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TAMPINES SECONDARY SCHOOL

Secondary Four Express/ Five Normal Academic
Preliminary Examination 2023

NAME					
CLASS				**REGISTER NUMBER**	

MATHEMATICS **4052/01**

24 August 2023

2 hours 15 minutes

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value of 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is **90**.

Mathematical Formulae

Compound Interest

Total amount = $P\left(1+\frac{r}{100}\right)^n$

Mensuration

Curved surface area of a cone = πrl

Surface area of a sphere = $4\pi r^2$

Volume of a cone = $\frac{1}{3}\pi r^2 h$

Volume of a sphere = $\frac{4}{3}\pi r^3$

Area of a triangle *ABC* = $\frac{1}{2}ab\sin C$

Arc length = $r\vartheta$, where ϑ is in radians

Sector area = $\frac{1}{2}r^2\vartheta$, where ϑ is in radians

Trigonometry

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

$$a^2=b^2+c^2-2bc\cos A$$

Statistics

Mean = $\frac{\Sigma fx}{\Sigma f}$

Standard deviation = $\sqrt{\frac{\Sigma fx^2}{\Sigma f}-\left(\frac{\Sigma fx}{\Sigma f}\right)^2}$

Answer **all** the questions

1 Tom invests \$4500 at a compound interest of 2.8 % per year for 5 years.

Calculate the total value of his investment at the end of 5 years.

Answer \$ _____ [2]

2 Given that $y = -4(3)^x$, which of the following represents the graph of y against x?

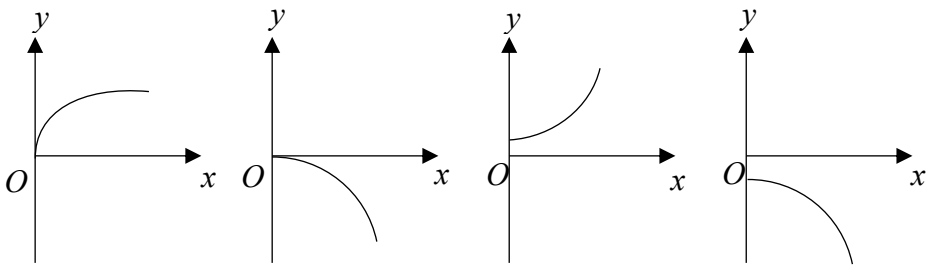


Diagram 1 Diagram 2 Diagram 3 Diagram 4

Answer Diagram _____ [1]

3 Given that $3 \times 27^n = 1$, find the value of n.

Answer: n = _____ [2]

4 The table below shows the distribution of ages of some students participating in a soccer tournament.

Age (in years)	13	14	15	16	17
Number of students	8	5	x	6	2

Given that the median age is 14 years, state the range of values of x.

Answer: _____ [2]

5 Box *A* contains four cards numbered 1, 4, 8 and 10. Box *B* contains three cards numbered 2, 5 and 7.

One card with number x is drawn from Box *A* and another card with number y is drawn from Box *B*. The possibility diagram shows the sum of the numbers x and y.

(a) Complete the possibility diagram.

+	**1**	**4**	**8**	**10**
2	3	6	10	12
5	6		13	15
7	8	11	15	

[2]

(b) Find, in simplest form, the probability that

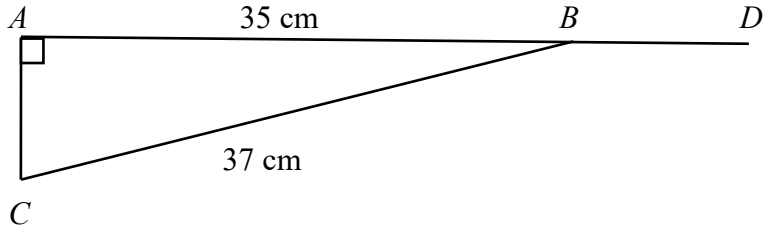
(i) $x + y$ is a prime number.

Answer: _____ [1]

(ii) $\frac{1}{2}x + \frac{1}{2}y \geq 8\frac{1}{2}$.

Answer: _____ [2]

6 In the diagram below, ABD is a straight line, $\angle DAC = 90°$, $AB = 35$ cm and $BC = 37$ cm.



(a) Find AC.

Answer: _____ cm [1]

(b) Write as a fraction

(i) $sin\ \angle ABC$,

Answer: _____ [1]

(ii) $cos\ \angle CBD$.

