

# **RAFFLES INSTITUTION** MATHEMATICS DEPARTMENT **2023 YEAR 4 RP MATHEMATICS TOPIC 3: DIFFERENTIATION (MATHS 1 & MATHS 2)**

# SUPPLEMENTARY WORKSHEET

### Name:

Class: Sec 4 ( )

Date:

#### 1 2021/Y4RP/M1/T2/Q2

- Find the derivative of  $(e^{6t})(e^{t-3}) + (e^{3t})^t$ . **(a)** [2]
- Given that  $y = \ln \frac{(1-x)^4}{\sqrt{(2x-3)^5}}$ , find  $\frac{dy}{dx}$ , leaving your answer as a simplified single **(b)**

fraction.

[Ans: (a)  $7e^{7t-3} + 6te^{3t^2}$  (b)  $\frac{3x-7}{(x-1)(2x-3)}$ ]

#### 2 2021/Y4RP/M1/T2/Q3

Given that  $y = e^{2x} (3x^2 - 1)$ , prove that  $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 6e^{2x}$ . [5]

#### 3 2021/Y4RP/M1/T2/Q4

Find the x-coordinate(s) of the point(s) on the curve,  $y = \sqrt{x} \left( 2\sqrt{x^5} + \frac{1}{2\sqrt{x^3}} \right)$  where x > 0, at which the tangent at the point is perpendicular to the line  $x-2y = \pi - 1$ , leaving your answer correct to 3 significant figures. [5] [Ans: x=0.408]

#### 4 2021/Y4RP/M1/T2/Q5

The equation of a curve is given by  $y = \frac{(5x+3)^2}{\sqrt{1-2x}}$  where  $x < \frac{1}{2}$ .

- Find  $\frac{dy}{dx}$ , giving your answer as a simplified single fraction. (i) [3]
- Find the range of values of x for which  $\frac{dy}{dr}$  is non-negative. (ii) [3]

[Ans: (i) 
$$\frac{(5x+3)(13-15x)}{\sqrt{(1-2x)^3}}$$
 (ii)  $-\frac{3}{5} \le x < \frac{1}{2}$ ]

#### 5 2021/Y4RP/M2/T1/Q1

A curve has the equation  $y = \frac{1}{3}\tan^3 2x - 7\ln(\sin x)$ . Obtain an expression of  $\frac{dy}{dr}$  in terms [3] 7 of tangent function.

[Ans: 
$$2\tan^4 2x + 2\tan^2 2x - \frac{1}{\tan x}$$
]

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## 6 <u>2021/Y4RP/M2/T1/Q2</u>

Given that  $y = \frac{\cos 2x}{e^{2x}}$ , show that  $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} = ky$ , where k is a constant to be determined. [5]

## 7 <u>2020/Y4RP/M1/T1/Q5</u>

Differentiate the following expressions with respect to x, simplifying your answer.

(a) 
$$\frac{4x^2 + 9}{\sqrt{x}}$$
 [2]

(b) 
$$\frac{2x+3}{(4x+1)^3}$$
 [3]

(c) 
$$(2x-5)\sqrt[3]{(1-3x)}$$
 [3]

[Ans: (a) 
$$\frac{3(4x^2-3)}{2\sqrt{x^3}}$$
 (b)  $-\frac{2(8x+17)}{(4x+1)^4}$  (c)  $\frac{7-8x}{\sqrt[3]{(1-3x)^2}}$ ]

### 8 <u>2020/Y4RP/M2/T1/Q2</u>

Differentiate  $\frac{e^{5x^2}}{\ln(2x)}$  with respect to x, leaving your answer in the simplest form. [3]

[Ans: 
$$\frac{e^{5x^2} \left(10x^2 \ln(2x) - 1\right)}{x \left[\ln(2x)\right]^2} J$$

### 9 <u>2020/Y4RP/M2/T1/Q1</u>

Differentiate  $\ln(\cos^5 3x)$  with respect to x, leaving your answer as a single trigonometric term. [2]

[Ans: 
$$-15 \tan 3x$$
]

[Ans: k = -8]

## 10 <u>2020/Y4RP/M2/T1/Q3</u>

Given that  $y = \tan 5x$ , show that  $\frac{d^2y}{dx^2}$  can be written in the form  $ky\frac{dy}{dx}$ , where k is a constant to be determined. [4] [Ans: k = 10]

(a) Differentiate 
$$2\sqrt{x^7} - \frac{\sqrt{x}}{2}$$
 with respect to x.

