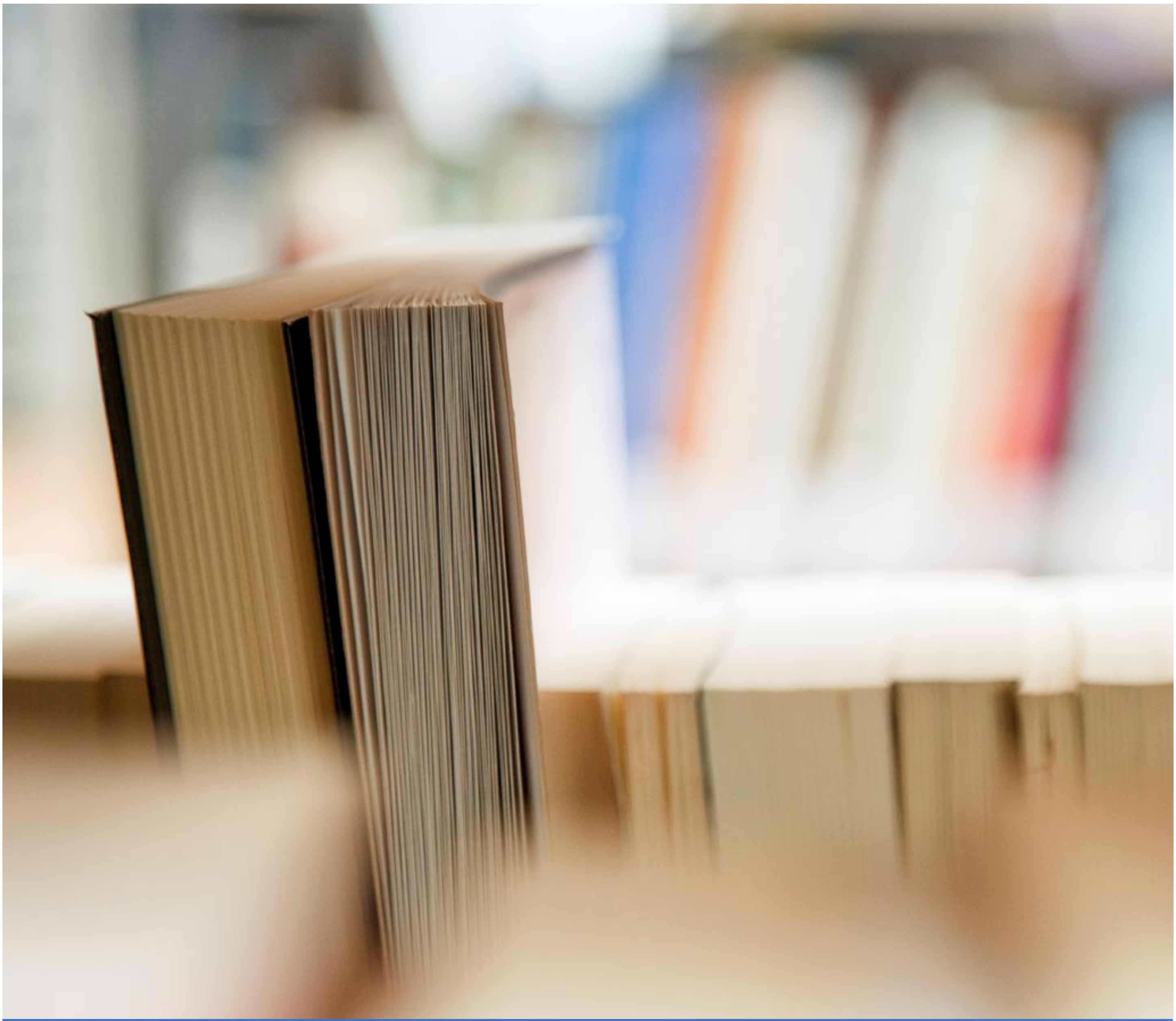


AWE ACADEMY

Summary Notes

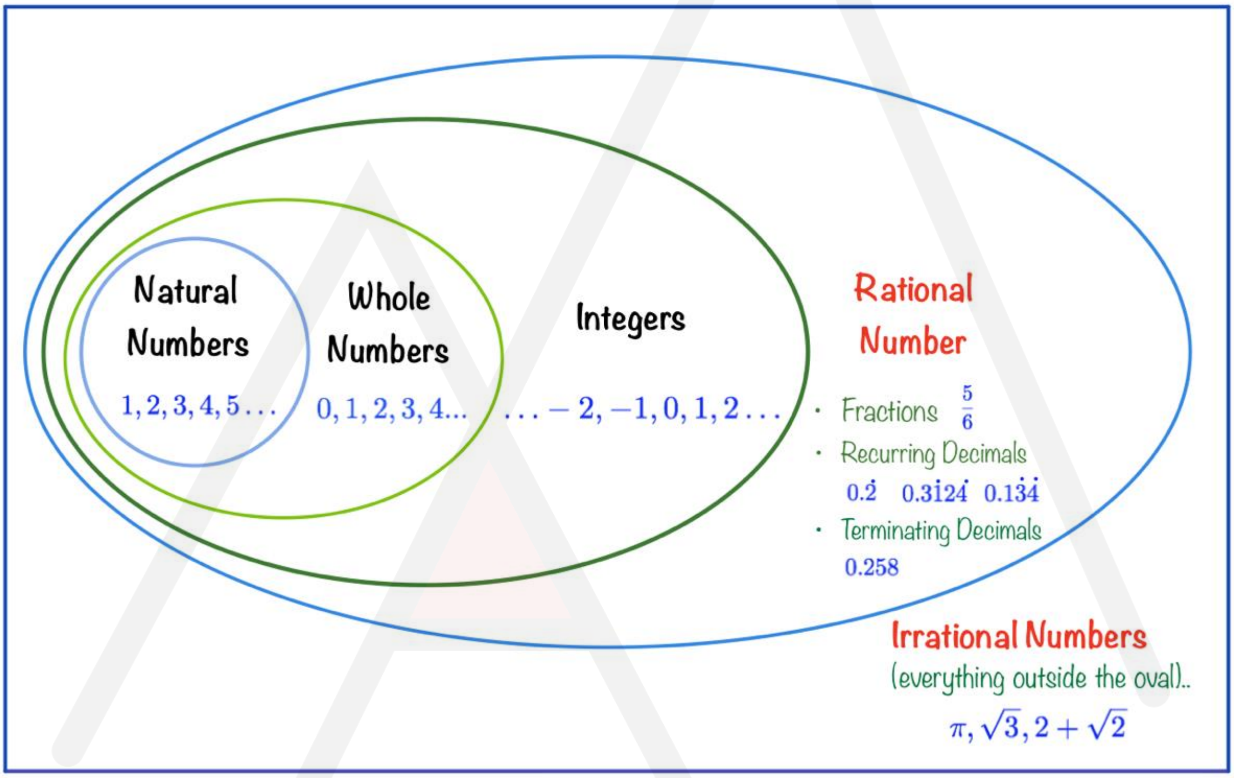
Canndii Wang

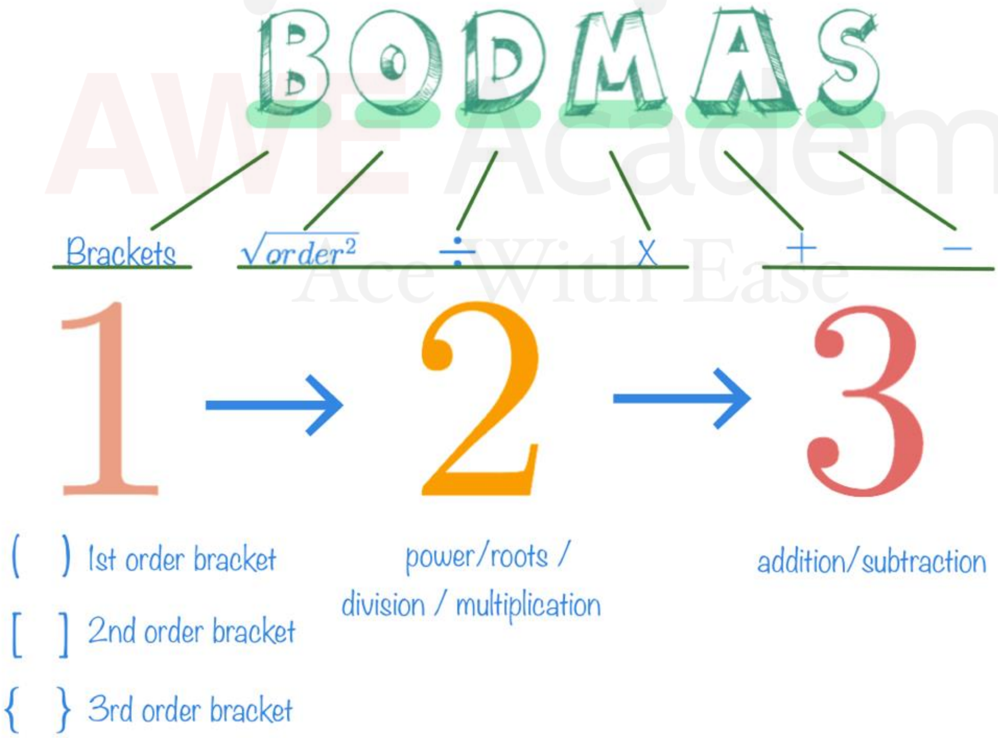
2021 version

Elementary Mathematics

REAL NUMBERS

(everything inside the rectangle)





Addition/Subtraction of Real Numbers

Same Sign

→ Adopt the common sign for the final answer

→ ADD the numerical value up ignoring the signs

Add

$+6 + 4 = +10$

Add

$-6 - 4 = -10$

Different Sign

→ Follow the sign of the GREATER numerical

→ Subtract the smaller number from the bigger number

Subtract

$-6 + 4 = -2$

Subtract

$+4 - 6 = +2$

Multiplication / Division of Real Numbers

Multiplication	Division
$- \times - = +$	$- \div - = +$
$+ \times + = +$	$+ \div + = +$
$- \times + = -$	$- \div + = -$
$+ \times - = -$	$+ \div - = -$

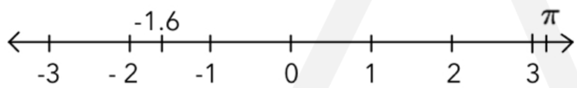
Relationship of Odd & Even numbers

Odd + Odd = Even	Odd × Odd = Odd
Even + Even = Even	Even × Even = Even
Odd + Even = Odd	Odd × Even = Even
Even + Odd = Odd	Even × Odd = Even

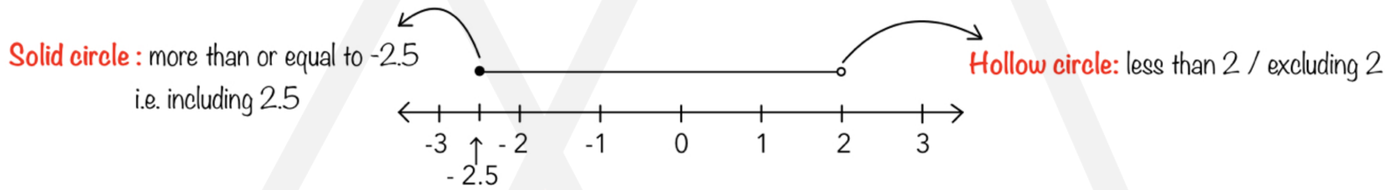
Odd − Odd = Even	Odd ÷ Odd = Odd
Even − Even = Even	Even ÷ Even = Even
Odd − Even = Odd	Odd ÷ Even = Fractions
Even − Odd = Odd	Even ÷ Odd = Fractions

Number line

1. All real numbers can be represented on the number line



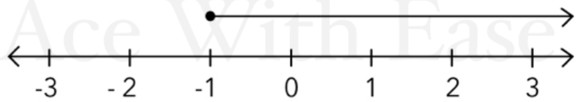
2. Real numbers greater than or equal to -2.5 and less than 3, i.e. $-2 \cdot 5 \leq n < 2$



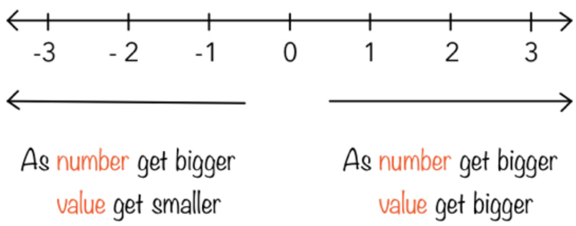
Note: smaller number always written on the LEFT,
bigger number to the RIGHT

$8 > n > -5$ ✗ $-5 < n < 8$ ✓

3. Real numbers greater than or equal to -1, i.e. $n \geq -1$



4. Value of numbers



PRIME FACTORISATION

Factors

Smallest element of numbers

All numbers can be written as a product of their factors

Example:

$$\begin{aligned} 48 &= 1 \times 48 \\ &= 2 \times 24 \\ &= 3 \times 16 \\ &= 4 \times 12 \\ &= 6 \times 8 \end{aligned}$$

Factors of 48

= 1, 2, 3, 4, 6,8, 12, 16, 24, 48

Prime Numbers

Numbers which can only be written as a product of 1 and itself.

NOTE: 1 is not a prime numbers

Prime Numbers	Non-Prime Numbers (Composite Numbers)
$2 = 1 \times 2$	$4 = 1 \times 4 = 2 \times 2$
$3 = 1 \times 3$	$6 = 1 \times 6 = 2 \times 3$
$5 = 1 \times 5$	$8 = 1 \times 8 = 2 \times 4$
$7 = 1 \times 7$	$9 = 1 \times 9 = 3 \times 3$
$11 = 1 \times 11$	$10 = 1 \times 10 = 2 \times 5$
$13 = 1 \times 13$ ...	$12 = 1 \times 12 = 2 \times 6 = 3 \times 4$ ...

Prime Factorisation

Express number as a product of prime factors

We can work this out by 3 methods:

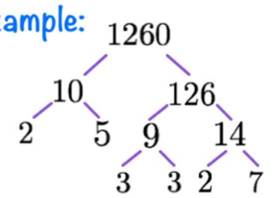
1. Splitting the factors into smaller portions

Example:

$$\begin{aligned} 1260 &= 10 \times 126 \\ &= 5 \times 2 \times 9 \times 14 \\ &= 5 \times 2 \times 3 \times 3 \times 2 \times 7 \\ &= 2 \times 2 \times 3 \times 3 \times 5 \times 7 \\ &= 2^2 \times 3^2 \times 5 \times 7 \end{aligned}$$

2. Branching out the factors into a smaller portions in a tree diagram

Example:



Prime factor = $2^2 \times 3^2 \times 5 \times 7$

3. Dividing it by prime numbers

Example:

2	1260
2	630
3	315
3	105
5	35
7	7
	1

Multiple

product of a number and another number

Example:

$5 \times 1 = 5$
$5 \times 2 = 10$
$5 \times 3 = 15$
$5 \times 4 = 20$
$5 \times 5 = 25$
$\dots$

Multiple of 5
$= 5, 10, 15, 20, 25, \dots$
(all divisible by 5)

Power

Number multiplied by itself, by the number of times indicated by index:

index/power

$$2^n = \underbrace{2 \times 2 \times \dots \times 2}_{n \text{ times}}$$

base

n times

Square: 2^{nd} power (Perfect Square)

Numbers: $2^2 = 4, 3^2 = 9, 4^2 = 16, \dots$

Variable: $x^2, x^2y^2z^8, 4a^4b^2c^2, \dots$

(!) For any term to be a square, the power of all the variables and numbers must be a multiple of 2.

Square Root: $\frac{1}{2}$ power

$$\sqrt{16} = 16^{\frac{1}{2}} = (4^2)^{\frac{1}{2}} = 4$$

$$\sqrt{16} = 16^{\frac{1}{2}} = \left[(-4)^2\right]^{\frac{1}{2}} = -4$$

(!) To find a square root of a number, change it to a indices with a power of 2 and divide its power by 2.

Cube: $3rd$ power (Perfect Cube)

Numbers: $3^3 = 27, 4^3 = 64, 5^3 = 125, \dots$

Variable: $x^3, 8a^6b^9c^{18}, \dots$

(!) For any term to be a cube, the power of all the variables and numbers must be a multiple of 3.

Cube Root: $\frac{1}{3}$ power

$$\sqrt[3]{81} = (81) = (3^3) = 3$$

(!) To find a cube root of a number, change it to a indices with a power of 3 and divide its power by 3.

Highest Common Factor (HCF)

- Largest common factors among 2 or more given numbers

Steps:

1. Rewrite numbers given as a product of their prime factors
2. Circle all the common prime factors with the lowest power
3. Multiply all the circled prime factors

Example:

- Find the HCF of 48 and 60

$$48 = 2 \times 2 \times 2 \times 2 \times 3$$
$$= 2^4 \times ③$$

$$60 = 2 \times 2 \times 3 \times 5$$
$$= ⓶^{2} \times 3 \times 5$$

HCF= $2^2 \times 3 = 6$

Lowest Common Multiple(LCM)

- Smallest common multiple among 2 or more given numbers.

Steps:

1. Rewrite numbers given as a product of their prime factors
2. Identity the highest power of each prime factor and circled them
3. Multiply all the circled prime factors

Example:

- Find the LCM of 48 and 60

$$48 = 2 \times 2 \times 2 \times 2 \times 3$$
$$= (2^4) \times ③$$

$$60 = 2 \times 2 \times 3 \times 5$$
$$= 2^2 \times 3 \times 5$$

LCM= $2^4 \times 3 \times 5 = 240$

ESTIMATION

To make an approximation is to round off a number to required degree of accuracy according to the following:

- A given place value
- A given number of decimal places
- A given number of significant places

Value Place

millions, hundred thousands, ten thousands, thousands, hundreds, tens, ones

1 2 3 4 5 6 7

Decimal Place

ones, tenth (1 d. p.), hundredth (2 d.p.), thousandth (3 d.p.)

0 . 0 0 1

Rounding off:

Round up if digit behind ≥ 5

Round down if digit < 5

(!) A number should only be rounded off from its original value not from the rounded values.

Example 1: Rounding off a specified place value.

Round off 28,524 to the nearest
a) 10
b) 100
c) 1 000
d) 10, 000

Answer:
a) 28524=28520 (nearest 10)
b) 28524 = 28 500 (nearest 100)
c) 28574=29000 (nearest 1000)
d) 28324=30 000 (nearest 10,000)

Example 2: Rounding off to a required number at decimal place.

Round off 8.4674 to
a) 3 decimal places
b) 2 decimal places
c) 1 decimal places
d) the nearest whole number

Answer:
a) 8.4764=8.467 (correct to 3d-p.)
b) 8.4764 = 8 .47 (correct to 2 d-p.)
c) 8.4764= 8.5 (correct to 1 d. p.)
d) 8 .4764 = 8 (correct to nearest whole number)

SIGNIFICANT FIGURE

Significant figures are the number of digits used to denote an exact value to a specified degree of accuracy.

Rules to identify the significant digits:

#1 All non-zero digit(s) are significant.

E. g. 24(2 s.f), 46.5 (3 s.f)

#2 Zero(s) between non-zero digit are significant.