Answer: _____ [1]

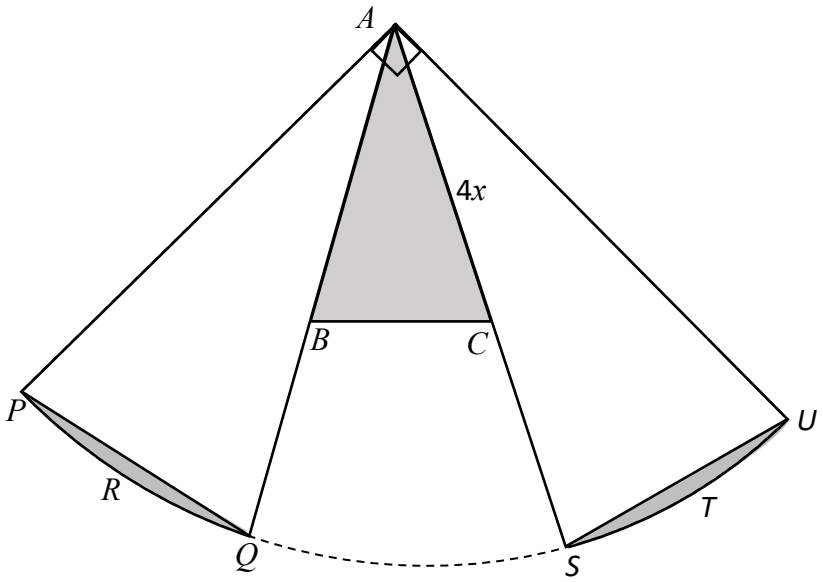
7 A worker works in a fast-food restaurant for 13 hours. The worker is paid \$$x$ for working at normal rate and \$$y$ for working at overtime rate. If he works for 10 hours at the normal rate and 3 hours at overtime rate, he will be paid \$124. On Saturdays, if he works for 8 hours at the normal rate and 5 hours at overtime rate, he will be paid \$133.

Form and solve two simultaneous equations to find the normal and overtime rate of pay.

Answer: Normal rate \$ _____

Answer: Overtime rate \$_____ [4]

8



The diagram shows an isosceles triangle ABC. Angle PAQ = angle BAC = angle SAU. $AC = 4x$.

Angle $PAU = 90°$. $APRQ$ and $AUTS$ are two identical sectors. B is the mid-point of AQ and C is the mid-point of AS.

Show that the shaded area can be expressed as $(p\pi + q)x^2$ where p and q are constants.

Answer:

[6]

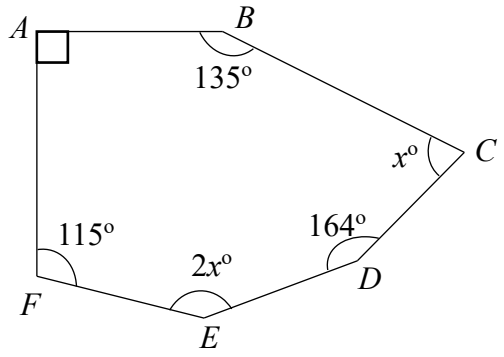
9 n is a positive integer.

Show that, for all n, $(6n+1)^2 - (6n-1)^2$ is a multiple of 24.

Answer:

[2]

10



Given the diagram above, find the value of x.

Answer: x = _____ [2]

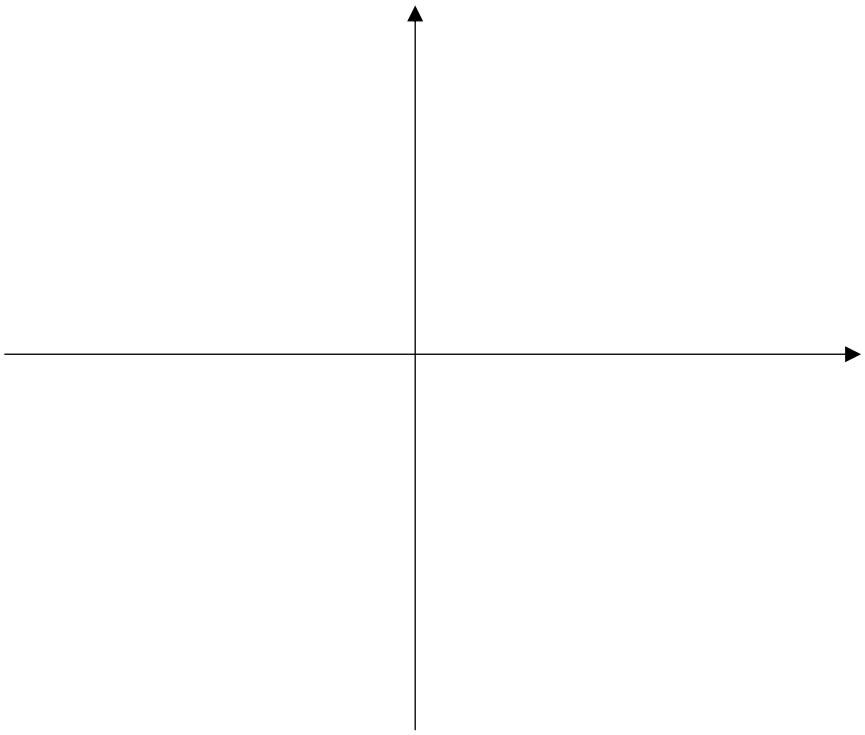
11 (a) Express $7-8x+x^2$ in the form of $p+(x+q)^2$.

