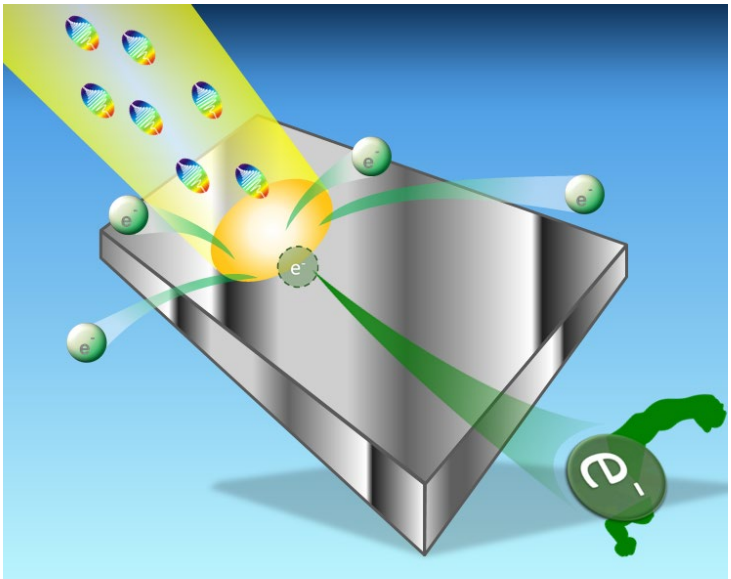
H2 Topic 19
Quantum Physics



The Nobel Prize in Physics 1921 was awarded to Albert Einstein "*for his services to Theoretical Physics, and especially for his discovery of the law of the photoelectric effect*."

Content

- Energy of a photon
- The photoelectric effect
- Wave-particle duality
- Energy levels in atoms
- Line spectra
- X-ray spectra
- The uncertainty principle

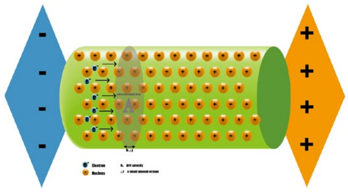
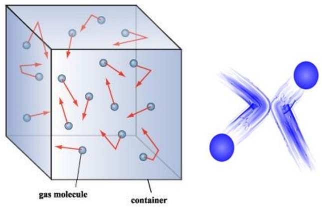
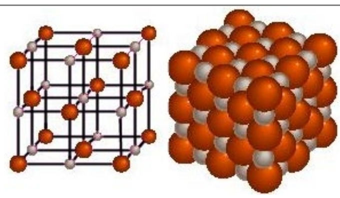
Learning Outcomes

Candidates should be able to:

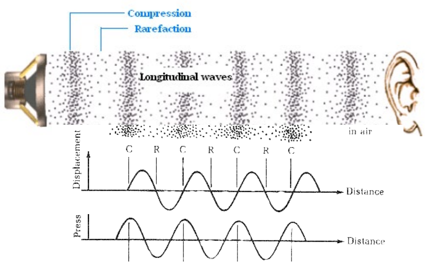
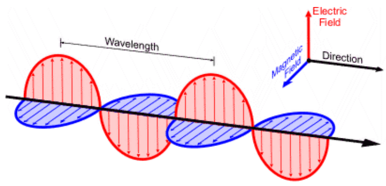
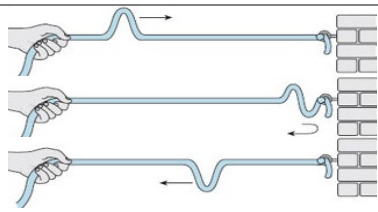
(a) show an appreciation of the particulate nature of electromagnetic radiation
(b) recall and use the equation $E = hf$ for the energy of a photon
(c) show an understanding that the photoelectric effect provides evidence for the particulate nature of electromagnetic radiation while phenomena such as interference and diffraction provide evidence for the wave nature
(d) recall the significance of threshold frequency
(e) recall and use the equation $\frac{1}{2}mv_{max}^2 = eV_s$, where V_S is the stopping potential
(f) explain photoelectric phenomena in terms of photon energy and work function energy
(g) explain why the stopping potential is independent of intensity whereas the photoelectric current is proportional to intensity at constant frequency
(h) recall, use and explain the significance of the equation $hf = \Phi + \frac{1}{2}mv_{max}^2$
(i) describe and interpret qualitatively the evidence provided by electron diffraction for the wave nature of particles
(j) recall and use the relation for the de Broglie wavelength $\lambda = h / p$
(k) show an understanding of the existence of discrete electronic energy levels in isolated atoms (e.g. atomic hydrogen) and deduce how this leads to the observation of spectral lines
(l) distinguish between emission and absorption line spectra
(m) recall and solve problems using the relation $hf = E_2 - E_1$
(n) explain the origins of the features of a typical X-ray spectrum
(o) show an understanding of and apply $\Delta p \Delta x \gtrsim h$ as a form of the Heisenberg position-momentum uncertainty principle to new situations or to solve related problems.

19.1 Particles and waves

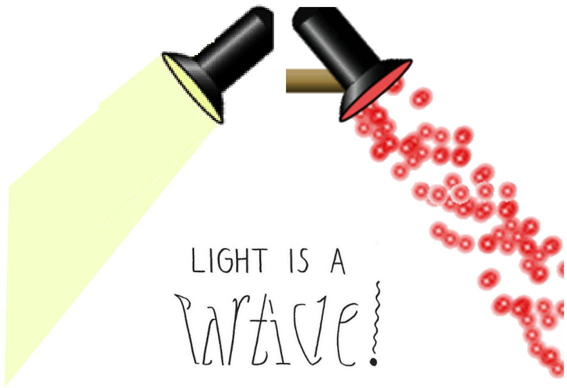
We often model matter using particles (small masses) that are hard, rigid and move according to Newton's Laws of motion. The macroscopic phenomena can then be explained by the random motion and collisions of the distribution of particles.

area	microscopic model	macroscopic phenomena	typical diagrams
electricity	flow of electrons	current	
gases	Kinetic Theory	pressure, volume, and temperature of gas	
solids	lattice structure	mechanical properties	

On the other hand, waves are used to explain the transport of energy without matter being transported. The medium of transport is often described as oscillating.

area	oscillating quantity	typical diagrams
sound	air pressure	
light (electromagnetic radiation)	electric field and magnetic field	
waves on a string	displacement (of string)	

19.2 Light can behave like waves or like particles



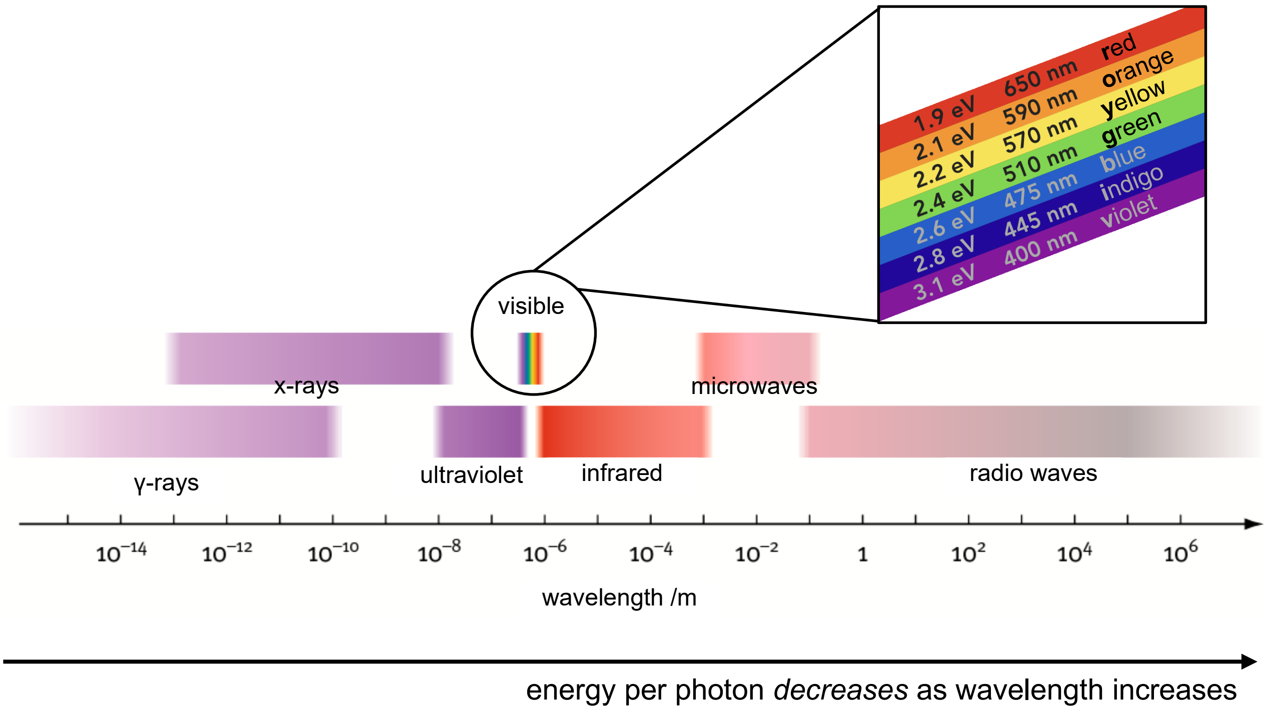
Light (or, in general, electromagnetic radiation) can undergo *superposition* and *interference*. These are *strong evidences that electromagnetic radiation behaves like waves.*

However, in order to explain the photoelectric effect, electromagnetic radiation needs to be treated as if they were particles.

19.3 Energy of a photon

A **photon** is a <u>discrete packet of energy</u> of <u>electromagnetic radiation.</u> Energy of one photon = Planck constant × frequency	$E = hf$

The Planck constant, *h*, is $h = 6.63 \times 10^{-34}$ J s .



Example 1
Determine the energy of one photon of frequency 5.5 x 10^{14} Hz.

$$E = hf$$
$$= (6.63 \times 10^{-34})(5.5 \times 10^{14})$$
$$= 3.6 \times 10^{-19} \text{ J}$$

Note: The energy in Joules for a photon is not a convenient order of magnitude, so the electronvolt (eV) is often used. One eV is the energy gained by an electron that is accelerated through the potential difference of 1 V:

1 eV = 1.6 x 10^{-19} C x 1 V = 1.6 x 10^{-19} J.

Example 2

Visible light has wavelengths spanning from 400 nm (violet) to 700 nm (red). Find the energy, in eV, of (i) a photon of red light and (ii) a photon of violet light.

(i) For red light:

$$E_{red} = hf = \frac{hc}{\lambda}$$
$$= \frac{(6.63 \times 10^{-34})(3 \times 10^{8})}{700 \times 10^{-9}}$$
$$= 2.84 \times 10^{-19} \text{ J}$$
$$= 1.78 \text{ eV}$$

(ii) For violet light:

$$E_{violet} = hf = \frac{hc}{\lambda}$$
$$= \frac{(6.63 \times 10^{-34})(3 \times 10^{8})}{400 \times 10^{-9}}$$
$$= 4.97 \times 10^{-19} \text{ J}$$
$$= 3.11 \text{ eV}$$

Note: Ultra*violet* causes skin cancer while infra*red* causes heating. Per photon, "*blue*-er" photons are more energetic than "*red*-der" photons.

Example 3

A 1.0 mW laser produces red light of wavelength 663 nm.

(i) Calculate the number of photons that the laser produces in one second.

(ii) The same 1.0 mW laser is then tuned to produce UV light of wavelength 221 nm instead. Calculate how many UV photons in a second that the laser now produces.

(i)

energy per photon

$$E = hf = \frac{hc}{\lambda}$$
$$= \frac{(6.63 \times 10^{-34})(3.0 \times 10^{8})}{663 \times 10^{-9}}$$
$$= 3.0 \times 10^{-19} \text{ J}$$

energy released in 1s

$$E_{\text{tot}} = Pt = 1.0\text{mW} \times 1\text{s} = 1.0 \times 10^{-3} \text{J}$$

number of photons per second

$$= \frac{E_{\text{tot}}}{E} = \frac{1.0 \times 10^{-3}}{3.0 \times 10^{-19}}$$
$$= 3.33 \times 10^{15} \text{ s}^{-1}$$

(ii)

energy per photon

$$E = hf = \frac{hc}{\lambda}$$
$$= \frac{(6.63 \times 10^{-34})(3.0 \times 10^{8})}{221 \times 10^{-9}}$$
$$= 9 \times 10^{-19} \text{ J}$$

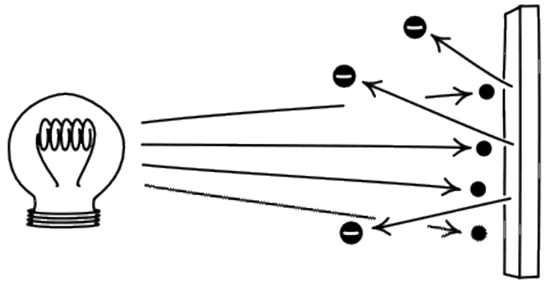
number of photons per second

$$= \frac{E_{\text{tot}}}{E} = \frac{1.0 \times 10^{-3}}{9.0 \times 10^{-19}}$$
$$= 1.11 \times 10^{15} \text{ s}^{-1}$$

Note: **1**. $E = hf = \frac{hc}{\lambda}$, because $f\lambda = c$ for photons. **2**. For a fixed power output (or intensity), there can be either (i) more photons each of lower energy (lower frequency or longer wavelength) or (ii) fewer photons each of higher energy (higher frequency or shorter wavelength).

19.4 The photoelectric effect

Photoelectric effect is the emission of electrons when electromagnetic radiation of high-enough frequency is incident on a metal surfaces



Some characteristics of the photo-emission:

- A single photon can only interact with (and pass on its energy to) a single electron. The energy transfer is all-or-nothing.
- Not all photons (of sufficient energy) get to interact with electrons. There is probability involved for a successful interaction – some photons will "miss" the electrons in the metal.
- The photoelectrons are emitted in all random directions with varying speeds.

