ST. ANDREW'S JUNIOR COLLEGE
JC 1 PHYSICS 2020
TOPIC 1: MEASUREMENT

Learning Outcomes: Candidates should be able to:

a.	recall the following base quantities and their SI units: mass (kg), length (m), time (s), current (A), temperature (K), amount of substance (mol).
b.	express derived units as products or quotients of the base units and use the named units listed in 'Summary of Key Quantities, Symbols and Units' as appropriate.
c.	use SI base units to check the homogeneity of physical equations.
d.	show an understanding of and use the conventions for labelling graph axes and table columns as set out in the ASE publication *Signs, Symbols and Systematics: The ASE Companion to 16–19 Science, 2000.*
e.	use the following prefixes and their symbols to indicate decimal sub-multiples or multiples of both base and derived units: pico (p), nano (n), micro (μ), milli (m), centi (c), deci (d), kilo (k), mega (M), giga (G), tera (T).
f.	make reasonable estimates of physical quantities included within the syllabus.
g.	distinguish between scalar and vector quantities, and give examples of each.
h.	add and subtract coplanar vectors.
i.	represent a vector as two perpendicular components.
j.	show an understanding of the distinction between systematic errors (including zero error) and random errors.
k.	show an understanding of the distinction between precision and accuracy.
l.	assess the uncertainty in a derived quantity by addition of actual, fractional, percentage uncertainties or by numerical substitution (a rigorous statistical treatment is not required).

1. SI Units (Le Système International d'Unités)

A quantity is something that can be measured. Time is an example of a quantity. When physicists quote time intervals, they would use a **numerical magnitude** and a **unit**. An example of a measured time interval would be 9.58 seconds where "9.58" represents a number (numerical magnitude) and "seconds" represents a unit in which time can be measured in. For a quantity to be meaningful, the unit is as important as the numerical magnitude: 9 seconds is certainly different from 9 hours.

1.1 Base Quantities and their Units

The SI system of units consists of **7 base units** in which all other units can be derived from using suitable combination of these base units.

Base Quantities	SI Base Units	
	Name	Symbol
Length	metre	m
Mass	kilogram	kg
Time	second	s
Amount of substance	mole	mol
Temperature	kelvin	K
Current	ampere	A
*Luminous intensity	candela	cd

Note: *Luminous intensity is not required in the syllabus.

1.2 Derived Quantities and their Units

A derived quantity is linked to other quantities via a valid equation.

A **derived unit** can be expressed in terms of **products or quotients** of base units.

Examples of derived quantities (with their units)

Derived Quantities	Equation	Derived Units
Area (A)	$A = l^2$	m^2
Volume (V)	$V = l^3$	m^3
Density (ρ)	$\rho = m / V$	$kg / m^3 = kg\ m^{-3}$
Velocity (v)	$v = l / t$	$m / s = m\ s^{-1}$
Acceleration (a)	$a = \Delta v / t$	$(m\ s^{-1}) / s = m\ s^{-2}$
Momentum (p)	$p = m \times v$	$(kg)\ (m\ s^{-1})$

The following table shows some derived units which are named after famous scientists.

Derived Quantities	Equation	Derived Unit		Derived Units in terms of base units
		Common Name	Symbol	
Force	$\Delta p / t$	Newton	N	$kg\ m\ s^{-2}$
Pressure	F / A	Pascal	Pa	$kg\ m\ s^{-2} / m^2 = kg\ m^{-1}\ s^{-2}$
Energy	$F \times d$	Joule	J	$kg\ m\ s^{-2}\ m = kg\ m^2\ s^{-2}$
Power	E / t	Watt	W	$kg\ m^2\ s^{-2} / s = kg\ m^2\ s^{-3}$
Frequency	$1 / t$	Hertz	Hz	$1 / s = s^{-1}$
Charge	$I \times t$	Coulomb	C	A s
Potential Difference	E / Q	Volt	V	$kg\ m^2\ s^{-3}\ A^{-1}$
Resistance	V / I	Ohm	Ω	$kg\ m^2\ s^{-3}\ A^{-2}$

(See APPENDIX for the various symbols for quantities.)

Worked Example 1

Express the newton in terms of SI base units.
Hint: Think of an equation with Force as the subject. Write down the equation.

Solution:
Since the Newton is the unit for force, think of an equation with force as the subject.
The most obvious one is: $F = m\,a$

Let [ ] denote 'unit of'
Since, [F] = N, [m] = kg, [a]= m s^{-2}
Thus, N = kg m s^{-2}

Common Error:
A common error in the solution is simply equating units to quantities.
Eg. N = $m\,a$
= kg x m s^{-2}

This will be penalised because while the Newton is the unit for force; Newton is NOT force.

Worked Example 2 (modified N02 P1 Q1)

The drag force F experienced by a steel sphere of radius r dropping at speed v through a liquid is given by

$$F = arv, \quad \text{where } a \text{ is a constant.}$$

What is a suitable SI unit for a?

Solution:

Make a the subject first, $a = \dfrac{F}{r\,v}$

Let [ ] denote 'unit of'

$\therefore \quad [a] = \dfrac{\text{N}}{\text{m m s}^{-1}}$

$= \text{N m}^{-2}\text{ s}$

Another possible answer will be

$$[a] = \frac{kg\,m\,s^{-2}}{m^2\,s^{-1}}$$

$$= \text{kg m}^{-1}\text{ s}^{-1}$$

{There are 2 possible answers here because question did not insist on SI base units.}

1.3 Concept of Homogeneity

Consider the equation $v = u + at$ from your O-level course. Since the equation is physically correct, it **must** be homogeneous or **dimensionally consistent**, then the three terms v, u and at must have the same SI base units:

$[v] = [u] = [at]$

$[v] = \text{m s}^{-1}$

$[u] = \text{m s}^{-1}$

$[at] = \text{m s}^{-2}\ \text{s} = \text{m s}^{-1}$

From this, we can conclude that this equation is homogeneous.

Note: Apart from checking if an equation is homogeneous, knowing that the equation is homogeneous lets you work out the units of an unknown term. For example, in the topic of Gravitational Fields (H2 Physics), you will come across the equation $F = GMm/r^2$, where G is a constant you have not come across yet, M and m are masses, F is a force and r is a distance. You can now work out the units of G, which is the gravitational constant (6.67×10^{-11} N m^2 kg^{-2}).

1.3.1 Limitations of Homogeneity

It is a common error to conclude that a homogeneous equation is physically correct. For example, the following equations are homogeneous but physically wrong.

Error	Example
1. Incorrect unit-less coefficient	$v = \mathbf{5}u + at$
2. Extra term	$v = u + at \mathbf{+} \sqrt{as}$
3. Incorrect sign	$v = \mathbf{-}u + at$

Worked Example 3

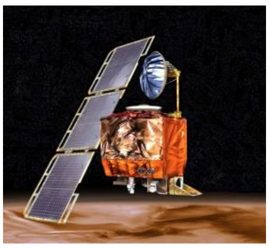
The motion of an object under constant acceleration a can be described by the equation $v^2 = u^2 + 2as$, where v is the final velocity and u is the initial velocity. Find the unit of s.

Solution:

If the equation is homogeneous, then $[v^2] = [u^2] = [2as] = (\text{m s}^{-1})^2 = \text{m}^2\ \text{s}^{-2}$.
$[s] = [as] / [a] = (\text{m}^2\ \text{s}^2) / (\text{m s}^{-2}) = \text{m}$

Interesting History: Nov. 10, 1999: Metric Math Mistake Muffed Mars Meteorology Mission.

A disaster investigation board reports that NASA's Mars Climate Orbiter burned up in the Martian atmosphere because engineers failed to convert units from British Imperial to metric.



Scan this QR code or go to
http://www.wired.com/thisdayintech/2010/11/1110mars-climate-observer-report/

Worked Example 4

The flow of fluid can be described by the Bernoulli equation $P + \frac{1}{2}\rho v^2 + \rho g h = \text{constant}$,

where P = pressure, ρ = density, v = velocity, g = gravitational acceleration, h = elevation.
Determine if the equation is a correct one.

Solution:

If the equation is homogeneous, $[P] = \left[\frac{1}{2}\rho v^2\right] = [\rho g h]$

Term 1: p = force/ area
$[P]$ = kg m s^{-2} / m^2 = kg m^{-1} s^{-2}

Term 2: $\left[\frac{1}{2}\rho v^2\right]$ = $[\rho v^2]$ = kg m^{-3} (m $s^{-1})^2$ = kg m^{-1} s^{-2}

Term 3: $[\rho g h]$ = (kg m^{-3})(m s^{-2})(m) = kg m^{-1} s^{-2}

Since all 3 terms have the same SI base units, we can conclude that the equation is homogeneous (but we cannot comment on the correctness. Refer to 1.3.1)

Remember: Presentation is very important. Many students will write h = m which is not correct. h is a quantity and m is a unit for the quantity. They are not the same.

Prefixes for SI Units

A unit can be made larger or smaller by placing a prefix before it.

Multiplying Factor	Prefix	Symbol	Example
10^{-12}	pico	p	**pico**metre (pm) – unit for wavelength of gamma-rays
10^{-9}	nano	n	**nano**metre (nm) – unit for wavelength of light
10^{-6}	micro	μ	**micro**metre (μm) – unit for size of bacteria and cells
10^{-3}	milli	m	**milli**gram (mg) – unit for sugar content in a blood sample
10^{-2}	centi	c	**centi**metre (cm) – unit for measuring length and width of a piece of paper
10^{-1}	deci	d	cubic **deci**meter (dm^3) – unit for bulk volume measurements in laboratory
10^{3}	kilo	k	**kilo**metre (km) – unit for distances between geographical locations
10^{6}	mega	M	**mega**ohms (MΩ) – unit for resistance of carbon resistor
10^{9}	giga	G	**giga**byte (GB) – unit for computer hard-disk space
10^{12}	tera	T	**tera**hertz (THz) – unit for frequencies of microwave communications

{Note that only the last three are in CAPITAL fonts}

Worked Example 5 (J05 P1 Q1)

Decimal sub-multiples and multiples of units are indicated using a prefix to the unit. For example, the prefix milli (m) represents 10^{-3}.
Which of the following gives the sub-multiples or multiples represented by pico (p) and giga (G)?

	pico (p)	**giga (G)**
A	10^{-9}	10^{9}
B	10^{-9}	10^{12}
C	10^{-12}	10^{9}
D	10^{-12}	10^{12}

1.4 Making Reasonable Estimates of Quantities

As students of physics, we ought to be aware of the magnitudes of physical quantities in our daily lives.

