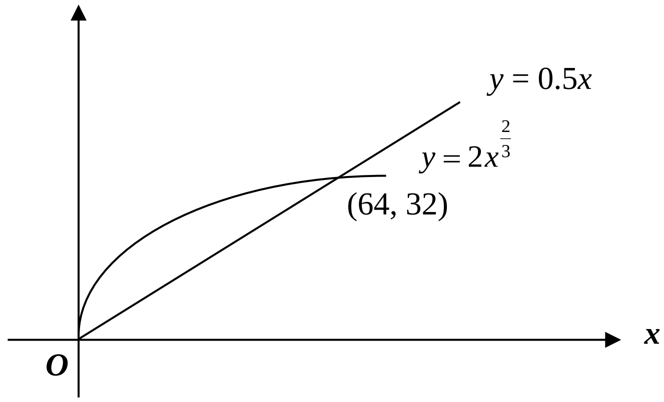
AMKSS 2019 Prelim 4E5N AM P1 Answer Scheme

	Answer	**Marks**
1(i) **[2]**	$f''(x) = 2px - 8$ $2p(2) - 8 = 4$ $p = 3$	M1 (differentiate) A1
1(ii) **[3]**	$f(x) = x^3 - 4x^2 + x + c$ $(3)^3 - 4(3)^2 + (3) + c = 0$ $c = 6$ $f(x) = x^3 - 4x^2 + x + 6$	M1 (correct integration without *c*) M1 (substitute point to find *c*) A1
2[4]	Gradient = $\dfrac{25-16}{16-10} = \dfrac{3}{2}$ $Y = \dfrac{3}{2}X + c$ $16 = \dfrac{3}{2}(10) + c$ $c = 1$ $Y = \dfrac{3}{2}X + 1$ $y^2 = \dfrac{3}{2}\sqrt{x} + 1$ $2^2 = \dfrac{3}{2}\sqrt{x} + 1$ $\sqrt{x} = 2$ $x = 4$	M1 (find gradient) M1 (substitute a point to find *c*) M1 (form equation) A1

3[7]	$\alpha+\beta=\frac{3}{2}$ $\alpha\beta=\frac{p}{2}$	M1
	$\alpha^3+\beta^3=-\frac{q}{8}$ $\alpha^3\beta^3=8$	M1
	$\left(\frac{p}{2}\right)^3=8$	M1 (substitution)
	$p^3=64$ $p=4$	A1
	$(\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2)=-\frac{q}{8}$	M1 (factorise cubic)
	$(\alpha+\beta)\left((\alpha+\beta)^2-3\alpha\beta\right)=-\frac{q}{8}$	
	$\left(\frac{3}{2}\right)\left(\left(\frac{3}{2}\right)^2-3(2)\right)=-\frac{q}{8}$	M1 (using their value of *p*)
	$q=45$	A1

4(i) (ii)(b) [4]		(i) B1 (correct shape passing through origin) (ii)(b) B1 (positive gradient straight line passing through origin) B1, B1 (indicate **<u>origin</u>** and (64, 32))
4(ii) (a) [2]	$2\sqrt[3]{x}=\sqrt{x}$ $2x^{\frac{1}{3}}=x^{\frac{1}{2}}$ $4x^{\frac{2}{3}}=x$ $2x^{\frac{2}{3}}=\frac{1}{2}x$ $y=\frac{1}{2}x$	M1 (change to index form) A1
4(ii) (b)	$2x^{\frac{1}{3}}=x^{\frac{1}{2}}$ $2x^{\frac{1}{3}}-x^{\frac{1}{2}}=0$ $x^{\frac{1}{3}}\left(2-x^{\frac{1}{6}}\right)=0$ $x=0$ or $x^{\frac{1}{6}}=2$ $x=64$ $y=0$ $\quad y=32$	Marks under 4(i) answer