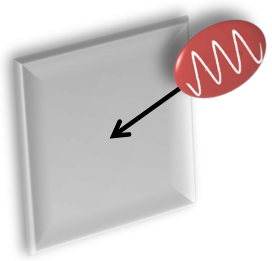
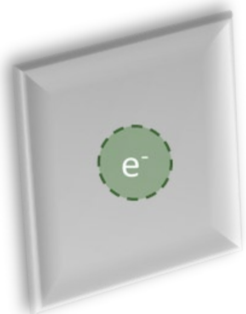
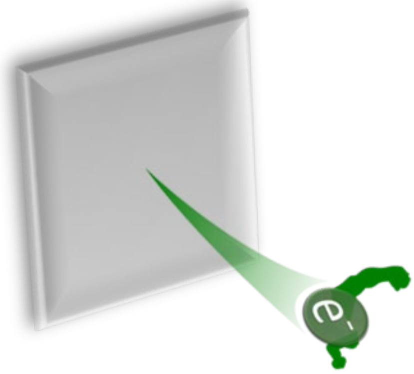
19.5 The photoelectric equation

Photoelectric equation is

$$hf = \Phi + \frac{1}{2}mv_{max}^2$$

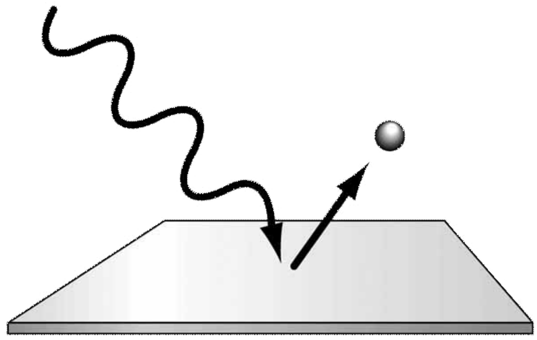
$$\begin{pmatrix}\text{energy provided by}\\ \text{a single photon}\end{pmatrix} = \begin{pmatrix}\text{work function energy}\\ \text{of a particular metal}\end{pmatrix} + \begin{pmatrix}\text{maximum kinetic energy}\\ \text{of a photoelectron}\end{pmatrix}$$

The photoelectric equation is a statement of the principle of conservation of energy.

		
A photon of energy *hf* is incident on the metal surface.	Photon meets an electron. Electron absorbs all energy of the one photon.	Some of the energy is used to escape from the metal. The rest remains as kinetic energy of the electron.

19.6 The work function energy Φ

Work function energy is the minimum energy needed to cause emission of electron from the surface of a metal



The work function energy Φ is unique to each type of metal. It is typically measured in eV.

The work function energy can be affected by the surface condition of the metal, such as it being oxidised.

Example 4

Light of wavelength 543 nm is incident on a clean sodium surface. The photoelectrons released are found to have negligible amount of kinetic energy. Determine the work function Φ for sodium metal in eV.

$$hf = \Phi + \frac{1}{2}mv_{\max}^2$$

$$\frac{hc}{\lambda} = \Phi_{\text{Na}} + E_{K,\max}$$

$$\begin{aligned}\Phi_{\text{Na}} &= \frac{hc}{\lambda} - E_{K,\max} \\ &= \frac{(6.63\times10^{-34})(3\times10^{8})}{543\times10^{-9}} - 0 \\ &= 3.66\times10^{-19}\text{ J} \\ &= 2.29\text{ eV}\end{aligned}$$

19.7 The threshold frequency and threshold wavelength

Threshold frequency is the minimum frequency of electromagnetic radiation to cause emission of electron from the surface of a metal

$$\boxed{\Phi = hf_0}$$

In other words, it is the frequency of electromagnetic radiation for electrons to be emitted from metal surface with zero kinetic energy.

$$hf = \Phi + \frac{1}{2}mv_{\max}^2$$

$$hf_0 = \Phi + 0$$

Threshold frequency is therefore unique to each type of metal and can be affected by the surface condition.

Threshold wavelength is the maximum wavelength of electromagnetic radiation to cause emission of electron from the surface of a metal

$$\boxed{\Phi = \frac{hc}{\lambda_0}}$$

In other words, it is the wavelength of electromagnetic radiation for electrons to be emitted from metal surface with zero kinetic energy.

$$\frac{hc}{\lambda} = \Phi + \frac{1}{2}mv_{\max}^2$$

$$\frac{hc}{\lambda_0} = \Phi + 0$$

Threshold wavelength is therefore unique to each type of metal and affected by the surface condition.

Example 5

Magnesium metal has a work function of 3.63 eV. Find the maximum wavelength of electromagnetic radiation needed for emission of electron to occur.

$$hf_0 = \Phi$$
$$\frac{hc}{\lambda_0} = \Phi$$

$$\lambda_0 = \frac{hc}{\Phi} = \frac{(6.63 \times 10^{-34})(3 \times 10^{8})}{(3.63)(1.6 \times 10^{-19})}$$
$$= 342 \times 10^{-9} \text{ m}$$

Example 6

The maximum kinetic energy of the electrons emitted from a metallic surface is 0.99 eV when the frequency of the incident radiation is 7.5 × 10^{14} Hz. Determine

(i) the work function of the metal in eV;

(ii) the threshold frequency.

(i)

$$hf = \Phi + \frac{1}{2}mv_{max}^2$$
$$\Phi = hf - E_{K,\,max}$$
$$= (6.63 \times 10^{-34})(7.5 \times 10^{14}) - (0.99)(1.6 \times 10^{-19})$$
$$= 3.39 \times 10^{-19} \text{ J}$$
$$= 2.12 \text{ eV}$$

(ii)

$$\Phi = hf_0$$
$$f_0 = \frac{\Phi}{h}$$
$$= \frac{3.39 \times 10^{-19}}{6.63 \times 10^{-34}}$$
$$= 5.11 \times 10^{14} \text{ Hz}$$

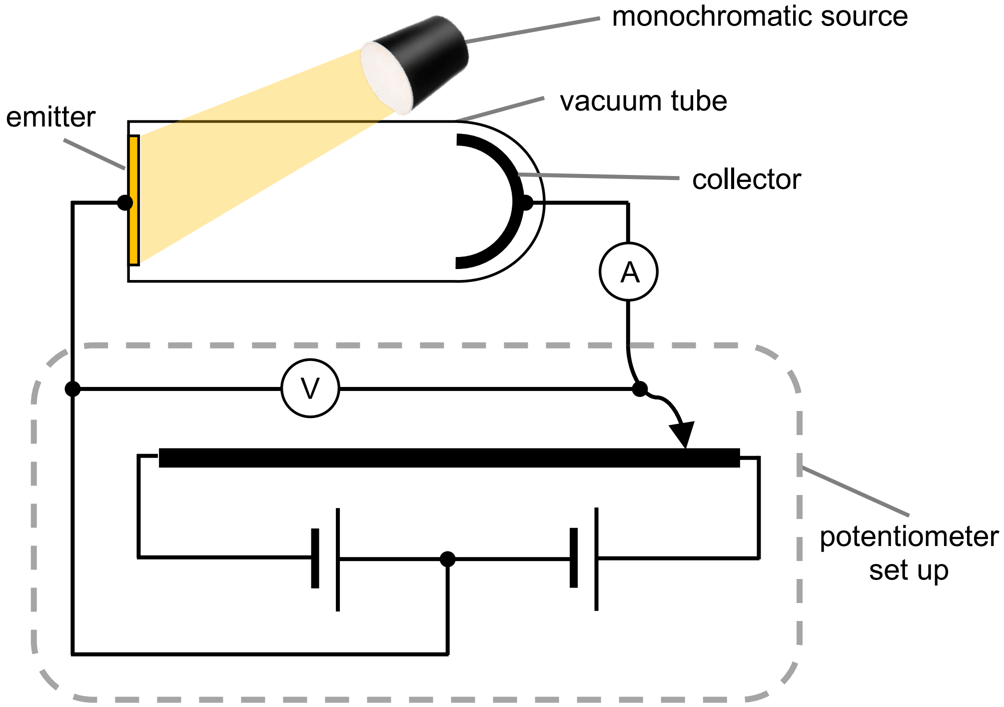
Example 7

Explain why, for any particular wavelength of electromagnetic radiation, most of the electrons are emitted with kinetic energies less than the maximum kinetic energy.

The photoelectrons only gain maximum kinetic energy when emitted from metal surface, thereby losing the minimum amount of energy before being emitted.

For electrons that are below the surface, some energy is required to bring the electron up to the surface before emission.

19.8 The photoelectric experimental set up



Apparatus	Function
Monochromatic source	Provide electromagnetic radiation (*light*) of a well-defined frequency/wavelength.
Vacuum tube	Allow movement of electrons without collision with ambient gas molecules. Sometimes otherwise called an evacuated tube.
Emitter	The metal surface where emission of electrons takes place.
Collector	When set at a positive potential relative to the emitter, the collector attracts the available photoelectrons. When set at a negative potential relative to the emitter, photoelectrons experience an electric force towards emitter and away from collector. This is useful for determining the maximum kinetic energy of photoelectrons.
Ammeter	Measures the photoelectric current.
Potentiometer set up	Varies the potential difference and polarity between emitter and collector. Allow for the two different settings that the collector works under (i.e., at a higher or lower potential than the emitter) in the experiment.

Example 8

Violet light of 400 nm is incident on a clean surface of potassium metal of work function 2.0 eV.

Determine the maximum speed of photoelectrons.

$$hf = \Phi + \frac{1}{2}mv_{max}^2$$

$$\frac{hc}{\lambda} = \Phi + \frac{1}{2}m_e v_{max}^2$$

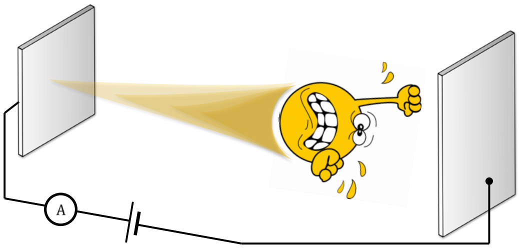
$$v_{max} = \sqrt{\frac{2\left(\frac{hc}{\lambda} - \Phi\right)}{m_e}} = \sqrt{\frac{2\left(\frac{(6.63\times10^{-34})(3\times10^{8})}{400\times10^{-9}} - (2.0)(1.6\times10^{-19})\right)}{9.11\times10^{-31}}} = 6.24\times10^{5}\ \text{ms}^{-1}$$

Note: Do check against logic and verify that the speed does not exceed the speed of light.

19.9 The stopping potential, V_s

The movement of electrons in the vacuum tube is invisible, and the velocities involved are large. A convenient way of measuring the maximum kinetic energy of the photoelectrons, is to “repel” them such that none of them can reach the collector, *that is, all the kinetic energy of the photoelectrons is converted into electric potential energy before any photoelectron can reach the collector plate.*

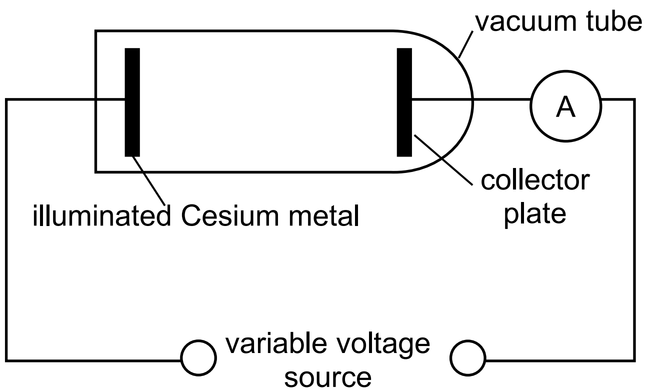
Stopping potential, V_s ,is the minimum potential difference between the emitting metal and collector that prevents the most energetic photoelectrons from reaching the collector plate, resulting in zero photoelectric current



$$\frac{1}{2}mv_{max}^2 = eV_s$$

Example 9

UV light of wavelength 350 nm falls onto Cesium metal of work function 2.1 eV. Find the stopping potential required for the ammeter to register zero current.



by conservation of energy,
max KE of electron
= gain in Electric PE of electron at V_s

$$\frac{1}{2}mv_{max}^2 = eV_s$$

$$hf = \Phi + \frac{1}{2}mv_{max}^2$$
$$= \Phi + eV_s$$

$$eV_s = hf - \Phi$$

$$V_s = \left(\frac{hc}{\lambda e} - \frac{\Phi}{e}\right)$$
$$= \left(\frac{(6.63\times10^{-34})(3\times10^{8})}{(350\times10^{-9})(1.6\times10^{-19})} - \frac{2.1\times1.6\times10^{-19}}{1.6\times10^{-19}}\right)$$
$$= 1.45 \text{ V}$$

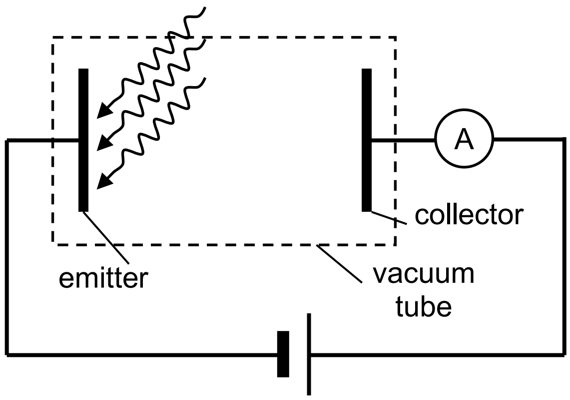
Note: The stopping potential is a negative voltage (relative to the emitter) as it needs to “repel” the photoelectrons.

Example 10

Light of wavelength 300 nm falls onto a metal emitter. The photon energy is enough to cause emission of photoelectrons. The ammeter reads 0.08 mA.

