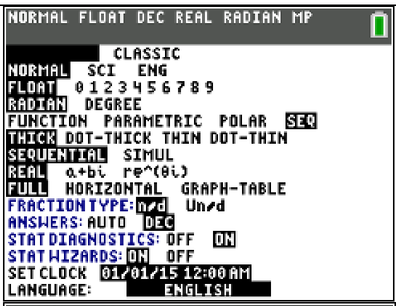
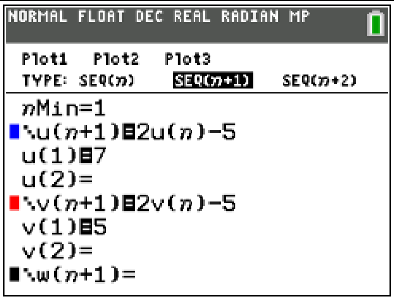
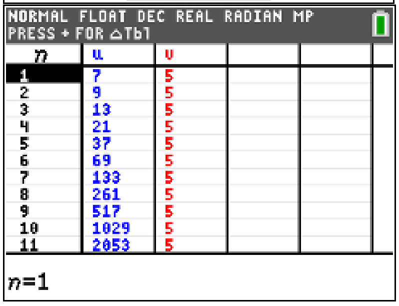
1i	<u>Method 1</u>: Let $y = ax^2 + bx + c$ be the required quadratic equation. $\frac{dy}{dx} = 2ax + b$ Since the quadratic curve have a minimum point at $(1,-2)$, $0 = 2a + b$ ----- (1) $-2 = a(1)^2 + b(1) + c$ $a + b + c = -2$ ------- (2) When $x = 2, \frac{dy}{dx} = 5$. $5 = 2a(2) + b$ $4a + b = 5$ ------ (3) Solving (1), (2) and (3) via GC, $a = \frac{5}{2}, b = -5, c = \frac{1}{2}$. Hence, $y = \frac{5}{2}x^2 - 5x + \frac{1}{2}$. <u>Method 2</u>: Since the quadratic curve have a minimum point at $(1,-2)$, it is has the equation of the form $y = k(x-1)^2 - 2$. $\frac{dy}{dx} = 2k(x-1)$. When $x = 2, \frac{dy}{dx} = 5$. $5 = 2k(2-1)$ $k = \frac{5}{2}$ Hence, $y = \frac{5}{2}(x-1)^2 - 2$ $y = \frac{5}{2}x^2 - 5x + \frac{1}{2}$.

2ai







(A) The sequence is increases by adding terms of a geometric progression (powers of 2). In particular, $u_{n+1} = u_n + 2^n$.

(B) The sequence is a constant sequence, where the value remains constant at 5.

2aii

<u>Method 1</u>:

$u_2 = 2u_1 - 5 = 2p - 5$

$u_3 = 2(2p-5) - 5 = 4p - 15$

$u_4 = 2(4p-15) - 5 = 8p - 35$

$u_5 = 2(8p-35) - 5 = 16p - 75$

Hence,

$101 = 16p - 75$

$p = 11$

<u>Method 2</u>:

$u_{n+1} = 2u_n - 5$

$u_n = \dfrac{u_{n+1} + 5}{2}$

Hence,

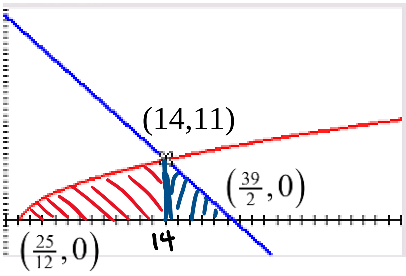
$u_4 = \dfrac{u_5 + 5}{2} = \dfrac{106}{2} = 53$