Worked Example 6
Give reasonable estimates for the following:

Physical Quantity	Reasonable Estimate
Mass of 3 cans (330 ml) of Pepsi	1 kg
Mass of a medium-sized car	1000 kg
Length of a football field	100 m
Reaction time of a young person	0.2 s

- When making an estimate, it is only reasonable to give the figure to **1 significant figure** since an estimate is not precise at all.
- Often, when making an estimate, we need to express a more complicated quantity in terms of other simpler quantities using a formula first. (See Example 7)
- Occasionally, students are asked to estimate the area under a graph. The usual method of counting squares within the enclosed area is used.

Worked Example 7
Estimate the average running speed of a typical 17-year-old's 2.4-km run.

Solution:

Average Speed = $\dfrac{\text{distance}}{\text{time}}$

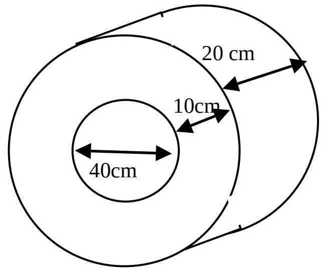
$= \dfrac{2400}{12.5 \times 60}$ assuming the person completes the run in 12.5 min

$= 3.2 \text{ m s}^{-1}$

$= 3 \text{ m s}^{-1}$ (to 1 sig. fig.)

Worked Example 8 (N08 P1 Q2)

Which estimate is realistic?

A *The kinetic energy of a bus travelling on an expressway is 30 000 J.* Speed Limit of bus = 60 km h^{-1} = 16.7m s^{-1} Estimated mass of a Double decker SBS bus = 10×10^3 kg KE = ½ $m v^2$ = 2.02 MJ <unrealistic> OR Alternatively, given that the speed of bus is 16.7 m s^{-1}, KE = 30 000 J = ½ m $(16.7)^2$ m = 215 kg (too small a value for a bus, even when it is empty) <unrealistic>	**B** *The power of a domestic light is 300 W.* A single fluorescent light bulb in the house usually runs at about 8 W to 40 W. Even an incandescent (filament) bulb (almost non-existent nowadays) runs at about 150 W. <unrealistic> (Do check the power rating on the cover of light bulb to find out the power rating of your bulbs.)
C *The temperature of a hot oven is 300 K.* 300 K = 27 ˚C. Not very hot. <unrealistic>	**D** *The volume of air in a car tyre is 0.03 m^3.* Making rough estimates[1], Diameter of inner rim = 40 cm Width of tyre = 20 cm Width of the side of the tyre = 10 cm  Volume of air $= \pi\left[\left(\frac{0.40+0.20}{2}\right)^2 - \left(\frac{0.40}{2}\right)^2\right] \times 0.20$ = 0.03 m^3 (1 sf) <realistic>

Worked Example 9 (N09 P2 Q1 (H1))

Give a **reasoned** estimate of the area of the island of Singapore, giving your answer in SI unit. [2]

Solution:
Singapore is about 20 km north-south and 50km east-west.
Area of such a rectangle is 1000 km^2
By estimating that Singapore is roughly 60% to 80% of the rectangle,
the area of the island of Singapore is about 700 km^2. (actual area is 715.8 km^2)

[1] For more information, you can refer to car websites like http://www.toyotasingapore.com.sg/cars/new_cars/vios/fullspecs.asp to find out more about the car specifications. For example, for the Toyota Vios tyres, 185/60 R15 shows that the width of the base of the wheel is 185 mm, the inner diameter is 15(=38.1 cm) and the width of the side of the wheel is 60% of 185 mm, which is 111mm. This gives a volume of 0.03 m^3.

1.5 Errors and Uncertainties

An error in a particular reading is a measure of the degree of uncertainty in that reading. As a result, the reading deviates from the true value. Often, the words "error" and "uncertainty" mean the same thing and are used interchangeably.

Sources of errors can come from man, machine, material, experimental technique or environment. Errors can be classified as either **random** or **systematic** errors.

1.5.1 Random Errors

What is a random error?

Random error is an error which causes measurements to be **sometimes larger** than the **true value and sometimes smaller** than the true value.

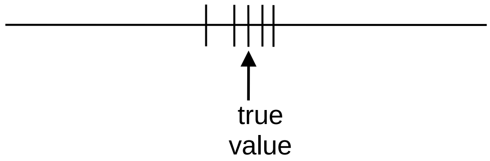


Fig. 1.5.1: Effect of random errors on readings (assuming no systematic errors)

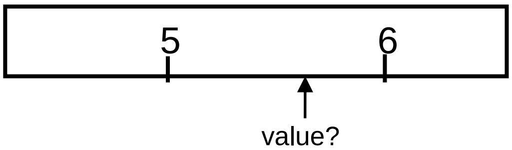
Random errors are **equally likely to be positive or negative** with respect to the true value. They can have **different magnitudes**.

Since random errors are equally likely to be positive or negative, we **can reduce random errors** by repeating the measurements and taking the average of the readings. (Contrast: Systematic errors cannot be reduced by averaging repeated readings.)

Common mistake:
Random error is a measurement that is sometimes larger than the true value and sometimes smaller than the true value.

Some possible causes of random errors are:

1. Readings taken from scale divisions which are too far apart.



2. Small fluctuations in temperature.

3. Parallax error — not placing the eye directly over the pointer.

4. Inability of an experimenter to repeat his action precisely e.g. human judgement in starting and stopping a stopwatch.

1.5.2 Systematic Errors

What is a systematic error?

Systematic error is an error which causes measurements to be **either, always** larger than the **true value, or always** smaller than the true value.

Fig. 1.5.2: Effect of a systematic error on readings (with the ever present random errors)

Since the same magnitude of error of the **same** sign is introduced in each reading, **systematic errors cannot be reduced by averaging repeated readings**.

Systematic errors are difficult to detect. Repeated readings cannot reveal the presence of a systematic error. However, systematic errors **can be** eliminated or greatly reduced if a faulty instrument is replaced, if the experimental technique/procedure is improved, or a different experimental approach is used. (Contrast: Random errors can never be eliminated.)

Some possible causes of systematic errors are:

1. **Zero error** of an instrument.

2. An incorrectly calibrated instrument e.g. a slow-running stop-watch.

3. Using the wrong constant e.g. taking g to be 10 m s^{-2} when it should be 9.81 m s^{-2}.

4. Wrong experimental procedure e.g. not subtracting background count rate when determining the count rate from a radioactive source.

1.5.3 The distinction between precision and accuracy

Precision*:* refers to the degree of agreement (scatter, spread) of **repeated** measurements of the same quantity. (This is regardless of whether or not the measurements are correct.)

It is a measure of the magnitude of the random errors present; **high precision means small random error**. Hence, readings which are precise have a small spread.

Don't mistake **precision of an instrument** with **precision of a set of measurements**. Precision of an instrument is related to the smallest division of the instrument.

Accuracy refers to the degree of agreement between the result of measurement(s) and the true value of the quantity.

It is a measure of the magnitude of the systematic error present; **high accuracy means small systematic error**.

If several readings of a quantity are taken, we take the average of the readings and compare it with the true value to see if it is accurate.
Note: This averaging does not improve the accuracy of the measurements.

Example 10 (N96 P1 Q3)
Four students each made a series of measurements of the acceleration of free fall *g*. The table shows the results obtained.

Which student obtained a set of results that could be best described as precise but not accurate?

Student	results, *g*/ m s^{-2}			
A	9.81	9.79	9.84	9.83
B	9.81	10.12	9.89	8.94
C	9.45	9.21	8.99	8.76
D	8.45	8.46	8.50	8.41

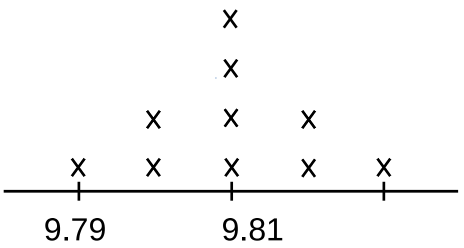
Solution:
For precise results, the spread (difference between largest and smallest values) should be small.
For not accurate results, the average should be far from the true value of 9.81.

For example for Student B,
Spread = largest value – smallest value = 10.12 – 8.94 = 1.18 (not precise)
Average = (9.81 + 10.12 + 9.89 + 8.94) / 4 = 9.69 (not accurate)

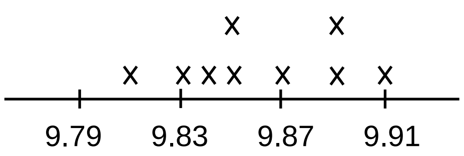
For precise but not accurate reading results, choose the option with the smallest spread but average value furthest from 9.81.

Ans: D

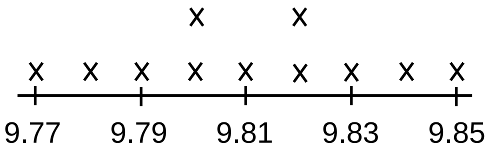
Worked Example 11
Consider a case where 4 students perform the same experiment to obtain the acceleration of free fall *g* and obtain 4 different sets of readings. Comment on their results with regards to accuracy and precision.



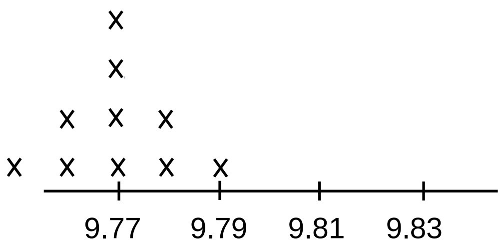
Precise and accurate
(g = 9.81)



NOT precise and NOT accurate
(g = 9.86)

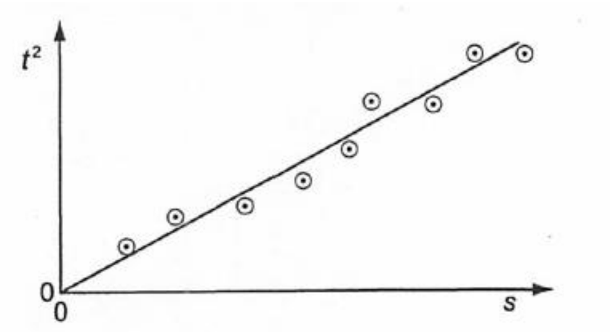


Accurate and NOT precise
(g = 9.81)



Precise and NOT accurate
(g = 9.77)

Worked Example 12 [2010 P1 Q1]
An object falls freely from rest and travels a distance *s* in time *t*. A graph of t^2 against *s* is plotted and used to determine the acceleration of free fall *g*.