(i) Calculate the number of electrons being emitted per second.

(ii) If the probability of a photon successfully causing photoemission is 2.3%, estimate the power of the light shining on emitter.



Current flows only when photoelectric effect is active. Since the collector plate is positive with respect to the emitter, we assume all emitted photoelectrons reach the collector plate.

(i)

total charge emitted per second

$= It = 0.08\times10^{-3}\times1 = 0.08\times10^{-3}\text{ C}$

no. of electrons emitted per sec

$$= \frac{\text{total charge}}{\text{charge per electron}}$$

$$= \frac{0.08\times10^{-3}}{1.6\times10^{-19}} = 5.0\times10^{14}\ \text{s}^{-1}$$

(ii)

In 1 second,

$$\frac{\text{no. of photons causing photoemission}}{\text{total number of photons}}$$

$$= \frac{\text{no. of photoelectrons emitted}}{\text{total number of photons}} = 0.023$$

total number of photons

$$= \frac{\text{no. of photoelectrons emitted}}{0.023}$$

$$= \frac{5.0\times10^{14}}{0.023}$$

power = energy emitted in 1 second

$$= n_{\text{photon}}E_{\text{photon}}$$

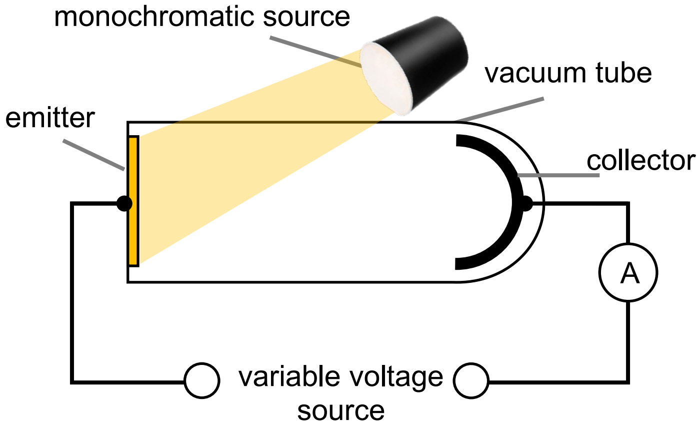
$$= n_{\text{photon}}\left(\frac{hc}{\lambda}\right)$$

$$= \frac{5.0\times10^{14}}{0.023}\left(\frac{(6.63\times10^{-34})(3\times10^{8})}{300\times10^{-9}}\right)$$

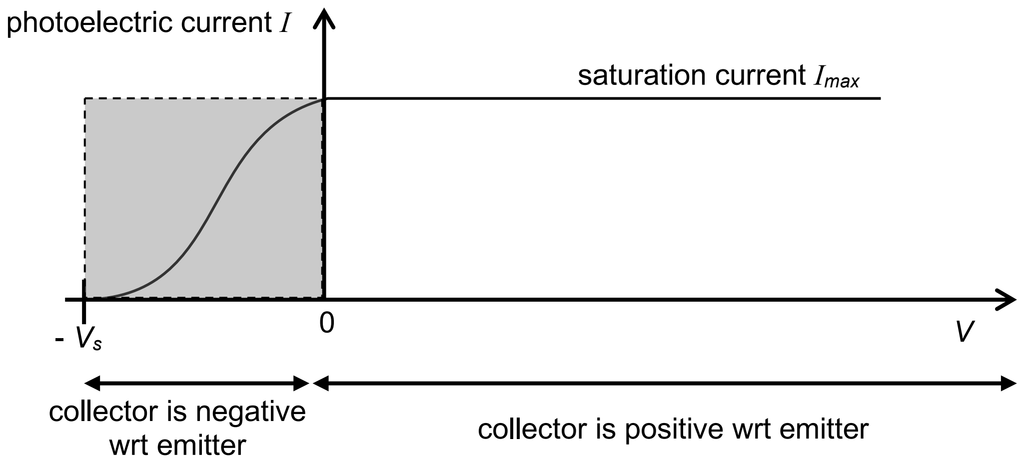
$$= 0.0144\text{ W}$$

19.10 The saturation current

By varying the potential difference as well as the polarity between the emitter and collector, we can investigate the change in the photoelectric current.



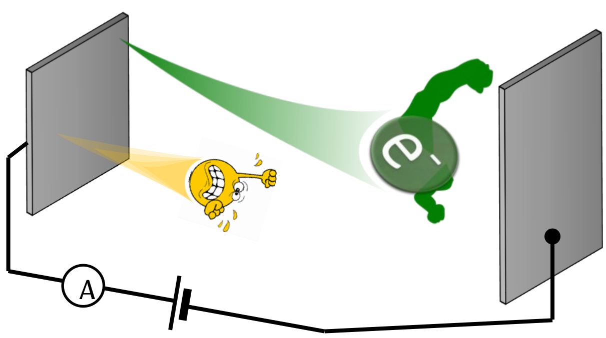
Experimentally, with the collector plate being at a slightly higher potential than the emitter, the photocurrent registered reaches a maximum value, called the saturation current (I_{max}). This corresponds to the case where all photoelectrons emitted are able to reach the collector plate.



What is the significance of the region in the grey dotted box?

The photoelectric current is in-between zero and the saturation current.

Only some photoelectrons are able to reach the collector plate.



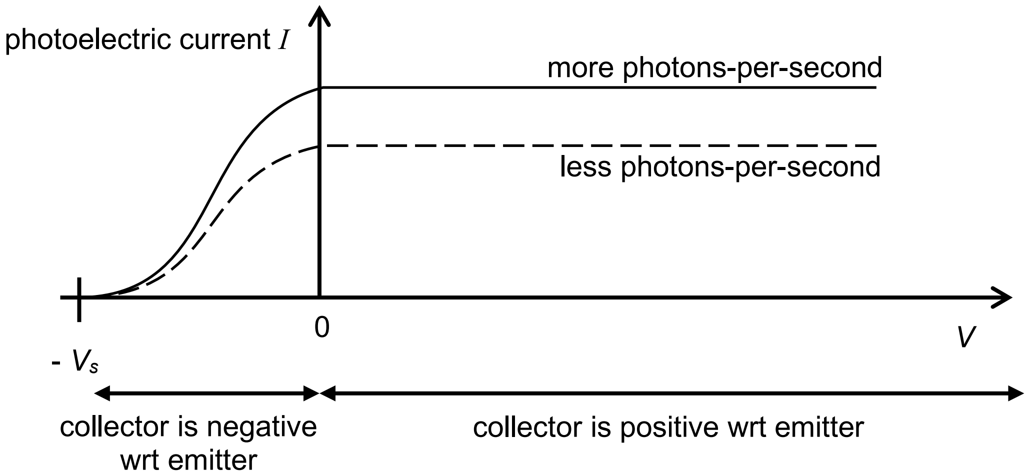
Explanation

When the collector plate is negative with respect to the emitter plate, electrons from the emitter plate loses kinetic energy (KE) and gains electric potential energy (EPE) as they move towards the collector plate. Hence, not all the emitted photoelectrons will make it to the collector plate. A photoelectron with a smaller KE will lose all its KE (and gained the corresponding amount of EPE), reach a state of being momentarily at rest, before turning back toward the emitter.

How will the graph change when the intensity of electromagnetic radiation increases?

If frequency remains constant, energy per photon remains constant, the maximum kinetic energy of photoelectrons remains constant, and so the stopping potential remains the same.
(the terms used in the corresponding photoelectric equation $hf = \Phi + \frac{1}{2}mv_{max}^2$ remains the same)

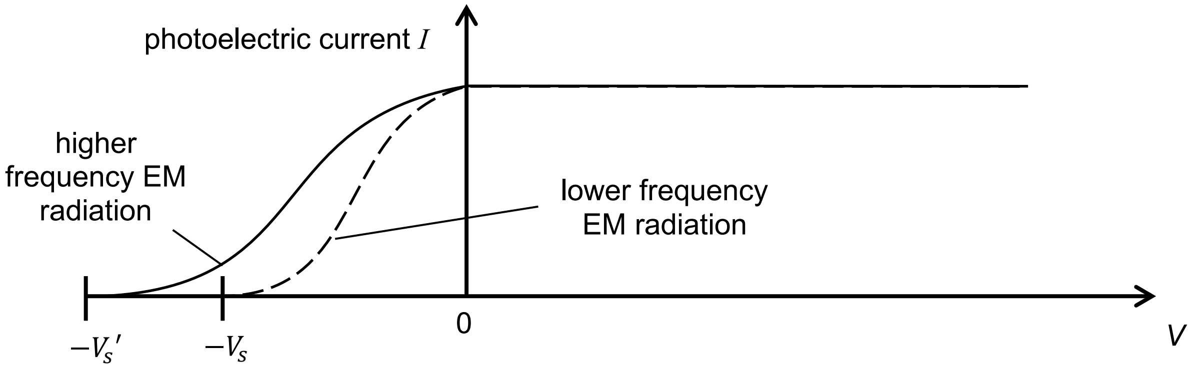
Since the energy per photon remains constant, with increasing intensity, there must be more photons reaching the emitter per second. Hence, more photoelectrons will be emitted per second, and the saturation current will be higher.



How will the graph change when the frequency of electromagnetic radiation increases, but the number of photons reaching the emitter per second remains the same?

Energy per photon, given by $E = hf$, increases. The maximum kinetic energy of photoelectrons increases. By the photoelectric equation $hf = \Phi + \frac{1}{2}mv_{max}^2 = \Phi + eV_s$, a larger stopping potential is required to prevent the most energetic photoelectrons from reaching the collector plate.

The number of photons reaching the emitter per second remains the same. The number of emitted electrons per second remains the same. The saturation current remains the same.

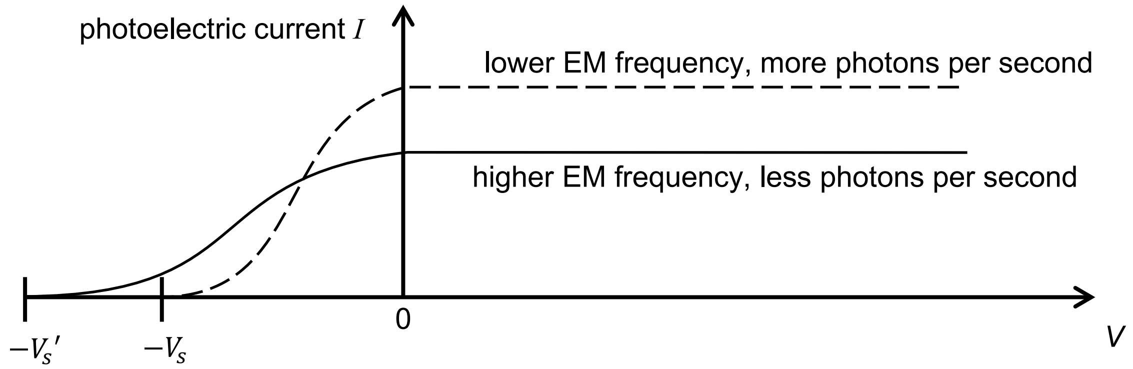


How will the graph change when the frequency of electromagnetic radiation increases while the power output of the light source remains?

The individual photon energy will increase but the number of photons reaching the emitter per second will decrease.

$$\text{power} = \text{no. of photons per unit time} \times \text{energy per photon}$$

The stopping potential will be of a larger magnitude, but the saturation current will decreases.



Example 11

In a photoelectric experiment, a metal with work function 1.8 eV is irradiated with light of wavelength 450 nm.

(i) Determine the stopping potential.

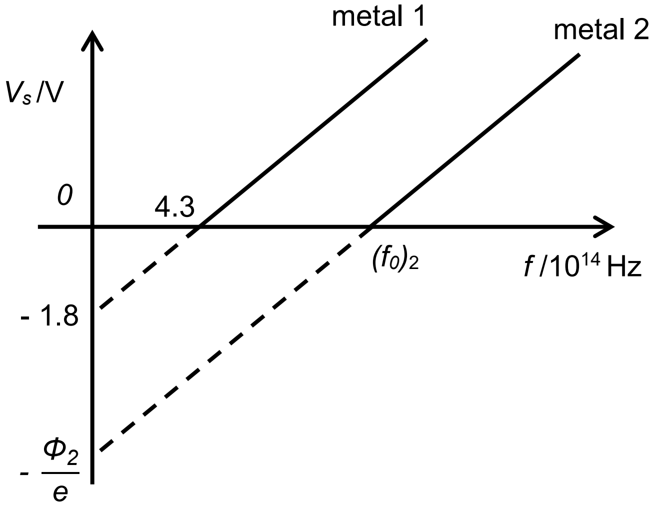
(ii) The metal is then replaced with another of a higher work function. Sketch the graph for both metals in the axes provided.