E.g 301 (3s.f.), 5.0001 (5s.f.)

#3 Zero(s) that come before the first non-zero digit are not significant.

E. g. 0.0007 (1 s.f.), 00101 (3 s.f.)

#4 Zero(s) following a non-zero digit after the decimal point are significant.

E.g. 0.60 (2 s.f.), 5.4000 (5 s.f.)

#5 Zero(s) following a non-zero digit in a whole number may or may not be significant depends on the estimation made.

E.g. 56700 can be 3 s.f. , 4 s.f. or 5 s.f.

Example 1: Identify digits which are significant

State the number of significant figures in each of the following.

a) 2345 → **4 s.f.**
b) 0.059 → **2 s.f.**
c) 8200 → **2 s.f.**
d) 4.002 → **4 s.f.**
e) 5.370 → **4 s.f.**
f) 42.0120 → **4 s.f.**
g) 234000 (to the nearest tens) → **5 s.f.**
h) 234000 (to nearest thousands) → **3 s.f.**

Example 2: Rounding off to a required number of significant figures

Round off each of the following to the number of significant figure as stated in the brackets.

a) 4586 (3 s.f.) → **4590**
b) 0.05758 (2 s.f.) → **0.058**
c) 3.401 (3 s.f.) → **3.40**
d) 234015 (1 s.f.) → **200,000**
e) 39.95 (3 s.f.) → **40.0**
f) 5.19974 (4 s.f.) → **5.200**

Estimation is a way of predicting the answer to a question. Estimation allow us to have a quick check as to whether an answer is roughly the right range without having to work it out exactly.

To estimate a calculation, round off each number to the number of significant figure as necessary and estimate from there.

To estimate to 1 significant figures, estimates to 2 significant figures in the working and then round off to 1 significant figures in the final answer.

To estimate to 2 significant figures, estimates to 3 significant figures in the working and then round off to 2 significant figures in the final answer.

Remember to work to 1 significant figure more than the required in the final answer.

INDICES

$$a^m$$

base ← a, index → m

Laws

same base:

1. $a^m \times a^n = a^{m+n}$ — E.g. $5^4 \times 5^5 = 5^9$
2. $a^m \div a^n = a^{m-n}$ — E.g. $3^5 \div 3^2 = 3^3$

same index/power:

3. $a^m \times b^m = (ab)^m$ — E.g. $4^4 \times 5^4 = (20)^4$
4. $a^m \div b^m = \left(\frac{a}{b}\right)^m$ — E.g. $2^5 \div 5^5 = \left(\frac{2}{5}\right)^5$

5. $(a^m)^n = a^{mn}$ — E.g. $(6^2)^3 = 6^6$

Properties

1. $a^0 = 1$ (power of zero) — E.g. $5^0 = 1$
2. $a^{-m} = \frac{a^{-m}}{1} = \frac{1}{a^m}$ — E.g. $7^{-2} = \frac{1}{7^2}$

   $a^m = \frac{a^m}{1} = \frac{1}{a^{-m}}$ — E.g. $3^7 = \frac{1}{3^{-7}}$

   (negative power)

3. $a^{\frac{1}{n}} = \sqrt[n]{a}$ — E.g. $4^{\frac{1}{4}} = \sqrt[4]{4}$
4. $a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$ — E.g. $10^{\frac{2}{3}} = \sqrt[3]{10^2} = \left(\sqrt[3]{10}\right)^2$

   (Fractional power)

5. If $a^n = a^m$, then $n = m$ — E.g. If $2^{-\frac{3}{2}} = 2^x$, then $x = -3$

Tips

1. Power only apply to those included in the brackets. Others remain untouched.

$3\left(a^2bc^3\right)^2 = 3a^6b^2c^6$ ✔ $3^2a^6b^2c^6$ ✘

2. Numbers with attached powers cannot be easily cancelled out.

$$\frac{4^{x+2}}{8^{2x+3}} \neq \frac{1^{x+2}}{2^{2x+3}}$$

Instead, put them to their lowest prime factor and use the laws of indices:

$$\frac{2^{2(x+2)}}{2^{3(2x+3)}} = 2^{2(x+2)-3(2x+3)} = 2^{-4x-5}$$

3. Numbers with attached powers cannot be easily added or subtracted together.

$$2^{15} - 2^{14} \neq 2$$

Instead, factorise it: $2^{15} - 2^{14} = 2^{14}(2-1) = 2^{14}$

4. Number without powers are just power 1.

Example: $6 = 6^1 \quad 8 = 8^1 \ldots$

Thus, when solving for unknown, write down the power, even when it is a 1.

$$36^x = 6$$
$$6^{2x} = 6^1$$
$$2x = 1$$
$$x = \frac{1}{2}$$

5. Always remove any coefficient first, then express the terms in their lowest prime factors.

$$\frac{1}{64} = 2\left(8^x\right)$$

$$\frac{1}{128} = 8^x$$ Remove Coefficient

$$128^{-1} = 8^x$$

$$2^{-7} = 2^{3x}$$ Express in Lowest Prime Factors

$$-7 = 3x$$
$$x = -\tfrac{7}{3}$$

6. For questions with x in the power, change the 1 to a common base with power 0.

$$3^{x+2} = 1$$
$$3^{x+2} = 3^0$$
$$x + 2 = 0$$
$$x = -2$$

STANDARD FORM

Standard form: $A \times 10^n \quad 1 \leq A < 10$ **n = integer**

*** A must be in 3 s. f. if need to be rounded off ***

Some useful significant figures to remember:

Name	Symbol / Value	Name	Symbol / Value
Tera (trillion)	$T : 10^{12}$	Pico (trillionth)	$p : 10^{-12}$
Giga (billion)	$G : 10^9$	Nano (billionth)	$n : 10^{-9}$
Mega (million)	$M : 10^6$	Micro (millionth)	$\mu : 10^{-6}$
Kilo (thousand)	$K : 10^3$	Milli (thousandth)	$m : 10^{-3}$

Division

→ move decimal point to the left

Example:

$$1 \cdot 72 \times 10^{-3} = 1 \cdot 72 \times \frac{1}{10^3} = 0 \cdot 00172$$

Workings:

$0.001 \cdot 72$

→
- move decimal pt to the left 3 times
- insert the 'O' accordingly

Multiplication

→ move decimal point to the right

Example:

$$1.72 \times 10^3 = 1720$$

Workings:

$1 \cdot 720.$

→
- move decimal pt to the right 3 times
- insert the 'O' accordingly

Basic knowledge

$$10^m \times 10^n = 10^{m+n}$$

$$10^m \div 10^n = 10^{m-n}$$

$$10^{-m} = \frac{1}{10^m}, m > 0$$

Division/ Multiplication of Standard Form

1. Group the 'A' portion together (multiply / divide).

2. Group the 10^n portion together (multiply / divide).

3. Make sure that for the final answer the 'A' portion is between 1 and 10 and in 3 s.f. whenever necessary

Example:

$$1.45 \times 10^3 \div 2.86 \times 10^{-5} = 1.45 \div 2.86 \times 10^3 \times 10^{-5}$$

$$= 0.50699\ldots \times 10^{-2}$$

shift the decimal pt to the right by 1, i. e. minus 1 from the power of 10^n

$$= 5.07 \times 10^{-2-1}$$

$$= 5.07 \times 10^{-3}$$

Addition /Subtraction of Standard Form

1. To solve addition and subtraction of standard form we need to use factorisation.

2. Make sure that both term have the same 10^n.

3. Final answer in the form of $A \times 10^n, 1 \leq A < 10$.

Example:

$$3 \cdot 76 \times 10^6 + 4 \cdot 87 \times 10^4 = 3 \cdot 76 \times 10^6 + 0 \cdot 0048 \times 10^6$$

$$= 10^6\,(3.76 + 0.0048)$$

$$= 3.76 \times 10^6$$

Either change both to 10^4 or 10^6

If change to 10^6 :

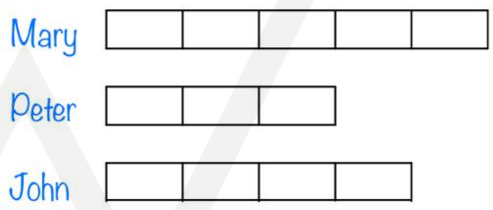
$0.0\,4.87 \times 10^{4+2} = 0.0048 \times 10^6$

Add 2 to the power of 10^n therefore shift 4.87 2 d. p. to the left.

RATIO

Ratio compares 2 or more quantities that are expressed in the same units ☐ : ☐ : ☐.

For example, money earned by Mary , Peter and John are in the ratio of 5:3:4.



It also shows relative proportions.

Ratio of a to b $= a : b$

$$= \frac{a}{b}$$

NOTE: A and B must be similar units.

Proportion of a $= \dfrac{a}{a+b}$

PROPERTIES

1. Can multiply throughout by a common factor, even for rational fractions

	Mary	:	Peter	:	John
	5	:	3	:	4
$\times 2$	10	:	6	:	8
$\times 10$	100	:	60	:	80
$\div 2$	50	:	30	:	40

	$\frac{3}{4}$	:	$\frac{5}{12}$	:	$\frac{2}{7}$
$\times 12$	9	:	5	:	$\frac{24}{7}$
$\times 7$	63	:	35	:	24

2. Ratio must be in similar units

✓ $15cm : 60cm = 1 : 4$

✗ $15cm : 5m \neq 3 : 1$

Change to similar units first:

✓ $15cm : 5m = 15cm : 500cm$
$= 3 : 100$

NOTE: Always express in simplest lowest form

PERCENTAGE

Percentage is just:

- a fraction with 100 as its denominator
- a proportion over 100

$$25\% = \frac{25}{100} = 0.25$$

They are the same

Percentage → Fraction / Decimal: Divide by 100%

Example:
$$33\% = \frac{33}{100} = 0.33$$
$$0.5\% = \frac{0.5}{100} = 0.005$$

Fraction / Decimal → Percentage: Multiply by 100%

Example:
$$\frac{2}{5} = \frac{2}{5} \times 100\% = 40\%$$
$$0.3 = 0.3 \times 100\% = 30\%$$

$$\text{Percentage change} = \frac{\text{new value} - \text{initial value}}{\text{initial value}} \times 100\%$$

initial value → relative 100%
compare new value against initial value

if % change > 0 : there is an increase in value

if % change < 0: there is an decrease in value

RATE

This compares the change in 2 quantities measured in different units, usually involve time.

$$\text{Average rate} = \frac{\text{total amount}}{\text{total time taken}}$$

Example: A man works 60 hours and earns \$900. Calculate the rate of pay.

$$\text{Rate of pay} = \frac{\text{total pay}}{\text{total work time}} = \frac{\$900}{60hrs} = \$15/hr$$

Other Variations:

$$\text{Average speed} = \frac{\text{total distance travelled}}{\text{total time taken}}$$

$$\text{Average cost} = \frac{\text{total cost}}{\text{total amount}}$$

MAPS & SCALES

This concept is just the application of ratio.

DISTANCE SCALE

$1 : n$

- 1 unit length on map represents n units on actual ground
- Representative fraction (RF) $= \frac{1}{n}$
- Map scale 1: 50000

  1 cm on map represent 50 000 cm actual length

Take Note: map unit and actual unit must be the same before putting them in ratio, unless otherwise stated

AREA SCALE

- This is the square of linear scale
- Linear scale $\longrightarrow 1 : n$

  Area scale $\longrightarrow (1)^2 : (n)^2$

Example: Map scale of $2cm : 1km$

Area scale is $(2cm)^2 : (1km)^2$

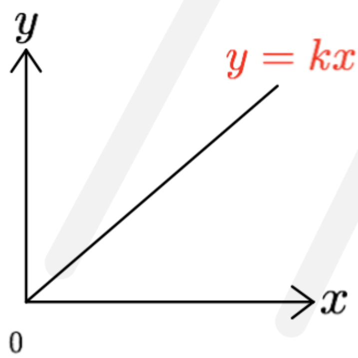
$4cm^2 : 1km^2$

Take Note: Change both the unit of the map and actual length to what is required in the question before squaring them.

PROPORTION

Direct Proportion

$$y \propto x$$



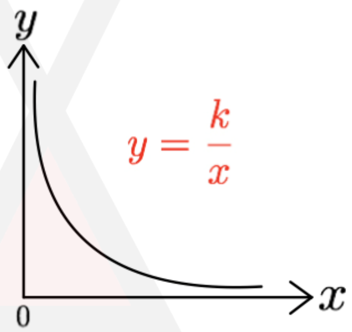
- y varies directly as x
- when x increases, y increases

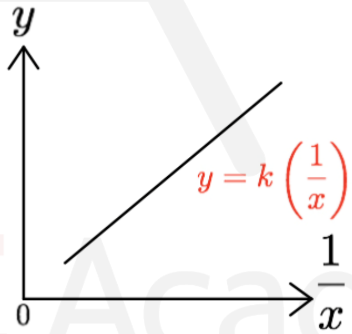
Formula: $y = kx$

where k is a constant

Indirect proportion/ Inverse proportion

$$y \propto \frac{1}{x}$$





- y varies indirectly as x
- when x increases, y decreases

Formula: $y = \frac{k}{x}$

where k is a constant

Example: People building houses

↑ People ↓ Time taken to build

Other Variation

$y \propto \frac{1}{x^2} \Rightarrow y = \frac{k}{x^2}$

$y \propto \frac{1}{x^3} \Rightarrow y = \frac{k}{x^3}$

$y \propto x^3 \Rightarrow y = kx^3$

where k is a constant

Area of circle, $A = \pi r^2$

$\rightarrow A \propto r^2$

since π is a constant

Volume of sphere, $V = \frac{4}{3}\pi r^3$

$\rightarrow V \propto r^3$

since $\frac{4}{3}\pi$ is a constant

FINANCIAL TRANSACTIONS

Profit / Loss

In any business, you either make a profit, make a loss or break even. This is determined by the value of:

Selling price - Cost price

Value > 0: Profit

Value < 0 : Loss

Value = 0 : Break even

① **Profit** of y%

$$= \frac{y}{100} \text{ x cost price}$$

$$= \frac{\text{selling price - cost price}}{\text{cost price}} \text{ x } 100\%$$

② **Selling price**

$$= \text{cost price} + \frac{y}{100} \text{ x cost price}$$

$$= \frac{100+y}{100} \text{ x cost price}$$

③ **Cost price**

$$= \frac{\text{selling price}}{100+y} \text{ x } 100$$

① **Loss** of y %

$$= \frac{y}{100} \text{ x cost price}$$

$$= \frac{\text{selling price - cost price}}{\text{cost price}} \text{ x } 100\%$$

② **Selling price**

$$= \text{cost price} - \frac{y}{100} \text{ x cost price}$$

$$= \frac{100-y}{100} \text{ x cost price}$$

③ **Cost price**

$$= \frac{\text{selling price}}{100-y} \text{ x } 100$$

For a discount of n% on selling price,

① Discount $= \frac{n}{100}$ x selling price

② Price after discount $= \frac{100-n}{100}$ x selling price

③ Selling price $= \frac{\text{Price after discount}}{100-n}$ x 100

④ $n\% = \frac{\text{original selling price - price after discount}}{\text{original selling price}}$ x 100

Interest Rate

SIMPLE INTEREST

Fixed interest every year based on principal (initial amount)

$$I = \frac{PRT}{100}$$

- I: Simple interest earned
- P: Principal (initial) amount
- R: Interest rate per annum
- T: Time period (years)

$$A = P + I$$

- A: Accumulated amount: principal + interest
- P: Principal (initial) amount
- I: Simple interest earned

COMPOUND INTEREST

Interest rate is based on the cumulated amount of money present, inclusive of the interest earned in the previous years.

$$A = P\left(1 + \frac{r}{100}\right)^n$$

- A: Accumulated amount: principal + interest
- P: Principal (initial) amount
- r: Interest rate per time period
- n: Number of time periods

same unit of time

Example: P = $100, R=1% per annum, T=5 years

Year	Interest	Total
0	$0	$100
1	1% × $100 = $1	$101
2	1% × $100 = $1	$102
3	1% × $100 = $1	$103
4	1% × $100 = $1	$104
5	1% × $100 = $1	$105

Example: P = $100, R=1% per annum, n=5 years

Year	Interest	Total
0	$0	$100
1	1% × $100 = $1	$101
2	1% × $101 = $1.01	$102.01
3	1% × $102 · 01 = $1.02	$103.01
4	1% × $103.03 = $1.03	$104.01
5	1% × $104.06 = $1.04	$105.01

Currency Exchange

(Currency 1) \$ X = (Currency 2) \$ Y $\longrightarrow$ (Currency 1) \$1 = (Currency 2) $\$\frac{Y}{X}$

$\longrightarrow$ (Currency 2) \$1 = (Currency 1) $\$\frac{X}{Y}$

Example: Exchange rate between USD and SGD was SGD 1.25 = USD 1

$$\text{SGD } 1 = \text{USD } \frac{1}{1\cdot 25} = \text{USD } 0.80$$

$$\text{SGD } 5 = 5 \times 0.80 = \text{USD } 4$$

$$\text{USD } 8 = 8 \times 1\cdot 25 = \text{SGD } 10$$

Tax Payment

Chargeable Income	Income Tax Rate (%)	Gross Tax Payable (\$)
First \$20000 Next \$10000	0 2	0 200
First \$30000 Next \$10000	– 3 · 5	200 350
First \$40000 Next \$40000	– 7	550 2800
First \$80000 Next \$40000	– 11.5	3350 4600
First \$120000 Next \$40000	– 15	7950 6000
First \$160000 Next \$40000	– 17	13950 6800
First \$200000 Next \$120000	– 18	20750 21600
First \$320000 Above \$320000	– 20	42350

For chargeable income of \$150000, \$7950 will be the amount taxed on the first \$120000.
The remaining \$30000 will be taxed at 15%, which is \$4500.
Hence, total taxed amount is \$12450.

REDUCTION OF TAXABLE AMOUNT

Chargeable income = annual gross income - tax reliefs

Disposable income = annual gross income-taxes paid

Utilities Payment

Total cost of utility = Usage x cost of utility per unit

Sample table on cost of each utility:

Utility	Category (per month)	Cost per unit (before GST)
Water	$\leq 40m^3$ $> 40m^3$	$\$1.1700/m^3$ $\$1.4000/m^3$
Gas	–	$\$0.2241/kWh$
Electricity	–	$\$0.2101/kWh$

NOTE: Make sure that the unit correspond, before doing the calculations.