The gradient of the graph is found to be 0.204 s^2m^{-1}.
Which statement about the value obtained for *g* is correct?
A It is accurate but not precise.
B It is both precise and accurate.
C It is neither precise nor accurate.
D It is precise but not accurate.

Solution:
Ans: A

Using the kinematics equation s = ut + ½ at^2, where initial velocity u = 0 (from rest) and acceleration a = g. The equation can be rearranged into t^2 = (2/g) s, where gradient = (2/g) = 0.204.

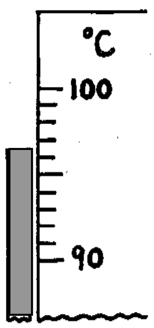
Using this value, g = 9.8039 m s^{-2}, which is close to the known value of g (9.81 m s^{-2}) and thus accurate. However, there is significant scattering of data points around the straight line hence the value is not precise.

Note:
The general equation for a straight line graph is y = mx + c, where m and c are constants and y and x are variables. It is important for students to know what variables to plot in order to obtain a straight line graph for a given equation.

1.5.4 Estimating Maximum Possible Error in a Single Measurement (important for practical)

1. The random error in any particular measurement is an <u>estimate</u> only and therefore, to be reasonable, can only be given to <u>one</u> significant figure.
2. The <u>maximum</u> possible random error in a scale measurement is normally estimated to <u>half</u> the smallest division.

Worked Example 13



Estimate the error in the thermometer reading below. Hence, write down the reading of the temperature to the correct degree of precision. (Assume for the purposes of this question that there is no systematic error.)

Smallest division = 1˚C
Error = half of smallest division = 0.5 ˚C
Hence, temperature = (96.5 ± 0.5) ˚C

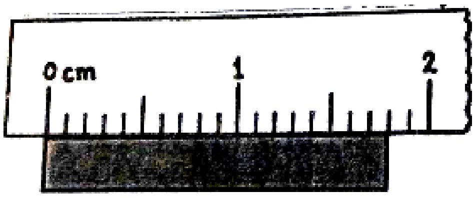
IMPORTANT!
The reading must always be recorded to the same degree of precision as the actual error.

Worked Example 14

In measuring length using a ruler, there is a maximum possible random error of **<u>0.5</u>** mm in judging the **<u>position of each end of the body against the scale</u>**. Therefore, the random error in measuring length is 2 x (½ smallest division) ie. one smallest division or **<u>1</u>** mm.

Write down the length of the shaded object to correct degree of precision using a ruler.

ℓ = (18 – 0) mm
$\Delta\ell$ = (0.5 + 0.5) mm
Therefore, l = (18 ± 1) mm = (1.8 ± 0.1) cm



1.5.5 Actual, Fractional and Percentage Errors

The length ℓ of a pencil is found to be $(\underbrace{14.5}_{\ell} \pm \underbrace{0.1}_{\Delta\ell})$ cm.

<u>actual</u> or absolute error	$\Delta\ell$ = 0.1 cm
<u>fractional</u> error	$\dfrac{\Delta\ell}{\ell} = \dfrac{0.1}{14.5} = 0.0069$
<u>percentage</u> error	$\dfrac{\Delta\ell}{\ell} \times 100\% = \dfrac{0.1}{14.5} \times 100\% = 0.69\%$

Hence, ℓ can be written as (14.5 ± 0.1) cm or 14.5 cm ± 0.69%.

IMPORTANT!

Actual error should always be written to **<u>one</u>** significant figure. However, fractional or percentage error may be recorded up to <u>2 significant figures.</u>

During measurement, the choice of instrument will affect the precision of the measurement. Below is an example of the measurement of the diameter of a sphere which is 1 cm.

Instrument	Precision	Reading	
Ruler	1 mm 0.1 cm 0.001 m	10 mm 1.0 cm 0.010 m	Lowest precision
Vernier callipers	0.1 mm 0.01 cm 0.0001 m	10.0 mm 1.00 cm 0.0100 m	
Micrometer screw gauge	0.01 mm 0.001 cm 0.00001 m	10.00 mm 1.000 cm 0.01000 m	Highest precision

Note: Please refer to the Appendix for explanation of using the Vernier callipers.

IMPORTANT!

The reading must always be recorded to the same degree of precision (same decimal place) as the actual error. Hence the number of '0's after the non-zero number cannot be dropped.

Worked Example 15

Are these measurement recordings correct?

Measurement	Is it correct?	Why?
v = (929.345 ± 0.012) Pa	No	The error should be recorded to 1 significant figure.
w = (929.345 ± 0.01) Pa	No	The reading is of higher precision than the error.
x = (929.35 ± 0.01) Pa		
y = (929 ± 10) Pa		
z = (9.29 ± 0.09) x 10^2 Pa		

Worked Example 16

A student measures the diameter of a cylindrical wooden pencil with a metre rule. How could he get the highest precision of the measurement?

A Use a micrometer with zero error and take one value of the diameter.

B Take the average value of several measurements of the diameter along different parts of the pencil using the metre rule.

C Take the average value of several measurements of the diameter along different parts of the pencil using vernier calipers without zero error.

D Take the average value of several measurements of the diameter along different parts of the pencil using vernier calipers with zero error.

Solution:

Ans: A

The precision of using a micrometer is higher than that of a metre rule or vernier callipers. Hence using a micrometer instead of a ruler can increase the precision of the measurement.

1.5.6 Number of Significant Figures for Calculated Quantities in general calculations without errors given for the measured quantities

Expression	Rule	Example
multiplication, division and trigonometric functions	number of **significant figures** in the answer should equal the least number of significant figures in any of the quantities involved	I = 1.7 A, P = 2.005 W, $V = P / I$ = 1.2 V (to 2 sig. fig.)
addition and subtraction	number of **decimal places** (NOT significant figures) in the answer should equal the least number of decimal places in any of the quantities involved	17.2911 J + 2.03 J = 19.32 J (to 2 d.p.)

Important:

Leaving the answer with 1 more significant figure (eg. V = 1.18 V) is tolerable for A-Level Exam. However leaving the answer with 1 less significant figure (eg. V = 1 V) or too many significant figures (eg. V = 1.17941 V) is not acceptable!

1.5.7 Error Computation

The result of an experiment is usually calculated using an expression containing different measured quantities.

To estimate the total random error in the results, first evaluate individually the error involved in each quantity. Then, proceed according to the following rules.

RULE 1: FOR ADDITION AND SUBTRACTION OF QUANTITIES

When two or more quantities are added or subtracted, **add** together their **actual** errors to obtain the **maximum possible** error in the result.

1. If Y= nA $\pm$ mB, ΔA and ΔB given, n & m are constants
   ΔY = nΔA + mΔB

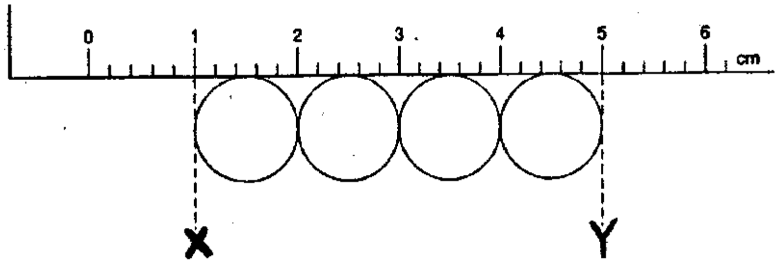
2. If Y= nA + mB, ΔY and ΔA given
   To find ΔB, we need to make B the subject first
   B = 1/m (Y – nA)
   ΔB = 1/m ΔY + n/m ΔA

Note that the error of A is not subtracted.

Worked Example 17

Quantity	Error
$S = a + b$	$\Delta S = \Delta a + \Delta b$
$Q = c - d$	$\Delta Q = \boldsymbol{\Delta c + \Delta d}$
$R = 3p - 2q$	$\Delta R = \boldsymbol{3\Delta p + 2\Delta q}$
$R = p + 2q$	$\Delta p = \boldsymbol{\Delta p + 2\Delta q}$

Example 18 (J94 P1 Q2 modified)
A student attempts to measure the diameter of a steel ball by using a metre rule to measure four similar balls in a row.



What is the diameter of a steel ball together with its associated uncertainty?

What if the measurement of the diameter of a steel ball is made by measuring just one ball?

RULE 2: FOR MULTIPLICATION AND DIVISION OF QUANTITIES

When two or more quantities are multiplied or divided, **add** together their **fractional** (or percentage) **errors** to obtain the **maximum** **fractional** (or percentage) **error** in the result. $Y = aX^nZ^m$, then $\frac{\Delta Y}{Y} = \lvert n \rvert \frac{\Delta X}{X} + \lvert m \rvert \frac{\Delta Z}{Z}$ (a, n & m are constants)   (Refer to appendix for the derivation)

Example 19

Quantity	Error
$V = 3AB$ (where A and B are variables)	$\frac{\Delta V}{V} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$
$D = \frac{M}{4\pi V}$	
Resistivity, $\rho = \frac{\pi R d^2}{4L}$	

IMPORTANT: Constants such as 3 and π, are ignored in the calculation of fractional (or percentage) error as they are considered error-free.

Example 20

Given that: $Q = \frac{3ab^2}{\sqrt{c}}$

The percentage errors of *a*, *b* and *c* are 1 %, 3 % and 2 % respectively. Calculate the percentage error in *Q*.

Solution:

$\frac{\Delta Q}{Q} = \frac{\Delta a}{a} + 2\frac{\Delta b}{b} + \frac{1}{2}\frac{\Delta c}{c}$ → $\frac{\Delta Q}{Q} \times 100 = \left(\frac{\Delta a}{a} \times 100\right) + 2\left(\frac{\Delta b}{b} \times 100\right) + \frac{1}{2}\left(\frac{\Delta c}{c} \times 100\right)$

= 1 + 2 (3) + ½ (2) = 8 %

Example 21

The acceleration of free fall, *g* is calculated using the formula $g = \frac{4\pi^2 L}{T^2}$, where *L* is the length of the pendulum, measured as (20.0 ± 0.1) cm and the value of g is found to be 9.81 ± 2%

Find the percentage uncertainty in the calculated value of T.

Solution:

Make T the subject first.

IMPORTANT:

It is a common error for students to state fractions and ratios in fractional form. The answer should always be in decimal form with the appropriate number of significant figures.

Worked Example 22 (N12 P1 Q2)
The equation connecting object distance *u*, image distance *v* and focal length *f* for a lens is

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}.$$

A student measures values of *u* and *v*, with their associated uncertainties.
These are *u* = 50 mm ± 3 mm, v = 200 mm ± 5 mm. He calculates the value of *f* as 40 mm.