(i)

$$hf = \Phi + \frac{1}{2}mv_{\max}^2$$
$$= \Phi + eV_s$$
$$V_s = \left(\frac{h}{e}\right)(f) - \frac{\Phi}{e}$$
$$= \left(\frac{h}{e}\right)\left(\frac{c}{\lambda}\right) - \frac{\Phi}{e}$$
$$= \left(\frac{6.63\times10^{-34}}{1.6\times10^{-19}}\right)\left(\frac{3\times10^{8}}{450\times10^{-9}}\right) - \frac{1.8\times1.6\times10^{-19}}{1.6\times10^{-19}}$$
$$= 0.96 \text{ V}$$

(ii) Threshold frequency for metal 1:

$$\Phi = hf_0$$
$$f_0 = \frac{\Phi}{h}$$
$$= \frac{(1.8)(1.6\times10^{-19})}{6.63\times10^{-34}}$$
$$= 4.3\times10^{14} \text{ Hz}$$



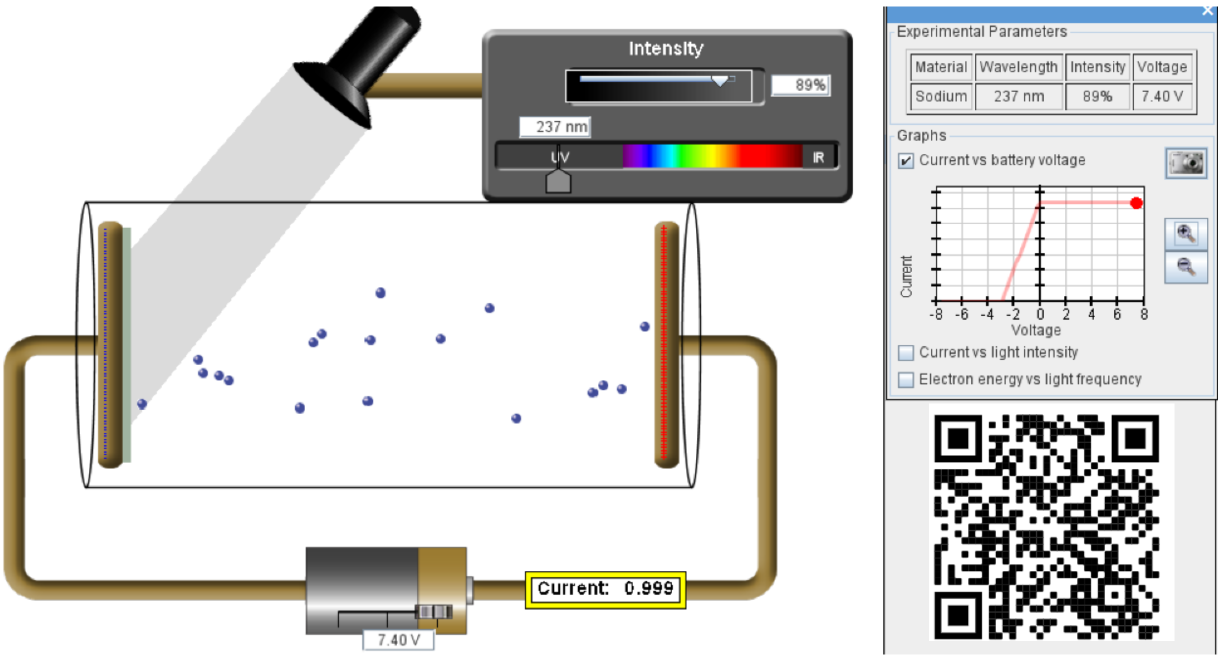
Note: Gradient is the same, as $\frac{h}{e}$ is constant. The y-intercept is $\frac{\phi}{e}$. Since there is no emission of electrons until the threshold frequency f_0 is reached, the x-intercept reflects f_0.

19.11 The particulate nature of electromagnetic radiation

The photoelectric effect provides evidence that electromagnetic radiation can behave like particles.

Evidence		**Failure of wave model's prediction**
(E)	**E**mission of electrons only when electromagnetic radiation is above a minimum (threshold) frequency regardless of intensity	Very intense light, regardless of frequency, should provide sufficient energy to cause emission of electrons.
(M)	**M**aximum kinetic energy of emitted electrons is affected by the frequency of electromagnetic radiation but is independent of intensity	Increased intensity of electromagnetic radiation should increase the maximum kinetic energy of the electrons.
(I)	**I**nstantaneous emission of electrons when electromagnetic radiation is above a minimum frequency even for very low intensity	Any frequency of electromagnetic radiation should cause emission of electron if exposure time is long enough and that the metal surface accumulates enough energy.

A virtual photoelectric experiment

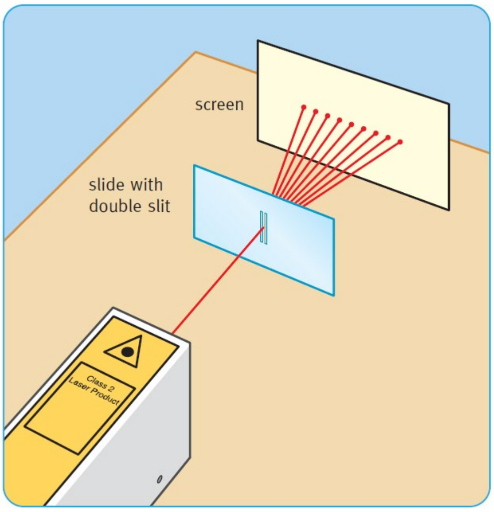
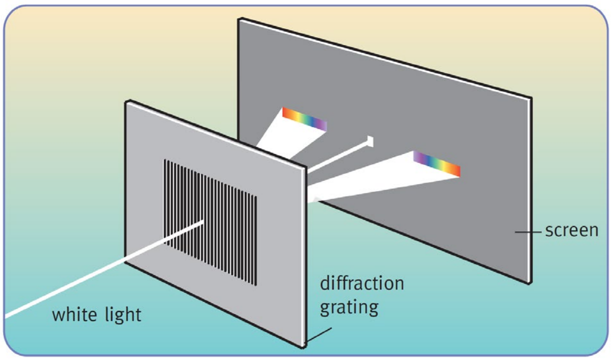


Explore the virtual photoelectric experiment using the QR Code. Adjust the intensity and the wavelength of the electromagnetic radiation from the lamp. Change the bias on the emitter and collector using the switch on the battery.

You should be able to explain the behaviour of the photoelectrons with what you learnt.

19.12 Waves undergo superposition and interference

Typical experimental set-ups involve double slits and diffraction grating:

Double slit interference	Diffraction grating
	
$\lambda = \dfrac{ax}{D}$	$d \sin\theta = n\lambda$

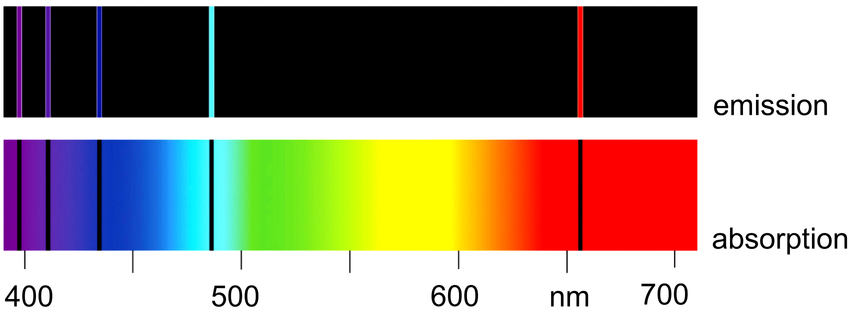
“White light” consists of light of all the wavelengths (400 to 700 nm) that are visible to human eyes. When white light passes through a diffraction grating, each wavelength is diffracted by a different angle. The resulting separation pattern, which looks like a rainbow, is a spectrum (plural: *spectra*).

19.13 Line Spectra

An absorption line spectra is a continuous spectrum crossed by dark lines

An emission line spectra is discrete bright lines of different colours on a dark background

Scan for the coloured image



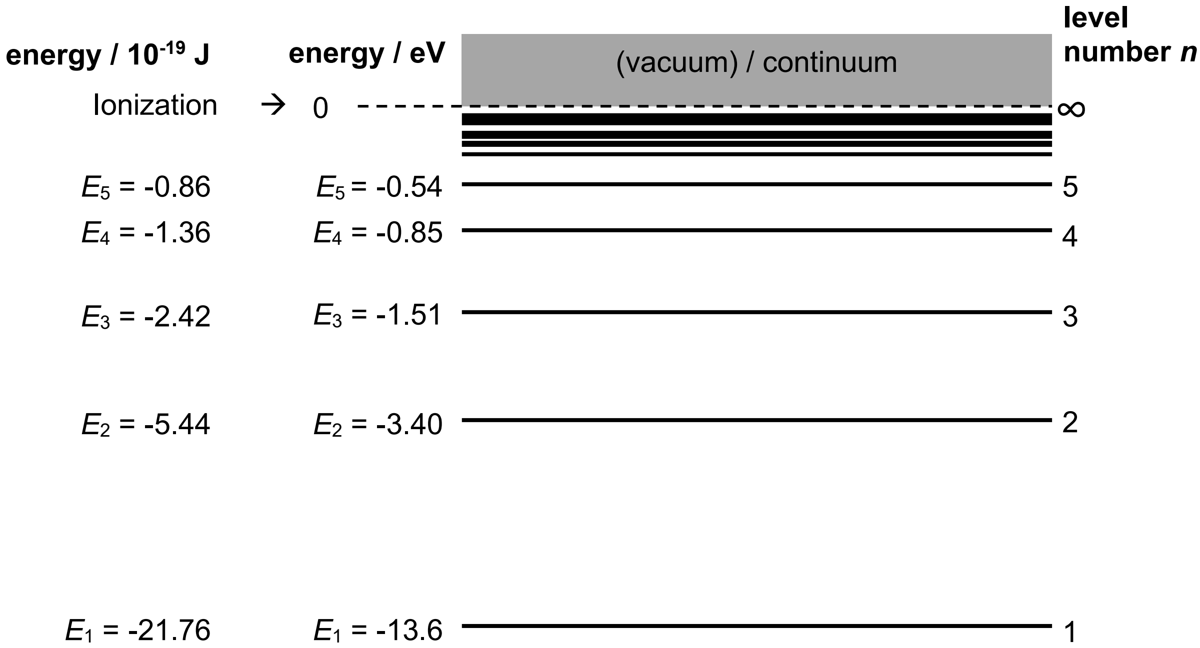
The *emission* line spectrum and the *absorption* line spectrum of hydrogen gas are shown above. Notice that an absorption spectrum is the *inverse* of the emission spectrum for the same element.

The positions of the lines are the same for neutral atoms belonging to the same element – the atoms can only *emit* or *absorb* light of certain wavelengths. These lines (representing wavelengths) are *unique* or *characteristic* of the element. In astrophysics (analyzing light from a distant star) or qualitative analysis of compounds, this feature is useful for identifying the presence of elements.

19.14 Line spectrum is evidence of discrete energy levels for electrons in isolated atoms

Each coloured/dark line corresponds to one wavelength or frequency represents a photon of a specific energy given by $E = hf$, that is emitted/absorbed when orbital electrons undergo specific energy changes when de-exciting/promoting between discrete energy levels in the atom (absorbed photons are then re-emitted in all directions shortly when the unstable, excited atom de-excites).

Energy levels of electrons in isolated atoms are typically represented by the following diagram:



(a) **All energy levels have negative values.**
The negatively charged electrons orbit the positively charged nucleus under the influence of an attractive electric force. This is similar to satellites orbiting the Earth under the gravitational force. The total energy (kinetic + potential energy) of such a bound system is negative.

(b) **The highest level of energy is taken to be zero**, corresponding to the case in which the electron is infinitely far from the nucleus.

(c) **The ground state is the most stable energy state.**
States with higher energy levels tend to deexcite to states with lower energy level.

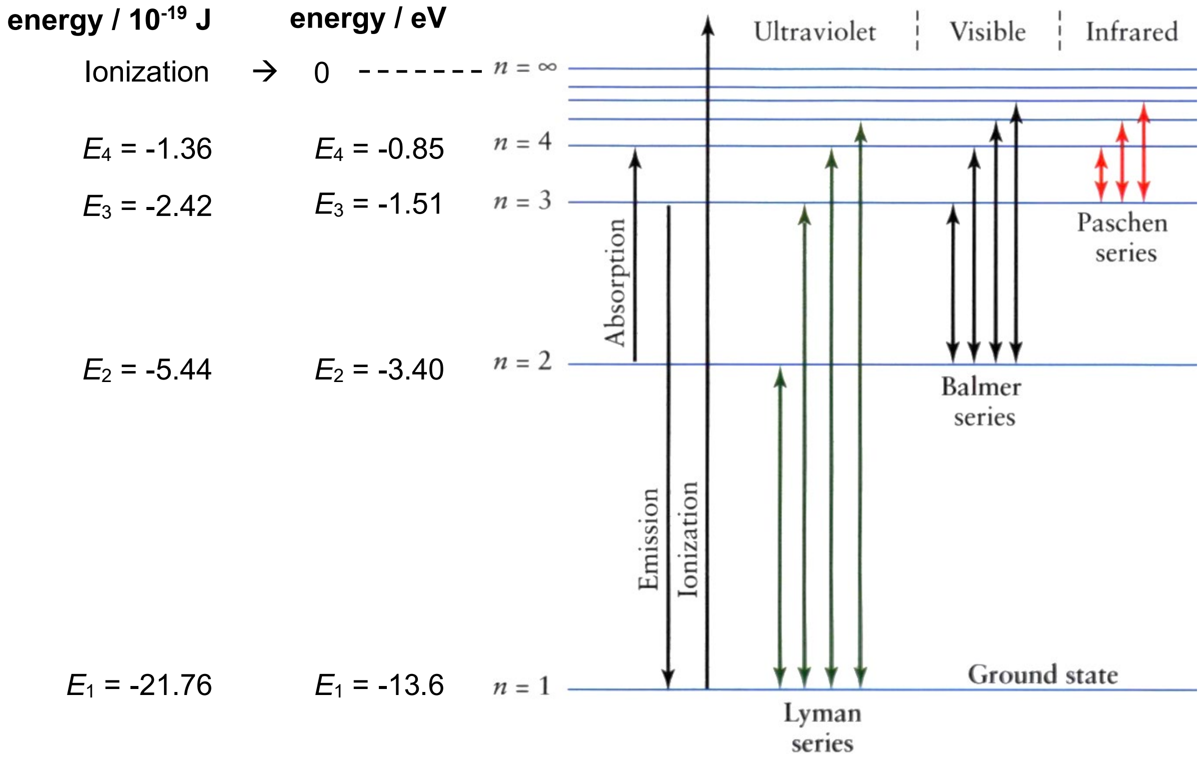
(d) **The ionisation energy is the minimum energy needed to remove an electron from the atom, or to move it to infinity**.
Energy required to remove from ground stated is $E_{\text{final}} - E_{\text{initial}} = 0 - E_1 = |E_1|$.

