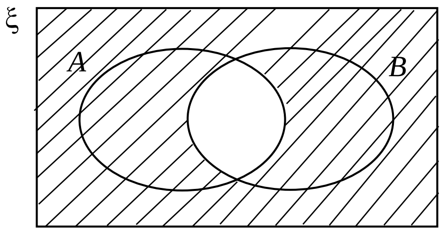
Paya Lebar Methodist Girls' School (Secondary)
Department of Mathematics
2017 Preliminary Examination
Mathematics Paper 1 (4048/1) Worked Solutions

Qns No.	Solution
1(a)	$3(3x+y)-2(x-5)$ $= 9x+3y-2x+10$ $= 7x+3y+10$
1(b)	$(2x-1)^2-36$ $= (2x-1+6)(2x-1-6)$ $= (2x+5)(2x-7)$ **OR** $(2x-1)^2-36$ $= 4x^2-4x+1-36$ $= 4x^2-4x-35$ $= (2x+5)(2x-7)$
2	$15ax+21bx-14by-10ay$ $= 3x(5a+7b)-2y(7b+5a)$ $= (5a+7b)(3x-2y)$
3(a)	
3(b)	$P = \{1, 2, 3\}$ $Q = \{2, 3, 4, 5, 6\}$ Elements in $P \cap Q$ are 2 and 3.
4	$(4n+3)^2-(16n^2+5)$ $= 16n^2+24n+9-16n^2-5$ $= 24n+4$ $= 4(6n+1)$ Since the expression is a multiple of 4, it is divisible by 4.
5	Let the width be x cm. $(x)(2x)(9)=288$ $18x^2=288$ $x^2=16$ $x=4$ $(-4$ is rejected) Length of base = 8 cm.

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6	$\frac{\sin\angle XZY}{17.4}=\frac{\sin 52°}{13.8}$ $\sin\angle XZY=\left(\frac{17.4\sin 52°}{13.8}\right)$ $= 83.5°$ or $96.5°$
7	$\left(\frac{45-h}{45}\right)^3=\frac{1}{3}$ $\frac{45-h}{45}=\sqrt[3]{\frac{1}{3}}$ $-h=45\left(\sqrt[3]{\frac{1}{3}}\right)-45$ $h=13.80\,(2\text{ dp})$ **OR** Let the height of small pyramid be x cm. $\left(\frac{x}{45}\right)^3=\frac{1}{3}$ $x=45\left(\sqrt[3]{\frac{1}{3}}\right)$ $x=31.2013$ $h=45-31.2013$ $= 13.80$ (2 dp)
8	• Title is biased – Does not allow reader to make their own judgement • The width of cylindrical bars are not equal – exaggerates the difference between the years
9	$\frac{3}{2x^2+7x-4}-\frac{2}{1-2x}$ $=\frac{3}{(2x-1)(x+4)}-\frac{2}{1-2x}$ $=\frac{3}{(2x-1)(x+4)}+\frac{2}{2x-1}$ $=\frac{3+2(x+4)}{(2x-1)(x+4)}$ $=\frac{2x+11}{(2x-1)(x+4)}$
10(a)	$\angle DAC=50°$ ($\angle$ at centre = 2$\angle$ at circumference)
10(b)	$\angle ADO=180°-70°$ ($\angle$s in the opp segs) $= 110°$
10(c)	$\angle ADO=180°-50°-110°$ ($\angle$ sum of Δ) $= 20°$ $\angle ADO=\frac{180°-100°}{2}$ (base $\angle$ of isos Δ)

Qns No.	Solution
	$= 40°$ $\angle ACO = 40° - 20°$ $= 20°$
11(a)	$15 - 12x + x^2$ $= x^2 - 12x + 15$ $= (x-6)^2 + 15 - \left(\frac{-12}{2}\right)^2$ $= (x-6)^2 - 21$
11(b)	Minimum value is -21
11(c)	Equation is $x = 6$.
12(a)	$P = 1.01 \times 10^5 + 10094d$ $2.22 \times 10^5 = 1.01 \times 10^5 + 10094d$ $d = \frac{2.22 \times 10^5 - 1.01 \times 10^5}{10094}$ = 12.0 m (3 sf) Depth of diver = 12.0 m
12(b)	$P_1 = 1.01 \times 10^5 + 10094d_1$ $P_2 = 1.01 \times 10^5 + 10094d_2$ $3.5 \times 10^5 = (1.01 \times 10^5 + 10094d_1) - (1.01 \times 10^5 + 10094d_2)$ $3.5 \times 10^5 = 10094(d_1 - d_2)$ $d_1 - d_2 = \frac{3.5 \times 10^5}{10094}$ = 34.7 m Difference in depths is 34.7 m
13	In Singapore, 10 grams of gold cost \$567.40 1 gram costs \$56.74 USD 1275.10 = \$1.382 × 1275.10 = \$1762.19 In Los Angeles, 31.10 grams of gold cost \$1762.19 1 gram of gold costs \$56.67 Gold is cheaper in Los Angeles. **OR** 1.382 Singapore dollars = 1 US dollar S\$567.40 = USD $\frac{567.40}{1.382}$ = USD 410.56 In Singapore, 10 grams of gold cost USD 410.56

