TEMASEK JUNIOR COLLEGE, SINGAPORE

Preliminary Examination 2014
Higher 2

MATHEMATICS

Paper 1

9740/01

1 September 2014

Additional Materials: Answer Paper
List of Formulae (MF 15)

3 hours

READ THESE INSTRUCTIONS FIRST

Write your *Civics Group* and *Name* on all the work that you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use a graphic calculator.
Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **6** printed pages.

TEMASEK JUNIOR COLLEGE, SINGAPORE
PASSION PURPOSE DRIVE

1 **(i)** Find the derivative of $\sqrt{4-x^2}$ with respect to x. [1]

(ii) Given the differential equation $\sqrt{4-x^2}\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}=1$, find y in terms of x. [4]

2 **(a)** The point A has coordinates $(3, a, b)$ where $a, b \in \mathbb{R}$. Given that A lies on the xy-plane and the magnitude of the position vector of A is 5, find the values of a and b. [3]

(b) The real numbers c and d are such that the vectors $\mathbf{m}=\mathbf{i}+d\mathbf{j}+c\mathbf{k}$ and $\mathbf{n}=c\mathbf{i}+d\mathbf{j}+\mathbf{k}$ are perpendicular to each other. Show that $\left|\mathbf{m}\times\mathbf{n}\right|=\left(c-1\right)^2$. [3]

3 Given $\mathrm{f}\left(z\right)=pz^2+qz+r$ where p, q and r are complex numbers such that $\mathrm{f}\left(1\right)=2\mathrm{i}$. The equation $\mathrm{f}\left(z\right)=0$ has roots $1-\mathrm{i}$ and $1-2\mathrm{i}$. Find p, q and r. [6]

4 Without the use of a graphic calculator, solve the inequality $2x+5\le\dfrac{10}{2-x}$. [3]

Hence find the solution to the inequality $2\cos\theta+5\le\dfrac{10}{2-\cos\theta}$, where $0\le\theta\le2\pi$. [3]

5 A souvenir company received an order to produce a souvenir that must satisfy all of the following conditions:

(1) The souvenir is a solid cuboid with a square base.
(2) The souvenir is made using $1\mathrm{m}^3$ of superior clay.
(3) The external surface of the souvenir must be coated with a special-mixed glow paint.

Find the dimensions of the souvenir, in m, such that the amount of special paint needed is the minimum. [7]

[Turn over

6 A contagious disease was found to infect a village with a population of 10000 people. Let P, in thousands, be the number of infected people t days after the start of the outbreak. The disease spread at a rate that was proportional to the product of the number of infected people and the number of non-infected people. It was found that when P reached half the initial population of the village, the disease was spreading at a rate of 1000 people per day.

Show that the spread of the disease can be modelled by the differential equation

$$\frac{\mathrm{d}P}{\mathrm{d}t} = \frac{25-(P-5)^2}{25}.$$ [2]

Given that 100 people were infected by the disease initially, find P in terms of t. [3]

Explain what would happen to the village population in the long term. [2]

7 A convergent geometric sequence of positive terms, G has first term a and common ratio r.

Write down, in terms of a and r, an expression for the nth odd-numbered term of G. [1]

If the sum of first n odd-numbered terms of G is equal to the sum of all terms of G after the nth odd-numbered term, show that $2r^{2n} + r^{2n-1} - 1 = 0$.

(i) Hence find the value of r when $n = 5$. [3]

(ii) In another sequence H, each term is the reciprocal of the corresponding term of G. If the nth term of G and H is denoted by u_n and v_n respectively, show that a new sequence whose nth term is $\ln\left(\frac{u_n}{v_n}\right)$, is an arithmetic progression. [4]

[Turn over

8 The functions f and g are defined by

$$\mathrm{f} : x \mapsto x^2 - 8x + 13, \qquad x \in \mathbb{R}, x \leq 4,$$

$$\mathrm{g} : x \mapsto a - \mathrm{e}^{-x}, \qquad x \in \mathbb{R}.$$

(i) Show that f^{-1} exists and express f^{-1} in a similar form, stating the domain clearly. [3]

(ii) Determine the largest integer value of a such that fg exists. [2]

(iii) For the largest value of a obtained in **(ii)**, find fg(x) and state the domain and the range of fg. [4]