$u_3 = \dfrac{u_4 + 5}{2} = \dfrac{58}{2} = 29$

$u_2 = \dfrac{u_3 + 5}{2} = \dfrac{34}{2} = 17$

$p = u_1 = \dfrac{u_2 + 5}{2} = \dfrac{22}{2} = 11.$

2bi	$v_3 = a+2b-7$ $v_4 = b+2(a+2b-7)-7 = 2a+5b-21$ Hence, $v_4 = 2v_3$ $2a+5b-21 = 2(a+2b-7)$ $b = -14+21$ $b = 7.$
2bii	$v_3 = a+2(7)-7 = a+7$ $v_4 = 2a+14$ $v_5 = (a+7)+2(2a+14)-7 = 5a+28.$
2ci	Let the sequence be $x_1, x_2, x_3, ...$ and $S_n = x_1+x_2+...+x_n$. $x_n = S_n - S_{n-1}$ $= n^3-11n^2+4n-\left((n-1)^3-11(n-1)^2+4(n-1)\right)$ $= 3n^2-3n+1-11(2n-1)+4$ $= 3n^2-25n+16$
2cii	$m^3-11m^2+4m = 3^3-11(3^2)+4(3)$ $m^3-11m^2+4m+60 = 0$ By GC, $m = -2, 3, 10$ $m = 10$ (as $m > 3$.)

3(i)	$\frac{dy}{dt}=6$ and $\frac{dx}{dt}=6t$. $\frac{dy}{dx}=\frac{6}{6t}=\frac{1}{t}$. At $(14,11)\Rightarrow 11=6t-1\Rightarrow t=2$. Hence the gradient of the tangent is $\frac{dy}{dx}=\frac{1}{2}$. Therefore, the gradient of the normal is $\frac{-1}{1/2}=-2$. Equation of the normal: $y-11=-2(x-14)$ $2x+y=39.$
3(ii)	 When $x=\frac{25}{12}, t=\frac{1}{6}$ and when $x=14, t=2$. Required area $=\int_{\frac{25}{12}}^{14} y\ dx+\frac{1}{2}(11)\left(\frac{39}{2}-14\right)$ $=\int_{\frac{1}{6}}^{2}(6t-1)(6t)\ dt+\frac{121}{4}$ $=\left[12t^3-3t^2\right]_{\frac{1}{6}}^{2}+\frac{121}{4}$ $=\left(12(2^3)-3(2^2)-12\left(\frac{1}{6^3}\right)+3\left(\frac{1}{6^2}\right)\right)+\frac{121}{4}$ $=\frac{2057}{18}$ units^2.
3(iiia)	Required area $=\frac{2057}{18}\times\underbrace{2}_{\text{Scale by factor of 2 parallel to } x\text{-axis}}\times\underbrace{3}_{\text{Scale by factor of 3 parallel to } y\text{-axis}}$ $=\frac{2057}{3}$ units^2.

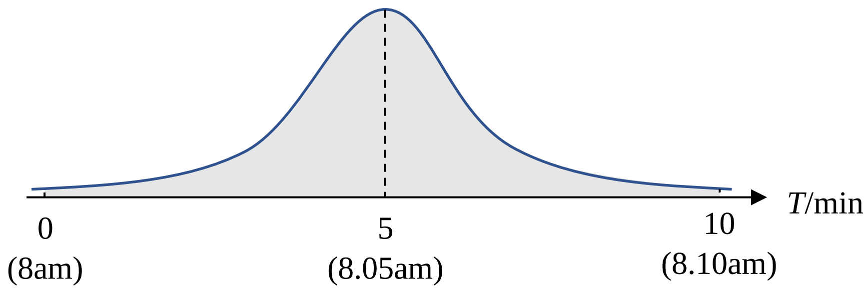
3(iiib)	Scale by a factor of 2 parallel to *x*-axis: Replace *x* with $\frac{x}{2}$. Scale by a factor of 3 parallel to *y*-axis: Replace *y* with $\frac{y}{3}$. Curve *D*; $\begin{cases} \frac{x}{2} = 3t^2 + 2 \\ \frac{y}{3} = 6t - 1 \end{cases} \Rightarrow \begin{cases} x = 6t^2 + 4 \\ y = 18t - 3 \end{cases}$ <u>Method 1</u>: Considering *x*, $t = \sqrt{\frac{x-4}{6}}$ (reject – ve as $t \geq \frac{1}{6}$.) Hence, $y = 18\sqrt{\frac{x-4}{6}} - 3$. <u>Method 2</u>: Considering *y*, $t = \frac{y+3}{18}$ Hence, $x = 6\left(\frac{y+3}{18}\right)^2 + 4$ $x = \frac{y^2 + 6y + 9}{54} + 4$ $54x = y^2 + 6y + 225.$