NUMBER PATTERN

There are essentially 3 types of patterns:

① Constant difference between terms

i. e. $1, 4, 7, 10, 13 \ldots$

$+3 \quad +3 \quad +3 \quad +3$

Formula nth term:

$$T_n = a + (n-1)d$$

a ↑ Ist term; d ↑ Constant difference

Example:

$83, 77, 71, 65, 59, \ldots$

$-6 \quad -6 \quad -6 \quad -6$

Constant difference : -6

Ist term : 83

$$T_n = 83 + (n-1)(-6)$$
$$= 83 - 6n + 6$$
$$= 89 - 6n$$

② Varying difference between terms

But constant difference between the amount of difference

i. e. $3, 6, 11, 18, 27, \ldots$

$+3 \quad +5 \quad +7 \quad +9$ ⟵ Varying difference

$+2 \quad +2 \quad +2$ ⟵ Constant difference

Formula nth term:

$$T_n = a + (n-1)d_1 + \frac{1}{2}(n-1)(n-2)d_2$$

a ↑ Ist term; d_1 ↑ Ist difference; d_2 ↑ Constant difference

Example:

$9, 16, 26, 39, 55, \ldots$

$+7 \quad +10 \quad +13 \quad +16$

$+3 \quad +3 \quad +3$

Ist term: 9

Ist difference: 7

Constant difference: 3

$$T_n = 9 + (n-1)(7) + \frac{1}{2}(n-1)(n-2)(3)$$
$$= \frac{3}{2}n^2 + \frac{5}{2}n + 5$$

③ Multiplied by constant difference

i. e. $2, 8, 32, 128, 512, \ldots$

$\times 4 \quad \times 4 \quad \times 4 \quad \times 4$

Formula nth term:

$$T_n = ar^{n-1}$$

a ↑ Ist term; r ↳ Multiplied Constant Difference

Example:

$200, 100, 50, 25, \ldots$

$\div 2 \quad \div 2 \quad \div 2$

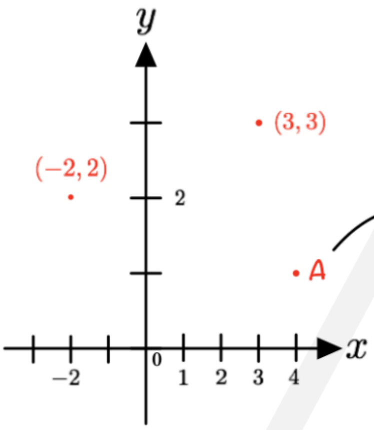
NOTE: $\div 2$ is equivalent to $\times \frac{1}{2}$

Constant difference : $\frac{1}{2}$

Ist term : 200

$$T_n = 200\left(\frac{1}{2}\right)^{n-1}$$

COORDINATE GEOMETRY



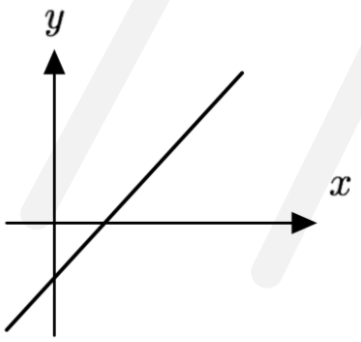
Location for point A is x=4, y=1.

Its coordinate is (4,1).

when plotting a graph choose a scale that is easy for you to plot your points and big enough

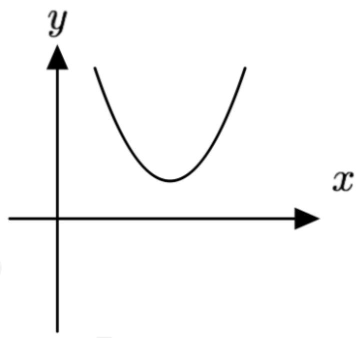
Graphs

<Linear>



The change is consistent with every unit increase.

<Non-Linear>



The change is not consistent with every unit increase.

Linear graphs

A linear graph is where the points can be joined to form a straight line.

(we need a minimum of 2 points to draw a straight line accurately)

A linear graph consists of all pairs of (x, y) that satisfy the linear equation which is in the form of:

$$y = mx + c$$

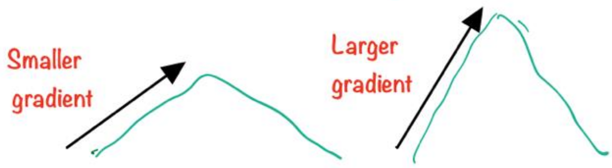
x and y coordinates of a point on the line

m: gradient of the line
:determine how steep the line is

y- intercept: the point where the line cut y-axis (0, y)
: the value of y when x = 0

Note: Any point on the line will fulfil the equation of y=mx+c

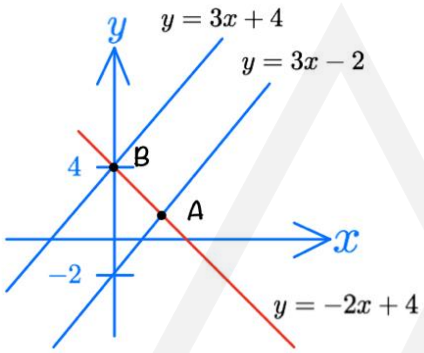
Gradient of Linear Graph



The steeper the hill the greater the m,

↑ m ↑ slope

effort needed to climb to climb a slope



Note: For lines that are // , they have same gradient but different y-intercept.

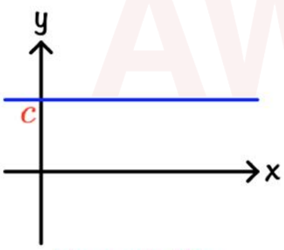
To find intersection point of 2 lines (A&B):

Point A → equate y = -2x +4 and y = 3x -2 to find out x-coordinate and sub the value into any of the 2 eqn to find the y- coordinate.

Point B → since B lies on the y-axis we know that the y-coordinate of the point straight from the graph which is 4. Sub the y-value back into either y = -2x + 4 or y = 3x + 4 to find x-coordinate.

4 types of linear graph and the effect of the gradient, m:

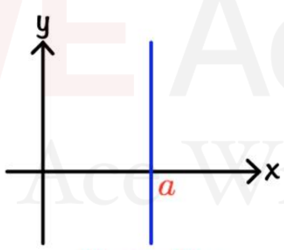
$m = 0$



Horizontal line
$y = c$

- passes through y - intercept at c, where can be positive or negative

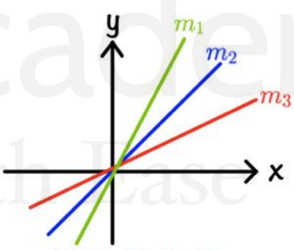
$m = \infty$



Vertical line
$x = a$

- passes through x - intercept at a, where a can be negative or positive

$m > 0$

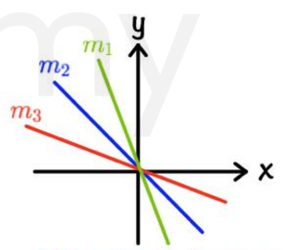


Upward sloping
$y = mx$
$m_1 > m_2 > m_3$

- Slanted line that pass through origin
- Slope upwards

↑ m ↑ Slope

$m < 0$



Downward slopping
$y = -mx$
$m_1 < m_2 < m_3$

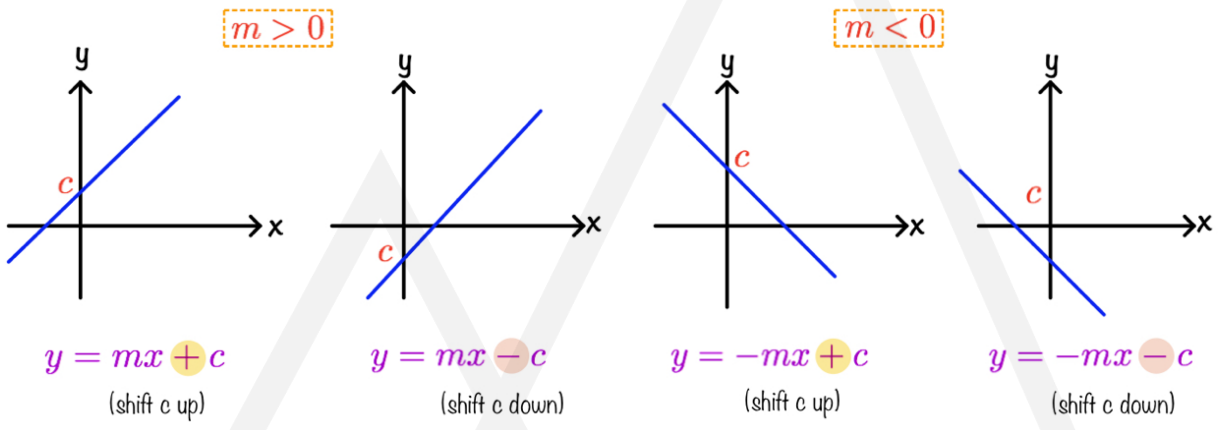
- Slanted line that pass through origin
- Slope downwards

↑ m ↓ Slope

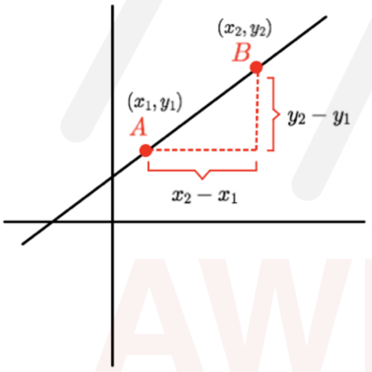
To obtain y = mx + c , we can shift y = mx up or down vertically by c

$$y = mx + c$$

Graph cuts y-axis at c (y - intercept)



Formula



Gradient of the line:

Step 1: select 2 points on the line

Step 2: Find vertical & horizontal

Step 3: Gradient $= \dfrac{\text{Vertical change}}{\text{Horizontal change}} = \dfrac{y_2 - y_1}{x_2 - x_1}$

Equation of the line:

$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{or} \quad y - y_1 = m(x - x_1)$$

Length at line segment AB

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \text{or} \quad \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

SKETCHING GRAPH

Type 1

completing the square

General formula: $a(x-h)^2+k$

$a>0$ ∪ *shaped*	min. pt.
$a<0$ ∩ *shaped*	max. pt.

h & f are constants

Step 1: Min. / Max pt

by observing the eqn:

$$a(x-h)^2+k$$

x → h, y → k

$$(h, k)$$

Step 2: x-intercept (cuts the x-axis)

$$\therefore y=0$$

Sub $y=0$ into equation to find out the x-value where the curve touch the x-axis

$$(x, 0)$$

Step 3: y-intercept (cuts the y-axis)

$$\therefore x=0$$

Sub $x=0$ into equation to find out the y-value where the curve touch the y-axis

$$(0, y)$$

Type 2

cross-method

General formula: $a(x-p)(x-q)$

$a>0$ ∪ *shaped*	min.pt.
$a<0$ ∩ *shaped*	max.pt

p & q are constants

Step 1: x-intercept

by observing the eqn:

$$a(x-p)(x-q)$$

p and q are the x-intercept

$$(p, 0)\ (q, 0)$$

Step 2: Min. / Max pt

x-value of min. / max. pt : $\dfrac{p+q}{2}$

midpoint of x-intercept

y-value of min. / max. pt : Sub x value back into eqn to find y

Step 3: y-intercept (cuts the y-axis)

$$\therefore x=0$$

Sub $x=0$ into equation to find out the y-value where the curve touch the y-axis

$$(0, y)$$

Example 1

Draw $y=(x-2)^2+6$

since a> 0, ∪ *shaped*

Step 1: Min. / Max pt $=(2,6)$

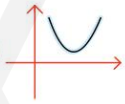
Step 2: x-intercept $\therefore y=0$

Sub $y=0$ *into eqn*

$$0=(x-2)^2+6$$

x value is undefined, no x-intercept

If $a>0$, graph is floating above x-axis



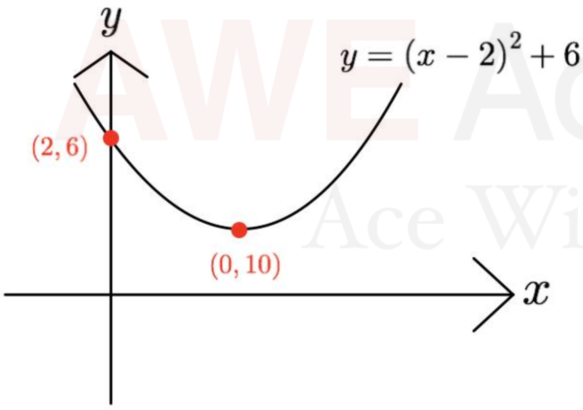
If $a<0$, graph is floating below x-axis

Step 3: y-intercept $\therefore x=0$

Sub $x=0$ *into eqn*

$$y=(0-2)^2+6=10$$

coordinate of y-intercept $(0,10)$



Example 2

Draw $y=-2(x-2)(x+4)$

since a<0 ∩ *shaped*

Step 1: x-intercept

2 and -4 are the x-intercept

coordinates of x-intercept is $(2,0)$ and $(-4,0)$

Step2 : Min. / Max pt

since a<0 ∩ *shaped* $\therefore$ max *pt*

max. pt x-values : $\dfrac{2+(-4)}{2}=-1$

max. pt y-values : *Sub* $x=-1$ into eqn

$$y=-2(-1-2)(-1+4)$$
$$=18$$

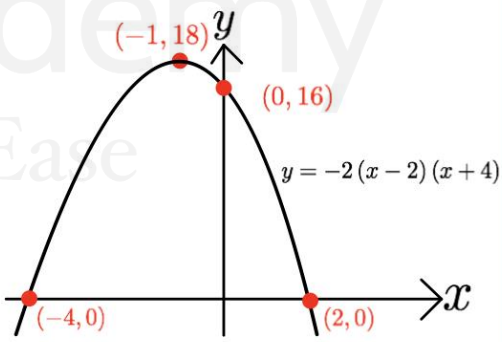
coordinates of max. pt: $(-1,18)$

Step 3: y-intercept $\therefore x=0$

Sub $x=0$ *into eqn*

$$y=-2(0-2)(0+4)=16$$

coordinate of y-intercept is $(0,16)$



Remember to label in your eqn, x-intercept, y-intercept and min. or max. pt

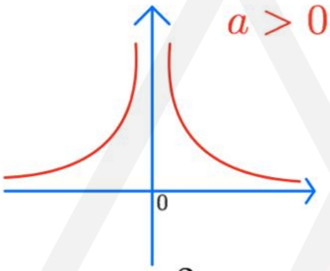
NON- LINEAR GRAPH

Graph of $y = ax^n$

where a and n are constants

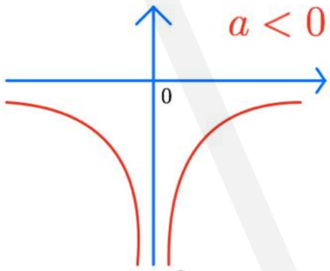
$n = -2, y = \dfrac{a}{x^2}$

$a > 0$

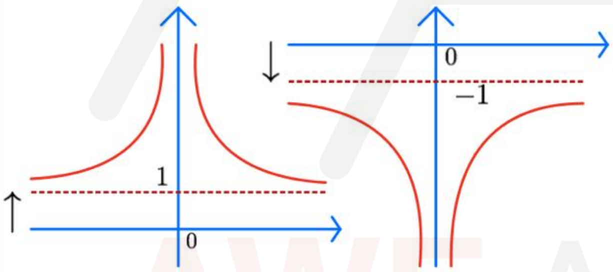


$$y = \frac{2}{x^2}$$

$a < 0$



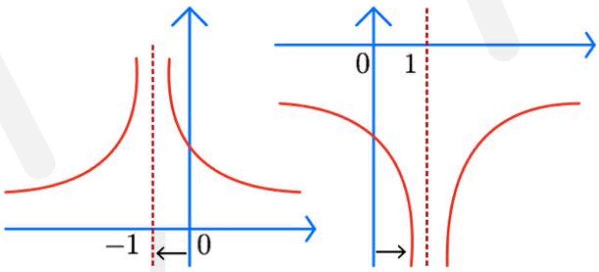
$$y = -\frac{2}{x^2}$$



$$y = \frac{2}{x^2} + 1 \qquad y = -\frac{2}{x^2} - 1$$

$y = \dfrac{a}{x^2} + b$ is obtained by shifting $y = \dfrac{a}{x^2}$ up or down by $|b|$ units.

$b > 0$: shift up
$b < 0$: shift down

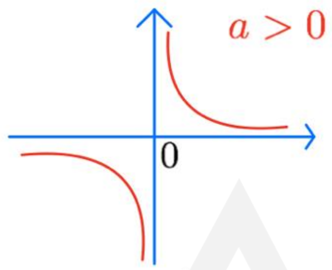


$$y = \frac{2}{(x+1)^2} \qquad y = -\frac{2}{(x-1)^2}$$

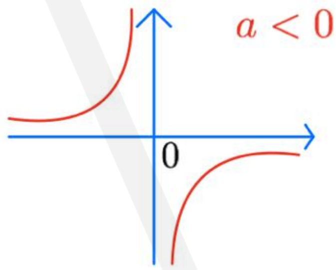
$y = \dfrac{a}{(x+c)^2}$ obtained by shifting $y = \dfrac{a}{x^2}$ left or right by $|c|$ units.

$c > 0$: shift left
$c < 0$: shift right

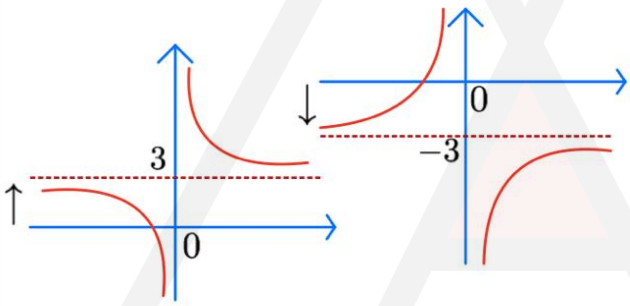
$n = -1 : y = \dfrac{a}{x}$



$y = \dfrac{1}{x}$



$y = -\dfrac{1}{x}$



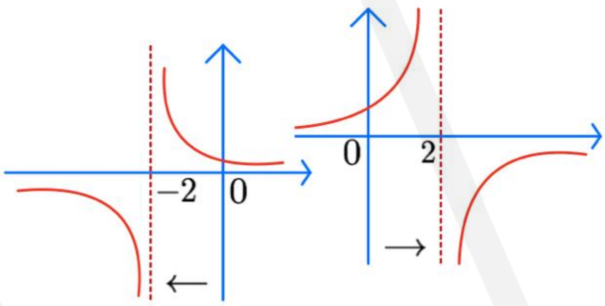
$y = \dfrac{1}{x} + 3$ $\quad$ $y = -\dfrac{1}{x} - 3$

$y = \dfrac{a}{x} + b$ is obtained by shifting

$y = \dfrac{a}{x}$ up or down by $|b|$ units.

$b > 0$: shift up

$b < 0$: shift down



$y = \dfrac{1}{x+2}$ $\quad$ $y = -\dfrac{1}{x-2}$

$y = \dfrac{a}{x+c}$ obtained by shifting

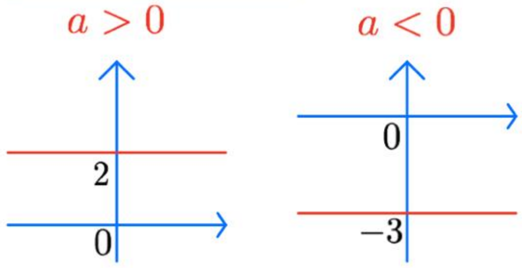
$y = \dfrac{a}{x}$ left or right by $|c|$ units.