Answer _____ [2]

(b) **Sketch** the graph of $y = 7 - 8x + x^2$ on the axes below.

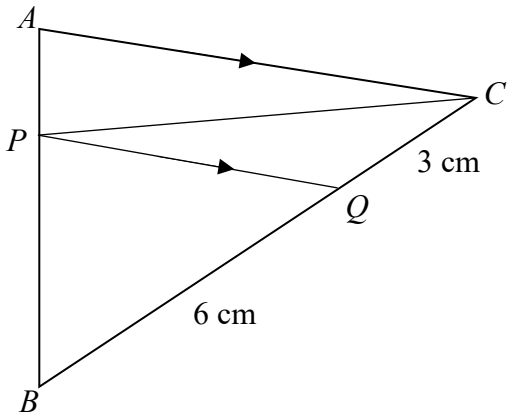
Indicate clearly the coordinates of the points where the graph crosses the axes and the minimum point on the curve.

Answer



[3]

12 In the diagram below, $QC = 3$ cm, $BQ = 6$ cm and PQ is parallel to AC.



(a) Stating your reasons clearly, show that triangle PQB is similar to triangle ACB.

Answer

[2]

(b) Find the value of $\frac{Area\ of\ triangle\ ABC}{Area\ of\ trapezium\ APQC}$.

Answer: _____ [2]

13 A group of 150 adults took part in a run.

The table below shows the distribution of the times taken to complete the run.

Time (t minutes)	$30 < t \leq 40$	$40 < t \leq 50$	$50 < t \leq 60$	$60 < t \leq 70$	$70 < t \leq 80$
Number of adults	25	62	35	22	6

(a) Calculate an estimate of the mean time.

Answer: _____ minutes [1]

(b) Calculate an estimate of the standard deviation.

Answer: _____ minutes [1]

(c) The standard deviation of a second group of adults taking part in the run is 8 minutes. One of the runners in the second group claims that all the runners in the second group run faster as the standard deviation of the second group is lower than the first group.

Explain if you agree and justify your reason.

Answer:

_____

_____

_____

_____ [1]

14 (a) Expand and simplify $(2x - 5q)(2x - 5q)$.

Answer: _____ [2]

(b) Given that $(2x - 5q)(2x - 5q) = 4x^2 + 40x + 100$. Find the value of q.

Answer: q = _____ [3]

15 Simplify $\left(\frac{3x}{4y^2}\right)^{-2}$.

Answer: _____ [2]

16 A map is drawn to a scale of 1 cm to 650 m.

(a) Express the scale in the form 1 : n.

Answer: _____ [1]

(b) A straight road has a length of 20.8 km. Find its length on the map in cm.

Answer: _____ cm [2]

(c) A sea port has an area of 60 cm^2 on the map. Find its actual area in km^2.

Answer: _____ km^2 [2]

17 Written as a product of its prime factors, $1512 = 2^3 \times 3^3 \times 7$ and $720 = 2^x \times 3^y \times 5$.

(a) Find the value of x and y.

Answer: $x =$ _____ [1]

Answer: $y =$ _____ [1]

(b) Find the lowest common multiple of 1512 and 720.

Answer: _____ [1]

(c) Find the smallest positive integer k such that $\frac{720}{k}$ is a square number.

Answer: $k =$ _____ [1]

18 (a) Factorise $12nm - 3n - 4m^2 + m$.

Answer: _____ [2]

(b) Using factorisation, solve $8x^2 - 26x + 15 = 0$.

Answer: _____ or _____ [3]

19 In store A, each cable costs \$12, each charger costs \$25 and each earpiece costs \$16.

In store B, each cable costs \$2 more, each charger costs \$4 less and each earpiece costs \$3 less.

This information can be represented by the matrix $\mathbf{Q} = \begin{pmatrix} 12 & 2 \\ 25 & -4 \\ 16 & -3 \end{pmatrix}$.

Ali and Mary go to the stores.

Ali buys 4 cables, 2 chargers and 3 earpieces.

Mary buys 6 cables and 3 earpieces.

(a) Represent their purchases in a 2×3 matrix **P**.

Answer: _____ [1]

(b) Evaluate the matrix $\mathbf{R} = \mathbf{PQ}$.

Answer: **R** =_____ [2]

(c) At which store would Ali spend more and by how much more?