Qns No.	Solution
	1 gram costs USD 41.06 In Los Angeles, 31.10 grams of gold cost USD 1275.10 1 gram of gold costs USD 41 Gold is cheaper in Los Angeles.
14	C(46, 24)
15(a)	$\tan \angle BCP = \frac{6}{5}$ $\angle BCP = 50.194°$ $\angle PCQ = 2 \times 50.194°$ $= 100.4°$ (1 dp)
15(b)	$\angle PBQ = 360° - 90° - 90° - 100.4°$ ($\angle$ sum of quad) $= 79.6°$ Perimeter of *APDQA* $= \frac{360° - 100.4°}{360°} \times 2\pi(5) + \frac{360° - 79.6°}{360°} \times 2\pi(6)$ = 52.0 cm (3 sf)
15(c)	Area of *APDQA* $= \frac{360° - 100.4°}{360°} \times \pi(5)^2 + \frac{360° - 79.6°}{360°} \times \pi(6)^2 + 2\left[\frac{1}{2}(5)(6)\right]$ = 175 cm^2
16(a)	$\angle DCB = 115°$ (alt $\angle$s, *AB* //*CD*)
16(b)	$\angle DCF = 360° - 125° - 115°$ ($\angle$s at a point) $= 120°$ Since $\angle DCF = \angle EFG = 120°$, therefore *EF* // *DC*. Since *DC* // *BA*, therefore *EF* // *BA*.
17(a)	$540 = 2 \times 2 \times 3 \times 3 \times 3 \times 5$ $= 2^2 \times 3^3 \times 5$
17(b)	540 is not a perfect cube as power of 2 and 5 are not multiples of 3.
17(c)	$m = 3$, $n = 5$
18(a)	Subst $A(-1, -3)$ into $y = ax^2 + bx$ $-3 = a(-1)^2 + b(-1)$ $-3 = a - b$ ------------ (1) Subst $B(3, 33)$ into $y = ax^2 + bx$ $33 = a(3)^2 + b(3)$ $33 = 9a + 3b$ ------------(2) (1) × 3: $-9 = 3a - 3b$ ------------(3)

Qns No.	Solution
	$(2)+(3)$: $24=12a$ $a=2$ Subst a = 2 into (1) $-3=2-b$ $b=5$
18(b)	Gradient of AB = $\dfrac{33-(-3)}{3-(-1)}$ = 9 Equation of AB is $y=9x$
19	For carpark A, Total charge = $\$2.20+4\times\$1.20+\$3$ = \$10 For carpark B, Total charge = $\$0.04(3\times60+45)$ = \$9 She should park in carpark B
20(a)	1 : 20 000 1 cm : 0.2 km Distance on map = $\dfrac{66}{0.2}$ = 330 cm
20(b)	Area of the lake in Town Y in map = $\dfrac{150}{100}\times0.34$ = 0.51 cm^2 1 cm : 0. 2 km 1 cm^2 : 0. 04 km^2 Actual area of lake in Town Y = 0.51 × 0.04 = 0.0204 km^2
21(a)	Bearing of C from B = $201°\pm1°$
21(b)	No. It is not the point of intersection of the 2 bisectors. Refer to attachment at the last page
22(a)	Let x be the probabilty of Bernice winning. $2x+2x+x=1$ $5x=1$ $x=\dfrac{1}{5}$ P(Ann winning) = $\dfrac{2}{5}$

Qns No.	Solution
	P(Bernice winning) = $\frac{1}{5}$ P(Carol winning) = $\frac{2}{5}$
22(b)	P(Bernice or Carol wins) = $\frac{1}{5}+\frac{2}{5}$ = $\frac{3}{5}$
22(c)	P(a player wins both games) = $\left(\frac{2}{5}\times\frac{2}{5}\right)+\left(\frac{1}{5}\times\frac{1}{5}\right)+\left(\frac{2}{5}\times\frac{2}{5}\right)$ = $\frac{9}{25}$
23(a)	$\overrightarrow{PQ}=\begin{pmatrix}k\\0\end{pmatrix}-\begin{pmatrix}-3\\4\end{pmatrix}$ $=\begin{pmatrix}k+3\\-4\end{pmatrix}$
23(b)	$\lvert\overrightarrow{OP}\rvert=\lvert\overrightarrow{OQ}\rvert$ $\sqrt{(-3)^2+4^2}=\sqrt{k^2}$ $k^2=25$ $k=5$ or -5
23(c)	$\overrightarrow{OR}=3\overrightarrow{PO}$ $\overrightarrow{OR}=3\begin{pmatrix}3\\-4\end{pmatrix}$ $=\begin{pmatrix}9\\-12\end{pmatrix}$ $R(9,-12)$
24(a)	Circumference of circle = $2\pi(6x)$ = $12\pi x$ cm
24(b)	Perimeter of shaded region = $2\pi(4x)+\frac{1}{2}[2\pi(2x)]+4x$ = $8\pi x+2\pi x+4x$ = $10\pi x+4x$ = $2x(5\pi+2)$ cm
24(c)	Area of shaded region = $\pi(4x)^2-\frac{1}{2}\left[\pi(2x)^2\right]$ = $16\pi x^2-2\pi x^2$ = $14\pi x^2$ cm2 Area of circle = $\pi(6x)^2$