(e) **Electrons can only exist in such energy levels**, and not in between.

(f) **Electrons can make transitions between energy levels by absorbing or emitting energy.**

Many scientists studied the hydrogen atom extensively as it is the simplest to study (1 central proton and 1 electron obeying Coulomb's Law), so much so that the entire series of spectra resulting from the energy level transitions of the lone election in hydrogen were named after the scientists who discovered them.

It was also discovered that the energy levels for hydrogen follows $E_n = -\dfrac{13.6 \text{ eV}}{n^2}$.



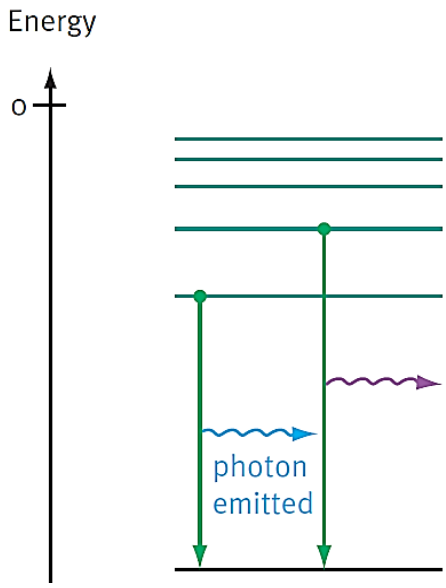
hydrogen spectra

19.15 De-excitation of electron in atom emits photon

An electron in an excited state (higher energy level) tends to deexcite. When an electron deexcites and transits to a lower energy level, the energy loss is emitted as a single photon which has energy that is equal to the difference between the initial and final energy levels.

a single photon is emitted by an atom when
a single electron deexcites
from a higher energy level to a lower energy level
in that atom

$$hf = E_{high} - E_{low}$$



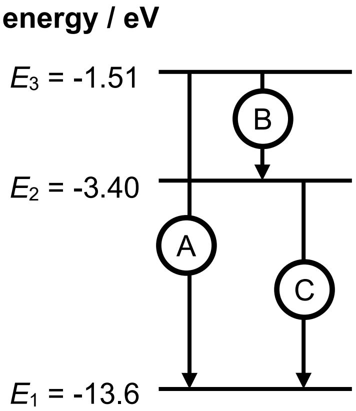
Example 12

An excited electron is at the E_3 energy state of a hydrogen atom.

(i) State the number of different possible de-excitation transitions possible.

(ii) Determine the wavelengths of the emitted photons with each possible transition, and identify which part of the EM spectrum these photons lie in.

3 transitions are possible:



$$E_{3\to1} = hf = \frac{hc}{\lambda_A}$$

$$\lambda_A = \frac{hc}{|E_1 - E_3|} = \frac{(6.63\times10^{-34})(3\times10^{8})}{|(-13.6-(-1.51))|(1.6\times10^{-19})}$$
$$= 103 \text{ nm (UV)}$$

$$E_{3\to2} = hf = \frac{hc}{\lambda_B}$$

$$\lambda_B = \frac{hc}{|E_2 - E_3|} = \frac{(6.63\times10^{-34})(3\times10^{8})}{|(-3.40-(-1.51))|(1.6\times10^{-19})}$$
$$= 658 \text{ nm (visible)}$$

$$E_{2\to1} = hf = \frac{hc}{\lambda}$$

$$\lambda_A = \frac{hc}{|E_1 - E_2|} = \frac{(6.63\times10^{-34})(3\times10^{8})}{|(-13.6-(-3.40))|(1.6\times10^{-19})}$$
$$= 122 \text{ nm (UV)}$$

19.16 Excitation of electron in atom requires energy input

If an atom is excited from a lower energy level to a higher energy level by absorbing a photon, the energy of the photon must be equal to energy difference between the two levels:

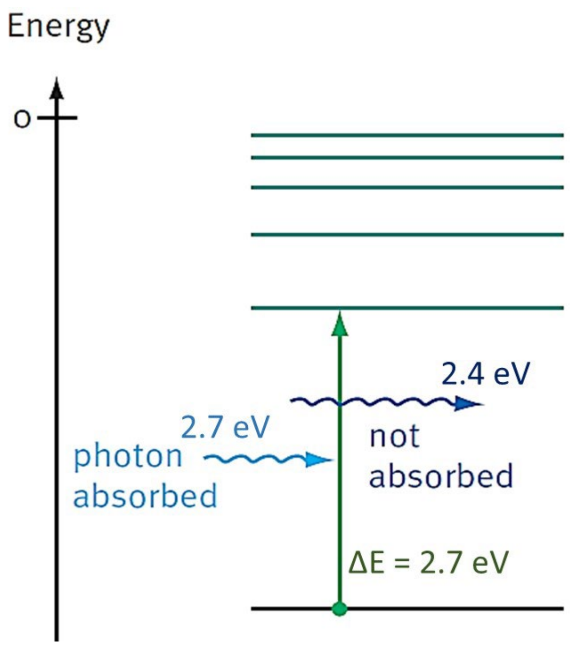
$$hf = \Delta E = E_{\text{final}} - E_{\text{initial}}$$

Energy required for electrons to transit from lower energy levels to higher energy levels is *absorbed by the atom* via 2 ways: by the absorption of a single photon, or by the collision of another high speed particle, such as an electron.

For photon absorption *by atom*, the energy of the photon ($E = hf$) must be just the correct amount for the electron to make the energy level transition.

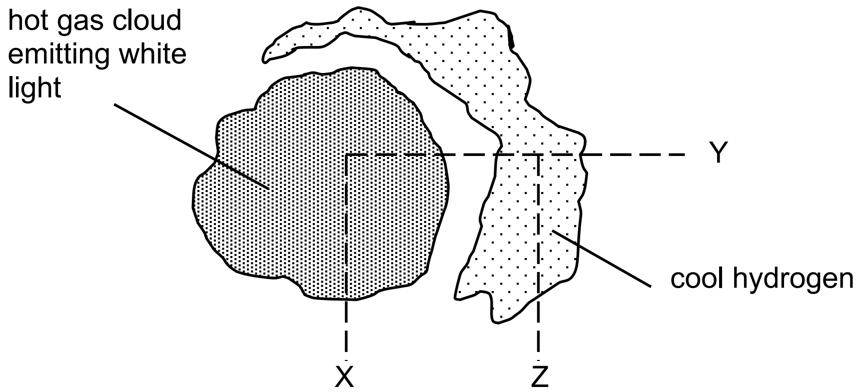
No such requirement exists for exciting electrons through bombarding *the atom* with high speed particles. *The atom* can absorb *some* of the kinetic energy required by the electron to make the transition, i.e. the kinetic energy of the bombarding particle can be *greater than or equal to* the energy difference between the energy levels involved in the transition.

In other words, the bombarding particle can lose any fraction of its kinetic energy during collision with the *atom*.

By Absorbing Photon	By High Speed Collisions
Cool gas is irradiated with electromagnetic radiation	Cool gas is bombarded by fast-moving particles
1 atom absorbs all the energy of 1 photon.	Atom absorbs ***part-of*** or ***all*** kinetic energy of a colliding particle. Particles can include ions, (other) atoms, or external electrons.
Photon energy must **exactly** be that of the difference in the electronic energy levels. $hf = \Delta E$ $= E_{final} - E_{initial}$	Kinetic energy of colliding particle *need not match* the energy level difference; but must be enough. $\frac{1}{2}mv^2_{colliding} \geq \lvert \Delta E \rvert$
 *A photon must have just the right amount of energy if it is to be absorbed by an atom.*	 *Neon signages (like these along the streets of Hong Kong) contain low pressure neon gas within a glass tube.* *A beam of electrons is passed through the tube. These electrons collide with, and transfer some/all of the kinetic energy to, the neon atoms, causing the excitation of these atoms. The amount of energy transferred to each excited neon atom is exactly equal to the energy difference between the two energy levels.* *When the excited atoms de-excite, photons of energy that is exactly the difference in energy levels are emitted and is seen by the human eye as coloured lights.*

Example 13

A cool region of hydrogen gas surrounds a hot gas cloud emitting white light in outer space. Describe and explain the type of spectrum observed from point X, Y and Z.



Point X:

- Continuous spectrum of wavelengths of electromagnetic radiation that are visible to the human eye.
- Appears as rainbow colours, which make up white light.

Point Y:

- Absorption spectrum, a continuous spectrum crossed by dark lines.
- The atoms in the cool hydrogen absorb photons of specific energy values that are equal to the difference between discrete energy levels when an orbital electron transits from a lower energy level to a higher energy level.
- The specific energy of an absorbed photon is $E = hf$, shows up as dark lines of specific frequencies, hence wavelengths, against white light spectrum.
- The absorbed photons are re-emitted in all directions, so only a small negligible fraction of the re-emitted photons are observed at Y, thus still appears as dark lines.

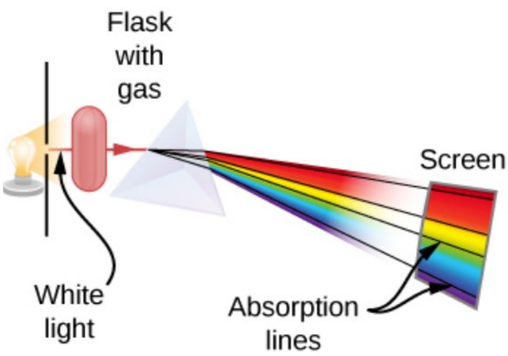
Point Z:

- Emission spectrum, discrete bright lines of different colours on a dark background.
- When cool hydrogen atoms absorb photons, they are excited and unstable. They shortly de-excite and re-emit photons in all directions.
- The photons have specific energy values that are equal to the difference in discrete energy levels when an orbital electron transits from a higher energy level to a lower energy level.
- The specific energy of a re-emitted photon is $E = hf$, and shows up as coloured lines of specific frequencies, hence wavelengths, on a dark background.

Example 14

Line spectra is evidence of discrete energy levels for electrons in isolated atoms.
Describe 2 experiments that prove this.

(a) Absorption spectrum of hydrogen



White light passes through a cold hydrogen gas that is contained in a glass flask.

Photons that correspond to the energy changes between energy levels are absorbed.

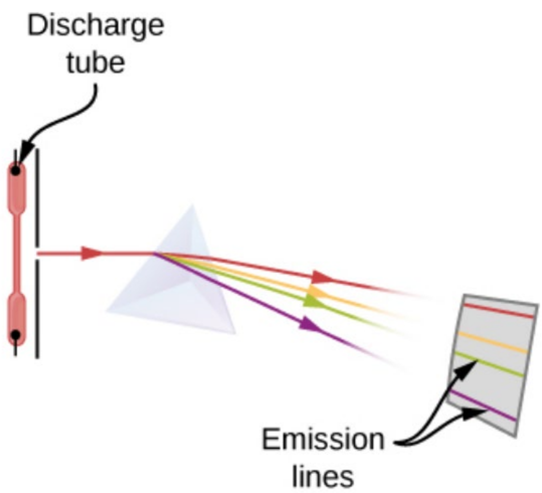
All other photons pass through unabsorbed.

As the atoms de-excite, photons are re-emitted in all directions, with much less intensity (negligible) in the original direction of travel.

A prism or diffraction grating separates the wavelengths.

This results in dark lines on a coloured background.

(b) Emission spectrum of hydrogen



Hydrogen gas is contained in a glass discharge tube with a high potential difference applied between the electrodes, which causes the gas to heat up and the atoms excited.

Upon de-excitation, photons are emitted as the electrons changes their energy level from higher to lower. These photon energies correspond to the energy difference between energy levels.

The light emitted from the gas passes through a prism or diffraction grating that separates its wavelengths.

This results in coloured lines on a dark background.

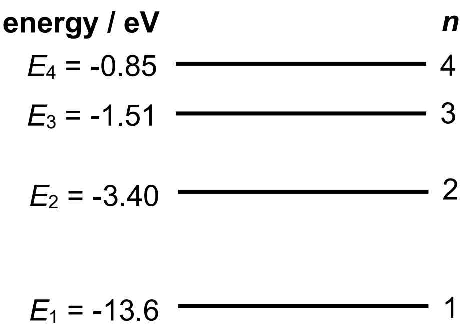
Each line corresponds to a specific photon energy. The discrete energy changes proves that atoms contain discrete electronic energy levels.

Example 15

The allowed energy levels of a hydrogen atom, in eV, are given by the formula $E_n = -\dfrac{13.6\text{ eV}}{n^2}$

(a) Draw a labelled energy level diagram for n = 1 to n = 4.
(b) Explain why a cool hydrogen atom will be unable to absorb a photon of 11.0 eV.
(c) State the number of absorption lines expected for electronic transitions between these energy levels for a cool, diffuse hydrogen gas.
(d) Given a hot hydrogen gas where the atoms are all excited to the n = 4 state, state the number of emission lines expected when the gas returns to a cool, diffuse state.

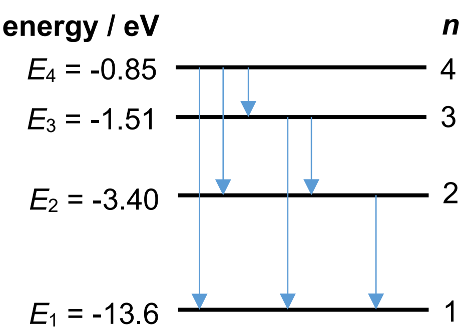
(a)



(b) An atom can only absorb a photon of energy that is exactly the difference in discrete energy levels of the atom.
A cool hydrogen atom has the electron at the ground state of n = 1.
None of the transitions from n = 1 to n = 2, 3 or 4, has an energy difference of 11.0 eV.