Answer: Store _____ [1]

\$ _____ [1]

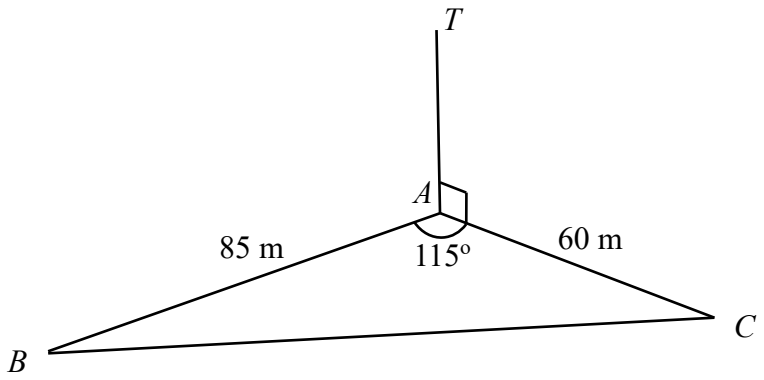
(d) Ali and Mary shop in store B.

John has a 10% discount voucher and Mary has a 5% discount voucher.

How much would they pay altogether for their items?

Answer: \$ _____ [2]

20 In the diagram, ABC are three points on horizontal ground.
$AB = 85$ m, $AC = 60$ m and angle $BAC = 115^o$.



(a) Calculate the length of BC.

Answer: _____ m [2]

(b) A girl standing at B is flying a drone T.

The drone, T, is vertically above A.

A string, BT, attached to the drone is at 35° to the horizontal.

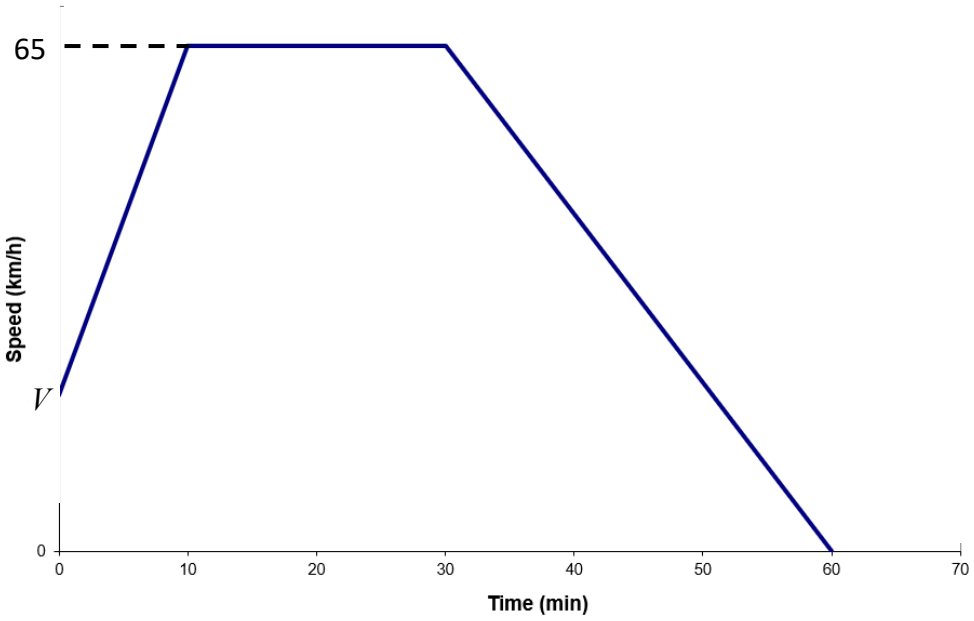
Calculate the angle of elevation of the drone when viewed from C.

Answer: _____ [3]

(c) Calculate the shortest distance from A to BC.

Answer: _____ m [3]

21 The diagram below shows the speed-time graph of the first 60 minutes of a car journey.



(a) The area beneath the speed-time graph represents the distance travelled by the car.

The car travels at the initial speed of V km/h. Given that the distance travelled for the first 30 minutes is 28.75 kilometres. Calculate the value of V.

Answer:

[3]

(b) Hence, find the deceleration of the car in km/h^2 for the last 30 minutes.

Answer: _____ km/h^2 [2]

(c) Find the average speed of the car for the whole journey.

Answer: _____ km/h [2]

22 The coordinates of the points P, Q and R are $(-1, 4)$, $(9, b)$, and $(3, 6)$ respectively.

The line of PQ is parallel to $y = \frac{b}{8}x + c$.

(a) Find PR.

Answer: _____ [2]

(b) Find the equation of the line PQ.

Answer: _____ [3]

(c) Given another equation of the line k is $2y = 4x + 4$, explain the relationship between line PQ and line k.

Answer: _____

_____

_____ [1]

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