k+3}.2^{k+1}t^2+2^{k+3}t+1\\&\equiv 1\left(\text{mod}\, 2^{k+3}\right)\end{aligned}$$

$\therefore P_{k+1}$ is true

Hence by Mathematical Induction, P_n is true for $n\in\mathbb{Z}^+$.

(b) $a^{2k+1}+1=(a+1)\left(a^{2k}-a^{2k-1}+a^{2k-2}-.....+1\right)$

$\therefore a+1$ divides $a^{2k+1}+1$.

Suppose n is not a power of 2, ie $n = 2^r.(2k+1)$ where k is positive integer and r is a non-negative integer.

Let $a = 2^{2^r}$ in the previous result .

$\left(2^{2^r}\right)^{2k+1} + 1 = \left(2^{2^r} + 1\right)\left(a^{2k} - a^{2k-1} + a^{2k-2} - ..... + 1\right)$

$\left(2^{2^r(2k+1)}\right) + 1 = \left(2^{2^r} + 1\right)\left(a^{2k} - a^{2k-1} + a^{2k-2} - ..... + 1\right)$

$2^n + 1 = \left(2^{2^r} + 1\right)\left(a^{2k} - a^{2k-1} + a^{2k-2} - ..... + 1\right)$

Therefore $\left(2^{2^r} + 1\right) | \left(2^n + 1\right)$ and since k is a positive integer , $\left(a^{2k} - a^{2k-1} + a^{2k-2} - ..... + 1\right) \neq 1$ or -1. So $2^n + 1$ is not prime.

So proof by contrapositive , if $2^n + 1$ is a prime then n is a power of 2.

7 Let $(n, n+r) = d$ and $(n, r) = e$

Then $d | n$ and $d | n+r \Rightarrow d | n+r-n \Rightarrow d | r \therefore d \leq e$

$e | n$ and $e | r \Rightarrow e | n+r \Rightarrow e \leq d$

Therefore $d = e$ ie $(n, n+r) = (n, r)$

If n is odd and r is odd,

$\Rightarrow$ 2n is even and n + r is even

$\Rightarrow 2 | (2n, n+r)$

Since n is odd, there is no even factor in n

ie $(2n, n+r) = 2(n, n+r)$ since we need only bother about odd factors in n + r

$= 2(n, r)$

If n is odd and r is even,

Then 2n is even and n + r is odd

$\Rightarrow 2 \nmid (2n, n+r)$

$\Rightarrow (2n, n+r) = (n, n+r) = (n, r)$

since we need only bother with odd factors in n and n + r

Let $k = 1, 2, ...2n,$ and $r = 1, 2, ...n.$

(i) If n is odd,

For even k, $(2n, k) \neq 1$ ie 2n and k are not coprime

So need only consider odd k.

For odd r, $(2n, r) = (n, r)$

For even r, $(2n, n+r) = (n, r)$ from above result.

So there is a one-one matching for $(2n, k) = (n, r) = 1$

ie $\phi(2n) = \phi(n)$.

(ii) If n is even,
For even k, $(2n,k)\neq 1$ ie 2n and k are not coprime
So need only consider odd k.

For odd r, $(2n,r)=(n,r)$
For odd r, $(2n,n+r)=(n,r)$ from above result.
So for every odd r value such that $(n,r)=1$, then $(2n,r)=1$ and $(2n,n+r)=1$
ie $\phi(2n)=2\phi(n)$.

(iii) $\phi(2^n)=\phi(2.2^{n-1})=2\phi(2^{n-1})=.....=2^{n-1}\phi\left(2^{n-(n-1)}\right)=2^{n-1}\phi(2)=2^{n-1}$
Since $\phi(2)=1$ since 1 is the only number coprime with 2.