(c) 3.
A cool hydrogen gas has atoms at ground state of n = 1.
The transitions are $n_{1\to2}$, $n_{1\to3}$ and $n_{1\to4}$.

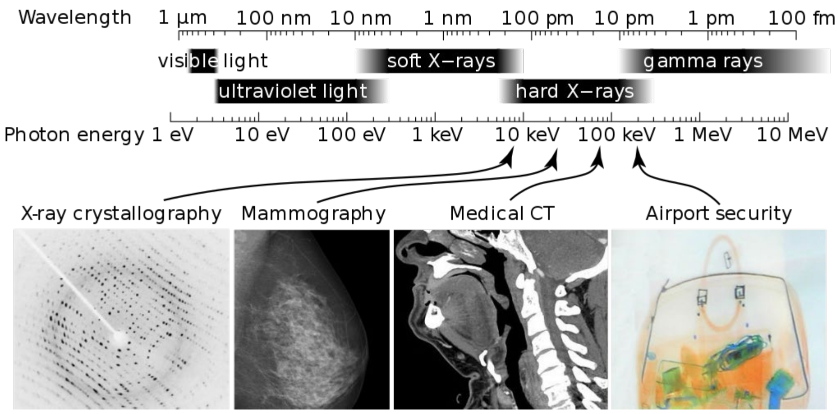
(d) 6. For de-exciting to ground state, there are 6 possible transitions:



19.17 X-rays

X-rays are very high-energy **photons** with **wavelengths ranging from 0.01 nm to 10 nm**. The wavelength is in the same order of magnitude as the interatomic spacing in crystals, making it suitable to study crystal structures.

X-rays are highly penetrating. This property is useful for non-invasive medical imaging of animal bodies because different types of body tissue absorb X-rays in different amounts. Because ionizing radiation is absorbed and over-exposure can cause dangerous changes to living tissue at a cellular level, the quantity and frequency of X-ray exposure has to be moderated.

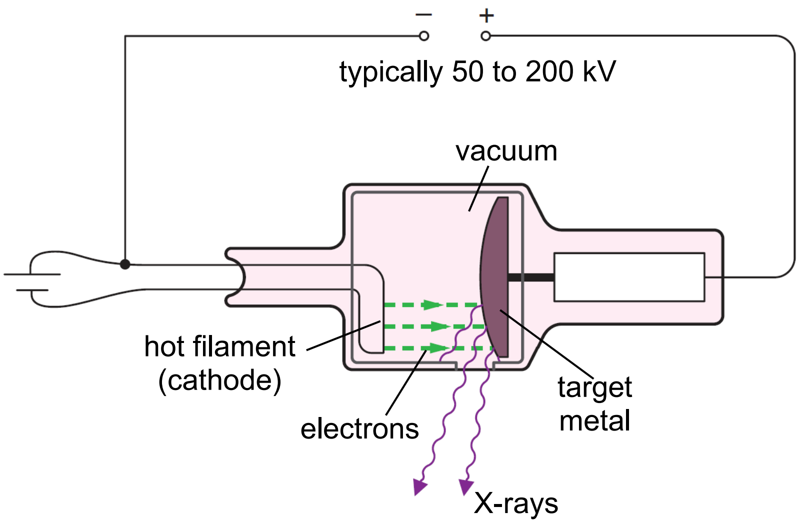


19.18 X-ray production is somewhat the reverse of photoelectric effect

For photoelectric effect, photons are incident on a metal surface, resulting in the emission of electrons.

In X-ray production, electrons strike a target metal, causing emission of photons, some of which are X-ray photons.

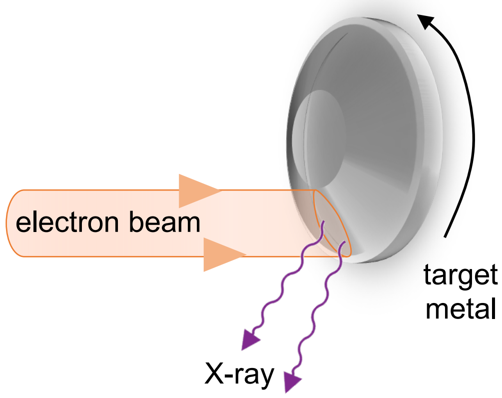
A typical set up for x-ray production is shown below.



A filament is heated up in order to emit electrons.

The electrons gain kinetic energy by accelerating through a large potential difference.

The high-speed electrons strike the target metal and lose their kinetic energy in the form of X-ray photons.

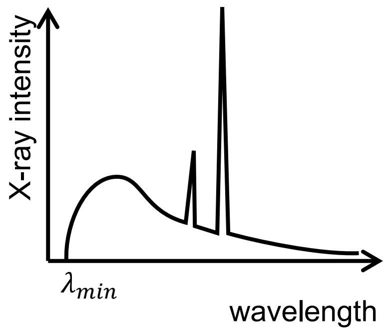
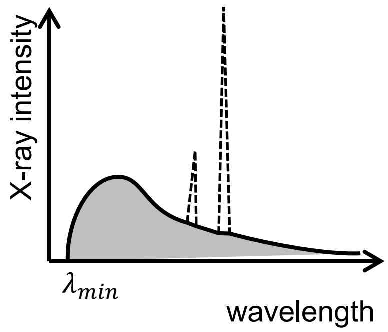
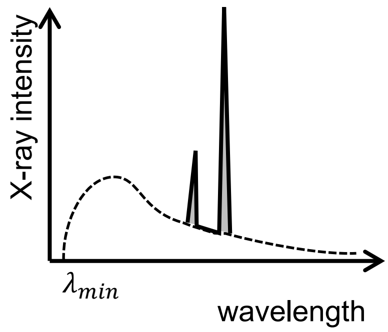


19.19 A typical X-ray spectrum has two components

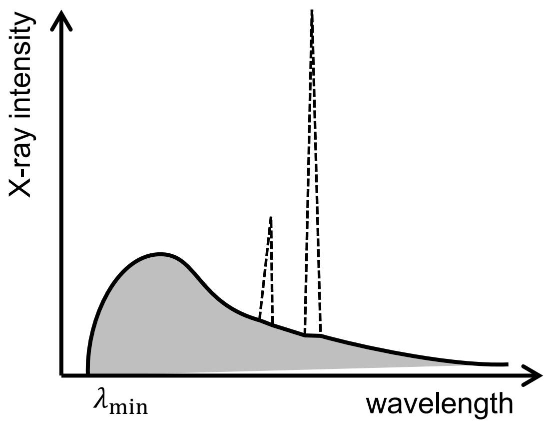
A typical X-ray spectrum spans a range of wavelengths and has two components, which are:

- a *continuous spectrum*
- *characteristic peaks*

These correspond to the two ways that a single bombarding electron can lose its kinetic energy when it strikes the target metal.

X-ray spectrum	continuous spectrum	characteristics peaks
		

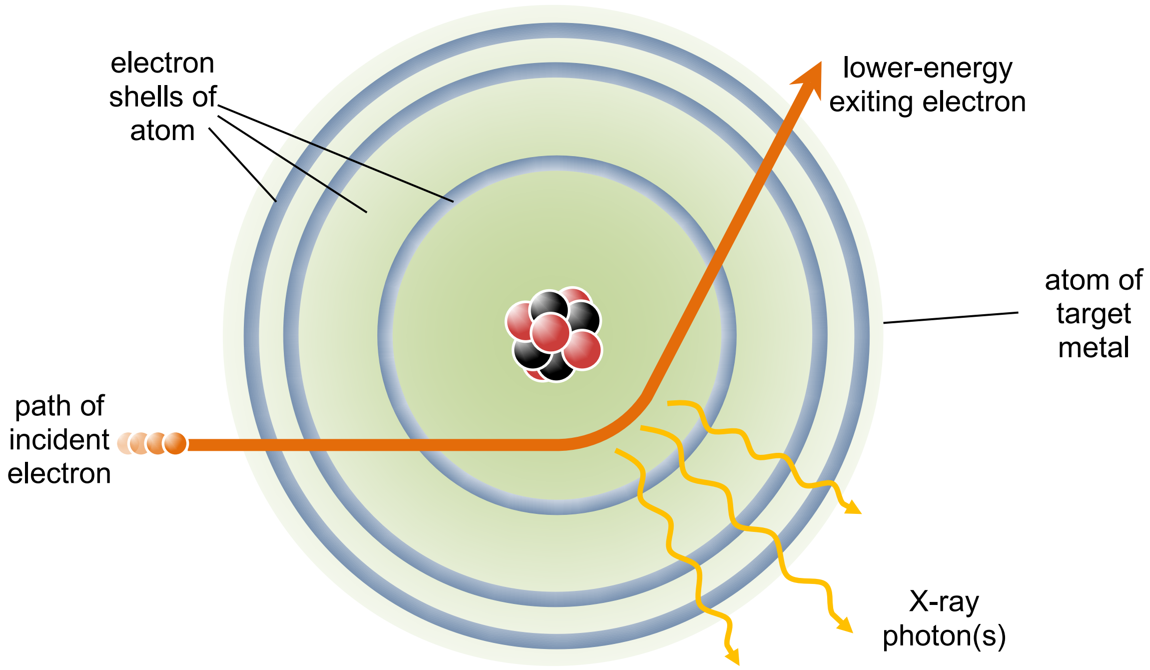
19.20 X-ray spectrum: The continuous spectrum



The continuous spectrum refers to the shaded portion in the diagram on the right.

In the experimental set up shown on page 25, electrons with very high kinetic energies can penetrate deep into the target metal. When the bombarding electrons come close to, and interact with, the nuclei in the metal target, they lose energy (experiencing slowing down) and emit photons in the X-ray energy range. i.e. X-ray photons. Thus, the continuous spectrum is known as Bremsstrahlung radiation, which stands for “braking radiation” in German.

X-ray photon is produced when electron is decelerated due to its interaction with a nucleus.
e.m. radiation is produced whenever charged particle is accelerated
larger acceleration results in larger photon energy
electrons hitting target have distribution of accelerations
hence distribution of wavelengths
Therefore, there is a continuous distribution of wavelengths.



One important quantity characterising the continuous spectrum is the shortest wavelength, λ_{min}, which corresponds to the most energetic X-ray photon being emitted (since $E = hc/\lambda$). λ_{min} occurs when all the kinetic energy of a single bombarding electron is converted into one X-ray photon.

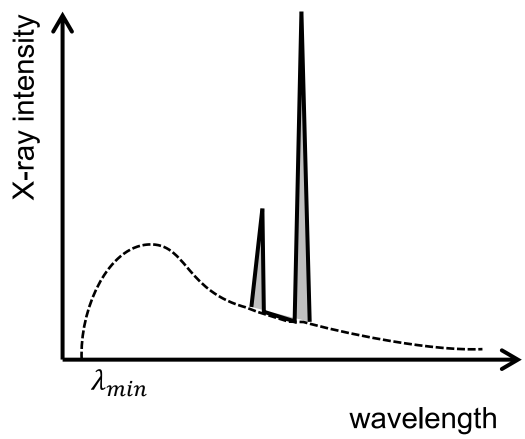
Electrons are accelerated through potential difference V
$\Rightarrow$ kinetic energy of electron $= qV$

$$\text{loss in KE} = E_{\text{photon}}$$

$$qV = \frac{hc}{\lambda_{\text{min}}}$$

Therefore, λ_{min} depends only the potential difference that is used to accelerate the bombarding electrons. It does not depend on the target metal.

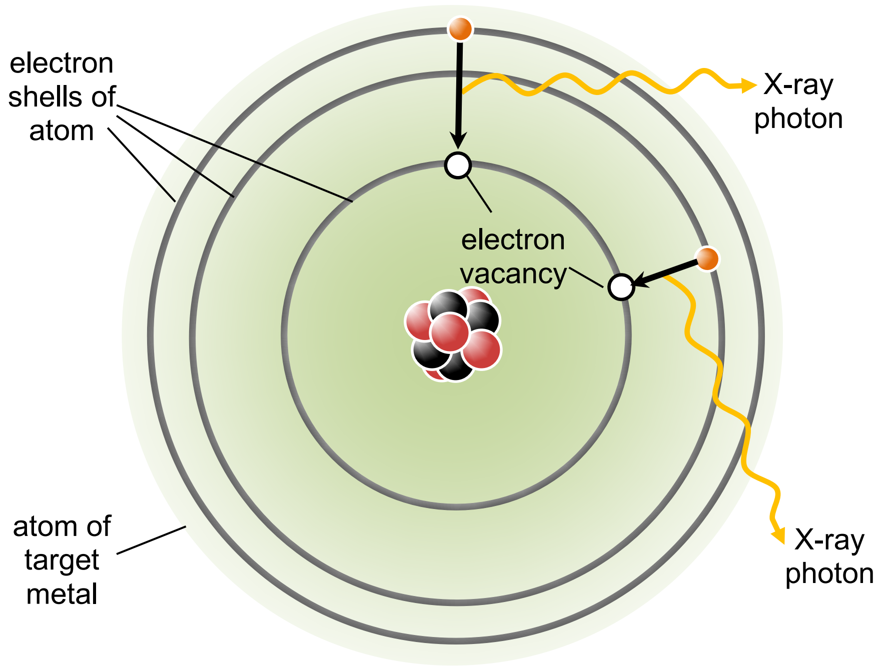
19.21 X-ray spectrum: The characteristic peaks



The characteristic peaks refer to the sharp, distinct peaks which protrude out from the smoothly-changing continuous spectrum.

They result from the similar mechanism as *emission spectra*: orbital electrons transit from a higher energy level to a lower energy level in the atom of the target metal, giving out energy in the form of photons in the process. The difference however lies in the electrons involved in both processes.