$c > 0$: shift left

$c < 0$: shift right

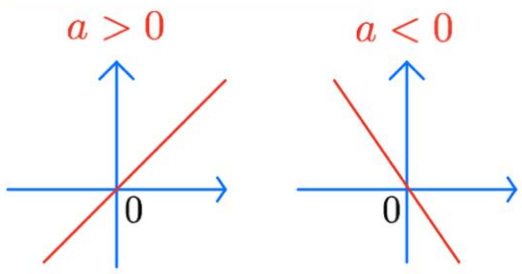
Linear Graph

$n = 0 : y = a$



$y = 2$ $\quad$ $y = -3$

$n = -1 : y = ax$

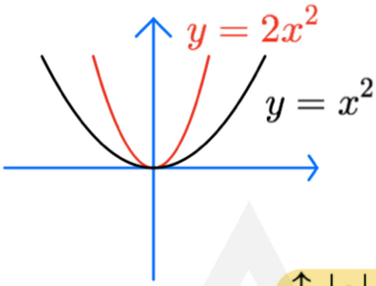


$y = x$ $\quad$ $y = -2x$

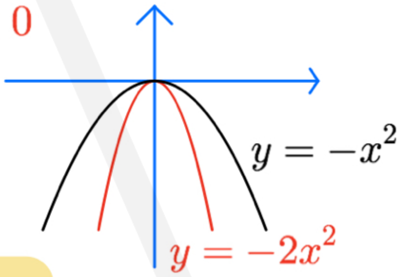
Quadratic Graphs

$n = 2 : y = ax^2$

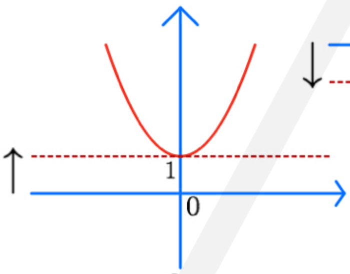
$a > 0$

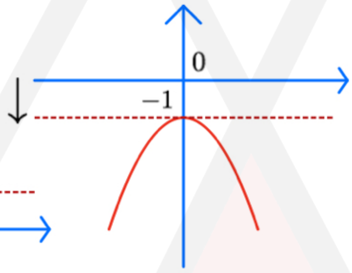


$a < 0$



$\uparrow |a| \longleftrightarrow \uparrow$ steepness of curve

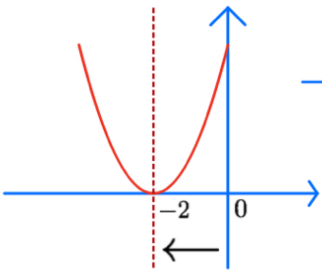


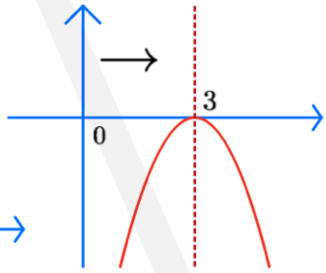


$y = 2x^2 + 1$ $\quad$ $y = -2x^2 - 1$

$y = ax^2 + b$ is obtained by shifting $y = ax^2$ up or down by $|b|$ units.

$b > 0$: shift up
$b < 0$: shift down





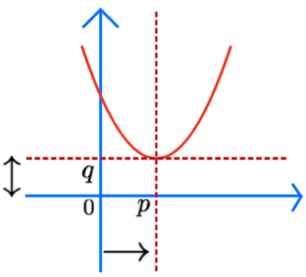
$y = (x + 2)^2$ $\quad$ $y = (x - 3)^2$

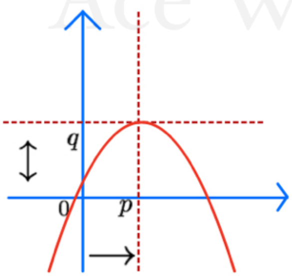
$y = a(x + 2)^2$ is obtained by shifting $y = ax^2$ left or right by $|c|$ units.

$c > 0$: shift left
$c < 0$: shift right

$$y = a(x - p)^2 + q$$

$a > 0$ $\quad$ $a < 0$



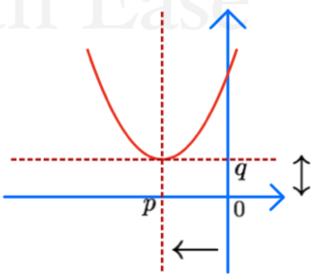


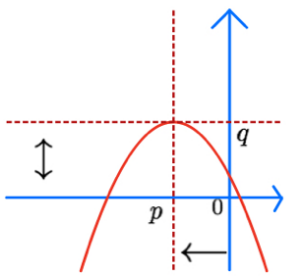
(min. pt) $\quad$ (max. pt)

- Min/max point at (p, q)
- Axis of symmetry at $x = p$
- Obtained by shifting $y = ax^2$ up or down by q, and shifting it right by p

$$y = a(x + p)^2 + q$$

$a > 0$ $\quad$ $a < 0$

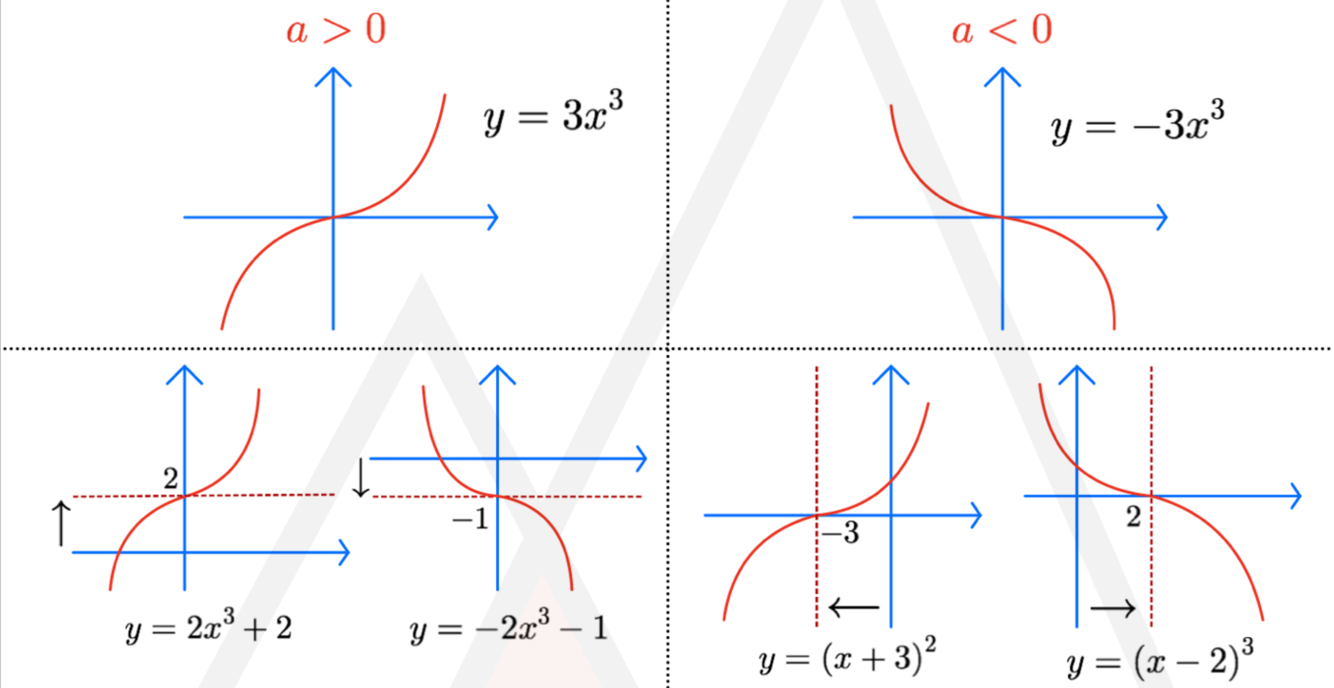




(min. pt) $\quad$ (max. pt)

- Min/max point at (p, q)
- Axis of symmetry at $x = p$
- Obtained by shifting $y = ax^2$ up or down by q, and shifting it left by p

$n = 3 : y = ax^3$



$y = ax^3 + b$ is obtained by shifting $y = ax^3$ up or down by $|b|$ units.

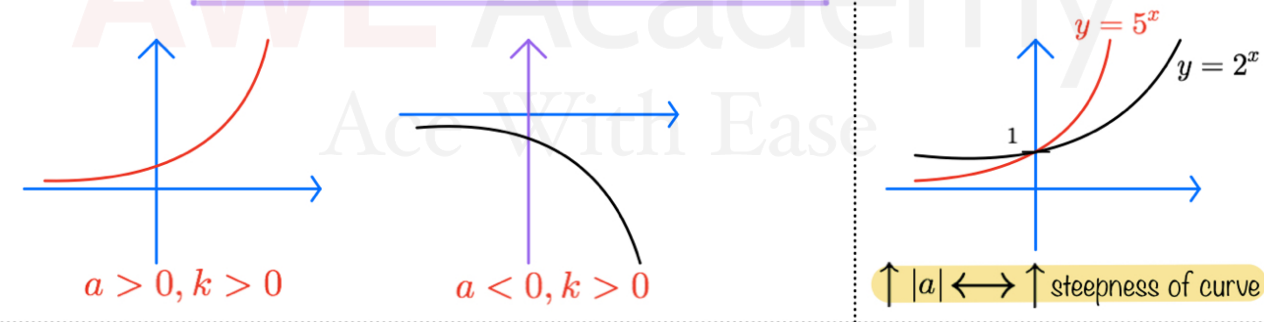
$b > 0$: shift up
$b < 0$: shift down

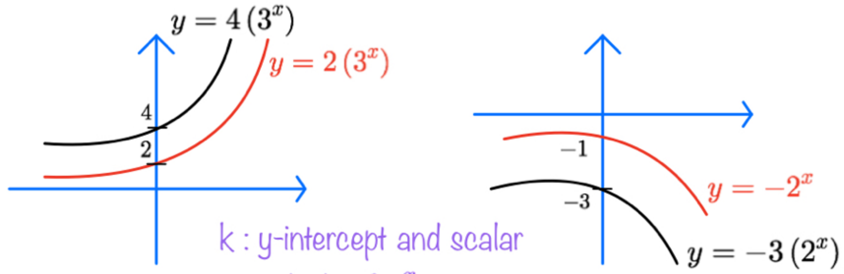
$y = a(x+c)^3$ is obtained by shifting $y = ax^3$ left or right by $|c|$ units.

$c > 0$: shift left
$c < 0$: shift right

Exponential Graphs

$y = ka^x$, where k and a are constants





k : y-intercept and scalar multiple of a^x

$y = k(a)^{x+n}$ is of the same graph type:

$y = k(a)^x (a)^n$
$y = [k(a)^n](a)^x$
$y = p(a)^x$

where $p = ka^n$ is constant

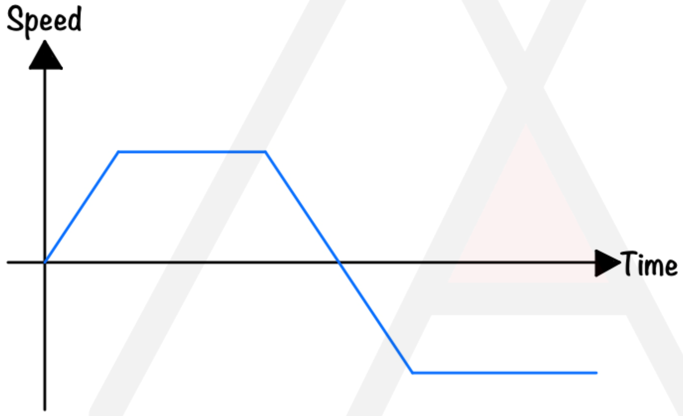
PRACTICAL GRAPH

Speed Time Graph

Shows the variation of an object's speed across time.

Linear

Acceleration / deceleration is constant.



gradient = $\frac{speed}{time}$

= acceleration or deceleration

area under the graph = distance travelled

Line above x-axis:

gradient > 0 : constant acceleration

gradient < 0 : constant deceleration

gradient $= 0$: constant speed

Line below x-axis:

= negative speed

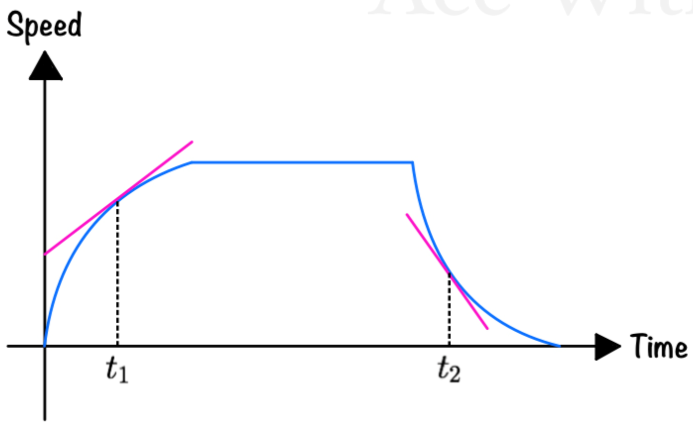
= opposite direction of travel

gradient > 0 : constant deceleration

gradient < 0 : constant acceleration

Non-Linear

Acceleration/deceleration is varying.



gradient of tangent at that point in time, t

= acceleration / deceleration at time t

gradient $= 0$: constant speed

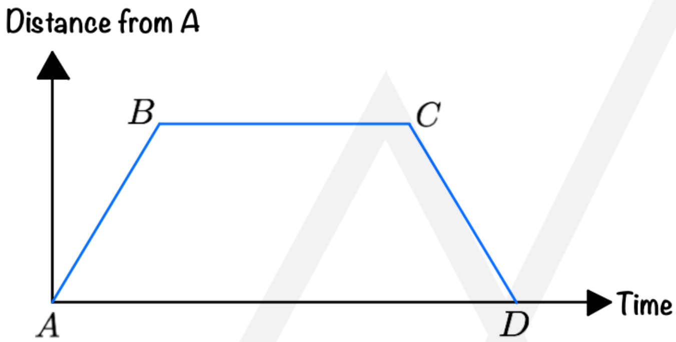
area under the graph = distance travelled

Distance Time Graph

Shows the distance of an object from starting point over time.

Linear

Speed is constant. No acceleration / deceleration.



$$\text{gradient} = \frac{\text{distance}}{time} = speed$$

gradient > 0 : moving away

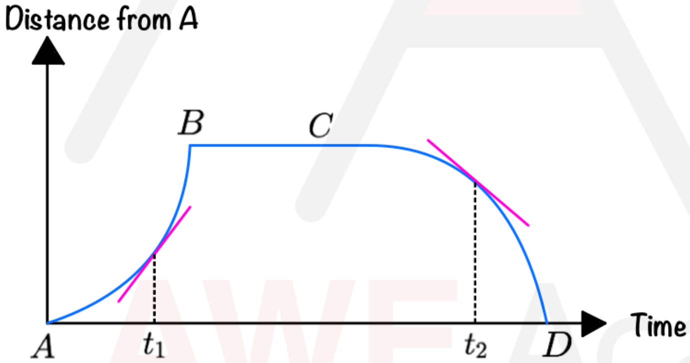
gradient < 0 : moving back towards A

gradient $= 0$: at rest

distance = add up distance at each point

Non-Linear

Speed is varying. There is acceleration / deceleration.



speed at time t =

gradient of tangent at that point in time

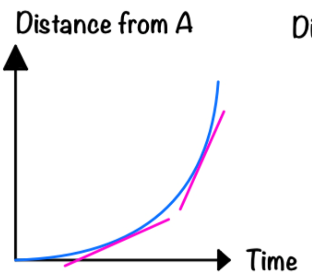
gradient > 0 : moving away

gradient < 0 : moving back towards A

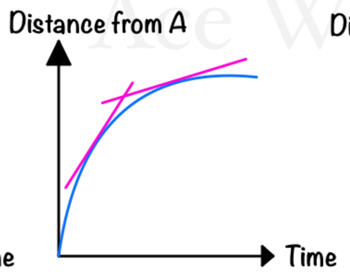
gradient $= 0$: at rest

distance = add up distance at each point

Away from A

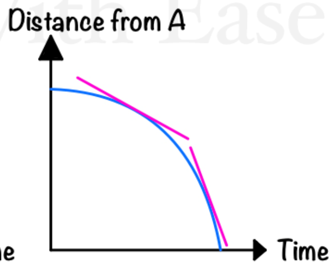


As t ↑
gradient ↑
steep (speed) ↑
∴ object accelerating

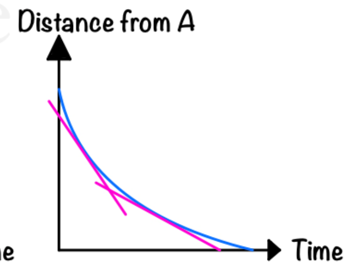


As t ↑
gradient ↓
steep (speed) ↓
∴ object decelerating

Towards A



As t ↑
gradient ↑
steep (speed) ↑
∴ object accelerating

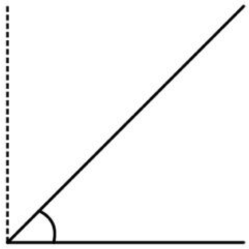


As t ↑
gradient ↓
steep (speed) ↓
∴ object decelerating

ANGLES

Types Of Angles

1. Acute angle



Angle less than 90°

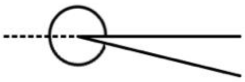
2. Right angle

Angle equal to 90°

3. Straight angle

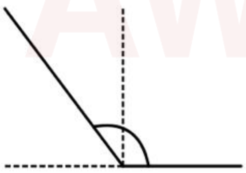
Angles equal to 180°

4. Reflex angle



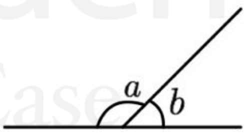
Angles greater than 180° but less than 360°

5. Obtuse angle



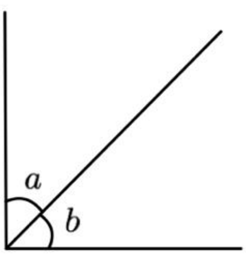
Angles greater than 90° but less than 180°

6. Supplementary angles



$a + b = 180^\circ$

7. Complementary angles



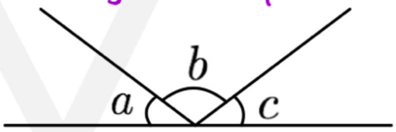
$a + b = 90^\circ$

Properties Of Angles

Below are the properties that you should look out for when solving questions involving angles in:

1. Circles
2. Polygons
3. Triangles
4. Quadrilaterals

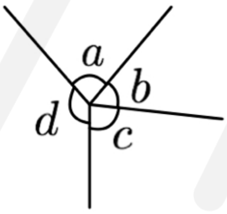
The sum of adjacent angles on a straight line is equal to $180°$



$$a + b + c = 180°$$

($\angle s$ on a straight line)

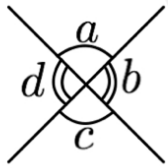
Angles at a point add up to $360°$



$$a + b + c + d = 360°$$

($\angle s$ at a point)

Vertically opposite angles are equal

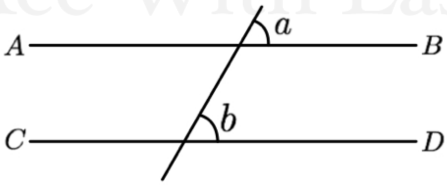


$a = c$ (vert. opp. $\angle$s)

$b = d$ (vert. opp. $\angle$s)

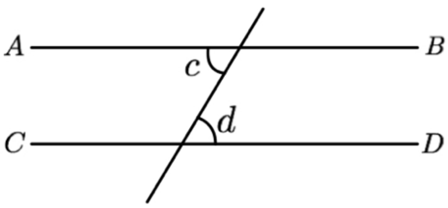
When two parallel lines are cut by a transversal (a line that intersect two or more line):

The corresponding angles are equal



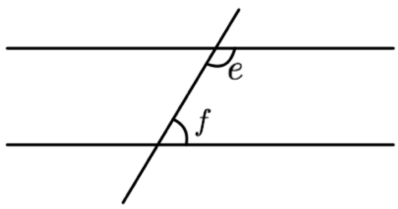
$a = b$ (corr. $\angle$s, AB // CD)

The alternate angles are equal



$c = d$ (alt. $\angle$s, AB // CD)

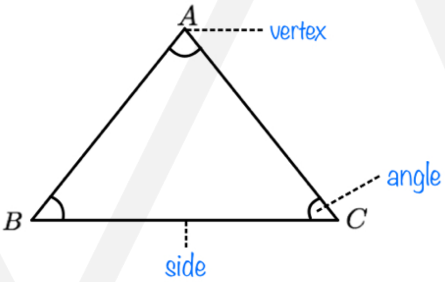
The interior angles are supplementary

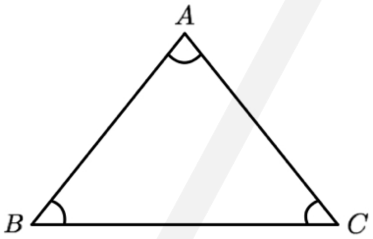


$e + f = 180°$ (int. $\angle$s, AB // CD)

TRIANGLES

A triangle is a plane figure with 3 straight sides.
It has 3 angles and 3 vertices.