(b) Differentiate  $\frac{3x+2}{\sqrt{5x-6}}$  with respect to x, simplifying your answer as a single fraction. [3]

[Ans: (a) 
$$7\sqrt{x^5} - \frac{1}{4\sqrt{x}}$$
 (b)  $\frac{15x - 46}{2\sqrt{(5x - 6)^3}}$ ]

### 12 <u>2019/Y4RP/M1/T2/Q3</u>

The curve  $y = \frac{p}{(3x^2 - q)^3}$  passes through the point A(-1, 3) where p and q are constants. Given that the gradient of the curve at point A is 9, find the values of p and q. [4]

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[2]

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## 13 <u>2019/Y4RP/M1/T2/Q4</u>

Given that  $y = (4x-1)^2 \sqrt{3-2x}$  where  $x < 1\frac{1}{2}$ , find  $\frac{dy}{dx}$ , simplifying your answer. Hence find the range of values of x for which the gradient of the curve is positive. [5]  $[Ans: \frac{5(4x-1)(5-4x)}{\sqrt{3-2x}}; \frac{1}{4} < x < 1\frac{1}{4}]$ 

### 14 <u>2019/Y4RP/M2/T1/Q3</u>

Given that  $y = e^{\sqrt{x}}$ , find the value of k such that  $4x \frac{d^2 y}{dx^2} + k \frac{dy}{dx} - y = 0.$  [5] [Ans: k = 2]

#### 15 <u>2019/Y4RP/M2/T1/Q1</u>

Differentiate the following with respect to *x*, simplifying your answer:

(i) 
$$\frac{\sin 3x}{x^2}$$
, [2]

(ii) 
$$\ln\left(\frac{e^{3x^2}}{\cos 3x}\right)$$
. [2]

[Ans: (i) 
$$\frac{3x\cos 3x - 2\sin 3x}{x^3}$$
 (ii)  $6x + 3\tan 3x$ ]

## 16 <u>2019/Y4RP/M2/T1/Q4</u>

- (i) By writing  $\sec x$  as  $(\cos x)^{-1}$ , show that  $\frac{d}{dx}(\sec x) = \sec x \tan x$ . [1]
- (ii) Hence find the following in terms of  $\sec x$ ,

(a) 
$$\frac{d}{dx}(\sec x \tan x)$$
, [2]

(b) 
$$\frac{\mathrm{d}}{\mathrm{d}x} \left[ \ln \left( \sec x + \tan x \right) \right].$$
 [2]

[Ans: (ii)(a)  $\sec x (2 \sec^2 x - 1)$  (b)  $\sec x ]$ 

### 17 <u>2018/Y4RP/CT/Q1(a) (M1/M2)</u>

Differentiate each of the following with respect to x, leaving your answer in the simplest form.

(a)  $(3+4x)e^{1-2x}$  [2]

[Ans: (a) 
$$-2(1+4x)e^{1-2x}$$
]

## 18 <u>2018/Y4RP/CT/Q1(b) (M2)</u>

Differentiate each of the following with respect to x, leaving your answer in the simplest form.

(b) 
$$5\tan^{5}(2x)$$
.

[Ans: (b)  $50\tan^4(2x)\sec^2(2x)$ ]

19 <u>2018/Y4RP/CT/Q6 (M2)</u> It is given that  $y = \sqrt{1 - \sin x}$ . (i) Find an expression for  $\frac{dy}{dx}$ , leaving your answer in the simplest form. [2]

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[2]

Show that  $\frac{d^2 y}{dx^2} = ky$ , where k is a constant to be determined. (ii) [4]

[Ans: (i) 
$$-\frac{\cos x}{2\sqrt{1-\sin x}}$$
 (ii)  $-\frac{1}{4}\sqrt{1-\sin x}$ ]

#### 20 2017/Y4RP/CT/Q1 (M2)

A curve has the equation  $y = x \tan(1-3x)$ . Find the gradient of the curve at the point where x = 1. [3]

<u>2017/Y4RP/CT/Q2 (M1/M2)</u> Given that  $y = \frac{e^{2x}}{\sqrt{4x+1}}$ , find  $\frac{dy}{dx}$ , simplifying your answer. 21 [3]