The electrons involved in X-ray production are those closest to the nucleus while the electrons involved in producing emission spectra are valence electrons. In particular, the high-speed electrons from an external source strike the electrons in the inner-most shell (K-shell), sometimes transferring enough energy to cause the ejection of these electrons from the K-shell, thereby leaving vacancies in K-shell. The electrons in the nearby shells (L-shell or M-shell) with higher energies deexcite to fill up the vacancies in the K-shell, in the process releasing X-ray photons. The energy of each X-ray photon released is exactly equal the energy difference between the 2 electron shells.



The electron shells of each type of metallic atoms have fixed energies. The X-ray photons thus also have fixed energy values, and thereby wavelengths that are *unique* to each type of target metal. The identity of metals can be found this way.

Note that the kinetic energy of the bombarding electrons must be enough to remove the K shell electrons (to infinity) before the characteristic peak can be produced. This energy needed is greater than the energies corresponding to either of the characteristic peaks.

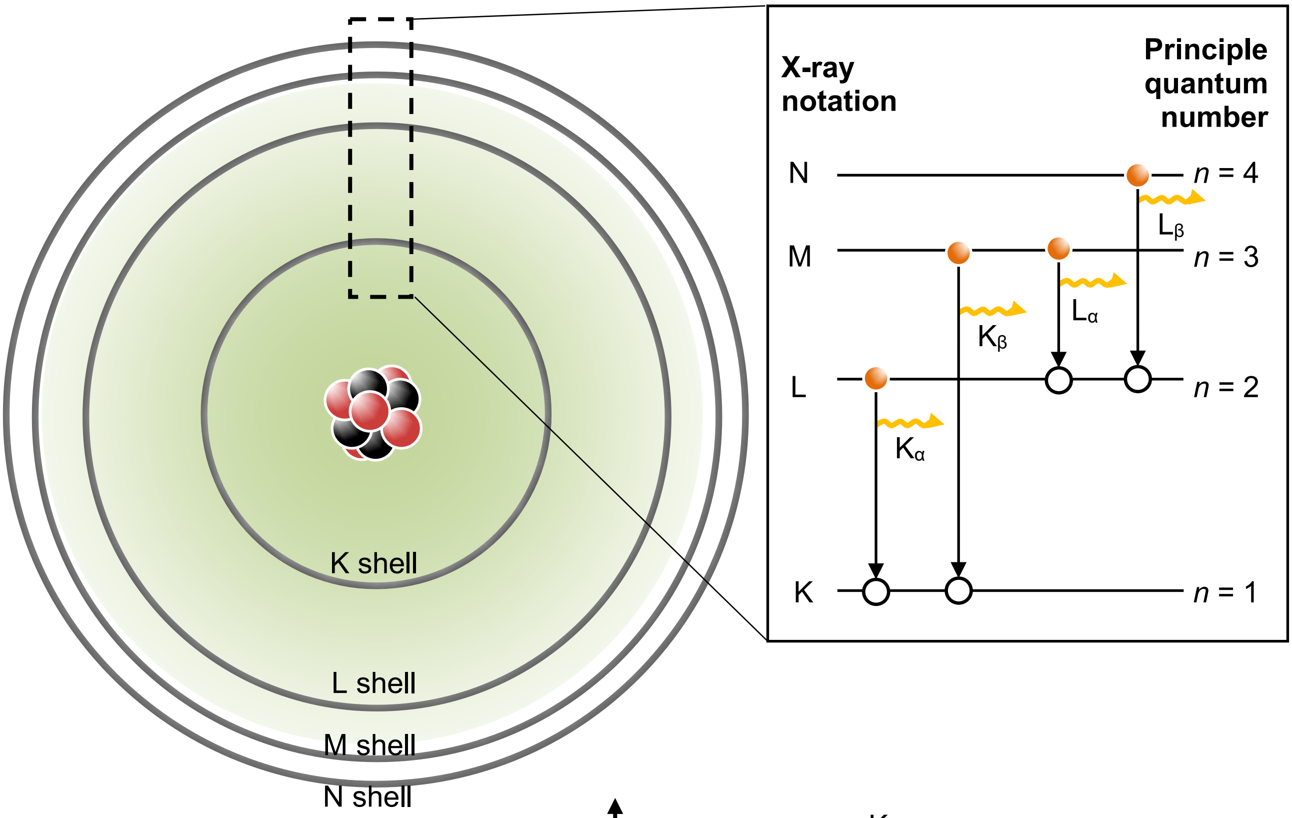
19.22 X-ray notation: Naming the characteristic peaks

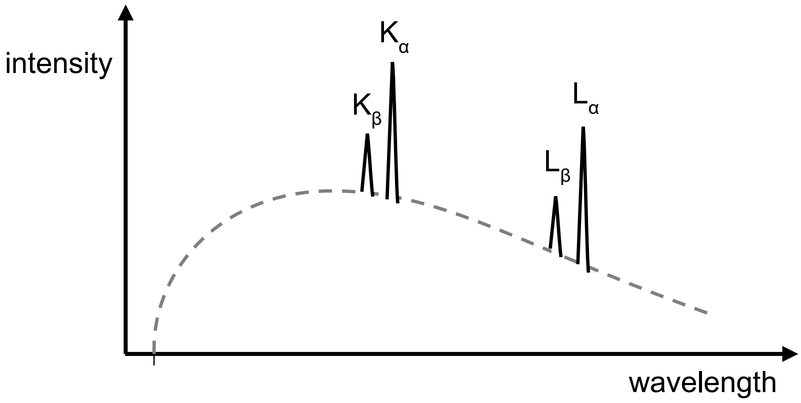
When a bombarding electron strikes the target metal, an innermost-shell (K shell) electron may be ejected. An electron in the next higher shell (L shell) can emit an X-ray photon and transit down into the K shell to fill the vacancy.

- The vacancy is first formed in the K shell, so the X-ray characteristic peak is labelled K.
- If the de-excitation comes from the L shell, the X-ray photon contributes to the K_α peak.
- If the de-excitation comes from the M shell, the X-ray photon contributes to the K_β peak.

To compare the K_α and K_β peaks,

- The energy difference between M shell and K shell is larger than that between L shell and K shell. So, the X-ray photons corresponding to the K_β peak is of higher energy (hence shorter wavelength) than those corresponding to the K_α peak (lower energy, hence longer wavelength).
- The L shell is closer to the K shell, hence electrons in L shell have a higher probability (than those in M shell) of deexciting to fill the vacancies in K shell. Hence, the K_α peak is higher in intensity than the K_β peak.

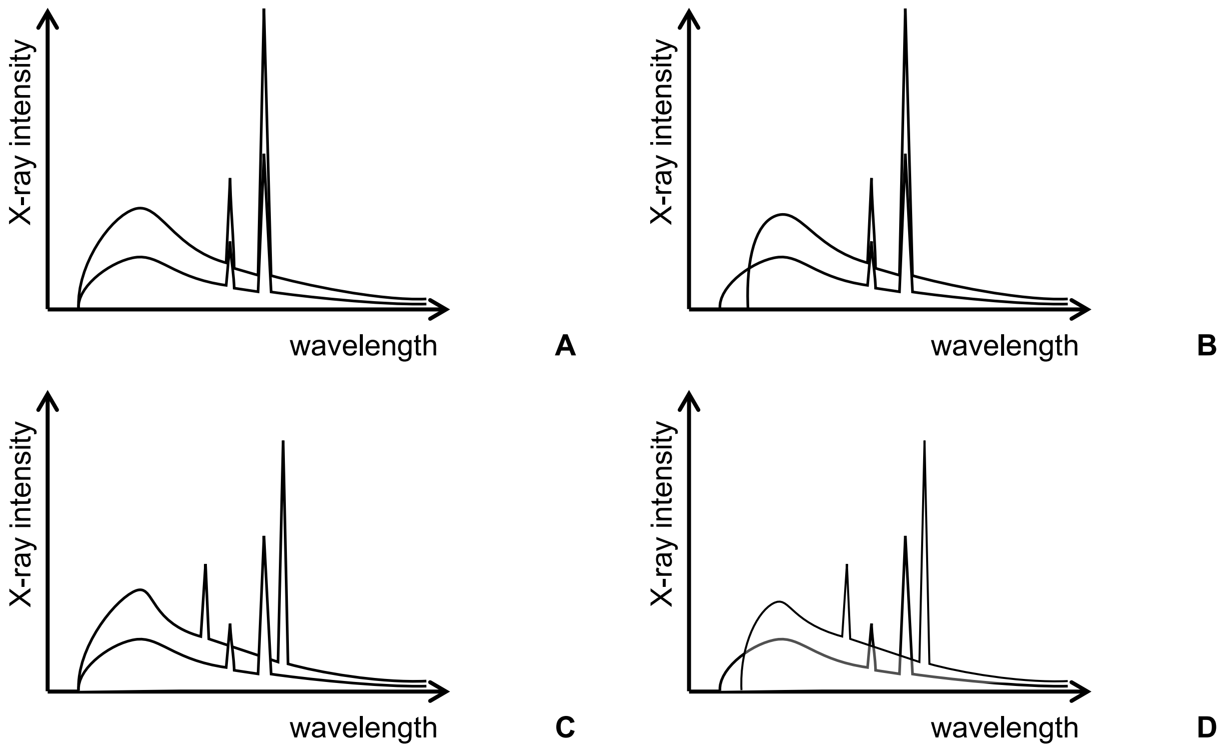




The external high-speed electrons can also happen to strike and remove the electrons in the L shell. M shell and N shell electrons will then deexcite to fill the vacancies in L shell, emitting X-ray photons in the process. These processes give rise to the L_α and L_β peaks.

Example 16

An X-ray tube operates at 200 kV with a tungsten target metal. The target metal is then replaced with an molybdenum version. Which of the following graphs correctly shows the possible spectra plotted on the same axis?

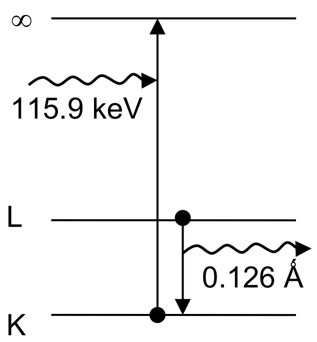


Operating voltage remains at 200 kV, so λ_{min} for the 2 spectra is the same value.

The metals are different so characteristic peaks should occur at different wavelengths. **C**.

Example 17

The energy of an incoming photon required to remove a K-shell electron in Uranium is 115.9 keV. If the K_α wavelength is 0.126 Å, find the wavelength of an incoming photon needed to remove an electron from L-shell completely. 1Å = 1 x 10^{-10} m.



$$E_{\text{X-ray photon emitted}} = hf = \frac{hc}{\lambda}$$

$$= \frac{(6.63\times10^{-34})(3\times10^{8})}{0.126\times10^{-10}}$$

$$= 1.58\times10^{-14}\ \text{J} = 98.66\ \text{keV}$$

$$E_{\text{L}\to\infty} = 115.9 - 98.66 = 17.24\ \text{keV}$$

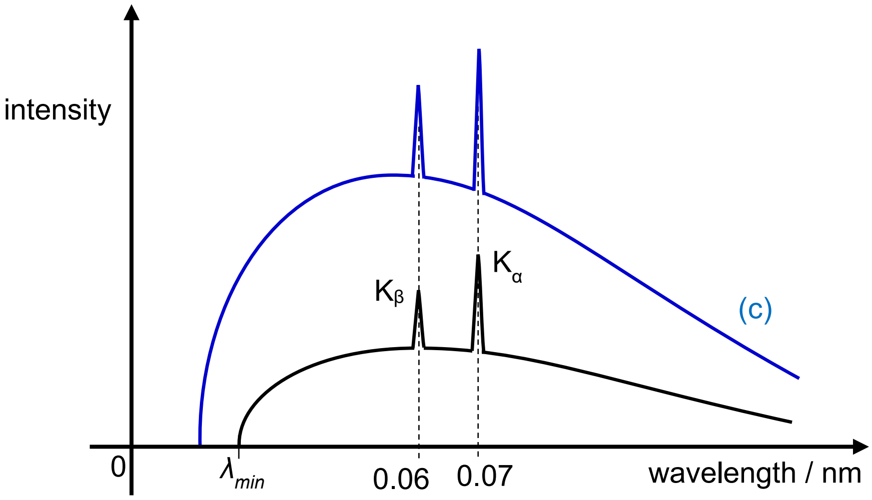
$$E_{\text{required}} = hf = \frac{hc}{\lambda}$$

$$\lambda_{\text{required}} = \frac{hc}{E_{\text{required}}} = \frac{(6.63\times10^{-34})(3\times10^{8})}{(17.24\times10^{3})(1.6\times10^{-19})}$$

$$= 7.21\times10^{-11}\ \text{m}$$

Example 18

The following is an X-ray spectrum showing 2 characteristic lines, K_α and K_β.



(a) Calculate the difference in the energy levels of the K and M shells of the target material.

Energy level difference between K and M shells is the photon energy corresponding to K_β.

$$\begin{aligned}\Delta E &= \left|E_K - E_M\right| \\ &= E_{photon} = \frac{hc}{\lambda} \\ &= \frac{(6.63\times10^{-34})(3\times10^{8})}{0.06\times10^{-9}} \\ &= 3.32\times10^{-15}\text{ J}\end{aligned}$$

(b) If the accelerating potential difference is 40 kV, determine the value for λ_{min}.

By Principle of Conservation of energy, loss in electric potential energy provides kinetic energy of one bombarding electron, which is then fully converted into one most-energetic photon.

$$qV = \frac{hc}{\lambda_{min}}$$

$$\lambda_{min} = \frac{hc}{qV} = \frac{(6.63\times10^{-34})(3\times10^{8})}{(1.6\times10^{-19})(40\times10^{3})}$$

$$= 3.11\times10^{-11}\text{ m} = 0.0311\text{ nm}$$

(c) Sketch on the same axes above, the spectrum that will be obtained when the accelerating potential difference is increased.