What is the uncertainty in this value?

A ± 2.1 mm **B** ± 3.4 mm **C** ± 4.5 mm **D** ± 6.8 mm

Solution:

$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}.$ *Rearranging and using Rule 2,*

$$f = \frac{uv}{u+v}$$

$$\frac{\Delta f}{f} = \frac{\Delta u}{u} + \frac{\Delta v}{v} + \frac{\Delta(u+v)}{u+v}$$

$$\frac{\Delta f}{40} = \frac{3}{50} + \frac{5}{200} + \frac{3+5}{50+200}$$

$$\Delta f = 4.68 \text{ mm}$$

Answer must be **WRONG** as it does not match any of the options.

→ Rule 2 cannot be used as the terms (*u* + *v)* and *uv* are **mutually dependent**. (ie, variables are repeated)

RULE 3: FOR EQUATIONS INVOLVING TERMS WHICH ARE DEPENDENT

Find the maximum and minimum of the quantity. Calculate the difference and divide it by 2 ie

$$\text{max error} = \frac{\text{max - min}}{2}$$

Using Rule 3 for **Example 22** again,

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$f = \left(\frac{1}{u} + \frac{1}{v}\right)^{-1}$$

$$f_{\text{max}} = \left(\frac{1}{u_{\text{max}}} + \frac{1}{v_{\text{max}}}\right)^{-1}$$

$$= \left(\frac{1}{50+3} + \frac{1}{200+5}\right)^{-1} = 42.11$$

$$f_{\text{min}} = \left(\frac{1}{50-3} + \frac{1}{200-5}\right)^{-1} = 37.87$$

$$\Delta \text{f} = \frac{42.11 - 37.87}{2} = 2.1$$

Ans: A

Note: Rules 1 and 2 are more commonly used in A-Level Exam.
Rule 3 is usually used only when the terms in the formula are not independent
such as $A = \frac{BC}{C+D}$

Worked example with visible thinking

An experiment uses the following equation to calculate the resistivity ρ of a copper wire.

$$R = \frac{4\rho l}{\pi d^2}$$

where d is the diameter and l is the length of the copper wire and R is its resistance.
Numerical values from one experiment were

d = (2.1 ± 0.1) mm
l = (20.000 ± 0.001) m
R = (0.10 ± 0.01) Ω.

(i) Calculate the resistivity of the copper together with its uncertainty. Show the method you used to determine the uncertainty.

resistivity = ……………………… ± ……………………….. Ω m [3]

(ii) Explain what the new resistance of the wire would be if it is stretched uniformly to twice its original length. [2]

Thought process:

1. *Is the equation such that the required quantity subject of the formula? If it is not, then the equation needs to be re-arranged.*
2. *Is the equation one that is multiplication/division or addition/subtraction? If it is multiplication/division, then form an equation in terms of fractional error using rule 2.*
3. *The absolute uncertainty must be expressed in one significant figure.*
4. *The required quantity must be expressed in the same precision as the uncertainty.*

<u>Solution</u>

(i)

$$\rho = \frac{R\pi d^2}{4l} = \frac{(0.10)\pi(0.0021)^2}{4(20)}$$

$$= 1.732 \times 10^{-8}\ \Omega\ \text{m}$$

$$\frac{\Delta\rho}{\rho} = \frac{\Delta R}{R} + \frac{2\Delta d}{d} + \frac{\Delta l}{l}$$

$$= \frac{0.01}{0.10} + \frac{2(0.1)}{2.1} + \frac{0.001}{20.000} \qquad \textbf{[1]}$$

$$= 0.19529$$

$$\Delta\rho = (0.19529)(1.732 \times 10^{-8})$$

$$= 0.338 \times 10^{-8}\ \Omega\ \text{m} \qquad \textbf{[1]}$$

$$\rho \pm \Delta\rho = 1.7 \times 10^{-8} \pm 0.3 \times 10^{-8}\ \Omega\ \text{m} \qquad \textbf{[1]}$$

or $17 \times 10^{-9} \pm 3 \times 10^{-9}\ \Omega\ \text{m}$

[*It is important to make ρ the subject of formula first. Otherwise, the following error can be committed >>*

$$\frac{\Delta R}{R} = \frac{\Delta\rho}{\rho} + \frac{2\Delta d}{d} + \frac{\Delta l}{l},$$

leading to $$\frac{\Delta\rho}{\rho} = \frac{\Delta R}{R} - \frac{2\Delta d}{d} - \frac{\Delta l}{l}$$

*This is **<u>fundamentally wrong</u>** as errors can never be subtracted*.]

(ii) When the length is stretched to twice its original length, its cross-sectional area is now halved. **[1]**

[*Do note that it is the area that is halved, not diameter. When the wire is stretched the volume is identical, ie V = (Area)x(Length)*]

As resistance is directly proportional to its length and inversely proportional to is cross-sectional area, the resistance is now
four times its original value. **[1]**

[*The formula for resistance is* $R = \frac{\rho l}{A}$, *in all of such questions asking about how one variable changes when another variable changes, one must look at all the other variables in the equation. In this case, many students ignored the fact that the cross-sectional area would also change when stretched.*

1.6 Scalar and Vector Quantities in Physics

There are two types of quantities in physics.
A **scalar** quantity has a magnitude only. It is completely described by a certain number and a unit.

Examples: distance, speed, mass, time, temperature, work done, kinetic energy, pressure, power, electric charge etc.

A **vector** quantity has both magnitude and direction. It can be described by an arrow whose length represents the magnitude of the vector and the arrow-head represents the direction of the vector.

Examples: displacement, velocity, moments (or torque), momentum, force, electric field etc.

Common Mistake:
Students tend to associate energy and pressure with vectors because of the vector components involved. For example, Work Done = Force x Displacement in the direction of Force. However, such considerations have no relevance on whether the quantity is a vector or scalar.

1.6.1 Meaning of negative sign in a quantity

For scalars with negative values, it simply means that the quantity has a value that is less than zero. For example, a temperature of -5 ˚C, an electric potential of −3 V and G.P.E. is -5 J.

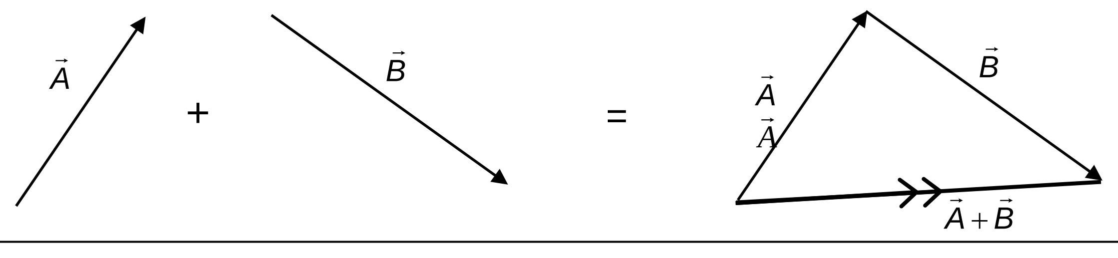
For vectors with negative values, it means that the quantity has the same magnitude but pointing in the opposite direction.

1.6.2.1 Vector Addition

When solving physics problems, it is often helpful to replace one vector quantity (eg. force) by a combination of two vectors. These two vectors must of course be equivalent to the given one.

Unlike scalar quantities that can be added easily using a calculator e.g. 1.8 kg + 2.2 kg = 4.0 kg, vector quantities have to be added with their directions taken into consideration.

The addition of 2 vectors can be performed using the triangle rule. Move the start of second vector to the end of the first vector. Resultant vector is drawn from the start of the first to the end of the second vector. Resultant is shown by the double arrow.

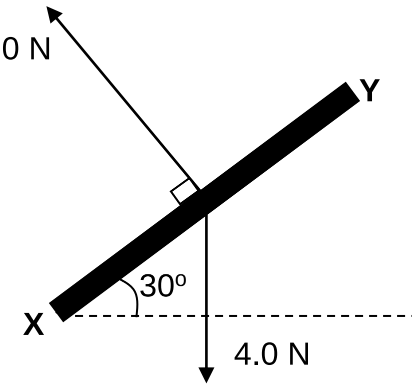


Three approaches for calculating the resultant vector are

1. By accurate scale drawing (Not common for A-Level)
2. By calculations using sine and cosine rules, or Pythagoras' theorem
3. By summing vector components

Note: Vector addition is commutative: $\vec{A}+\vec{B}=\vec{B}+\vec{A}$
This means that the order of addition of vectors is not important.

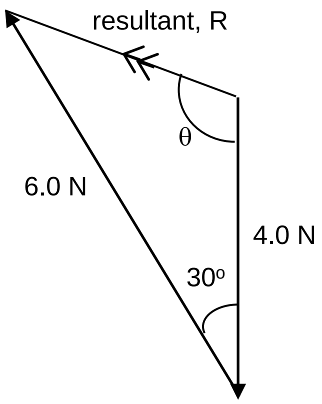
Worked Example 23 (By accurate scale drawing)
In the diagram below, XY represents a flat kite of weight 4.0 N. At a certain instant, XY is inclined at 30º to the horizontal and the wind exerts a steady force of 6.0 N at right angles to XY so that the kite flies freely.

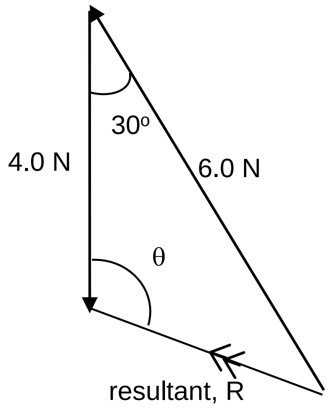


Draw a scale diagram to find the magnitude and direction of the resultant force acting on the kite.

Solution:

Scale: 1 cm ≡ 1.0 N





The 2 diagrams above illustrate the commutative law of addition of vectors

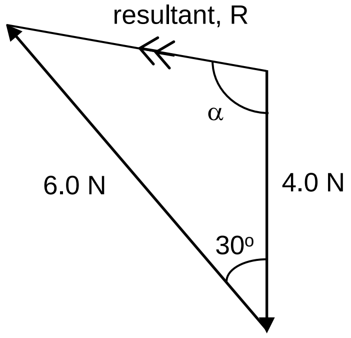
R = 3.2 N (≡ 3.2 cm)

at θ = 112° to the 4 N vector.

Scale drawing is not as accurate as calculation, so don't use this as far as possible.

Solve Example 23 again but use the calculation method using sine and cosine rules, or Pythagoras' theorem.