8 **(i)** $1-t^2+t^4-t^6+\ldots+t^{4n}-\dfrac{t^{4n+2}}{1+t^2}=\dfrac{1\left(1-(-t^2)^{2n+1}\right)}{1-(-t^2)}-\dfrac{t^{4n+2}}{1+t^2}$

$$=\frac{1+t^{4n+2}-t^{4n+2}}{1+t^2}=\frac{1}{1+t^2} \text{ (shown)}$$

(ii) Since $1+t^2\geq 1$, $\dfrac{t^{4n+2}}{1+t^2}\leq t^{4n+2}$

$$\int_0^x \frac{t^{4n+2}}{1+t^2}\,\mathrm{d}t\leq\int_0^x t^{4n+2}\,\mathrm{d}t$$

$$0\leq P_n(x)\leq\frac{x^{4n+3}}{4n+3}$$

$$\therefore\quad 0\leq P_n(x)\leq\frac{1}{4n+3}\quad(\text{since } 0\leq x\leq 1)$$

As $n\to\infty$, $\dfrac{1}{4n+3}\to 0$. By Squeeze Theorem, $P_n(x)\to 0$.

Therefore, $\displaystyle\int_0^x\frac{1}{1+x^2}\,\mathrm{d}x=\int_0^x 1-t^2+t^4-t^6+\ldots\mathrm{d}t+0$

$$\Rightarrow\quad \tan^{-1}x=x-\frac{x^3}{3}+\frac{x^5}{5}-\frac{x^7}{7}+\ldots=\sum_{r=0}^{\infty}\frac{(-1)^n}{2n+1}x^{2n+1}$$

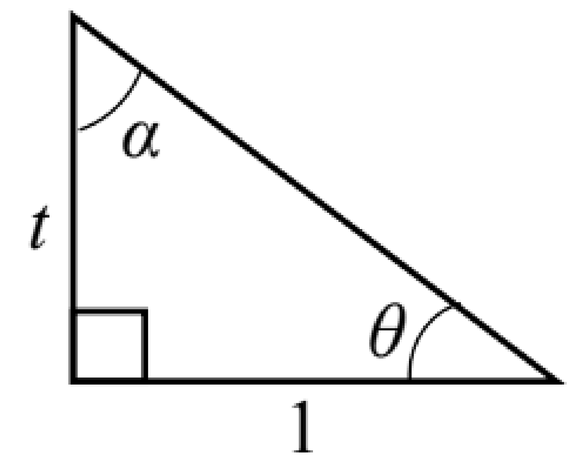
(iii) $\displaystyle\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|=\lim_{n\to\infty}\frac{(2n+1)|x|^{2n+3}}{(2n+3)|x|^{2n+1}}$

$$=|x|^2\lim_{n\to\infty}\frac{2n+1}{2n+3}=|x|^2$$

By ratio test, the series converges for $-1<x<1$ and diverges for $|x|>1$. Coupled with the fact that the series converges for $x=\pm 1$, we conclude that the expansion is valid for $-1\leq x\leq 1$.

(iv) Referring to the triangle, $\tan\theta = t$ and $\tan\alpha = \frac{1}{t}$.

Since $\theta + \alpha = \frac{\pi}{2}$, we obtain $\tan^{-1} t + \tan^{-1}\frac{1}{t} = \frac{\pi}{2}$.



$$\tan^{-1} 2021 = \frac{\pi}{2} - \tan^{-1}\frac{1}{2021}$$
$$\approx \frac{\pi}{2} - \left(\frac{1}{2021} - \frac{1}{2021^3(3)}\right) \approx 1.57030$$

(v) $\frac{\pi}{4} = \tan^{-1} 1$

$$= \tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3}$$
$$= \left(\frac{1}{2} - \frac{1}{3(2^3)} + \frac{1}{5(2^5)} - \ldots\right) + \left(\frac{1}{3} - \frac{1}{3(3^3)} + \frac{1}{5(3^5)} - \ldots\right)$$

Multiplying throughout by 4 yields the required result.