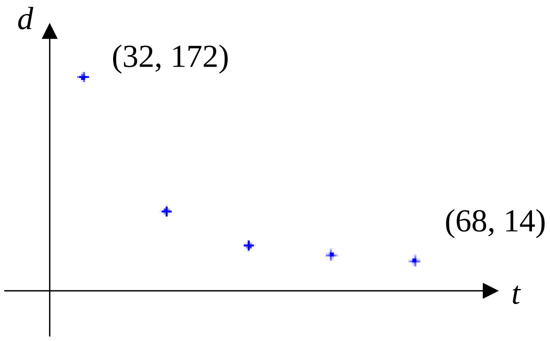
4(i)	Let O be the centre of the square, M be a midpoint of one of the sides of the square and P be the top vertex of the pyramid. We have $OM=\frac{a}{2}, PM=h, OP=H, \angle MOP=90°$. Also, $30=a+2h \Rightarrow h=15-\frac{a}{2}$. By Pythagoras theorem, $h^2=\left(\frac{a}{2}\right)^2+H^2$ $H^2=\left(h-\frac{a}{2}\right)\left(h+\frac{a}{2}\right)$ $=(15-a)(15)$ (sub $h=15-\frac{a}{2}$) $=225-15a$ (Shown)
4(ii)	Let V denote the volume of the pyramid. $V=\frac{1}{3}\left(a^2\right)\sqrt{225-15a}$ $V^2=\frac{1}{9}a^4\left(225-15a\right)=25a^4-\frac{5}{3}a^5$ $2V\frac{\mathrm{d}V}{\mathrm{d}a}=100a^3-\frac{25}{3}a^4$. At max V, $\frac{\mathrm{d}V}{\mathrm{d}a}=0$; $100a^3-\frac{25}{3}a^4=0$ $a^3\left(100-\frac{25}{3}a\right)=0$ $a=0$ or $a=12$ Clearly, $a=0, V=0$, which is not the maximum. If $a=12, V=\frac{1}{3}(12)^2\sqrt{225-15(12)}=144\sqrt{5}$ cm^3. This is the maximum volume.
4(iiia)	Let A be the required area of the 4 triangular faces.

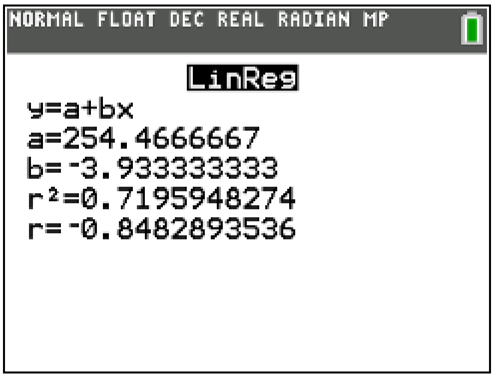
	$A=4\left(\frac{ah}{2}\right)$ $=2a\left(15-\frac{a}{2}\right)$ $=30a-a^2$ $=225-(a-15)^2$ $\leq 225-0$ (as $(a-15)^2 \geq 0$ for all a.) $=225\text{ cm}^2$. The largest area is 225 cm^2 when $a-15=0 \Rightarrow a=15$.
4(iiib)	When $a=15, H^2=225-15(15)=0 \Rightarrow H=0$. As the pyramid's height is 0 cm, this implies the pyramid is flat (2-dimensional).