Solution:



Using cosine rule, $a^2 = b^2 + c^2 - 2bc \cos A$

$$R^2 = 4^2 + 6^2 - 2(4)(6)(\cos 30°)$$

$$R = 3.2 \text{ N}$$

Using sine rule,

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{6}{\sin \alpha} = \frac{3.23}{\sin 30°}$$

$$\alpha = 68° \text{ or } 112°$$

$$= 112° \text{ to the 4 N vector or } 68° \text{ west of north}$$

1.6.2.2 Vector Subtraction

The subtraction of one vector from another must be carried out in the same way as vector addition. In this case, we add a **negative vector** to have the same effect as subtracting a vector.

In other words, $\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$

Note: Vector $(-\vec{B})$ has the same magnitude as vector $\vec{B}$ but in the **opposite** direction.

Example 24 (Vector Subtraction) (N85 P1 Q1)
A particle moves eastwards with an initial velocity of 15 m s^{-1}, as shown in Fig.1. At a later time, its velocity is 15 m s^{-1} at an angle of 60° North of East (Fig.2).

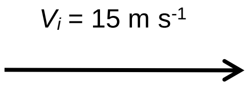


Fig.1

V_f =15 m s^{-1}

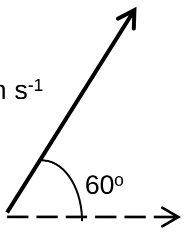


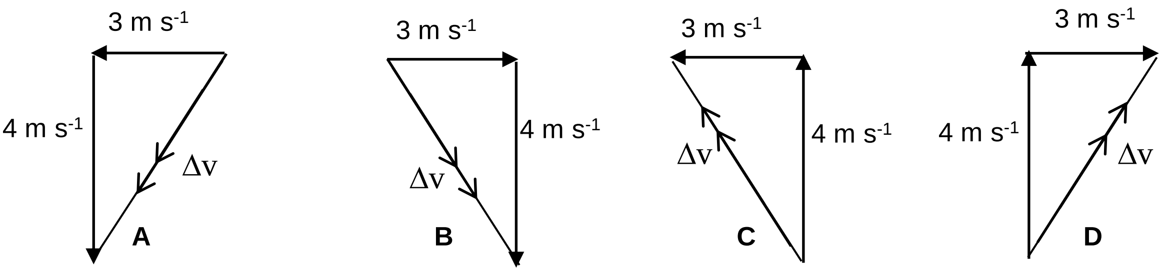
Fig.2

What is the change in velocity that has taken place in this interval?

Solution:

Worked Example 25 (Vector Subtraction)

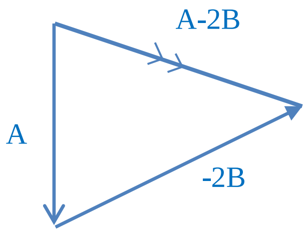
A ship travelling 3 m s^{-1} due East changes its course to 4 m s^{-1} due South.
What is the change in velocity Δv of the ship?



Ans: A

Worked Example 26
Given the following 2 vectors A and B as shown below, draw the resultant for A - 2B.

Solution:



1.6.2.3 Relative velocity

For example, you are standing stationary in a MRT station watching the MRT travelling from east to west at 20 m s^{-1}. However, the observer on the MRT, will see the MRT as stationary and see you (at the station) as moving from west to east at 20 m s^{-1}.

By convention, we are using V_{AB} as **relative velocity** of A with respect to B, where velocities of A and B are with respect to the ground. Hence,

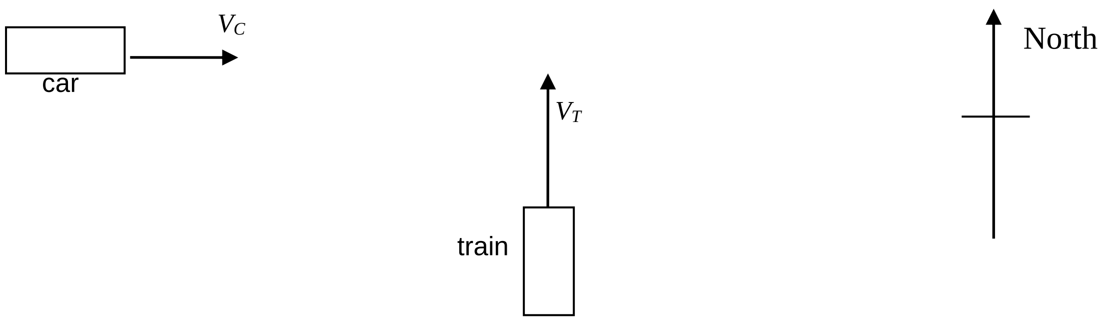
$$\boxed{V_{AB} = V_A - V_B}$$

Referring to the above example, state the relative velocity of MRT with respect to the station, V_{MS}, and relative velocity of station relative to MRT, V_{SM}. Taking westward as positive,

$V_{MS} = V_M – V_S = 20 – 0 = 20$ m s^{-1} westward

$V_{SM} = V_S – V_M = 0 – 20 = -20$ m s^{-1} (ie. eastward)

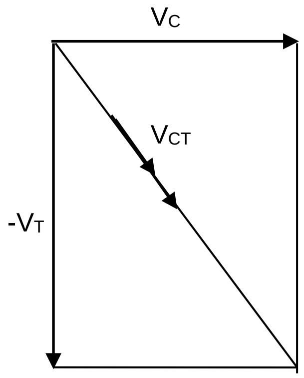
Worked Example 27 H1 N16/1/5 (modified)
A passenger in a train travelling due north at speed V_T sees a car travelling due east at speed V_C.



Sketch the diagram that shows the velocity V_R of the car relative to the passenger in the train.

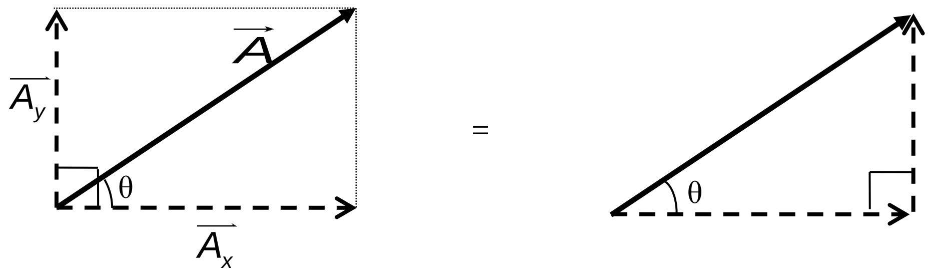
Hence we are looking for V_{CT}, or relative velocity of car with respect to train, Using vector subtraction,

$V_{CT} = V_C - V_T = V_C + (- V_T)$



1.6.2.4 Vector Resolution into two Perpendicular Components

We can resolve a single vector into two perpendicular components.
The vector $\vec{A}$ is resolved into two perpendicular components of A_x and A_y. Note that we either have vector A or its two components, A_x and A_y.



Here, $A_x = A \cos\theta$ and $A_y = A \sin\theta$

Alternatively, vector A can be resolved into other perpendicular components such as B_x and B_y as shown in Fig. 3.2.

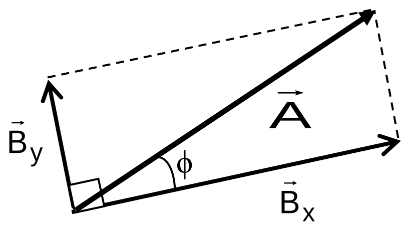
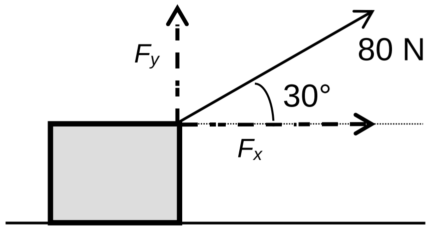


Fig. 3.2

Here, $B_x = A \cos \phi$ and $B_y = A \sin \phi$

Worked Example 28
A box is pulled along the floor by means of a rope making an angle of 30° with the horizontal, the pull of the rope being 80 N. Calculate the effective part of the pull in moving the box, and also the force tending to lift it off the floor.

Solution:



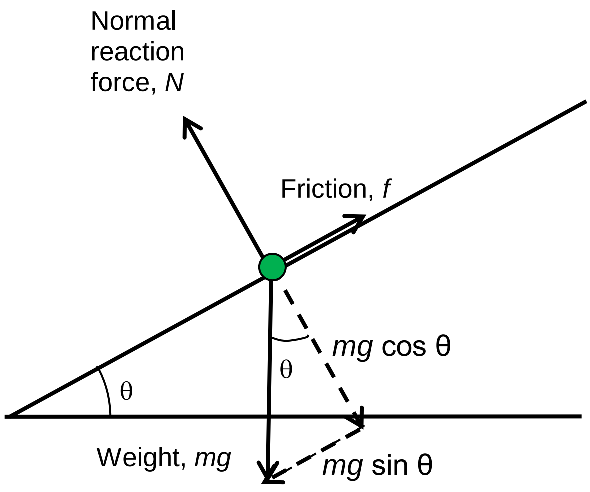
Since the box is moving horizontally, the effective part of pull is horizontal.

Effective part of pull in moving box,
F_x = 80 cos 30° = 69 N

Force tending to lift box off the floor,
F_y = 80 sin 30° = 40 N

Note: We do not always resolve vectors into the horizontal and vertical directions. In fact we can resolve any vector into any pair of perpendicular vectors. The choice of which pair of perpendicular vectors depends on the problem (see Example 29).

Worked Example 29 (important)
A body of mass, *m*, is resting on a rough slope inclined at an angle of θ to the horizontal.
Express the normal reaction force, *N*, and friction, *f*, in terms of *m*, *g* and θ.



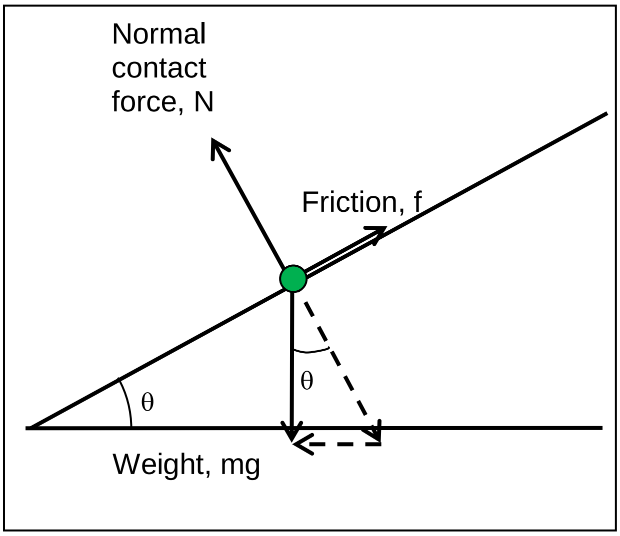
Since *N* is perpendicular to the slope and *f* is parallel to the slope, by finding the components of *mg* parallel and perpendicular to the slope, we can find *N* and *f*.