5[5]	$\sqrt{18}x = 2x + 6$ $\sqrt{18}x - 2x = 6$ $x\left(\sqrt{18} - 2\right) = 6$ $x = \dfrac{6}{\sqrt{18} - 2}$ $= \dfrac{6}{\sqrt{18} - 2} \times \dfrac{\sqrt{18} + 2}{\sqrt{18} + 2}$ $= \dfrac{6\sqrt{18} + 12}{18 - 4}$ $= \dfrac{18\sqrt{2} + 12}{14}$ $= \dfrac{9\sqrt{2} + 6}{7}$ $a = 9,\ b = 6$	M1 (factorise) M1 (correct conjugate) M1 (expand) A1, A1
6(i) [1]	Let $\mathrm{f}\left(x\right) = 2x^3 + 11x^2 + 12x - 9$ $\mathrm{f}\left(\dfrac{1}{2}\right) = 2\left(\dfrac{1}{2}\right)^3 + 11\left(\dfrac{1}{2}\right)^2 + 12\left(\dfrac{1}{2}\right) - 9 = 0$ Since remainder = 0, $\left(2x - 1\right)$ is a factor.	M1 (must give conclusion)
6(i) [3]	$2x^3 + 11x^2 + 12x - 9 = \left(2x - 1\right)\left(x^2 + kx + 9\right)$ $2k - 1 = 11$ $k = 6$ $2x^3 + 11x^2 + 12x - 9 = \left(2x - 1\right)\left(x^2 + 6x + 9\right)$ $= \left(2x - 1\right)\left(x + 3\right)^2$	M1 (using comparing coeff or long division) M1 A1 (no mark if no working)
6(iii) [4]	$\dfrac{15x + 17}{\left(2x - 1\right)\left(x + 3\right)^2} = \dfrac{A}{2x - 1} + \dfrac{B}{x + 3} + \dfrac{C}{\left(x + 3\right)^2}$ $15x + 17 = A\left(x + 3\right)^2 + B\left(2x - 1\right)\left(x + 3\right) + C\left(2x - 1\right)$ Let $x = -3$, $-28 = -7C$ $C = 4$ Let $x = \dfrac{1}{2}$, $24\dfrac{1}{2} = 12\dfrac{1}{4}A$ $A = 2$ Comparing coeff. of x^2, $0 = 2 + 2B$ $B = -1$ $\dfrac{15x + 17}{2x^3 + 11x^2 + 12x - 9} = \dfrac{2}{2x - 1} - \dfrac{1}{x + 3} + \dfrac{4}{\left(x + 3\right)^2}$	M1 (or combine fractions) A1 A1 A1

7(i) **[3]**	$\text{LHS} = (\operatorname{cosec}\theta - \cot\theta)^2$ $= \left(\dfrac{1}{\sin\theta} - \dfrac{\cos\theta}{\sin\theta}\right)^2$ $= \left(\dfrac{1-\cos\theta}{\sin\theta}\right)^2$ $= \dfrac{(1-\cos\theta)^2}{\sin^2\theta}$ $= \dfrac{(1-\cos\theta)^2}{1-\cos^2\theta}$ $= \dfrac{(1-\cos\theta)^2}{(1-\cos\theta)(1+\cos\theta)}$ $= \dfrac{1-\cos\theta}{1+\cos\theta}$ $= RHS$ Alternative method: $\text{LHS} = (\operatorname{cosec}\theta - \cot\theta)^2$ $= \operatorname{cosec}^2\theta - 2\operatorname{cosec}\theta\cot\theta + \cot^2\theta$ $= \dfrac{1}{\sin^2\theta} - \dfrac{2}{\sin\theta}\times\dfrac{\cos\theta}{\sin\theta} + \dfrac{\cos^2\theta}{\sin^2\theta}$ M1 $= \dfrac{1-2\cos\theta+\cos^2\theta}{\sin^2\theta}$ $= \dfrac{1-2\cos\theta+\cos^2\theta}{1-\cos^2\theta}$ M1 $= \dfrac{(\cos\theta-1)^2}{1-\cos^2\theta}$ $= \dfrac{(\cos\theta-1)(\cos\theta-1)}{(1-\cos\theta)(1+\cos\theta)}$ $= \dfrac{-(\cos\theta-1)}{(1+\cos\theta)}$ M1 $= \dfrac{1-\cos\theta}{1+\cos\theta}$ $= RHS$	M1 (change to sin, cos) M1 (apply $\sin^2\theta = 1-\cos^2\theta$) M1 (factorise)
7(ii) **[3]**	$(\operatorname{cosec}\theta - \cot\theta)^2 = \dfrac{1}{4}$ $\dfrac{1-\cos\theta}{1+\cos\theta} = \dfrac{1}{4}$ $4 - 4\cos\theta = 1 + \cos\theta$ $\cos\theta = \dfrac{3}{5}$ Basic $\angle = 0.9272952180$ $\theta = 0.9272952180, 5.355890089$ $= 0.927, 5.36$ rad(3sf)	M1 A1, A1