[Ans:  $\frac{8xe^{2x}}{\sqrt{(4x+1)^3}}$ ]

#### 22 2017/Y4RP/CT/Q3 (M2)

Given that  $y = \cos^2 2x$ , find  $\frac{dy}{dx}$  and  $\frac{d^2y}{dx^2}$ , simplifying your answers. Hence show that  $\left(\frac{d^2 y}{dx^2}\right)^2 + 16\left(\frac{dy}{dx}\right)^2 - 128y = k\cos 4x$ , where k is a constant to be determined. [5]

[Ans: 
$$\frac{dy}{dx} = -2\sin 4x$$
,  $\frac{d^2y}{dx^2} = -8\cos 4x$ ]

[Ans: (i)  $\frac{1+6x^2-8x^3}{(4x^3+1)(1-2x)}$ ]

#### 23 2016/Y4RP/CT/Q1(i) (M1/M2)

Differentiate each of the following with respect to x, expressing your answers as a single fraction.

(i) 
$$\ln \sqrt{\frac{4x^3 + 1}{1 - 2x}}$$
, [4]

Differentiate each of the following with respect to x, expressing your answers as a single fraction.

(ii) 
$$\frac{(3x+1)^4}{\tan^2 5x}$$
. [3]

[Ans: (ii) 
$$\frac{2(3x+1)^3 [6 \tan 5x - 5(3x+1) \sec^2 5x]}{\tan^3 5x}$$
]

## 25

 $\frac{2016/Y4RP/CT/Q4 \text{ (M2)}}{\text{It is given that } y = \sqrt{1 + \cos x}.}$ 

(i) Find 
$$\frac{dy}{dx}$$
, leaving your answer in its simplest form. [1]

(ii) Show that 
$$\frac{d^2 y}{dx^2} = -\frac{1}{4}y.$$
 [4]  
[Ans: (i)  $-\frac{\sin x}{2\sqrt{1+\cos x}}$ ]

## 26 <u>2015/Y4RP/CT/Q2 (M2)</u>

Given that  $y = \ln(1 + \sin x)$ ,

(a) write down an expression for 
$$\frac{dy}{dx}$$
 in terms of x, [1]

(b) show that 
$$\frac{d^2 y}{dx^2} = -\sec x \left(\frac{dy}{dx}\right)$$
. [3]

[Ans: (a) 
$$\frac{dy}{dx} = \frac{\cos x}{1 + \sin x}$$
]

#### 27 <u>2015/Y4RP/CT/Q4(a) (M2)</u>

For each of the following functions, find  $\frac{dy}{dx}$ , expressing your answers as a single fraction

(a) 
$$y = \frac{e^{2x+1}}{\cos^3 4x}$$
, [4]  
[Ans: (a)  $\frac{2e^{2x+1}(\cos 4x + 6\sin 4x)}{\cos^4 4x}$ ]

# 28 <u>2015/Y4RP/CT/Q4(b) (M1/M2)</u>

For each of the following functions, find  $\frac{dy}{dx}$ , expressing your answers as a single fraction

(b) 
$$y = \ln\left(\sqrt[5]{\left(\frac{10x-1}{7-5x}\right)^2}\right).$$
 [4]  
[Ans: (b)  $\frac{26}{(10x-1)(7-5x)}$ ]

# 29 <u>2014/Y4RP/CT/Q1(a) (M2)</u>

Find  $\frac{dy}{dx}$  for each of the following functions, expressing your answers as a single fraction.

(a) 
$$y = x \csc^2 x$$
, [3]

[Ans: (a) 
$$\frac{\sin x - 2x \cos x}{\sin^3 x} J$$

# 30 <u>2014/Y4RP/CT/Q1(b) (M1/M2)</u>

Find  $\frac{dy}{dx}$  for each of the following functions, expressing your answers as a single fraction.

(b) 
$$y = \ln\left(\sqrt[3]{\frac{2x}{4-3x}}\right).$$
 [3]

[Ans: (b)  $\frac{4}{3x(4-3x)}$ ]

## 31 <u>2014/Y4RP/CT/Q2 (M1/M2)</u>

If 
$$2y = (xe^x)^2$$
, show that  $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = e^{2x}$ . [4]

## 32 <u>2013/Y4RP/MYCT/Q1(a) (M1)</u>

Differentiate the following with respect to x, simplifying your answers as a single fraction.