$N = mg \cos \theta$
$f = mg \sin \theta$

Common mistake:
A major difficulty of students is the vector resolution of the vector (mg in this case), as they tend to draw the wrong right-angled "triangle".

Since we are "breaking" up the mg force, it must always be the **hypotenuse** of the right-angled "triangle" of the two perpendicular components. The mg force (hypotenuse) must always be larger than the resolved components, ie. $mg > mg \sin \theta$, and $mg > mg \cos \theta$.
You can also check that the two components are perpendicular to each other.

Example of common mistake in resolving *mg*:



1.6.2.5 Addition of Vectors by the Components Approach

Another approach to add vectors is to first resolve the vectors into two perpendicular components (usually in the x and y directions) and then sum all the x-components to obtain vector $\boldsymbol{R_x}$. Similarly, sum all the y-components to obtain vector $\boldsymbol{R_y}$.

$\boldsymbol{R_x} = \boldsymbol{A_x} + \boldsymbol{B_x} + \boldsymbol{C_x} + \ldots$
$\boldsymbol{R_y} = \boldsymbol{A_y} + \boldsymbol{B_y} + \boldsymbol{C_y} + \ldots$

The vectors, $\boldsymbol{R_x}$ and $\boldsymbol{R_y}$ are then added "vectorially" using the Pythagoras theorem to obtain the magnitude of resultant $\boldsymbol{R}$,

i.e. $$\boxed{|R| = \sqrt{R_x^2 + R_y^2}}$$

The direction, θ of the resultant (with respect to the horizontal) is obtained using the formula,

$$\boxed{\tan\theta = \frac{|R_y|}{|R_x|}} \quad .$$

Direction may be described with respect to the vertical, depending on the diagram drawn.

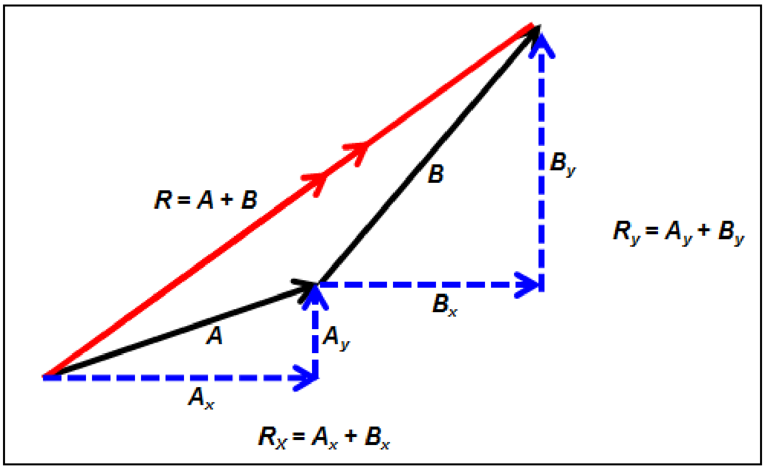
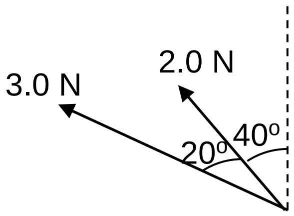


Diagram showing vector addition by the component approach
R = A + B

$R_x = A_x + B_x$
$R_y = A_y + B_y$

Worked Example 30

Use the components approach to find the magnitude and direction of the resultant force acting on the body.



R_x = 3.0 sin 60° + 2.0 sin 40°
= 3.884 N

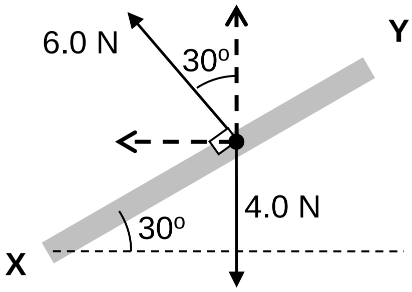
R_y = 3.0 cos 60° + 2.0 cos 40°
= 3.032 N

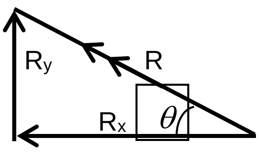
$$R = \sqrt{(3.884)^2 + 3.032^2}$$
$$= 6.9\,\text{N (to 2 sig. fig.)}$$

θ = tan^{-1} 3.032 / 3.884 = 38° above horizontal

Worked Example 31

Use the components approach to find the magnitude and direction of the resultant force acting on the kite of **Example 23** on pg 21.





Taking rightwards as the positive direction,
R_x = - 6.0 sin 30°
= - 3.0 N
(R_x is negative since it acts to the left)

Taking upwards as the positive direction,
R_y = 6.0 cos 30° - 4.0
= 1.2 N

$$R = \sqrt{(-3.0)^2 + 1.2^2}$$
$$= 3.2\,\text{N}$$

tan θ = 1.2 / 3.0
θ = 22°

R is at an angle 112° to the 4 N vector.
(90° + 22°)

SUMMARY

Base units: are (a choice of well-defined) units by which all other units are expressed.
Derived unit: a unit expressed as a product and/or quotient of the (7) base units.

Random error	Systematic error
An error which causes **measurements** to be **sometimes** larger than the true value and sometimes smaller than the true value	An error which causes **measurements** to be either, **always** larger than the true value, or always smaller than the true value.
Can be reduced e.g. by taking the average of repeated readings.	Cannot be revealed by repeated measurements, but can be eliminated for e.g., by checking the instrument in which the error is suspected, against a known reliable instrument.

Accuracy	Precision
Refers to the degree of agreement between the result of a measurement and the true value of the quantity.	Refers to the degree of agreement (scatter, spread) of repeated measurements of the same quantity.
A measure of the magnitude of the systematic error present; **high accuracy means small systematic error**	A measure of the magnitude of the random errors present; **high precision means small random error**

Error Computation Rules

Addition and Subtraction	Product and Division	All Mixed Formulae
If $Y = nA \pm mB$, then $\Delta Y = n\Delta A + m\Delta B$	If $Y = aX^nZ^m$, then $\frac{\Delta Y}{Y} = \lvert n\rvert \frac{\Delta X}{X} + \lvert m\rvert \frac{\Delta Z}{Z}$	max error = ½ (max - min)
Constants are included in computation of error.	Constants are not included in computation of error.	

Actual error **must** be recorded to only **1 sig. fig.**, & the **number of decimal places (dp)** in the error will determine the dp of the calculated quantity.

Scalar Quantity	Vector Quantity
Has only magnitude and no direction	Has both magnitude and direction. Only completely described if both are known.
E.g. distance, speed, energy	E.g. displacement, velocity, force

Vector Addition and Subtraction

1. By accurate scale drawing
2. By calculation using sine and cosine rules, or Pythagoras' theorem
3. By summing vector components
   - Vector resolution into desired components (where the original vector must be the **hypotenuse** of the two perpendicular vector components)
   - Sum up the respective perpendicular vector components. i.e.
     $R_x = A_x + B_x + C_x + \ldots$
     $R_y = A_y + B_y + C_y + \ldots$
   - The magnitude of the resultant $\lvert R\rvert = \sqrt{{R_x}^2 + {R_y}^2}$.

CONCEPT WORKSHEET

Question 1:
5 darts were aimed at the bull's eye dartboard. Indicate, using crosses, to illustrate

	Accurate and precise
	Accurate but not precise
	Precise but not accurate
	Not precise and not accurate

Question 2:
Express, correctly, the following measurements and their errors.

i) 123456789 ± 12345

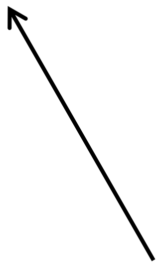
ii) 1234567.89 ± 5678

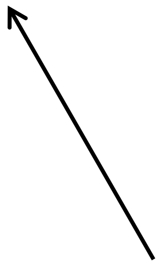
iii) 12345.6789 ± 12.345

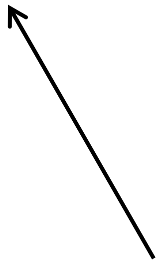
iv) 123.456 ± 0.789

v) 0.01234567 ± 0.00034567

Question 3
Resolve this vector in 3 different ways.







APPENDIX

SUMMARY OF KEY QUANTITIES, SYMBOLS AND UNITS (H2 Physics Only)

The following list illustrates the symbols and units that will be used in question papers.

Quantity	***Usual symbols***	***Usual unit***
Base Quantities		
mass	m	kg
length	l	m
time	t	s
electric current	I	A
thermodynamic temperature	T	K
amount of substance	n	mol
Other Quantities		
distance	d	m
displacement	s, x	m
area	A	m^2
volume	V, v	m^3
density	ρ	kg m^{-3}
speed	u, v, w, c	m s^{-1}
velocity	u, v, w, c	m s^{-1}
acceleration	a	m s^{-2}
acceleration of free fall	g	m s^{-2}
force	F	N
weight	W	N
momentum	p	N s
work	w, W	J
energy	E,U,W	J
potential energy	E_p	J
kinetic energy	E_k	J
heating	Q	J
change of internal energy	ΔU	J
power	P	W
pressure	p	Pa
torque	T	N m
gravitational constant	G	N kg^{-2} m^2
gravitational field strength	g	N kg^{-1}
gravitational potential	ϕ	J kg^{-1}
angle	θ	°, rad
angular displacement	θ	°, rad
angular speed	ω	rad s^{-1}
angular velocity	ω	rad s^{-1}
period	T	s
frequency	f	Hz
angular frequency	ω	rad s^{-1}
wavelength	λ	m
speed of electromagnetic waves	c	m s^{-1}
electric charge	Q	C
elementary charge	e	C
electric potential	V	V
electric potential difference	V	V
electromotive force	E	V
resistance	R	Ω
resistivity	ρ	Ω m
electric field strength	E	N C^{-1}, V m^{-1}
permittivity of free space	ε_o	F m^{-1}
magnetic flux	Φ	Wb
magnetic flux density	B	T