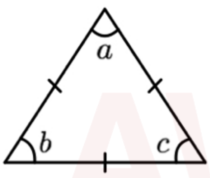
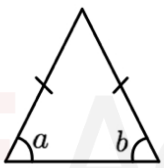
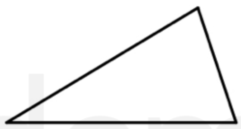


How to name an triangle?

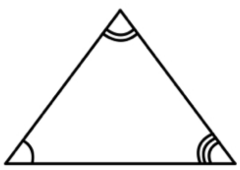
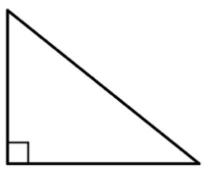
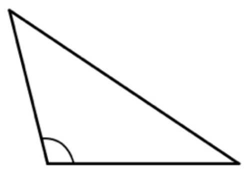
To name a triangle, use the vertices in a clockwise or anti-clockwise order.

The triangle is called $\triangle ABC$ or $\triangle ACB$

Types Of Triangles

Equilateral triangle	Isosceles triangle	Scalene triangle
		
Three equal sides All angles equal to 60° $a = b = c = 60^\circ$	Two equal sides Two base angles are equal $a = b$	No equal sides All angle are different in size

Group by: **Number of equal sides**

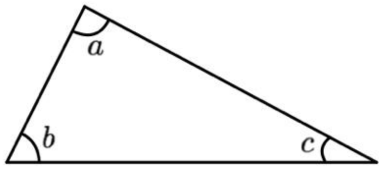
Acute-angled triangle	Right-angled triangle	Obtuse-angled triangle
		
All three angles are acute Could be : Equilateral, Isosceles, Scalene	One right angle Could be : Isosceles, Scalene	One obtuse angle Could be : Isosceles, Scalene

Group by: **Types of angels**

Whenever you see triangles:
1. Identify them by marking info given in questions.
2. Add information you know based on its properties.
3. Proceed to find other unknown angles.

Properties Of Triangles

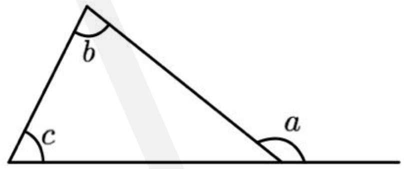
The angle sum of a triangle is 180°



$$a + b + c = 180^\circ$$

(∠sum of Δ)

The exterior angle of a triangle is equal to the sum of the interior opposite angle.



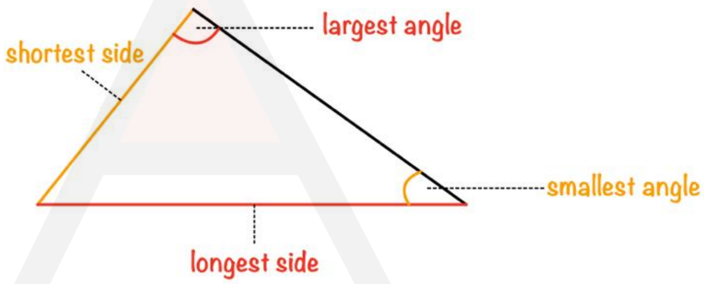
$$a = b + c$$

(ext.∠of Δ)

In a triangle:

The longest side is opposite the largest angle

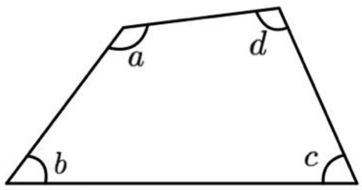
The shortest side is opposite the smallest angle



QUADRILATERALS

A quadrilateral is a plane figure have 4 straight lines and 4 angles.

The sum of angles in a quadrilateral is equal to 360°



$$a + b + c + d = 360^\circ$$

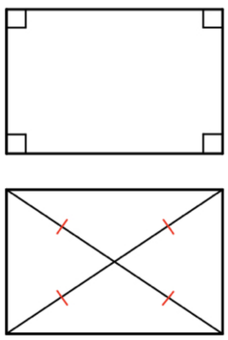
(∠ sum of quad.)

How to name a quadrilateral ?

Quadrilateral is named in a clockwise or an anticlockwise order

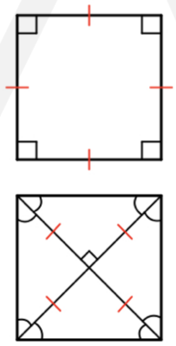
Properties Of Quadrilaterals

Rectangle



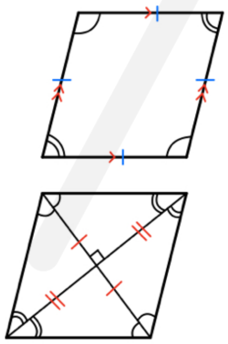
- Two pairs of parallel opposite sides.
- Opposite sides are equal in length.
- All four angles are right angles.
- Diagonals are equal in length.
- Diagonals bisect each other.

Square



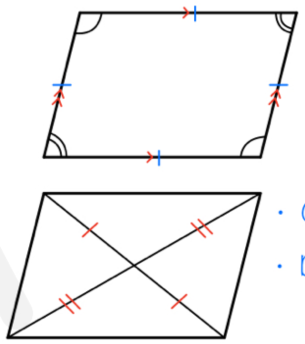
- Two pairs of parallel opposite sides.
- Four equal sides.
- All four angles are right angles.
- Diagonals are equal in length.
- Diagonals bisect each other at right angle.
- Diagonals bisect the interior angles.

Rhombus



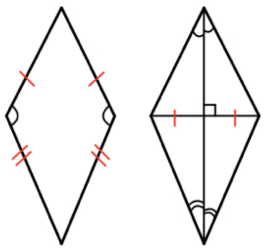
- Two pairs of parallel opposite sides.
- Four equal sides.
- Opposite angles are equal.
- Diagonals bisect each other at right angle.
- Diagonals bisect the interior angles.

Parallelogram



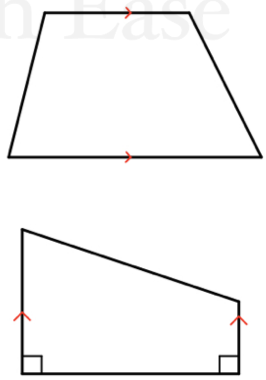
- Two pairs of parallel opposite sides.
- Opposite sides are equal in length.
- Opposite angles are equal.
- Diagonals bisect each other.

Kite



- No parallel sides.
- Two pair of equal adjacent sides.
- One pair of equal opposite angles.
- Diagonals intersect at right angles.
- One diagonals bisects the interior angles.

Trapezium

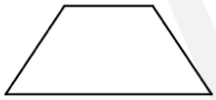
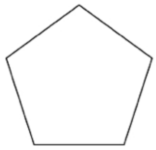
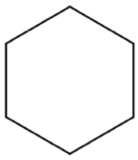
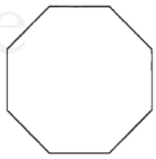
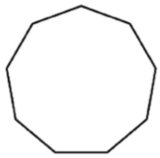
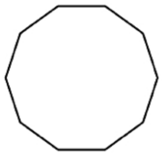


- One pair of parallel opposite sides.
- Angles between parallel sides are supplementary.

POLYGONS

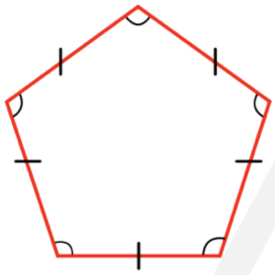
A polygon is a closed plane figure with three or more straight sides.

Types of polygons

Number of sides	Name of polygon
3	Triangle 
4	Quadrilateral 
5	Pentagon 
6	Hexagon 
7	Heptagon 
8	Octagon 
9	Nonagon 
10	Decagon 

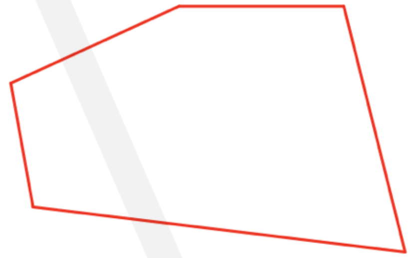
Angles properties of Polygons

Regular polygon

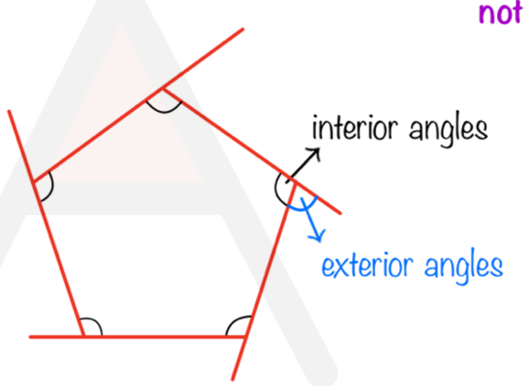


- All sides are equal
- Interior and exterior angles are all equal

Irregular Polygon



- Sides may not be equal
- Interior and exterior angles may not be equal.



For n-sided polygons (regular or irregular):

Sum of interior angles $= (n-2) \times 180°$

Sum of exterior angles $= 360°$

Interior $\angle$ + Exterior $\angle = 180°$

Specially for regular polygons:

Each interior angles $= \dfrac{(n-2) \times 180°}{n}$

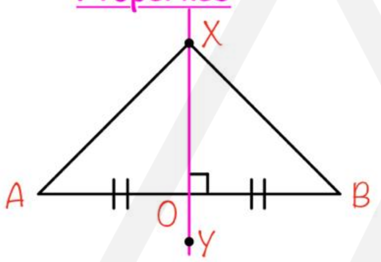
Each exterior angles $= \dfrac{360°}{n}$

BISECTORS

Perpendicular Bisector

- Bisects line
- Perpendicular to line
- Divides the line into 2 equal line segment

Properties

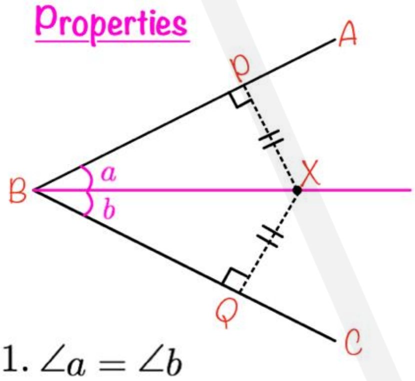


1. $XY \perp AB$
2. $OA = OB$
3. Any point on bisector equidistant to A and B; $AX = BX$

Angle Bisector

- Divides angle into a equal portions

Properties



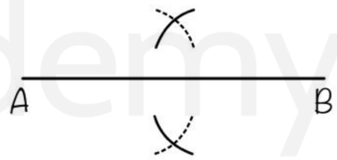
1. $\angle a = \angle b$
2. Any point on bisector equidistant to 2 sides; $PX = QX$

Perpendicular Bisector: Construction

1.

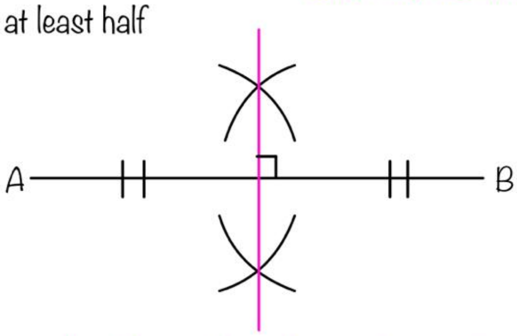
To draw a perpendicular bisector of AB, put compass at A and mark arc above & below the line.
(compass's width should be at least half the length of the line AB)

2.



Using the same extension of compass, put compass at B and mark arc above and below the line intersecting previous arc.

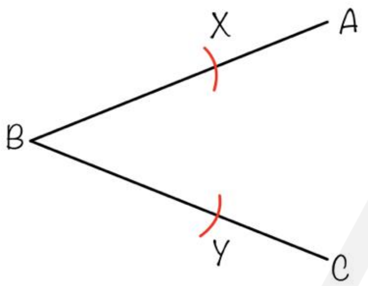
3.



Draw a line down joining the two intersections.

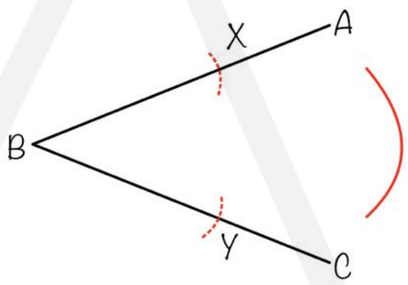
Angle Bisector: Construction

1.



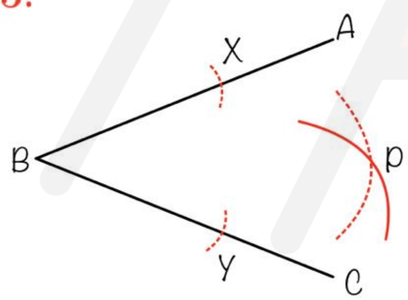
Compass at B, draw arcs to cut line AB at X and BC at Y (using the same compass extension / radius)

2.



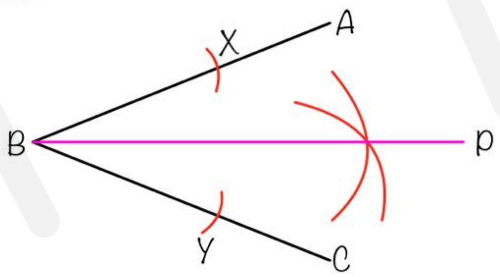
Compass at X, draw arc between lines AB and BC

3.



Compass at Y, draw another arc to intersect the previous arc at P (using same compass extension / radius as step 2)

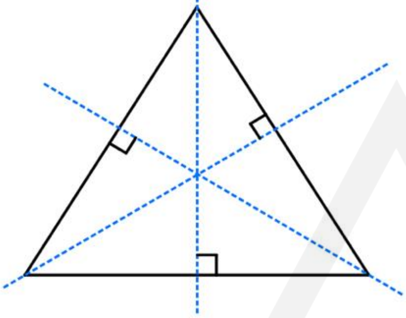
4.



Draw a line to connect B and P

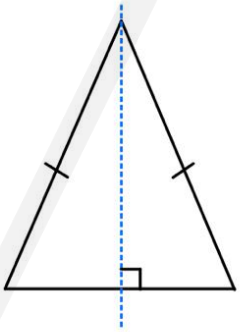
SYMMETRY PROPERTIES

EQUILATERAL TRIANGLE



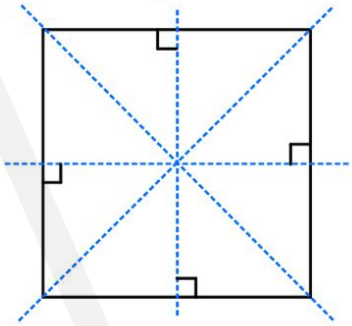
3 lines of symmetry

ISOSCELES TRIANGLE



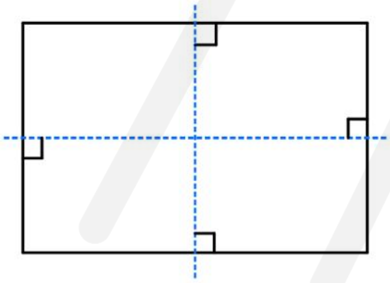
1 lines of symmetry

SQUARE



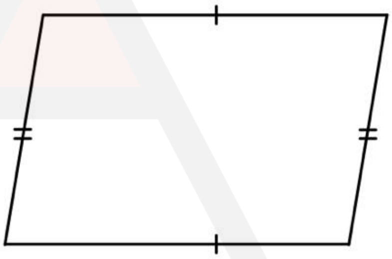
6 lines of symmetry

RECTANGLE



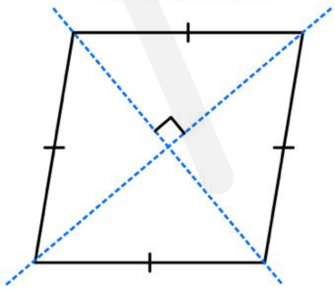
2 lines of symmetry

PARALLELOGRAM



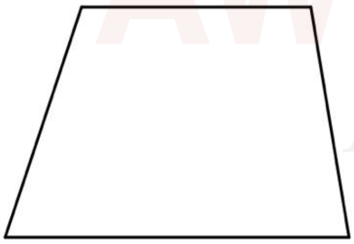
No lines of symmetry

RHOMBUS



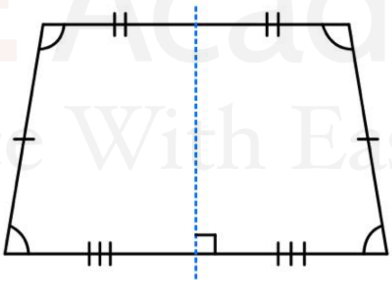
2 lines of symmetry

TRAPEZIUM



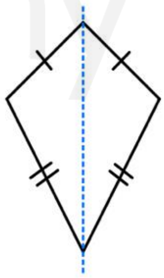
No lines of symmetry

REGULAR TRAPEZIUM



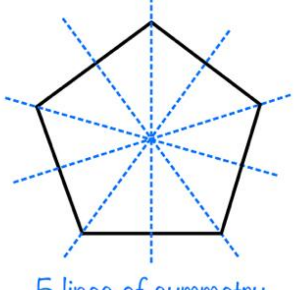
1 lines of symmetry

KITE



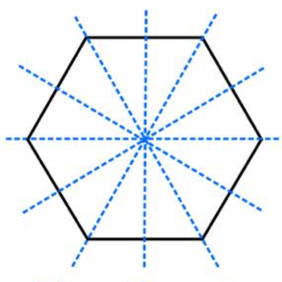
1 lines of symmetry

REGULAR PENTAGON



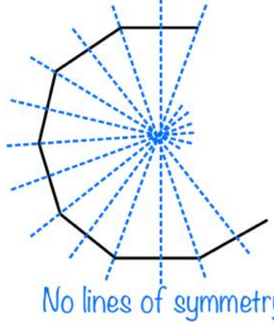
5 lines of symmetry

REGULAR HEXAGON



6 lines of symmetry

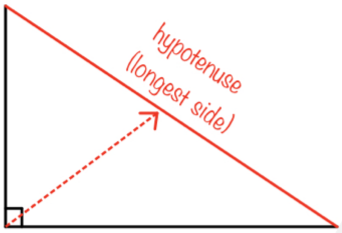
REGULAR N-GON



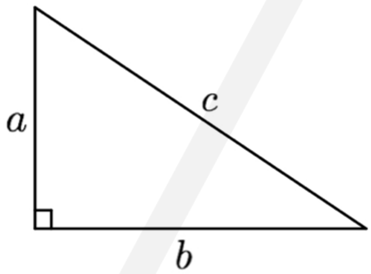
No lines of symmetry

TRIANGLES

Pythagoras' Theorem



In a right-angled triangle, the side that is opposite to the right angle, which is the longest side of the triangle is called the hypotenuse.