Qns No.	Solution
	$= 36\pi x^2$ cm2 Fraction not shaded = $\dfrac{36\pi x^2 - 14\pi x^2}{36\pi x^2}$ $= \dfrac{22\pi x^2}{36\pi x^2}$ $= \dfrac{11}{18}$
25(a)	$\mathbf{N} = \begin{pmatrix} 3 & 2 & 1 \\ 1 & 0 & 4 \end{pmatrix}$
25(b)	**T** = **NM** $= \begin{pmatrix} 3 & 2 & 1 \\ 1 & 0 & 4 \end{pmatrix}\begin{pmatrix} 1.6 & -0.1 \\ 2.8 & -0.25 \\ 0.7 & 0.2 \end{pmatrix}$ $= \begin{pmatrix} 11.1 & -0.6 \\ 4.40 & 0.4 \end{pmatrix}$
25(c)	The elements represent the amount Ben spent in bookstore A and the difference in amount spent in the two bookstores respectively.
25(d)	Charles has to pay = \$4.40 + \$0.40 = \$4.80
25(e)	Increased priced payment = $\dfrac{110}{100} \times \$11.10$ = \$12.21 Amount Ben has to pay after discount = $\dfrac{70}{100} \times \$12.21$ = \$8.55

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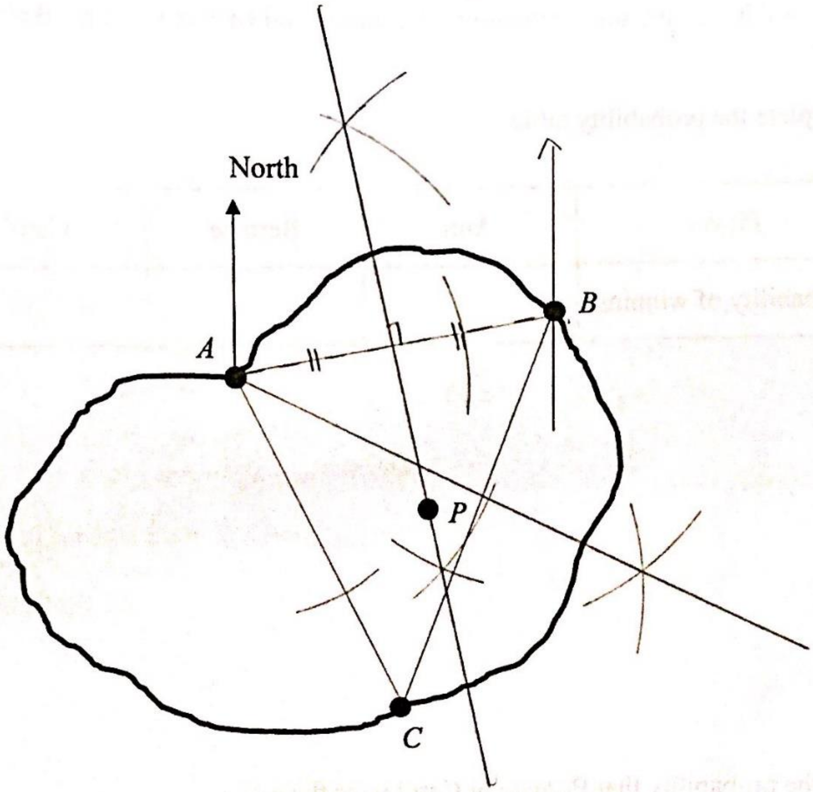
For Examiner's Use



The drawing above represents the map of an island.
The points A, B and C mark the positions of three hotels.

(a) Measure the bearing of C from B.

Answer ______________________ ° [1]

(b) An airport will be built on the island.
The location of the airport has to be equidistant from Hotel A and Hotel B.
It must also be equidistant from AB and AC.
Using geometrical constructions, explain whether the point P is a suitable location for the airport.

Answer ____

____ [3]