9(i) $B_2 = 2$ and $B_3 = 5$

(ii) Consider the set $\{1, 2, 3, \cdots, j, \cdots, n, n+1\}$. The object *j* can either be alone in a box or be in a box containing *k* other objects.
If *j* is alone, then we would need to distribute the remaining *n* objects into boxes. This gives B_n ways.
If *j* is in a box containing *k* other objects, then we need to choose *k* objects from the set of *n* objects to be together with object *j*. There are $\binom{n}{k}$ ways to do so. Thereafter, we need to distribute the remaining $(n-k)$ objects into boxes, which gives us B_{n-k} ways. By Multiplicative Principle, there is a total of $\binom{n}{k}B_{n-k}$ ways.
By Addition Principle,

$$B_{n+1} = \binom{n}{0}B_n + \binom{n}{1}B_{n-1} + \binom{n}{2}B_{n-2} + \ldots + \binom{n}{n-1}B_{n-(n-1)} + \binom{n}{n}B_{n-n}$$
$$= \binom{n}{n}B_n + \binom{n}{n-1}B_{n-1} + \binom{n}{n-2}B_{n-2} + \ldots + \binom{n}{1}B_1 + \binom{n}{0}B_0 \quad \because \binom{n}{r} = \binom{n}{n-r}$$

$$= \sum_{k=0}^{n} \binom{n}{k} B_k$$

(iii) $S_{n,n-1} = \binom{n}{2} = \frac{n(n-1)}{2}$

(iv) $B_n = \sum_{k=1}^{n} S_{n,k}$

(v) Given that *a* and *b* are both alone each in their own boxes, all we need to do is just to distribute the remaining *n* objects into boxes, which gives us B_n ways.

(vi) Given that *a* and *b* are both in the same box, we first leave *b* out and consider distributing the objects in the set $\{1,2,3,\cdots,n,a\}$ into boxes. This gives B_{n+1} ways. Now, all we need to do is to just put *b* into the box that *a* is in, giving us only 1 way to do so. Thus, the answer is B_{n+1}.

(vii) Now, *a* and *b* are in different boxes and that both *a* and *b* are not alone in their respective boxes. Let us now leave a and b out and start to distribute the objects in the set $\{1,2,3,\cdots,n\}$ into boxes. We know from (iii) that there number of ways is $B_n = S_{n,1} + S_{n,2} + S_{n,3} + ... + S_{n,n}$. Consider the case $S_{n,k}$ where we are distributing *n* objects in the set $\{1,2,3,\cdots,n\}$ into *k* boxes. Now, we want to distribute *a* and *b* into these *k* boxes which have already been filled up by items from $\{1,2,3,\cdots,n\}$. This gives $k\times(k-1)$ ways. Multiplying up, we get $k(k-1)S_{n,k}$. Then we sum up, we will get $\sum_{k=1}^{n} k(k-1)S_{n,k}$

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(viii) Take note that the cases in (v), (vi) and (vii) do not make up the number of ways to distribute objects from $\{1,2,3,\cdots,n,a,b\}$ into boxes. We are missing one last case where *a* and *b* are in different boxes and only one of them is alone in a box. We shall distribute *n* objects in the set $\{1,2,3,\cdots,n\}$ into boxes like what happened in (vii). First, we consider the case $S_{n,k}$ where we are distributing *n* objects in the set $\{1,2,3,\cdots,n\}$ into *k* boxes. Then, we will put *a* alone into a new box and try to put *b* into one of the *k* boxes. This gives us $kS_{n,k}$ ways and upon summing up, we get $\sum_{k=1}^{n} kS_{n,k}$. Upon permutation of *a* and *b*, we get $2\sum_{k=1}^{n} kS_{n,k}$.

So, $B_{n+2} = B_{n+1} + B_n + \sum_{k=1}^{n} k(k-1)S_{n,k} + 2\sum_{k=1}^{n} kS_{n,k}$.

It is obvious that $2\sum_{k=1}^{n} kS_{n,k}$ is even. In addition, $\sum_{k=1}^{n} k(k-1)S_{n,k}$ is also even because the product of 2 consecutive numbers must be even.

So, $2\sum_{k=1}^{n} kS_{n,k} + \sum_{k=1}^{n} k(k-1)S_{n,k} \equiv 0 \pmod 2$.

This implies that $B_{n+2} = B_{n+1} + B_n + \sum_{k=1}^{n} k(k-1)S_{n,k} + 2\sum_{k=1}^{n} kS_{n,k}$

$$\equiv B_{n+1} + B_n \pmod{2}$$