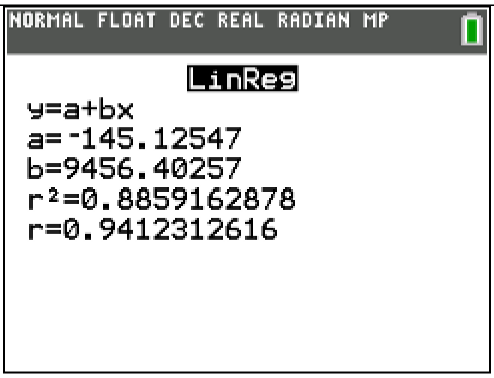
5(i)	Her possible points are 0, 4, 10 and 25.
5(ii)	Let X denote Tina's score. $\mathrm{E}(X)=(0)\mathrm{P}(X=0)+4\mathrm{P}(X=4)+10\mathrm{P}(X=10)+25\mathrm{P}(X=25)$ $=0+\frac{4(2r)(2r-1)}{(3r+1)(3r)}+\frac{10(2(r)(2r))}{(3r+1)(3r)}+\frac{25r(r-1)}{(3r+1)(3r)}$ $=\frac{16r-8+40r+25r-25}{3(3r+1)}$ $=\frac{81r-33}{3(3r+1)}$ $=\frac{27r-11}{3r+1}.$ $\mathrm{E}(X^2)=(0)\mathrm{P}(X=0)+4^2\mathrm{P}(X=4)+10^2\mathrm{P}(X=10)+25^2\mathrm{P}(X=25)$ $=0+\frac{16(2r)(2r-1)}{(3r+1)(3r)}+\frac{100(2(r)(2r))}{(3r+1)(3r)}+\frac{625r(r-1)}{(3r+1)(3r)}$ $=\frac{64r-32+400r+625r-625}{3(3r+1)}$ $=\frac{1089r-657}{3(3r+1)}$ $=\frac{363r-219}{3r+1}.$ Hence, $\mathrm{Var}(X)=\mathrm{E}(X^2)-(\mathrm{E}(X))^2$ $=\frac{363r-219}{3r+1}-\left(\frac{27r-11}{3r+1}\right)^2$ $=\frac{(3r+1)(363r-219)-(729r^2-594r+121)}{(3r+1)^2}$ $=\frac{1089r^2-294r-219-729r^2+594r-121}{(3r+1)^2}$ $=\frac{360r^2+300r-340}{(3r+1)^2}.$

5(iii)	$\frac{360r^2+300r-340}{(3r+1)^2}=38$ $360r^2+300r-340=342r^2+228r+38$ $18r^2+72r-378=0$ $r^2+4r-21=0$ $r=3$ ($r=-7$ is rejected as $r>1$.)

6(i)	 Note: s.d. is 1.2 min, so $5 > 3.6 = 3\times 1.2$. This means, by empirical rule, that 0 to 10 should cover almost all the distribution.
6(ii)	$T \sim \mathrm{N}\left(5, 1.2^2\right)$ $\mathrm{P}(T > 6) = 0.202328$ $= 0.202$ (3 s.f.)
6(iii)	We need to consider he time he takes to leave home, then the amount of time he takes to walk to work. $\mathrm{E}(T+W) = 21+5 = 26,\ \mathrm{Var}(T+W) = 1.2^2 + 3^2 = 10.44$ $T+W \sim \mathrm{N}(26, 10.44)$ $\mathrm{P}(T+W > 30) = 0.107864$ $= 0.108$ (3 s.f.)
6(iv)	$\mathrm{E}(T+D) = 19+5 = 24,\ \mathrm{Var}(T+D) = 1.2^2 + 6^2 = 37.44$ $T+D \sim \mathrm{N}(24, 37.44)$ $\mathrm{P}(\text{weather is fine} \mid \text{late}) = \dfrac{\mathrm{P}(\text{weather is fine} \cap \text{late})}{\mathrm{P}(\text{late})}$ $= \dfrac{\mathrm{P}(\text{weather is fine})\mathrm{P}(T+W>30)}{\mathrm{P}(\text{weather is fine})\mathrm{P}(T+W>30) + \mathrm{P}(\text{weather is not fine})\mathrm{P}(T+D>30)}$ $= \dfrac{(0.7)(0.107864)}{(0.7)(0.107864)+(0.3)(0.163400)}$ $= 0.6063435$ $= 0.606$ (3 s.f.)