19.23 Wave-particle duality

From the Photoelectric Experiment, we concluded that light can behave like particles. Conversely, electrons, which are normally considered as point-like particles, can display wave-like behaviour. Electron beams can undergo superposition and interference, which are typical wave-like behaviour. Louis de Broglie, a French Physicist, suggested that if light can behave as particles, then electrons and protons, conventionally treated as particles, should in some situations also exhibit wave-like behavior. He was awarded the Nobel Prize for Physics in 1929 for this discovery.

The **de Broglie wavelength** associated with a particle moving with momentum *p* is

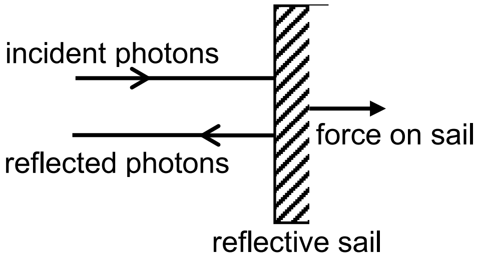
$$\lambda = \frac{h}{p},$$

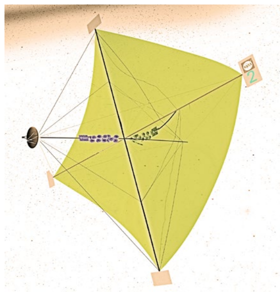
where *h* is the Planck constant, $h = 6.63 \times 10^{-34}$ J s.

The above equation holds for massive particles (electrons, neutrons, etc.) as well as massless particles (photons). For example, the momentum carried by a photon of wavelength λ is given by, according to the equation above, $p = \frac{h}{\lambda}$. *Photons do not have mass, but can carry momentum!*

Example 19

A lightweight spacecraft can be propelled via a photon sail by the Sun. As photons bounce off an ultra-thin reflective sail, radiation pressure acts across the surface area of the sail, giving thrust.





A photon sail has an area of 32 m^2 and exhibits perfect reflectance. Determine the magnitude of thrust if energy from the Sun reaches the sail perpendicularly with a total intensity of 1400 Wm^{-2}.

Light from the Sun has many different frequencies, and so there is a wide range of photons each of different photon energies.

$$E_{1\text{ photon}} = hf = \frac{hc}{\lambda} = \left(\frac{h}{\lambda}\right)c = p_{1\text{ photon}}c$$

For perfect reflectance, the change in momentum for each photon is:

$$|\Delta p| = |p_{\text{final}} - p_{\text{initial}}| = |2p|$$

By Newton's 2$^{\text{nd}}$ Law,

$$|F_{\text{net}}| = \left|\frac{dp_{\text{total}}}{dt}\right| = \left|N_{\text{photons}}\left(\frac{2p_{1\text{ photon}}}{t}\right)\right|$$

$$= \left(\frac{2}{t}\right)\left(\frac{N_{\text{photons}}E_{1\text{ photon}}}{c}\right) = \left(\frac{2}{t}\right)\left(\frac{E_{\text{total}}}{c}\right)$$

$$= \left(\frac{2}{c}\right)\left(\frac{E_{\text{total}}}{t}\right) = \left(\frac{2}{c}\right)(I(\text{Area}_{\text{sail}}))$$

$$= \left(\frac{2}{3\times10^{8}}\right)(1400)(32)$$

$$= 2.99\times10^{-4}\ \text{N} \approx 30\ \text{mN}$$

By Newton's 3$^{\text{rd}}$ Law, force by photons on sail = force by sail on photons = 30 mN

Example 20

Find the de Broglie wavelength associated with

(i) Mr Lim (mass = 100 kg) sprinting with speed = 10 ms^{-1}.

(ii) an electron (mass = 9.11 × 10^{-31} kg) accelerated through 30 V from rest.

(i)

$$\lambda_{\text{Lim}} = \frac{h}{p} = \frac{h}{mv}$$

$$= \frac{6.63\times10^{-34}}{(100)(10)} = 6.63\times10^{-37} \text{ m}$$

(ii)

By Principle of Conservation of Energy,
loss in EPE = gain in KE

$$qV = \frac{p^2}{2m_e}$$

$$p_{\text{electron}} = \sqrt{2m_e qV}$$

$$\lambda_{\text{electron}} = \frac{h}{p_{\text{electron}}} = \frac{h}{\sqrt{2m_e qV}}$$

$$= \frac{6.63\times10^{-34}}{\sqrt{2(9.11\times10^{-31})(1.6\times10^{-19})(30)}}$$

$$= 2.24\times10^{-10} \text{ m}$$

Notes: "Everyday" masses (such as humans and bullets) have extremely small de Broglie wavelengths – the wave-like behaviour is effectively not observable.

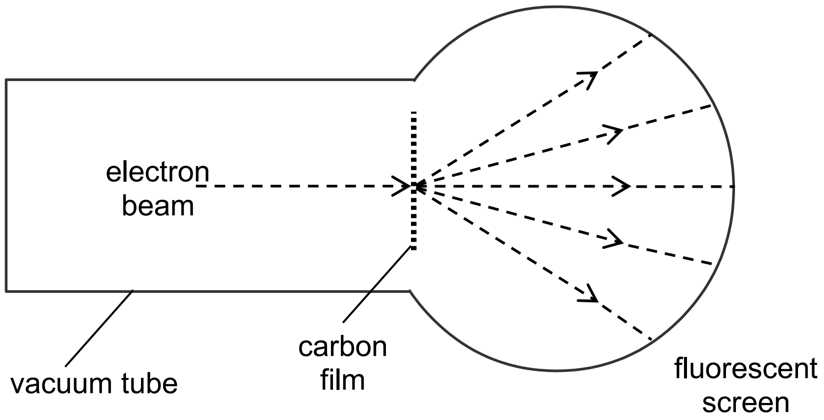
Consequently, "everyday" masses cannot undergo appreciable diffraction because the associated wavelengths are orders of magnitude smaller than dimensions of everyday life.

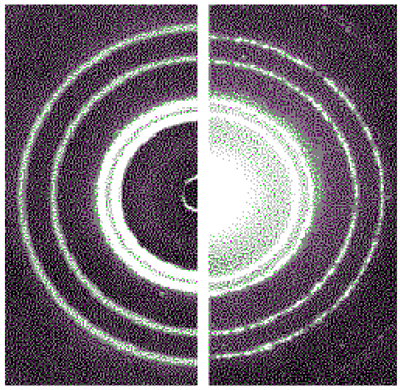
By choosing an appropriate accelerating potential, the de Broglie wavelength of electrons can be made to be in the X-ray region, which is of the same order of magnitude as the inter-atomic spacing in solid crystals, and a noticeable diffraction pattern can be observed if X-ray is incident on solid crystals. Therefore, it is very common to study crystals using X-rays or electron beams.

19.24 Electron diffraction is evidence of electrons having wave-like behaviour

Electron diffraction

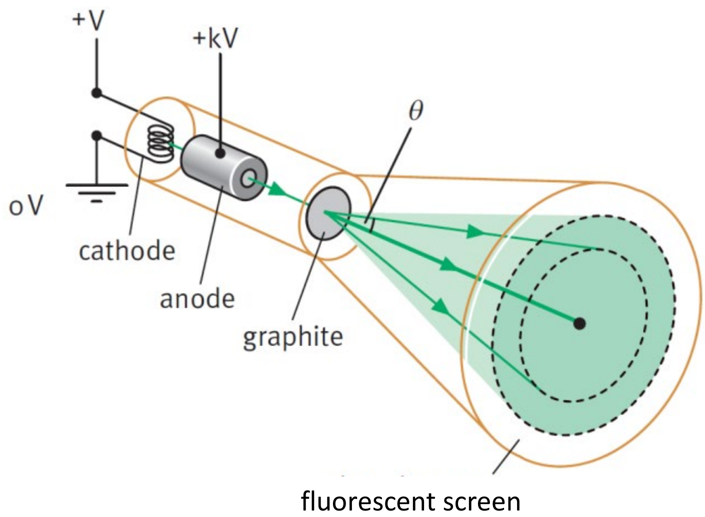
A beam of electrons is directed at a thin carbon film. A pattern of concentric rings is observed on a fluorescent screen, which is similar to diffraction pattern produced by using visible light.

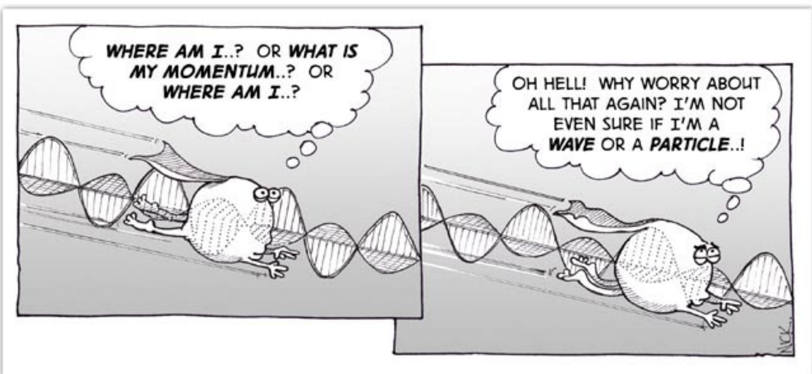




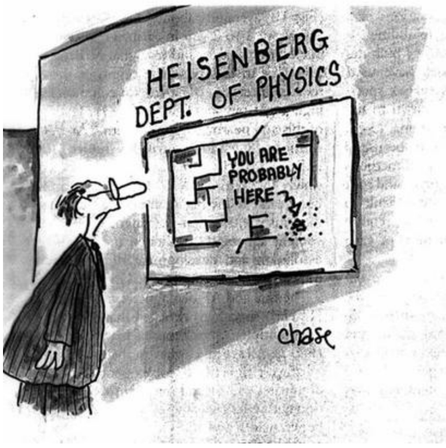
(EM *waves*) electrons (*particles*)

Note: A thin carbon film has many tiny crystals arranged in a regular manner, similar in concept to a diffraction grating with many slits regularly spaced out.





19.25 Heisenberg Uncertainty Principle



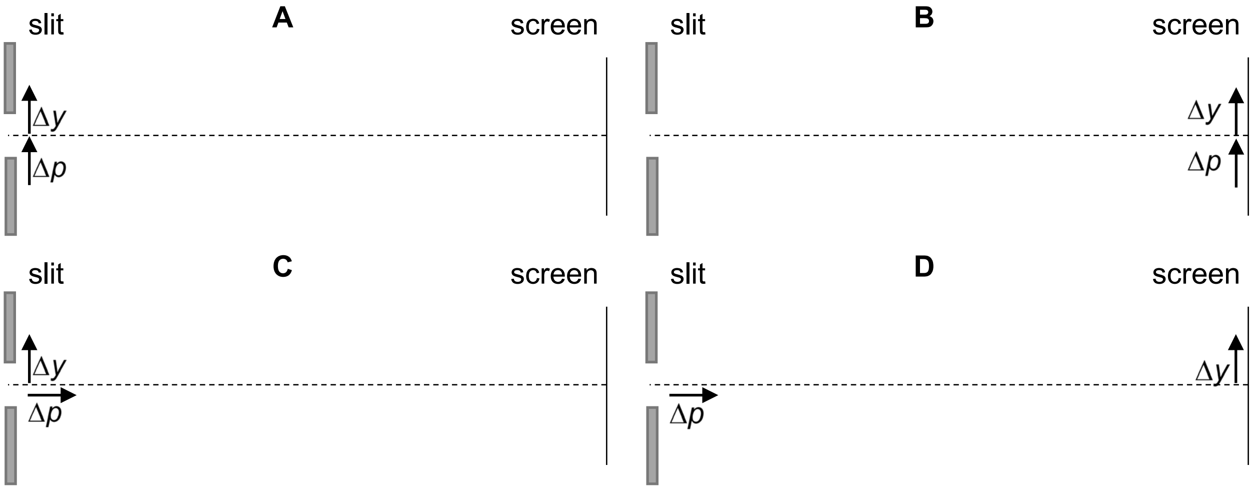
If a measurement of position is made with uncertainty Δx and a simultaneous measurement of momentum is made with uncertainty Δp. The product of the two uncertainties can never be smaller than the Planck constant

$$\boxed{\Delta p \Delta x \gtrsim h}$$

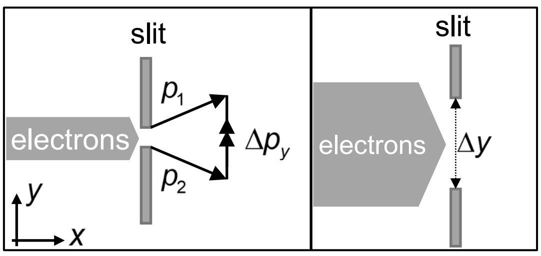
The uncertainty principle limits the precision that complementary quantities (such as position along *x*-axis and the momentum in the *x* direction) can have. For example, if we know very well the position along the *x*-axis of a particle (Δx is very small), then we cannot be very certain what momentum in the *x*-direction the particle possesses, since $\Delta p_x \gtrsim h/\Delta x$ is huge if Δx is very small. Note that this has nothing to do with the precision of the apparatus/method used to perform the measurements. The uncertainty principle is a fundamental principle of nature. However, we may still know p_y or p_z very well, even if we know the *x* position very well, since the uncertainty principle applies only to position and momentum along the same direction.

Example 21

For an electron passing through a single slit, the uncertainty in its position Δy and the uncertainty in its momentum Δp are related by $\Delta p \Delta y \gtrsim h$. Which diagram shows the positions where in the equation above are defined correctly?



Note: The uncertainty principle applies when measuring complementary quantities along the same dimension i.e. $\Delta p_y \Delta y \gtrsim h$.



Most will find the idea for Δy intuitive – that the slit width where the electron could have been anywhere in between. For momentum, the electrons initially have momentum in the *x* direction. After passing through the slit, the electrons spread out in the *y* direction (up/down). The uncertainty therefore is their momentum in *y* direction.

Answer: A