(a) 
$$\frac{7-2x}{\sqrt{2x+5}}$$
, [3]  $2x+17$ 

[Ans: (a)  $-\frac{2x+17}{\sqrt{(2x+5)^3}}$ ]

## 33 <u>2013/Y4RP/MYCT/Q1(b) (M1/M2)</u>

Differentiate the following with respect to x, simplifying your answers as a single fraction.

**(b)** 
$$\ln \sqrt{\frac{x}{(2x+1)^3}}$$
. [3]

[Ans: (b) 
$$\frac{1-4x}{2x(2x+1)}$$
]

## 34 <u>2013/Y4RP/MYCT/Q2 (M2)</u>

If 
$$y = \frac{\sin x + \cos x}{\cos x}$$
, show that  $\frac{d^2 y}{dx^2} - 2(y-1)\frac{dy}{dx} = 0.$  [4]

# 35 <u>2013/Y4RP/MYCT/Q3 (M2)</u> Differentiate $2e^{2x-1}\sin^4 4x$ with respect to x, simplifying your answer.

Differentiate  $2e^{2x-1}\sin^4 4x$  with respect to x, simplifying your answer. [3] [Ans:  $4e^{2x-1}\sin^3 4x(8\cos 4x + \sin 4x)$ ]

## 36 <u>2012/Y4RP/T2/Q1 (M1)</u>

Differentiate the following with respect to *x*, simplifying your answers.

(a) 
$$\frac{7x^2 - 3x}{\sqrt{x}},$$
 [2]

[Ans: (a) 
$$\frac{3(7x-1)}{2\sqrt{x}}$$
]

#### 37 2012/Y4RP/T2/Q1 (M1/M2)

Differentiate the following with respect to *x*, simplifying your answers.

(c) 
$$\frac{e^{2x}}{1-e^{2x}}$$
, [2]

(d) 
$$\ln \sqrt{\frac{5x+1}{1-2x}}$$
. [2]

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[Ans: (c) 
$$\frac{2e^{2x}}{(1-e^{2x})^2}$$
 (d)  $\frac{5}{2(3x+1)(1-2x)}$ ]

## 38 <u>2012/Y4RP/T2/Q1 (M2)</u>

Differentiate the following with respect to x, simplifying your answers. (b)  $x \sin^3 x$ ,

 $x \sin x$ ,

[Ans: (b)  $\sin^2 x (3x \cos x + \sin x)$ ]

[2]

# 39 <u>2009/Y4RP/T3/Q2 (M1)</u>

Differentiate the following with respect to *x*, simplifying your answers.

(a) 
$$\frac{(1-2x)^3}{\sqrt{4x-3}}$$
, [3]

 $[Ans:(a) \ \frac{4(1-2x)^2(4-5x)}{\sqrt{(4x-3)^3}}]$ 

## 40 <u>2009/Y4RP/T3/Q2(b) (M2)</u>

Differentiate the following with respect to x, simplifying your answers.

(b) 
$$3x^2 \cos^3\left(2x + \frac{\pi}{3}\right)$$
, [3]

[Ans: (b) 
$$6x\cos^2\left(2x+\frac{\pi}{3}\right)\left[\cos\left(2x+\frac{\pi}{3}\right)-3x\sin\left(2x+\frac{\pi}{3}\right)\right]$$
]

# 41 <u>2009/Y4RP/T3/Q2(c) (M1/M2)</u>

Differentiate the following with respect to x, simplifying your answers.

(c) 
$$\ln \sqrt{\frac{(1-x)^3}{2x}}$$
 [3]

[Ans: (c)  $\frac{2x+1}{2x(x-1)}$ ]

## 42 <u>2008/Y4RP/T3/Q1(a) (M1)</u>

Differentiate the following expressions with respect to x, giving your answers in the simplest form possible:

(a) 
$$\frac{2}{\sqrt{3-x}}$$
, [2]  
[Ans: (a)  $\frac{1}{\sqrt{(3-x)^3}}$ ]

## 43 <u>2008/Y4RP/T3/Q1(b) (M1/M2)</u>

Differentiate the following expressions with respect to x, giving your answers in the simplest form possible:

(b) 
$$\ln(2x+1)^3$$
, [2]

[Ans: (b) 
$$\frac{6}{2x+1}$$
]

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