Quantity	***Usual symbols***	***Usual unit***
permeability of free space	μ_o	$H\ m^{-1}$
force constant	k	$N\ m^{-1}$
Celsius temperature	θ	°C
specific heat capacity	c	$J\ K^{-1}\ kg^{-1}$
molar gas constant	R	$J\ K^{-1}\ mol^{-1}$
Boltzmann constant	k	$J\ K^{-1}$
Avogadro constant	N_A	mol^{-1}
number	N, n, m	
number density (number per unit volume)	n	m^{-3}
Planck constant	h	J s
work function energy	Φ	J
activity of radioactive source	A	Bq
decay constant	λ	s^{-1}
half-life	$t_{1/2}$	s
relative atomic mass	A_r	
relative molecular mass	M_r	
atomic mass	m_a	kg, u
electron mass	m_e	kg, u
neutron mass	m_n	kg, u
proton mass	m_p	kg, u
molar mass	M	kg
proton number	Z	
nucleon number	A	
neutron number	N	

Derivation of fractional error for product and quotient: (Not in syllabus)

Given an equation:

$$Y = aX^n Z^m \qquad (1)$$

Where *X, Y* and *Z* are variables with a given amount of uncertainty,
And *a, n* and *m* are numerical constants.
Differentiating (1) by partial fraction yields,

$$dY = (m)aX^n Z^{m-1} dZ + (n)aZ^m X^{n-1} dX \qquad (2)$$

$$\frac{(2)}{(1)} \Rightarrow \qquad \frac{dY}{Y} = \frac{(m)aX^n Z^{m-1}}{aX^n Z^m} dZ + \frac{(n)aX^{n-1} Z^m}{aX^n Z^m} dX$$

$$\frac{dY}{Y} = n\frac{dX}{X} + m\frac{dZ}{Z} \qquad \text{(Shown)}$$

For a quotient like, $Y = a\dfrac{X^n}{Z^m}$, you get a negative ,- $m\dfrac{dZ}{Z}$, for the fractional error.

We need to understand that in the computation of the fractional error, we need to consider the "worst case" scenario to find the highest possible error, so we need to **add** up all the fractional errors.

Thus for, $\qquad Y = a\dfrac{X^n}{Z^m}$

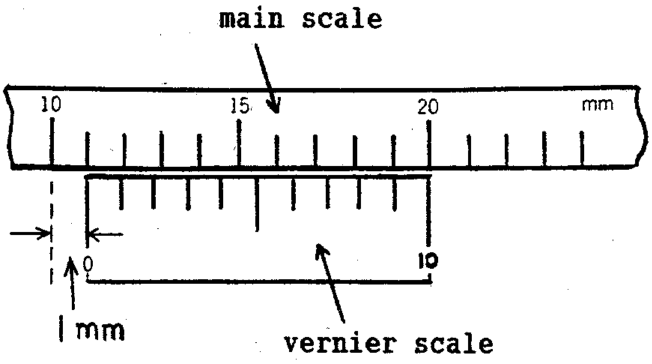
The fractional error, $\qquad \dfrac{dY}{Y} = n\dfrac{dX}{X} + m\dfrac{dZ}{Z}$

Therefore a more generic equation for the fraction error for product or quotient is

$$\frac{dY}{Y} = |n|\frac{dX}{X} + |m|\frac{dZ}{Z}$$

Measuring with Vernier callipers:

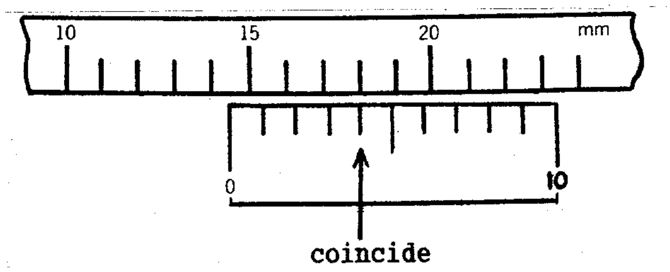
1. This Vernier scale is divided into 10 divisions and is of the same length as 9 divisions on the main scale.



2. This means that one division on the Vernier scale is shorter than one division on the main scale by 0.1 mm.

How to read?

Ex.1



1. The answer lies between 14 and 15 mm on the main scale.

2. Now, read from the Vernier scale the division which coincides with a division on the main scale.
   i.e. 4 div. x 0.1 mm = 0.4 mm
3. Hence, the reading is 14 + 0.4 = 14.4 mm
   = 1.44 cm

TUTORIAL 1: MEASUREMENT

Quantities and Units

(L1)1 (J96/P1/Q1)
Which list of SI units contains only **base** units?

A kelvin, metre, mole, ampere, kilogram
B kilogram, metre, second, ohm, mole
C kilogram, newton, metre, ampere, mole
D newton, kelvin, second, volt, mole [1]

(L1)2 (N01/P2/Q3)
The unit of a physical quantity may be shown with a prefix. For example, the prefix *micro* (μ) has the decimal equivalent 10^{-6}, so that 1 microampere (1 μA) can be written as 10^{-6} A. Complete the table below to show each prefix with its corresponding decimal equivalent. [3]

prefix	decimal equivalent
pico	……………….
micro	10^{-6}
giga	………………..
…………	10^{12}

(L2)3 (N95/P1/Q1)
The energy of a photon of light of frequency *f* is given by *hf*, where *h* is the Planck constant. What are the base units of *h*?

A kg m s^{-1} **B** kg m^2 s^{-1} **C** kg m^2 s^{-2} **D** kg m^2 s^{-3}

[1]

(L2)4 (J03/P1/Q2)
The unit of work, the joule, may be defined as the work done when the point of application of a force of 1 newton is moved a distance of 1 metre in the direction of the force.

Express the joule in terms of the base units of mass, length and time, the kg, m and s.

A kg m^{-1} s^2 **B** kg m^2 s^{-2} **C** kg m^2 s^{-1} **D** kg s^{-2}

[1]

Estimation

(L2)5 Make a reasoned estimate (to 1 sf) for the following quantities, with an appropriate SI unit:

(a) Weight of a human adult [1]

(b) Height of a 10-storey HDB block [1]

(c) Mass of water in an Olympic-size swimming pool [2]
(Hint: Use Density = Mass/Volume)

(d) Volume of a standard basket ball, and hence the average density of a basket ball. [2]

(e) The acceleration of a train on the Singapore rapid transit system. (H1N09/2/1) [2]

(f) The power of a car travelling along an expressway. (H1 N09/2/1) [2]

Errors and Uncertainties

(L1)6 (J95/P1/Q2)
When comparing systematic and random errors, the following pairs of properties of errors in an experimental measurement may be contrasted:

P_1: error can possibly be eliminated
P_2: error cannot possibly be eliminated

Q_1: error is of constant sign and magnitude
Q_2: error is of varying sign and magnitude

R_1: error will be reduced by averaging repeated measurements
R_2: error will not be reduced by averaging repeated measurements

Which properties apply to random errors?

A P_1, Q_1, R_2 **B** P_1, Q_2, R_2 **C** P_2, Q_2, R_1 **D** P_2, Q_1, R_1

[1]

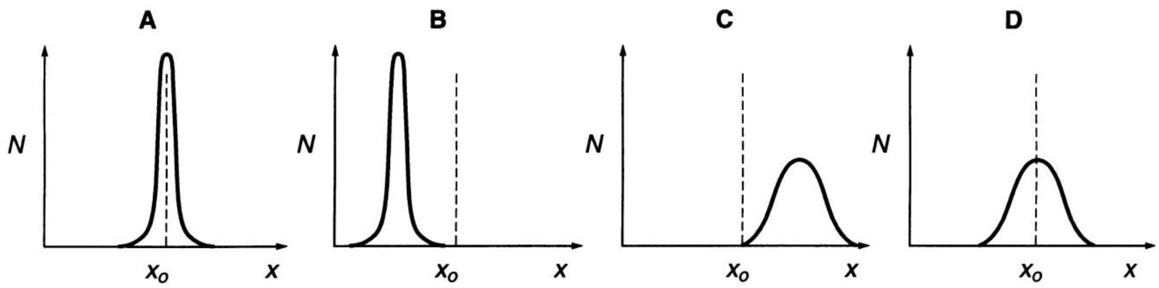
(L2)7 (J92/P1/Q1)
Which of the following experimental techniques reduces the systematic error of the quantity being investigated?

A timing a large number of oscillations to find a period
B measuring the diameter of a wire repeatedly and calculating the average
C adjusting an ammeter to remove its zero error before measuring a current
D plotting a series of voltage and current readings for an ohmic device on a graph and using its gradient to find resistance

[1]

(L2)8 (N97/P1/Q2)
A quantity x is measured many times and the number N of measurements giving a value x is plotted against x. The true value of the quantity is x_o.

Which graph best represents precise measurements with poor accuracy?



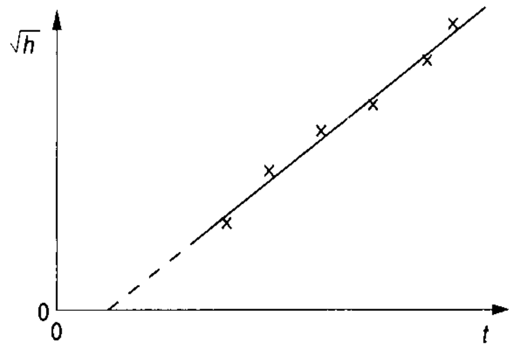
[1]

(L2)9 (N02/P1/Q2)
An object of mass 1.000 kg is placed on four different balances. For each balance the reading is taken five times. The table shows the values obtained together with the means. Which balance has the smallest systematic error but is not very precise?

balance	reading/kg					mean/kg
	1	2	3	4	5	
A	1.000	1.000	1.002	1.001	1.002	1.001
B	1.011	0.999	1.001	0.989	0.995	0.999
C	1.012	1.013	1.012	1.014	1.014	1.013
D	0.993	0.987	1.002	1.000	0.983	0.993

[1]

(L2)10 (N99/P3/Q1)
A student measures the time t for a ball to fall from rest through a vertical distance h. Knowing that the equation $h = ½at^2$ applies, the student plots the graph shown.



Which of the following is a possible explanation for the intercept?

A Air resistance has not been taken into account for larger values of h.
B There is a constant delay between starting the timer and releasing the ball.
C There is an error in the timer that consistently makes it run fast.
D The student should have plotted h against t^2.