8(i) **[3]**	Let $\angle EAC = \theta$ $\angle ACD = \theta$ (alternate angles) $\angle ABC = \theta$ (angles in alternate segments) $\angle DAC = \angle CAB$ (common angle) Triangles *ABC* and *ACD* are similar (all corresponding angles are equal / AA)	 M1 M1 A1 (+ statement to get 3 marks)
8(ii) **[3]**	$\angle AEC = \angle ABC = \theta$ (base angles of isosceles triangle) $\angle BCA = 2\theta$ (exterior angle of triangle) $\angle BCD = 2\theta - \theta$ $= \theta$ $= \angle ACD$ CD bisects $\angle ACB$.	M1 M1 A1
9(i) **[3]**	$\pi r^2 + 2\pi rh = 75\pi$ $r^2 + 2rh = 75$ $2rh = 75 - r^2$ $h = \dfrac{75 - r^2}{2r}$ $V = \pi r^2 h$ $= \pi r^2 \left(\dfrac{75 - r^2}{2r}\right)$ $= \dfrac{\pi}{2}\left(75r - r^3\right)$	M1 (form eqn) M1 (make *h* subject) M1 (substitute)
9(ii) **[3]**	$\dfrac{dV}{dr} = \dfrac{\pi}{2}\left(75 - 3r^2\right)$ $\dfrac{\pi}{2}\left(75 - 3r^2\right) = 0$ $r^2 = 25$ $r = 5 \quad or \quad r = -5$(rej) $V = \dfrac{\pi}{2}\left(75(5) - (5)^3\right) = 125\pi$ cm^3	B1 M1 (equate to 0) A1

10(i) **[1]**	$v = 3t^2 + 8t - 3$	B1
10(ii) **[6]**	$v = \int (6t+2)\,dt$ $= 3t^2 + 2t + c$ $-2 = 3(0)^2 + 2(0) + c$ $c = -2$ $v = 3t^2 + 2t - 2$ $s = \int (3t^2 + 2t - 2)\,dt$ $= t^3 + t^2 - 2t + c_1$ $0 = (0)^3 + (0)^2 - 2(0) + c_1$ $c_1 = 0$ $s = t^3 + t^2 - 2t$ $t^3 + 4t^2 - 3t - 14 = t^3 + t^2 - 2t$ $3t^2 - t - 14 = 0$ $(3t-7)(t+2) = 0$ $t = 2\frac{1}{3}$ or $t = -2$ (rej)	M1 (integrate without c) A1 M1 (integrate without c_1) A1 M1 (equate) A1
10(iii) **[2]**	When $t = 2\frac{1}{3}$, P: $v = 3\left(2\frac{1}{3}\right)^2 + 8\left(2\frac{1}{3}\right) - 3 = 32 > 0$ Q: $v = 3\left(2\frac{1}{3}\right)^2 + 2\left(2\frac{1}{3}\right) - 2 = 19 > 0$ *P* and *Q* are travelling in the same direction at the point of collision.	M1 A1

11(i) **[3]**	$\frac{3x+9}{5-x}=-\frac{1}{2}x+\frac{1}{2}$ $\frac{3x+9}{5-x}=\frac{-x+1}{2}$ $6x+18=(-x+1)(5-x)$ $6x+18=-5x+x^2+5-x$ $x^2-12x-13=0$ $(x-13)(x+1)=0$ $x=13$ or (rej) $\quad x=-1$ $y=1$ $P(-1,1)$	M1 (substitution) M1 (correct quadratic) M1 (solving)
11(ii) **[7]**	$y=\frac{3x+9}{5-x}$ $=\frac{24-3(5-x)}{5-x}$ $=\frac{24}{5-x}-3$ $\frac{3x+9}{5-x}=5$ $3x+9=25-5x$ $8x=16$ $x=2$ $\int_{-1}^{2}\left(\frac{24}{5-x}-3\right)dx$ $=\left[-24\ln(5-x)-3x\right]_{-1}^{2}$ $=\left[-24\ln(5-2)-3(2)\right]-\left[-24\ln(5-(-1))-3(-1)\right]$ $=7.635532333$ units^2 Area of rectangle $5\times(2-(-1))=15$ units^2 Area of shaded region $=15-7.635532333$ $=7.36446767$ $=7.36$ units^2 (3sf.)	M1 (or other valid method) B1 (correct x-coordinate of Q) M1, M1 (for ln and $-3x$) M1 Evaluate from –1 to 2 B1 (for rectangle) A1

12(i) **[3]**	$y=\frac{4}{3}x-16$ Gradient of normal = $-\frac{3}{4}$ $y=-\frac{3}{4}x+c$ $0=-\frac{3}{4}(12)+c$ $c=9$ $y=-\frac{3}{4}x+9$	B1 M1 (substitute point *A*) A1
12(ii) **[5]**	$\sqrt{(a-12)^2+(b-0)^2}=5$ $\sqrt{(a-12)^2+\left(-\frac{3}{4}a+9\right)^2}=5$ $a^2-24a+144+\frac{9}{16}a^2-\frac{27}{2}a+81=25$ $\frac{25}{16}a^2-\frac{75}{2}a+200=0$ $a^2-24a+128=0$ $(a-8)(a-16)=0$ $a=8$ or $a=16$ $b=3$ $b=-3$ (rej)	M1 (use distance formula or eqn of circle) M1 (substitute using equation of normal) M1 (form quadratic, RHS = 0) A1, A1 (must reject or else deduct 1 mark)