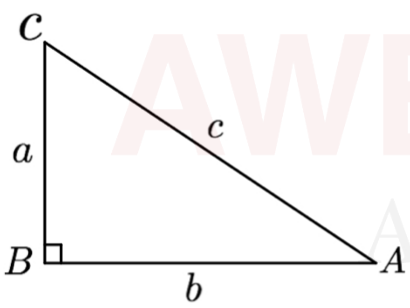
Pythagoras' Theorem:

The Pythagoras' Theorem states that, for any right-angled triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the length of the other two sides.

For right-angled triangle,

$c^2 = a^2 + b^2$

where c is the length of the hypotenuse



Converse of Pythagoras' Theorem:

The converse of Pythagoras' Theorem is also true and can be used to show that a particular triangle is a right-angled triangle.

In ΔABC, if $c^2 = a^2 + b^2$

then we can conclude

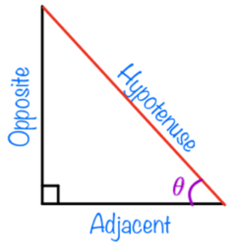
1. ΔABC is a right-angled triangle, and

2. $\angle ABC = 90^\circ$

Trigonometry Ratio

Acute θ in a right-angled triangle

TOA CAH SOH



Opposite → side opposite to θ the angle

Adjacent → side adjacent the θ angle

Hypotenuse → side opposite the right-angled, the longest side

$$\sin\theta = \frac{\text{Opposite}}{\text{Hypotenuse}}$$

$$\cos\theta = \frac{\text{Adjacent}}{\text{Hypotenuse}}$$

$$\tan\theta = \frac{\text{Opposite}}{\text{Adjacent}}$$

What does this mean?

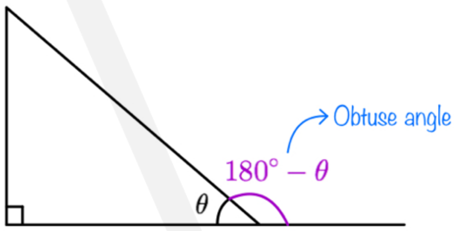
If you have either

a) length of 2 sides, or

b) 1 angle and 1 length,

You can find the unknown angle and length with trigonometry.

Obtuse θ in a right-angled triangle



$$\sin(180^\circ - \theta) = \sin\theta$$

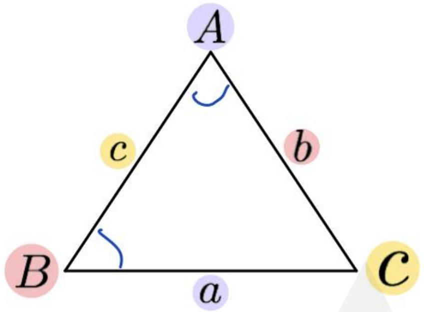
$$\cos(180^\circ - \theta) = -\cos\theta$$

$$\tan(180^\circ - \theta) = -\tan\theta$$

What does this mean?

We do not need to worry about an obtuse angle not being inside of a right-angled triangle.
As long as it is an exterior angle to a right-angled triangle, it can still be found easily.

Cosine Rule/Sine Rule



Sine Rule

$$\frac{a}{\sin \angle A} = \frac{b}{\sin \angle B} = \frac{c}{\sin \angle C}$$

OR

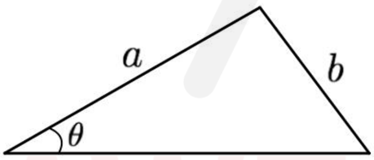
$$\frac{\sin \angle A}{a} = \frac{\sin \angle B}{b} = \frac{\sin \angle C}{c}$$

We can use Sine Rule to find an angle or a side, with the following conditions:

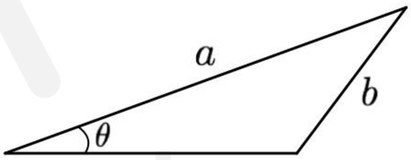
1. any 2 angles and 1 side. (finding unknown sides)
2. any 2 sides and 1 angles that is opposite to one of those sides. (finding unknown angles)

Ambiguous Case

1. 2 sides and 1 non-included angle is given.
2. Angle is acute.
3. Side opposite of given angle is less than other given side.



1st possibility



2nd possibility

Example

Given $\angle B = 25°$, $AC = 6cm$, $BC = 12cm$
Find $\angle A$.

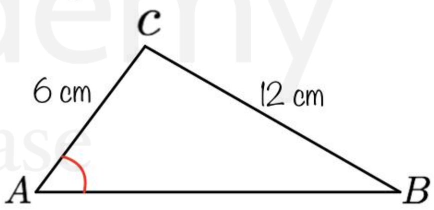
By Sine Rule, $\dfrac{\sin 25°}{6} = \dfrac{\sin \angle A}{12}$

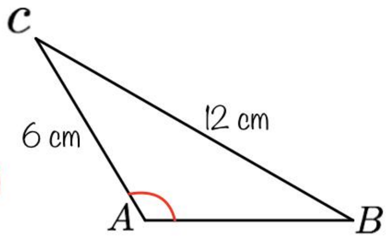
$$\sin A = \frac{12 \sin 25°}{6} = 0 \cdot 845$$

$$\angle A = \sin^{-1}(0.845)$$

$= 57 \cdot 7°$ (acute) or $180° - 57 \cdot 7°$ (obtuse)

$= 57 \cdot 7°$ or $122 \cdot 3$ (1d.p.)



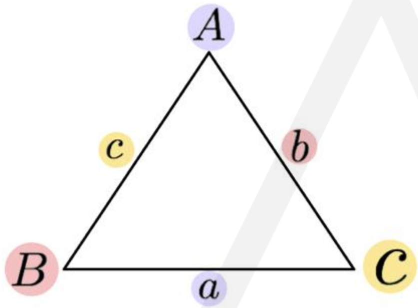


Area of triangle (non night-angled triangle) using sine rule

Given 2 sides and the included angle:

Area of $\triangle ABC = \frac{1}{2}ab \sin \angle C = \frac{1}{2}ac \sin \angle B = \frac{1}{2}bc \sin \angle A$

Cosine Rule



$$a^2 = b^2 + c^2 - 2bc\cos\angle A$$
$$b^2 = a^2 + c^2 - 2ac\cos\angle B$$
$$c^2 = a^2 + b^2 - 2ab\cos\angle C$$

Use to **find the remaining side** of a triangle when given 2 sides and the included angle.

From $a^2 = b^2 + c^2 - 2bc\cos\angle A$

$$2bc\cos\angle A = b^2 + c^2 - a^2$$

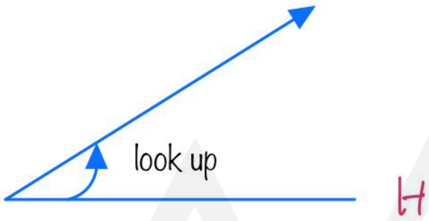
$$\cos\angle A = \frac{b^2 + c^2 - a^2}{2bc}$$

Use the formula to **find an angle** if all 3 sides are known.

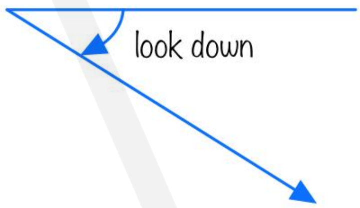
Angle Of Elevation & Depression

First you need to know:

Angle of elevation



Angle of depression

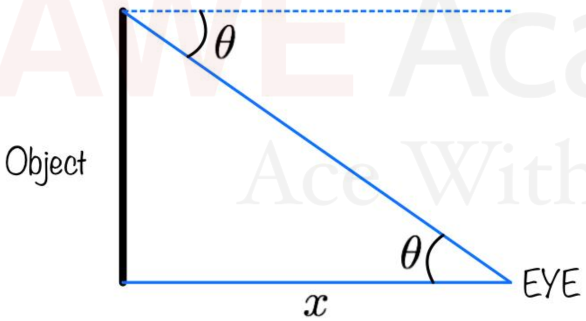


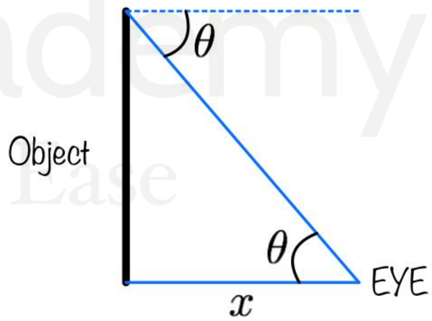
Then, use Pythagoras' Theorem and trigonometry to solve problems.

ONE USEFUL TIP WHEN WORKING IN 3D QUESTION:

Isolate the triangles by drawing it and label accordingly

* Greatest possible elevation or depression occurs when it has the **SHORTEST DISTANCE** between the base of the object and the eye. *





Shorter the distance x ⟶ larger the $\angle$ of elevation/depression, θ

$\angle$ of elevation = $\angle$ of depression (alternate $\angle$)

Bearings

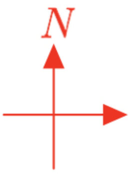
For bearings, you just have to remember 4 things:

1. Where is NORTH direction pointing at ?
2. Calculate clockwise direction from north direction.
3. Write in 3 digits i.e. 050°
4. Calculate from point of interest

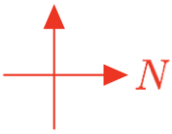
NOTE

Bearing of A from B

- Stand at B
- Face North
- Where is A from that position?

NORTH 

is not always in this direction.

It could be 

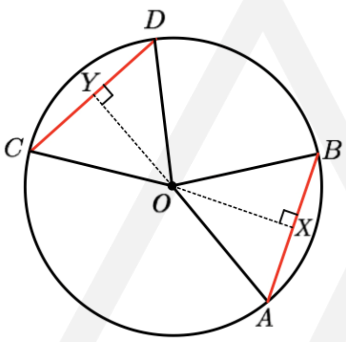
Hence adjustment must be made.

CIRCLES

→ classified into 2 categories: Symmetry and Angles

Properties: Symmetry

Rule 1: Chords which are equidistant from the centre are equal



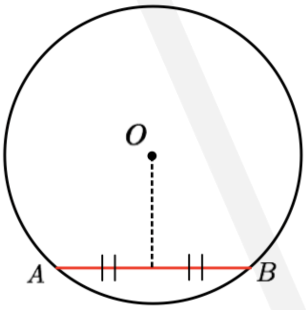
- Equal chords are equidistant from centre of circle.

  If $AB = CD$, then $OX = OY$

- Chords that are equidistant from centre O are equal in length.

  If $OX = OY$, then $AB = CD$

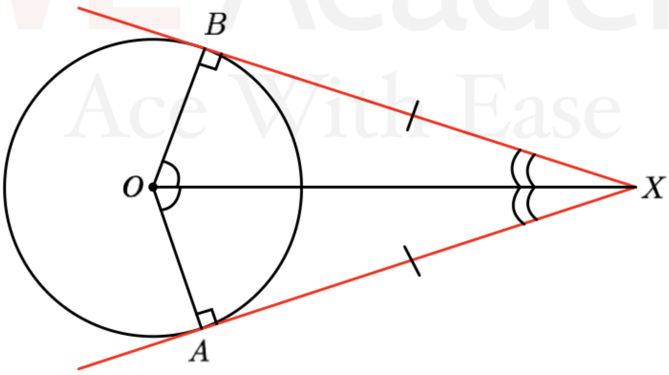
Rule 2: A line segment drawn from the centre of the circle bisect the chords into 2 equal parts and is perpendicular to the chord.



If $OX \perp AB$, then $AM = MB$

Conversely, if $AM = MB$, then $OX \perp AB$

Rule 3: Tangent from external points are equal



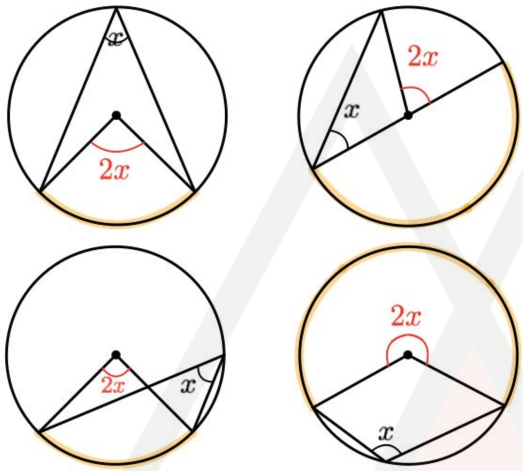
If XA and XD are tangent to the circle then,

1. $XA = XB$ (tangent from external pt.)
2. $\angle XOA = \angle XOB$
3. $\angle BXO = \angle AXO$

Properties: Angles

Rule 1: <u>Angle at centre is twice angle at circumference subtended by the same arc.</u>

($\angle$at centre circle $= 2 \times \angle$at circumference)

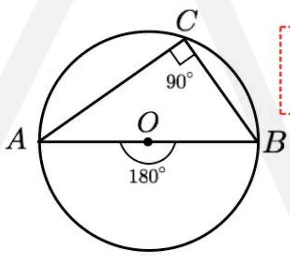


At

The angle at the circumference and the angle at the centre must be form within the same arc.

Rule 2: <u>The angle in semicircle is a right angle.</u>

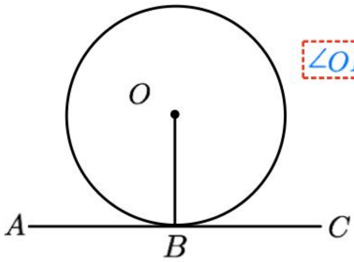
(rt. $\angle$ in semicircle)



This is actually Rule 1 with angle at centre $= 180°$

Rule 3: <u>Tangent is perpendicular to radius at point of contact.</u>

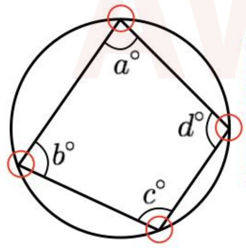
(tan $\perp radius$)



$\angle OBA = \angle OBC = 90°$

Rule 4: <u>Opposite angles in a cyclic quadrilateral add up to $180°$</u>

($\angle$in opposite segment are supplementary)

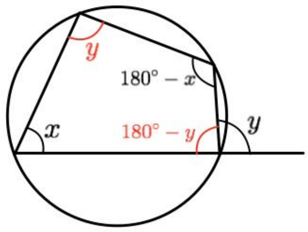


A cyclic quadrilateral is a 4 sided polygon with vertices on circumference of circles

$a° + b° = 180°$

$b° + d° = 180°$

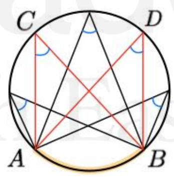
(Exterior $\angle$ of cyclic quadrilateral = interior opposite $\angle$)



Rule 5: <u>Angles in the same segment of circle are equal</u>

($\angle s$ in the same segment)

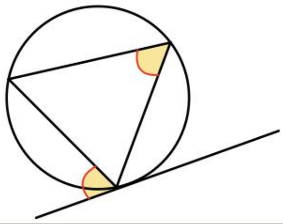
$\angle ACB = \angle ADB$



In another words, all angles at the circumference that are form by criss-crossing from the same 2 points on the arc

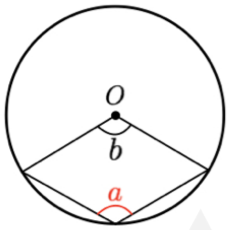
Rule 6: <u>The angle measure between a chord of a circle and a tangent is equal to the measure of angle in the alternate segment</u>

(Alternate Segment Theorem)



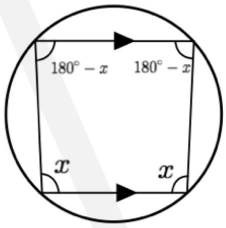
Things to look out for

1. ∠ in opposite segment DOES NOT refer to:



$\angle a + \angle b \neq 180^\circ$

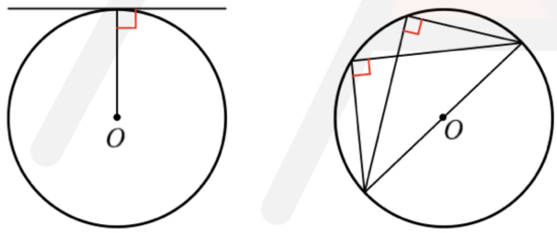
2. Trapezium in a circle



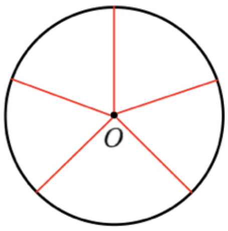
- angles are supplementary between 2 parallel lines.
- opposite angles are supplementary

3. Identity right angles in a circle:

- from tangents
- semi-circle

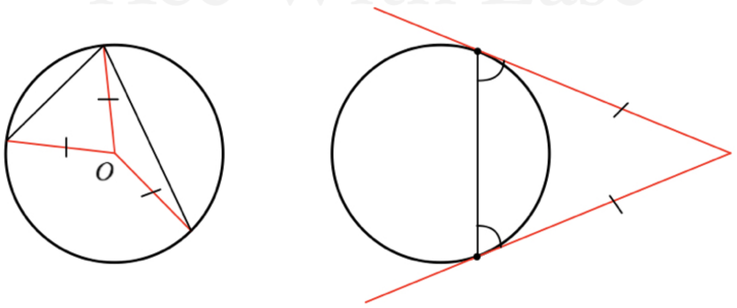


4. Identify common lengths especially when centre is given, such as the radii of the circle.



5. Identity isosceles triangles

- from same radius in a circle
- from extended tangents



VECTORS

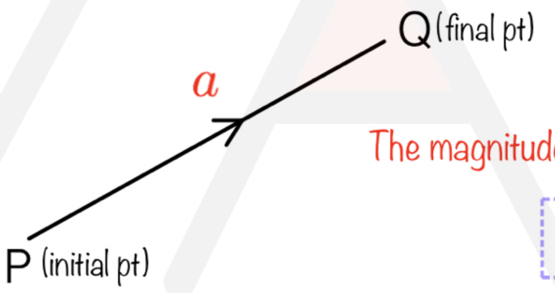
Vector Notation

A **vector quantity** is a quantity that possesses both magnitude and direction.