[1]

Error Computation

(L1)11 A rectangular plot of land has length (56.8 ± 0.1) m and width (32.3 ± 0.1) m. Calculate its perimeter and its uncertainty. [2]

(L1)12 (N81/P2/Q5)

In an experiment, the external diameter d_1 and internal diameter d_2 of a tube are found to be (64 ± 2) mm and (47 ± 1) mm respectively. The percentage error in $(d_1 - d_2)$ expected from these readings is at most

A 0.3% **B** 1% **C** 5% **D** 6% **E** 18%

[1]

(L2)13 (N98/P1/Q2)

The density of the material of a rectangular block was determined by measuring the mass and linear dimensions of the block. The table shows the results obtained, together with their uncertainties.

mass	$= (25.0 \pm 0.1)$ g
length	$= (5.00 \pm 0.01)$ cm
breadth	$= (2.00 \pm 0.01)$ cm
height	$= (1.00 \pm 0.01)$ cm

The density was calculated to be 2.50 g cm^{-3}. What was the uncertainty in this result?

A $\pm$ 0.01 g cm^{-3} **B** $\pm$ 0.02 g cm^{-3} **C** $\pm$ 0.05 g cm^{-3} **D** $\pm$ 0.0525 g cm^{-3}

[1]

(L2)14 (J03/P1/Q5)

A student makes measurements from which she calculates the speed of sound as 327.66 m s^{-1}. She estimates that her result is accurate to $\pm$3%.

Which of the following gives her result expressed to the appropriate number of significant figures?

A 327.7 m s^{-1} **B** 328 m s^{-1} **C** 330 m s^{-1} **D** 300 m s^{-1}

[1]

(L2)15 In a simple electrical circuit, the potential difference across a resistor is measured as (2.00 ± 0.01) V. The resistor is marked as having a value of $4.7\ \Omega \pm 2\%$.

If these values were used to calculate the current flowing through the resistor, what would be the percentage uncertainty in the value obtained?

[Hint: Resistance is defined as the ratio of the potential difference across a component to the current flowing through it, ie. $R = V / I$]

A 1.5% **B** 2.5% **C** 3.5% **D** 4.5%

[1]

(L2)16 Given the following measurements:

$X = (2.48 \pm 0.02)$ m, $Y = (3.05 \pm 0.01)$ m, $Z = (1.59 \pm 0.05)$ m

Express in terms of $P \pm \Delta P$, where $P = \dfrac{3X + 2Y}{Z}$. [3]

(L2) 17 Use Rule 3 for error computation to determine the error in s from $s = ut + \frac{1}{2}at^2$, given that $u = 4.0 \pm 0.1\ \text{m s}^{-1}$, $t = 3.32 \pm 0.01$ s and $a = 5.8 \pm 0.2\ \text{m s}^{-2}$. Express your answer in $s \pm \Delta s$.

[3]

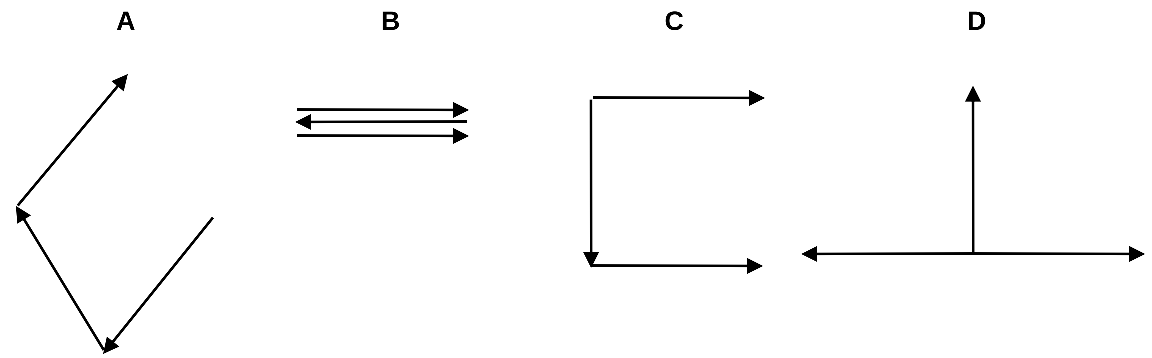
Vectors and Scalars

(L1)18 (J00/P1/Q1)
Which pair includes a **vector** quantity and a **scalar** quantity? [1]

A displacement; acceleration
B force; kinetic energy
C power; speed
D work; potential energy

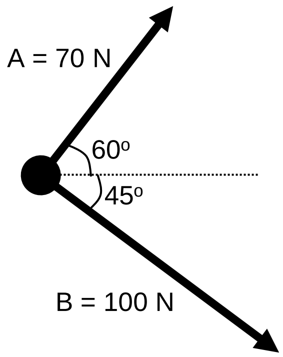
(L1)19 (N05/P1/Q1)
Each diagram shows three vectors of equal magnitude.

In which diagram is the magnitude of the resultant vector different from the other three? [1]



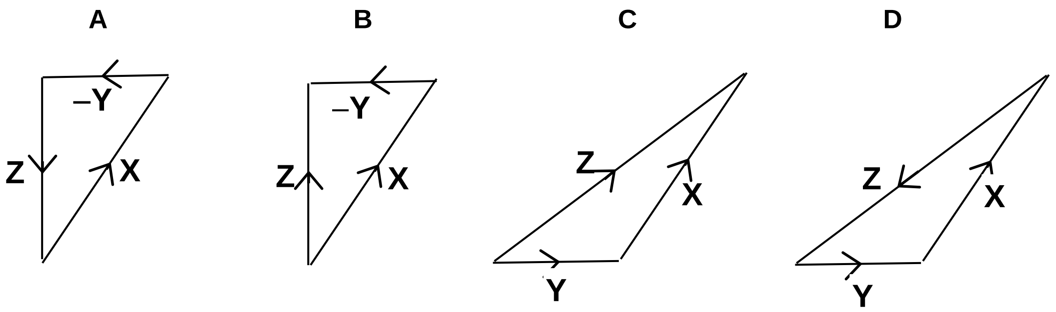
(L2)20 Determine the magnitude and direction of the resultant of the two forces shown in the figure below using

(a) sine and cosine rule method; [2]
(b) components method. [2]



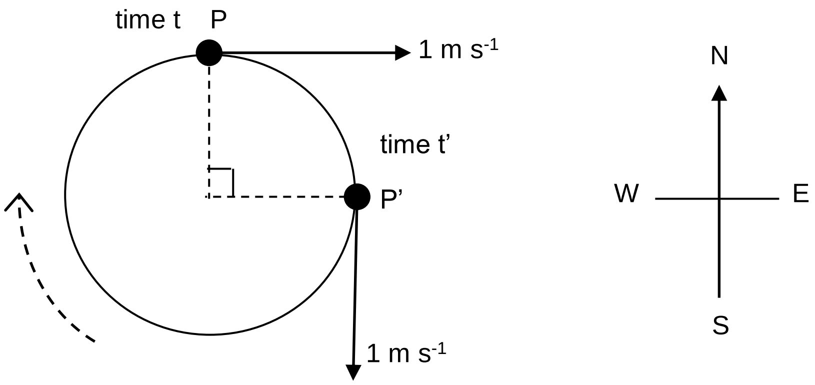
(L2)21 (J02/P1/Q2)
The diagram shows two vectors **X** and **Y**.

In which vector triangle does the vector **Z** show the magnitude and direction of vector **X** – **Y**?



[1]

(L2)22 A body rotates steadily clockwise at a constant speed of 1 m s^{-1}. At time t the body is at point P as shown. At time t', the body has rotated through 90° and it is now at point P'.



What is the change in the velocity of the body between t and t'?

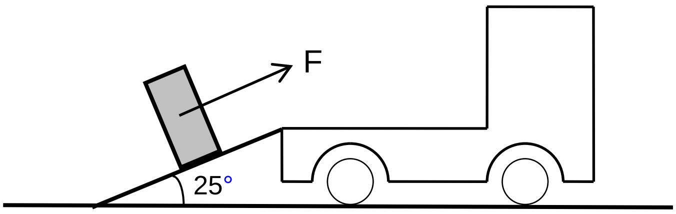
A	Zero	
B	1.4 m s^{-1}	south west
C	1.4 m s^{-1}	north east
D	2.0 m s^{-1}	south east

[1]

(L2)23 N16/1/3

An aircraft flies with an airspeed of 700 km h^{-1} through a 250 km h^{-1} jet-stream wind from the west. The pilot wishes to fly directly north from Australia towards Changi airport in Singapore. To achieve this, the pilot points the air craft away from the north direction. Determine the speed of the aircraft in the direction of the north relative to the ground. [1]

(L2)24 With the aid of an inclined plane, a 200 kg box is pushed up a flat-bed lorry. Assuming that the frictional force is negligible, find the minimum force needed to push the box up the inclined plane.



If a longer inclined plane is used, state whether more, equal or less force is needed to push the box up the incline. Give your reason. [2]

Numerical Answers

11	(178.2 ± 0.4) m
16	8.5 ± 0.3
17	45 ± 2 m
20	106 N, 5.5° below horizontal
23	654 km h^{-1}
24	829 N

TUTORIAL 1: MEASUREMENT SOLUTIONS

<u>Level 1 Solutions</u>

<table>
<tr><td>(L1)1</td><td>A

(There are only 7 base units: only kg, m, s, A, K, mol. And cd)</td><td>[1]</td></tr>
<tr><td></td><td></td><td></td></tr>
<tr><td>(L1)2</td><td>
<table>
<tr><th>prefix</th><th>decimal equivalent</th></tr>
<tr><td>pico</td><td><u>10^{-12}</u></td></tr>
<tr><td>micro</td><td>10^{-6}</td></tr>
<tr><td>giga</td><td><u>10^{9}</u></td></tr>
<tr><td><u>tera</u></td><td>10^{12}</td></tr>
</table>
</td><td>[3]</td></tr>
<tr><td></td><td></td><td></td></tr>
<tr><td>(L1)6</td><td>C as per definition</td><td>[1]</td></tr>
<tr><td></td><td></td><td></td></tr>
<tr><td>(L1)11</td><td>Perimeter, P = 2 x lengths + 2 x width = 178.2 m.
Absolute error, ΔP = 2 x Δlength + 2 x Δwidth = 0.4 m.
Hence, answer = (178.2 $\pm$ 0.4) m</td><td>
[1]
[1]</td></tr>
<tr><td></td><td></td><td></td></tr>
<tr><td>(L1)18</td><td>B force is a vector, kinetic energy is scalar</td><td>[1]</td></tr>
<tr><td></td><td></td><td></td></tr>
<tr><td>(L1)19</td><td>C Options A, B and D has a resultant of 1 unit (arrow) each.
The resultant for option C is √22 + 12 = √5 units</td><td>[1]</td></tr>
</table>

- End of tutorial solutions -