9 Given that $\ln y = \mathrm{e}^x$, show that $\dfrac{\mathrm{d}^2 y}{\mathrm{d}x^2} = \dfrac{\mathrm{d}y}{\mathrm{d}x}\left(\mathrm{e}^x + 1\right)$. [2]

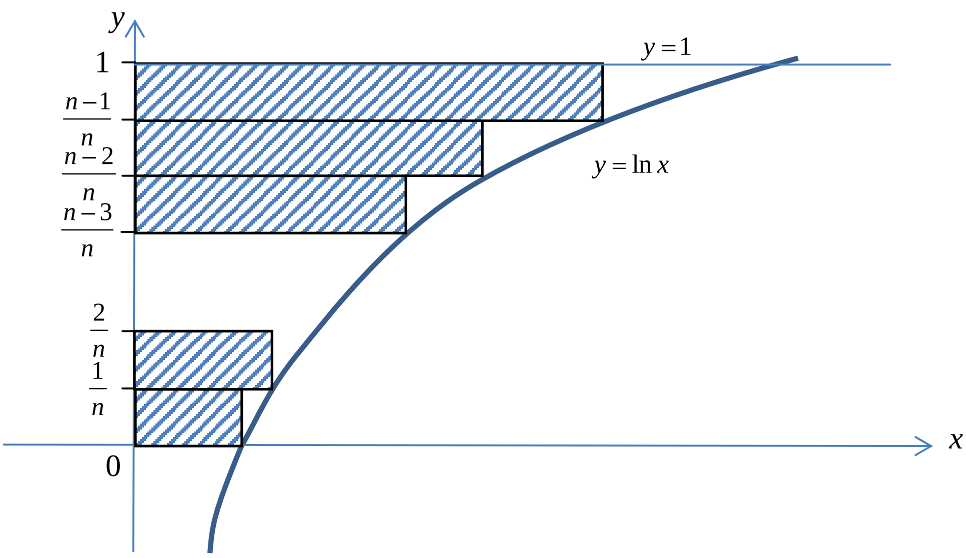
(i) Find the Maclaurin's series for $y = \mathrm{e}^{\mathrm{e}^x}$, up to and including the term in x^3. [4]

(ii) Find the first three non-zero terms of the Maclaurin series for $y = \mathrm{e}^{x+\mathrm{e}^x}$. [2]

Hence find in terms of e, the approximate area bounded by the curve $y = \mathrm{e}^{x+\mathrm{e}^x}$, the x-axis, the y-axis and the line $x = 0.5$. [2]

[Turn over

10



The region R is bounded by the x-axis, the y-axis, the line $y = 1$ and the curve $y = \ln x$ where $x \in \mathbb{R}, x > 0$.

The area of R may be approximated by the total area, A, of n rectangles each of height $\frac{1}{n}$, as shown in the above diagram.

Show that $A = \frac{1}{n}\left(\frac{1-\mathrm{e}}{1-\mathrm{e}^{\frac{1}{n}}}\right)$. [4]

Another finite region S is bounded by the x-axis, $x = \mathrm{e}$ and the curve $y = \ln x$ where $x \in \mathbb{R}, x > 0$.

Explain how A can be used to approximate the area of region S and state, with a reason, whether it is an underestimation or overestimation. [3]

Find the exact volume of the solid formed when region S is rotated completely about the y-axis. [3]

[Turn over

11(i) Prove by the method of induction that

$$\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1). \qquad [4]$$

(ii) It is given that $\mathrm{f}(r) = r^4$. Show that

$$\mathrm{f}(r) - \mathrm{f}(r-1) = ar^3 + br^2 + ar - 1,$$

for constants a and b to be determined. [2]

Hence find a formula for $\sum_{r=1}^{n} r^3$, leaving your answer in a fully factorised form. [6]

12 Two planes p_1 and p_2 have equations $ax - 3y - z = b$ and $4x + y + bz = 2a$ respectively. They intersect at the line l which contains the point $A(1, 0, -1)$.

(i) Find the values of a and b. [2]

(ii) Without the use of a graphic calculator, find a vector equation of the line l. [2]

Given that the point $N(-4, -6, 12)$ is the foot of perpendicular from point $B(1, c, d)$ to the line l, show that $6c - 13d = -217$. [3]

Another plane p_3 is parallel to the plane p_2 and contains B. Given that the distance between planes p_3 and p_2 is $\frac{5}{\sqrt{21}}$. Find the values of c and d. [5]

Hence write down two possible equations of plane p_3. [2]