A vector is presented by a **directed line segment**.

The arrow on the line segment indicates the direction of the vector.

The **length of the line** segments represents the **magnitude of the vector**.



The magnitude of $\overrightarrow{PQ}$ is denoted by $\left|\overrightarrow{PQ}\right|$ or $|a|$

$$\left|\overrightarrow{PQ}\right| = \text{Length of } PQ$$

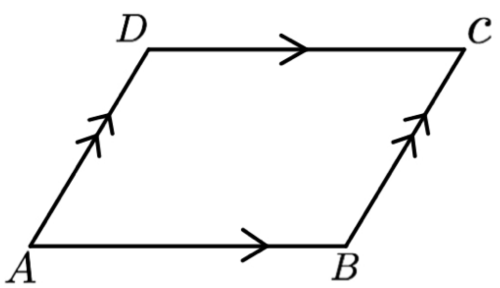
- **Equal vectors**

Two vectors are equal if they have same magnitude and the same direction.

Example: If $\overrightarrow{AB} = \overrightarrow{CD}$

$\Longrightarrow \left|\overrightarrow{AB}\right| = \left|\overrightarrow{CD}\right|$ and $AB // CD$

Example: If ABCD is a parallelogram, then the opposite sides are equal vectors.

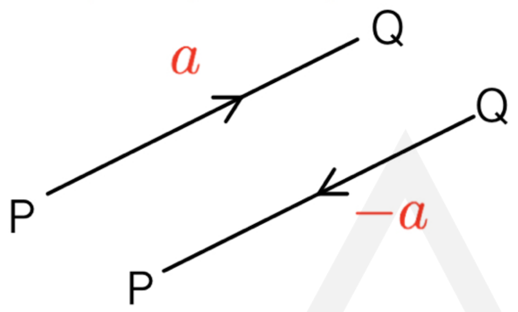


$$\overrightarrow{AB} = \overrightarrow{DC}$$
$$\overrightarrow{AD} = \overrightarrow{BC}$$

- **Negative Vectors**

Two vectors are negative vectors of each other if they have the same magnitude but in opposite directions.

$\overrightarrow{PQ}$ and $\overrightarrow{QP}$ are negative vectors of each other.



$$\overrightarrow{PQ} = -\overrightarrow{QP}$$
$$\overrightarrow{QP} = -\overrightarrow{PQ}$$
$$a = -(-a)$$

- **Zero Vectors**

The **zero vector** or **null vector** is a vector with zero magnitude and no direction.

Note:

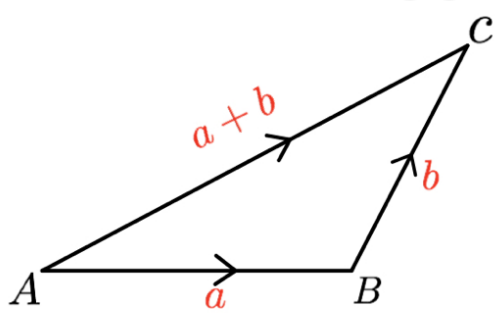
$$\overrightarrow{PQ} + \overrightarrow{QP} = 0$$
$$a + (-a) = 0$$

Addition of Vectors

To add vectors, use either the **triangle law of addition** or the **parallelogram law of addition.**

- **Triangle Law of Addition**

If two vectors a and b are represented by the sides $\overrightarrow{AB}$ and $\overrightarrow{BC}$ of a triangle, then $a + c$ is represented by the third side $\overrightarrow{AC}$



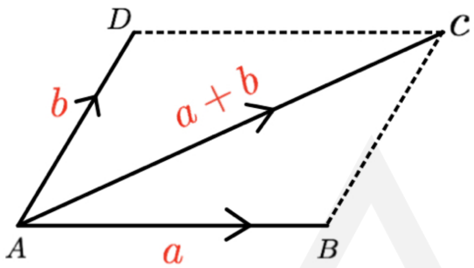
$$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$$
$$\overrightarrow{AC} = a + b$$

$$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$$

head — must be the same pt — tail — head tail

- ### Parallelogram Law of Addition

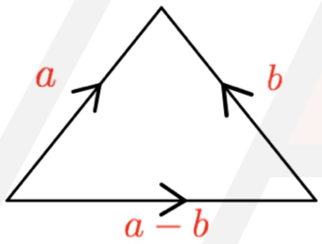
If two vectors a and b are represented by the adjacent side $\overrightarrow{AB}$ and $\overrightarrow{AD}$ of a parallelogram, then $a+b$ is represented by the diagonal of the parallelogram $\overrightarrow{AC}$



$$\overrightarrow{AB}+\overrightarrow{BC}=\overrightarrow{AC}$$
$$\overrightarrow{AC}=a+b$$

Subtraction of Vectors

To subtract 2 vectors, add the first vector to the negative of the second vector



$$a-b=a+(-b)$$

Scalar Multiplication of a Vector

When a vector a is multiplied by a scalar k, the resulting vector ka has a magnitude k times of a, $|ka| = k\,|a|$

ka is parallel to a and is in
1. the same direction as a if k is positive $(k>0)$
2. the opposite direction of a if k is negative $(k<0)$

Vector a is parallel to b only if

$$a=kb$$

$\overrightarrow{AB}=\overrightarrow{CD}$, then $AB//CD$ and $AB=CD$

$\overrightarrow{PQ}=k\overrightarrow{PR}$, then $PQ//PR$ and $PQ=kPR$

The pts P, Q, R lie on a straight line (i.e. they are collinear)

Scalar multiplication of vectors obey the following rules

If a and b are vectors, and m and n are real numbers then
1. $m(na)=n(ma)=mn(a)$
2. $(m+n)a=ma+na$
3. $m(a+b)=ma+mb$

Law of Column Vector Column

For any two column vectors $a = \begin{pmatrix} p \\ q \end{pmatrix}$ and $b = \begin{pmatrix} r \\ s \end{pmatrix}$

1. if $a = b$, then $\begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} r \\ s \end{pmatrix}$

2. $a + b = \begin{pmatrix} p \\ q \end{pmatrix} + \begin{pmatrix} r \\ s \end{pmatrix} = \begin{pmatrix} p + r \\ q + s \end{pmatrix}$

   $a - b = \begin{pmatrix} p \\ q \end{pmatrix} - \begin{pmatrix} r \\ s \end{pmatrix} = \begin{pmatrix} p - r \\ q - s \end{pmatrix}$

3. $ma = m\begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} mp \\ mq \end{pmatrix}$ where m is a scalar

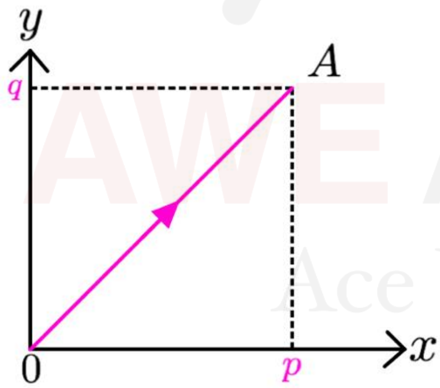
4. $ma + nb = m\begin{pmatrix} p \\ q \end{pmatrix} + n\begin{pmatrix} r \\ s \end{pmatrix} = \begin{pmatrix} mp + nr \\ mq + ns \end{pmatrix}$

Position Vector

Any point on a Cartesian plane can be represented as a position vector.

The position vector of point A on the plane is the vector from the origin O, to that given point A.

For any point A (p, q), the position vector of A with reference to the Origin O is given by:



coordinates of A (p, q)

$$\overrightarrow{OA} = \begin{pmatrix} p \\ q \end{pmatrix}$$

$$\left|\overrightarrow{OA}\right| = \sqrt{p^2 + q^2}$$

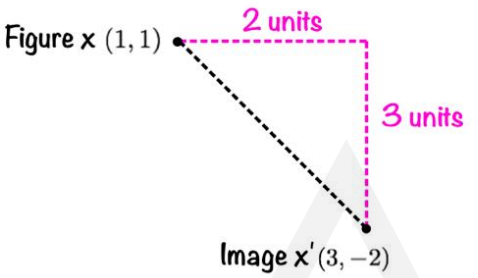
For any 2 points A and B,

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$$

Translation

Translation of $\begin{pmatrix} x \\ y \end{pmatrix}$: move x units horizontally and y units vertically

Example:



Every point move through:

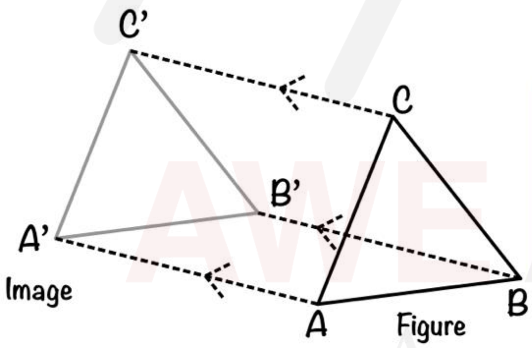
1. same direction
2. same distance

⟹ Figure **congruent** to image

↓

SAME SIZE
SAME SHAPE

Figure + Translation Vector = Image



coordinate of A
coordinate of B + Translation Vector = coordinate of A'
coordinate of C

coordinate of A + Translation Vector = coordinate of A'
coordinate of B + Translation Vector = coordinate of B'
coordinate of C + Translation Vector = coordinate of C'

MENSURATION

Conversions

For length:

To convert $m^2 \rightarrow cm^2$
(bigger unit to smaller unit, '×')

i.e. $\times 1000$

To convert $cm^2 \rightarrow m^2$
(smaller unit to bigger unit, '÷')

i.e. $\div 1000$

For circles:

Angles can be measured either in degree or radians, where

$$180^0 = \pi rad$$

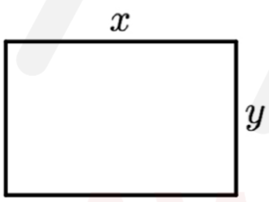
degree ⟶ radian

i.e. $\times \frac{\pi}{180°}$ rad

radian ⟶ degree

i.e. $\div \frac{180°}{\pi}$

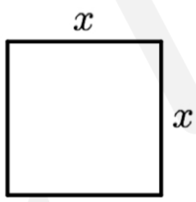
Polygons



Rectangle

Area = xy

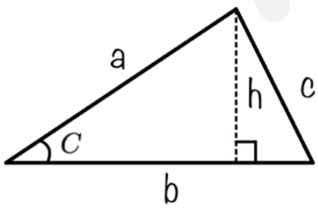
Perimeter = $2(x+y)$



Square

Area = x^2

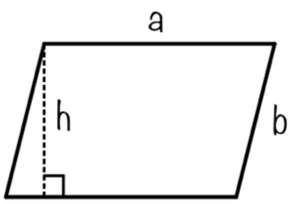
Perimeter = $4x$



Triangle

Area = $\frac{1}{2} \times b \times h = \frac{1}{2} ab \sin C$

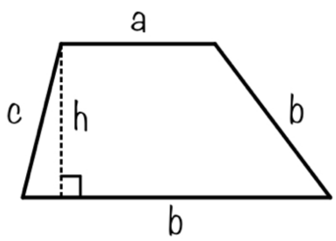
Perimeter = $a+b+c$



Parallelogram

Area = $a \times h$

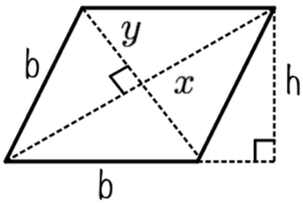
Perimeter = $2(a+b)$



Trapezium

Area = $\frac{1}{2}(a+b)h$

Perimeter = $a+b+c+d$

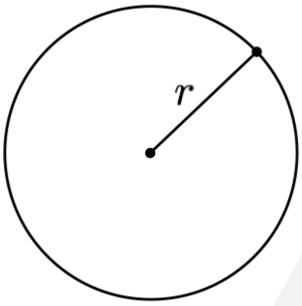


Rhombus

Area = $bh = xy$

Perimeter = $4b$

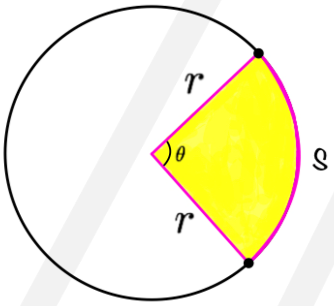
Circle



Area of circle, $A = \pi r^2$

Circumference of circle, $P = 2\pi r = \pi d$ (since d = 2r)

Sector



Perimeter of sector, $p = s + 2r$

where (by taking proportions)

arcs s = $\dfrac{\theta}{360^\circ} \times 2\pi r$ (where θ is in degrees)

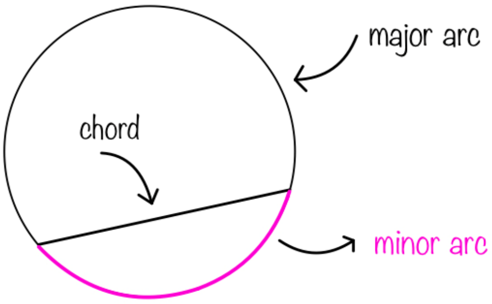
arcs s = $\dfrac{\theta}{2\pi} \times 2\pi r = r\theta$ (where θ in radians)

Area of circle, $A = \pi r^2$. By taking proportions,

Area of sector, $A = \dfrac{\theta}{360^\circ} \times \pi r^2$ (θ is in degrees)

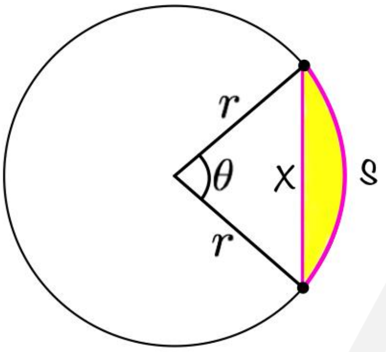
$= \dfrac{\theta}{2\pi} \times \pi r^2 = \dfrac{1}{2} r^2 \theta$ (θ is in radians)

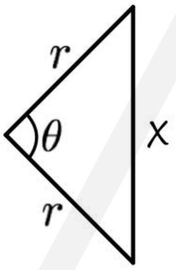
NOTE:



Arc: part of a continuous curve

Segment





Perimeter of segment, $P = x + s$

where (by using cosine rule),
$x^2 = r^2 + r^2 - 2r^2 \cos\theta$

$x = \sqrt{2r^2 - 2r^2 \cos\theta}$ and

$s = \dfrac{\theta}{360°} \times 2\pi r$ (where θ is in degrees)

$s = \dfrac{\theta}{2\pi} \times 2\pi r = r\theta$ (where θ in radians)

Area of segment,

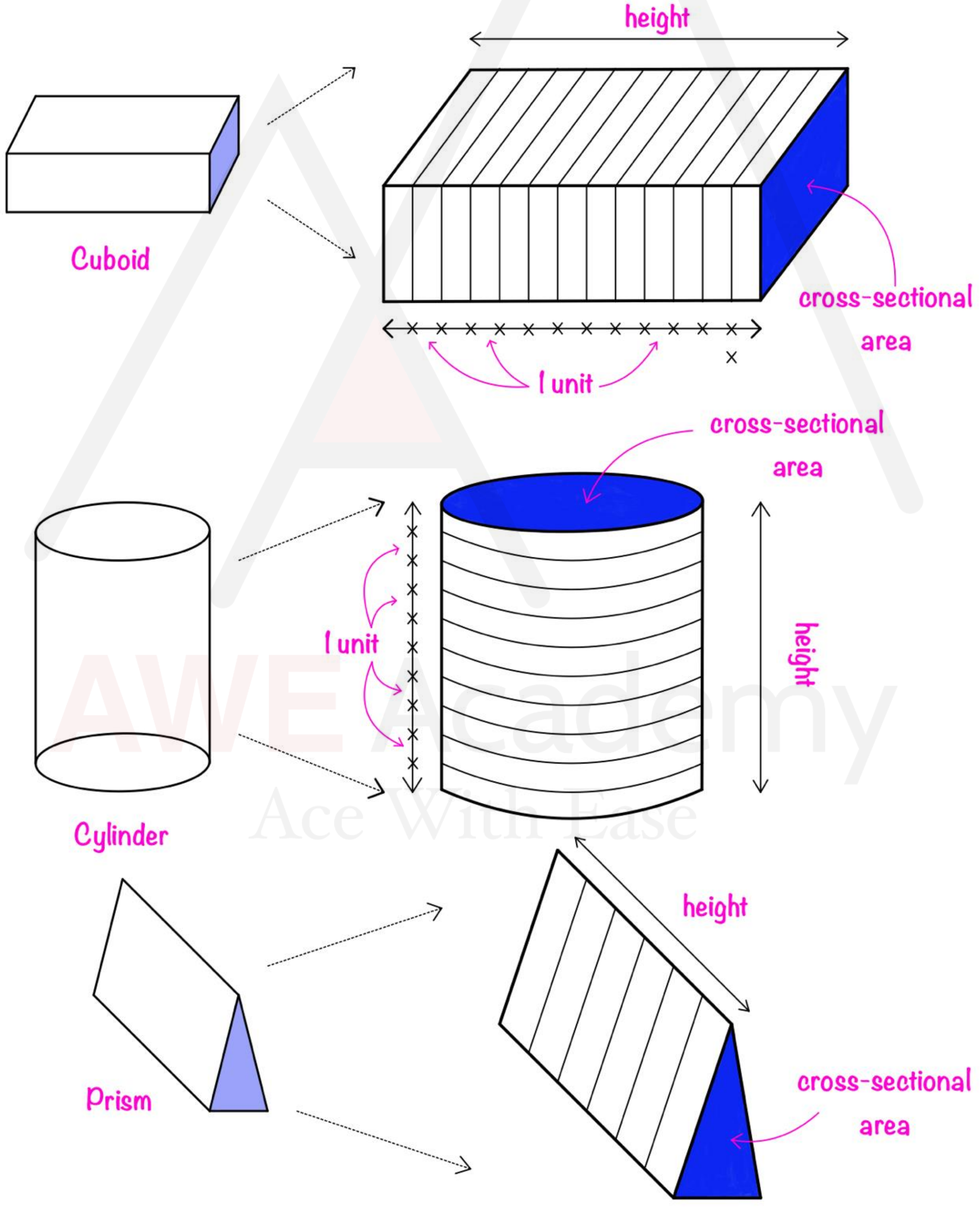
A = area of sector - area of triangle

$= \left(\dfrac{\theta}{360°} \times \pi r^2\right) - \dfrac{1}{2} r^2 \sin\theta$

$= \dfrac{1}{2} r^2 \theta - \dfrac{1}{2} r^2 \sin\theta$

Regular/Uniform Solids

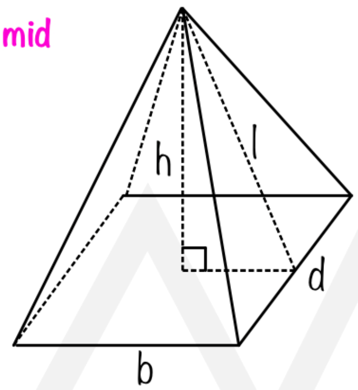
Volume = sum of all the cross-sectional areas
= cross-sectional area x height



Irregular/Non-Uniform Solids

May need to use Pythagoras' theorem or sine /cosine rule to find dimensions.