<table>
<tr><td></td><td></td></tr>
<tr><td>7(i)</td><td>




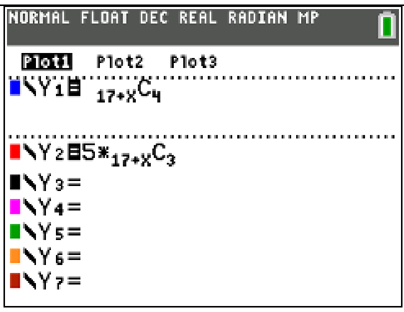
$r = -0.848$ (3 s.f.)
</td></tr>
<tr><td>7(ii)</td><td>


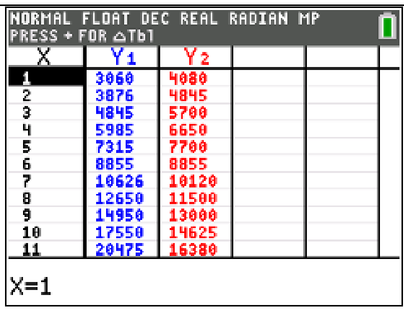
$d = -a + bu$

$a = 145.12547 = 145$ (3 s.f.)
$b = 9456.40257 = 9460$ (3 s.f.)
$r = 0.941$ (3 s.f.)
</td></tr>
<tr><td>7(iii)</td><td>As $t = 86$ is outside the data range $32 \le t \le 68$, Mr. Lim’s model does not fit the additional data point.</td></tr>
</table>

7(iv)	$T=\frac{5}{9}(t-32)$ $t=\frac{9}{5}T+32$ Hence, $d=-145.12547+9456.40257u$ $d=-145.12547+9456.40257\left(\frac{1}{t}\right)$ $d=-145.12547+\frac{9456.40257}{\frac{9}{5}T+32}$ $d=-145+\frac{47300}{9T+160}$ (3 s.f.)

8(i)	$\mathrm{P}(R=1)=\dfrac{\binom{17}{1}\times\binom{11}{11}}{\binom{28}{12}}=\dfrac{1}{1789515}=5.59\times10^{-7}$ (3 s.f.) $\mathrm{P}(R=2)=\dfrac{\binom{17}{2}\times\binom{11}{10}}{\binom{28}{12}}=\dfrac{88}{1789515}=4.92\times10^{-5}$ (3 s.f.) Therefore, $\mathrm{P}(R=1)<\mathrm{P}(R=2)$. (Shown)
8(ii)	$\mathrm{P}(R=4)=15\times\mathrm{P}(R=3)$ $\dfrac{\binom{17+r}{4}\times\binom{11}{8}}{\binom{28+r}{12}}=\dfrac{15\times\binom{17+r}{3}\times\binom{11}{9}}{\binom{28+r}{12}}$ $\binom{17+r}{4}\times\binom{11}{8}=15\times\binom{17+r}{3}\times\binom{11}{9}$ <u>Method 1</u>: $\binom{17+r}{4}\times\binom{11}{8}=15\times\binom{17+r}{3}\times\binom{11}{9}$ $\dfrac{(17+r)!}{4!(13+r)!}\times\dfrac{11!}{3!8!}=15\times\dfrac{(17+r)!}{3!(14+r)!}\times\dfrac{11!}{2!9!}$ $\dfrac{1}{4\times3}=\dfrac{15}{9(14+r)}$ $14+r=20$ $r=6$ <u>Method 2</u>: $\binom{17+r}{4}\times\binom{11}{8}=15\times\binom{17+r}{3}\times\binom{11}{9}$ $\binom{17+r}{4}=5\binom{17+r}{3}$ By GC,





X	Y1	Y2
1	3060	4080
2	3876	4845
3	4845	5700
4	5985	6650
5	7315	7700
6	8855	8855
7	10626	10120
8	12650	11500
9	14950	13000
10	17550	14625
11	20475	16380

At $r=6$, $\binom{17+r}{4} = 8855 = 5\binom{17+r}{3}$

Hence, $r = 6$.

9(i)	Each pen has an <u>equal chance of being sampled</u> and that pens are <u>chosen for the sample independently</u>.
9(ii)	Let X be the number of faulty pens out of a sample of 10 pens. $X \sim \mathrm{B}(10, 0.06)$ $\mathrm{P}(X \le 2) = 0.9811622$ $= 0.981$ (3 s.f.)
9(iii)	More than 5% is rejected is equivalent to less than or equal to 95% accepted in this scenario. Let Y be the number of accepted boxes out of 75 boxes. $Y \sim \mathrm{B}(75, 0.9811622)$ $\mathrm{P}(Y \le 75 \times 0.95) = \mathrm{P}(Y \le 71.25)$ $= \mathrm{P}(Y \le 71)$ $= 0.05345256$ $= 0.0535$ (3 s.f.)
9(iv)	Let W the number of faulty pens in a sample of 5. $W \sim \mathrm{B}(5, 0.06)$ Denote W_1 the number in the first sample and W_2 is the number in the second sample. $\mathrm{P}(\text{box accepted})$ $= \mathrm{P}(W_1 = 0) + \mathrm{P}(W_1 = 1)\mathrm{P}(W_2 \le 1) + \mathrm{P}(W_1 = 2)\mathrm{P}(W_2 = 0)$ $= 0.733904 + (0.234225)(0.968129) + (0.0299010)(0.733904)$ $= 0.98261$ $= 0.983$ (3 s.f.)
9(v)	The manager would prefer to use the alternative method as on average, <u>less testing would need to be done</u> as compared to the first test. This meant less resources, such as time and money would be spent on testing.

10(i)	Let X be the percentage of carbon, by weigh, in a round steel bar. Let μ be the percentage mean of the proportion of carbon, by weight, in a round steel bar. Test H0: $\mu = 1.5$ Against H1: $\mu \neq 1.5$ At 5% level of significance. Under H0, we have $\overline{X} \sim \mathrm{N}\left(1.5, \frac{0.09^2}{15}\right)$. $Z = \frac{\overline{X} - 1.5}{0.09/\sqrt{15}} \sim \mathrm{N}(0,1)$. For H0 to be rejected, $z_{\text{cal}} < z_{\text{crit}_1}$ or $z_{\text{crit}_2} < z_{\text{cal}}$ $\frac{\overline{x} - 1.5}{\frac{0.09}{\sqrt{15}}} < -1.95996$ or $1.95996 < \frac{\overline{x} - 1.5}{\frac{0.09}{\sqrt{15}}}$ $\overline{x} - 1.5 < -0.045545$ or $0.045545 < \overline{x} - 1.5$ $\overline{x} < 1.45445$ or $1.545545 < \overline{x}$ Taking into account of the context that $\overline{x}$ is a percentage, we must also have $0 \le \overline{x} \le 100$. Therefore, $0 \le \overline{x} < 1.45$ or $1.55 < \overline{x} \le 100$ (3 s.f.).
10(ii)	Unlike previously, the manager <u>does not know that the distribution of the amount of carbon in the flat bars is normal</u>. So, he takes a <u>sufficiently larger sample ($40 \ge 30$) that allows the Central Limit Theorem to apply</u>.
10(iii)	Unbiased estimate of population mean $= \frac{\sum x}{40} = \frac{10.16}{40} = 0.254$. Unbiased estimate of population variance $= \frac{1}{39}\left(\sum x^2 - \frac{\left(\sum x\right)^2}{40}\right)$ $= \frac{1}{39}\left(2.586342 - \frac{(10.16)^2}{40}\right)$ $= 1.46205 \times 10^{-4}$ $= 1.46 \times 10^{-4}$ (3 s.f.).

10(iv)	Let X be the percentage of carbon, by weigh, in a flat steel bar. Let μ be the percentage mean of the proportion of carbon, by weight, in a flat steel bar. Test H$_0$: $\mu = 0.25$ Against H$_1$: $\mu > 0.25$ At 2.5% level of significance. Under H$_0$, as $n = 40 \geq 30$ is large, by CLT, we have; $\overline{X} \sim \mathrm{N}\left(0.25, \dfrac{s^2}{40}\right)$ approximately. $Z = \dfrac{\overline{X} - 0.25}{s/\sqrt{40}} \sim \mathrm{N}(0,1)$ approximately. With $\overline{x} = 0.254, s^2 = 1.46205 \times 10^{-4}$. By GC, p-value $= 0.0182$ (3.s.f) < 0.025. We reject H$_0$. There is sufficient evidence at 2.5% level of significance that the mean amount of carbon in the flat bars is more than 0.25%.