Pyramid



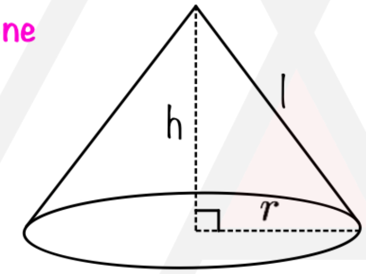
Volume $= \frac{1}{3} \times$ base area $\times$ height

$= \frac{1}{3} \times bd \times h$

Surface area = Area of all surfaces

(4 triangle + 1 square / rectangle)

Cone



Volume $= \frac{1}{3} \times$ base area $\times$ height

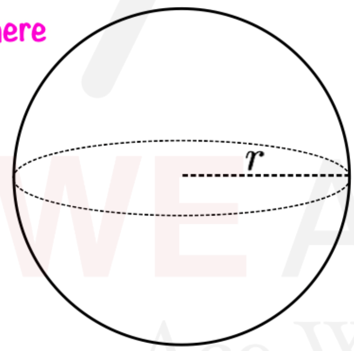
$= \frac{1}{3} \times \pi r^2 \times h$

$= \frac{1}{3}\pi r^2 h$

Surface area $= \pi r^2 + \pi r l$

area of circle base (πr^2); curved surface area ($\pi r l$)

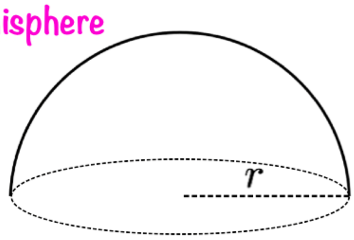
Sphere



Volume $= \frac{4}{3}\pi r^3$

Surface area $= 4\pi r^2$

Hemisphere



Volume $= \frac{2}{3}\pi r^3$

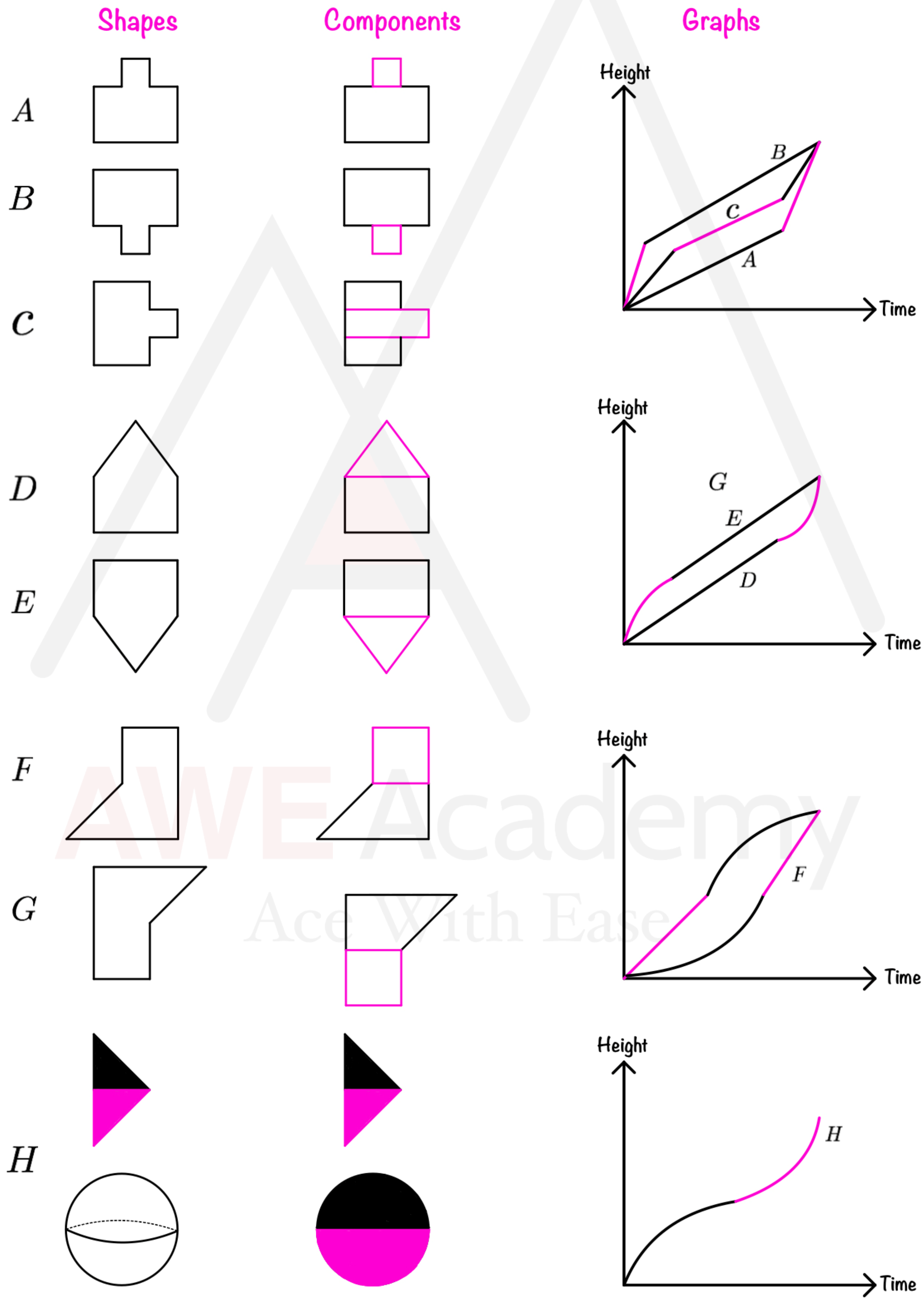
Surface area $= 2\pi r^2 + \pi r^2$

$= 3\pi r^2$

area of circle base (πr^2)

> **NOTE:** When finding surface area, check if
> 1) Solid is open or closed
> 2) Question is asking for outer or inner surface area

Combination of shapes / solids:



CONGRUENCE & SIMILARITY

Congruence

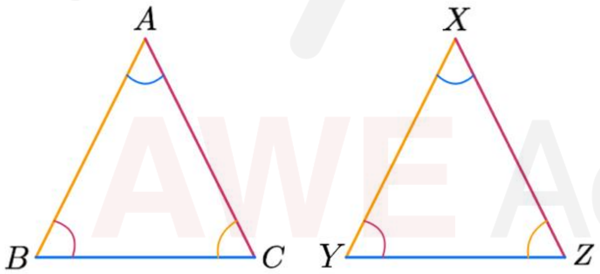
Same shape
Same size
Same object

Since the 2 objects in comparison are exactly the same:

1. their corresponding sides are equal
2. their corresponding angles are equal
3. their areas are equal

$$\triangle ABC \equiv \triangle XYZ$$

$$A \leftrightarrow X$$
$$B \leftrightarrow Y$$
$$C \leftrightarrow Z$$



$\angle ABC = \angle XYZ$
$\angle BCA = \angle YZX$
$\angle CAB = \angle ZXY$

and

$AB = XY$
$BC = YZ$
$AC = XZ$

Similarity

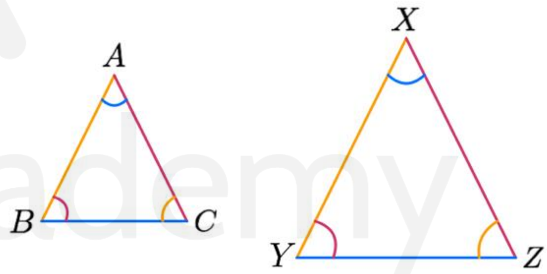
Same shape
Different size

The 2 objects in comparison are essentially the same, but of different sizes, hence:

1. their corresponding angles are equal
2. ratio of corresponding sides are equal

$\triangle ABC$ is similar to $\triangle XYZ$

$$A \leftrightarrow X$$
$$B \leftrightarrow Y$$
$$C \leftrightarrow Z$$



$\angle ABC = \angle XYZ$
$\angle BCA = \angle YZX$
$\angle CAB = \angle ZXY$

and

$$\frac{AB}{XY} = \frac{BC}{YZ} = \frac{AC}{XZ}$$

Test for congruent △

To show that 2 triangles are congruent, we just need to use **any of the 5 rules** below to prove it.

Rule 1 : SSS

→ 3 corresponding sides equal

Rule 2 : SAS

→ 2 sides equal, 1 included angle equal

Rule 3/4 : ASA , AAS

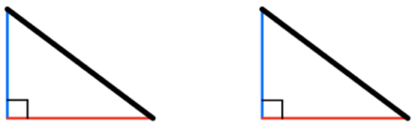
→ 2 angles equal, 1 corresponding side equal

ASA

AAS

Rule 5 : RHS

→ hypotenuse and 1 side of right-angled triangle equal



Test for similarity △

To show that 2 triangles are similar, we just need to use **any of the 3 rules** below to prove it.

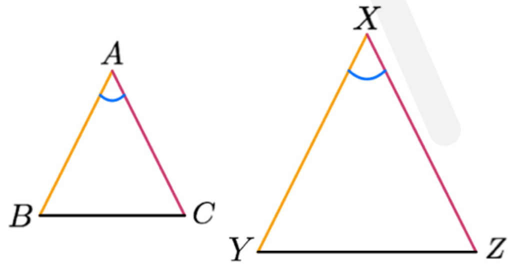
Rule 1 : 2 corresponding angles are equal

Rule 2 : 3 corresponding sides in the same ratio

$$\frac{AB}{XY} = \frac{BC}{YZ} = \frac{AC}{XZ}$$

Rule 3 : 2 corresponding sides in same ratio, 1 included angle equal

$$\frac{AB}{XY} = \frac{AC}{XZ} \text{ and } \angle BAC = \angle YXZ$$



How to identify which triangles are similar or congruent?

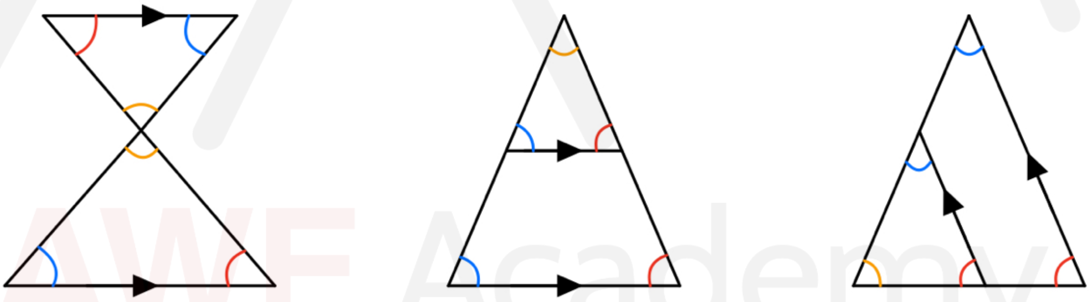
Step 1 : Identity the values of unknown sides and angles by applying knowledge about:

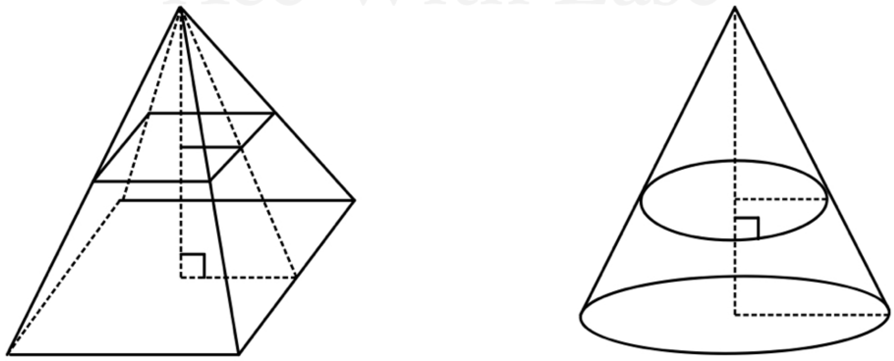
- Angles :
  - → vertically opposite angles
  - → alternate angles, etc.
- Triangles :
  - → Isosceles
  - → Equilateral, etc.

Step 2 : Match the sides and angles

Step 3: Use the relevant tests: congruence / similarity

Some examples on similarity to "capture" and identity at a glance.



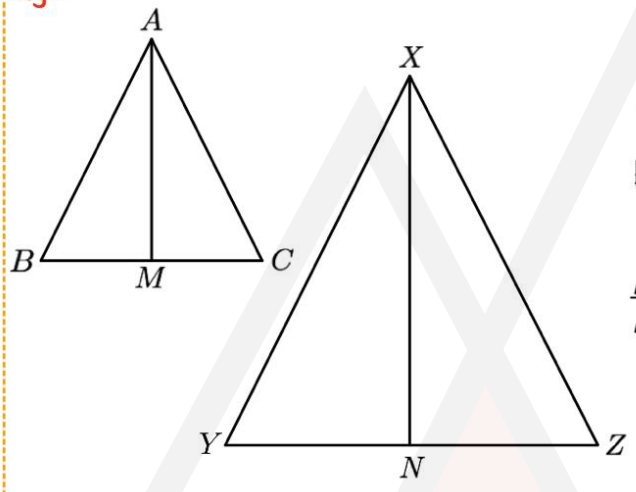


Similarity : Area

For similar figures A and B:

If $\frac{\text{length of A}}{\text{length of B}}\left(\frac{l_A}{l_B}\right) = \frac{\text{another length of A}}{\text{another length of B}}\left(\frac{h_A}{h_B}\right)$, then $\frac{\text{area of A}}{\text{area of B}} = \left(\frac{l_A}{l_B}\right)^2 = \left(\frac{h_A}{h_B}\right)^2$

Why?



Area of $\triangle ABC = \frac{1}{2}(AM)(BC)$

Area of $\triangle XYZ = \frac{1}{2}(XN)(YZ)$

Because the figures are similar, $\frac{AM}{XN} = \frac{BC}{YZ}$

$$\frac{\text{Area of } \triangle ABC}{\text{Area of } \triangle XYZ} = \frac{\frac{1}{2}(AM)(BC)}{\frac{1}{2}(XN)(YZ)}$$

$$= \frac{AM}{XN} \times \frac{BC}{YZ}$$

$$= \left(\frac{AM}{XN}\right)^2 \text{ or } \left(\frac{BC}{YZ}\right)^2$$

Similarity: Volume

Likewise for volume of similar solids A and B: If $\frac{l_A}{l_B} = \frac{h_A}{h_B}$, then $\frac{\text{volume of A}}{\text{volume of B}} = \left(\frac{l_A}{l_B}\right)^3 = \left(\frac{h_A}{h_B}\right)^3$

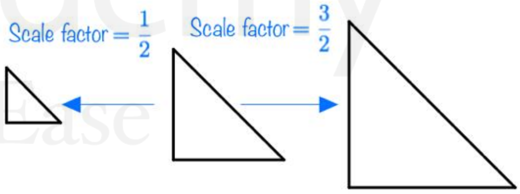
Scale Factor

For enlargement at figure to form image :

$$\text{Scale factor} = \frac{\text{length of image}}{\text{corresponding length of figure}}$$

$$\text{Length of figure} = \frac{\text{corr. length of figure}}{\text{scale factor}}$$

$$\text{Length of image} = \text{scale factor} \times \text{corr. length of figure}$$



Therefore, under enlargement with scale factor, the figure and its image are SIMILAR!

$$\Rightarrow \frac{\text{length of image}}{\text{corr. length of figure}} = \text{Scale factor}$$

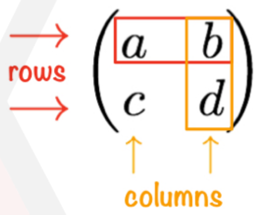
$$\Rightarrow \frac{\text{area of image}}{\text{area of figure}} = \left(\frac{\text{length of image}}{\text{corr. length of figure}}\right)^2 = (\text{scale factor})^2$$

MATRICES

Matrices: array of numbers used to represent/showcase information over time.
It is written as follows:

$$\begin{pmatrix}1&2&3\\4&5&6\\7&6&9\end{pmatrix},\ \begin{pmatrix}1\\2\end{pmatrix},\ (1,2),\ \begin{pmatrix}1&2\\3&4\end{pmatrix},\ \begin{pmatrix}1&2&3\\4&5&6\end{pmatrix},\ \begin{pmatrix}1&2\\3&4\\5&6\end{pmatrix}$$

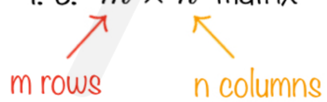
where

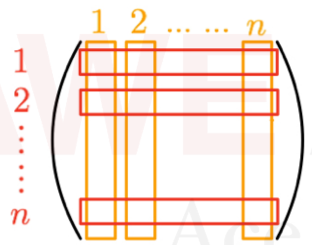


Order of matrix: size of the matrix

written in the form (number of rows) x (number of columns)

i. e. $m \times n$ matrix





Example:

2×3 matrix

⇒ 2 rows, 3 columns

⇒ $\begin{pmatrix}1&2&3\\4&5&6\end{pmatrix}$

Elements: each number / value in the matrix

In a 2×2 matrix, $\begin{pmatrix}a&b\\c&d\end{pmatrix}$, there are 4 elements: a, b, c, d

To simplify things later, we introduce this notation for each elements:

a_{ij}, where a_{ij} element in i^{th} row, j^{th} column.

Example:

In a 2×2 matrix, $\begin{pmatrix}a_{11}&a_{12}\\a_{21}&a_{22}\end{pmatrix}$

Matrix equality : 1. matrices of same order (i.e. same size)

2. corresponding elements equal

Example:

If $\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, then $a = 1, b = 2, c = 3, d = 4$

Addition/Subtraction

NOTE: Matrices Must Be Of Same Order

Addition:

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} + \begin{pmatrix} e & f \\ g & h \end{pmatrix} = \begin{pmatrix} a+e & b+f \\ c+g & d+h \end{pmatrix}$$

Just add/subtract corresponding elements

Subtraction:

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} - \begin{pmatrix} e & f \\ g & h \end{pmatrix} = \begin{pmatrix} a-e & b-f \\ c-g & d-h \end{pmatrix}$$

Special Matrices

Zero matrix: all elements are zero

$$O = \begin{pmatrix} 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & & \ddots & \vdots \\ 0 & \cdots & \cdots & 0 \end{pmatrix}$$

NOTE: $AO = 0$

Identity matrix: diagonals elements = 1, all other elements = 0

$$I = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & & \ddots & \vdots \\ 0 & \cdots & \cdots & 1 \end{pmatrix}$$

NOTE : $AI = A$

Multiplication

Scalar Multiplication

$$k\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} ka & kb \\ kc & kd \end{pmatrix}$$

Just Multiply Constant Into Every Element

where k is a constant

Matrix Multiplication

Condition: number of columns of 1^{st} matrix = number of rows of 2^{nd} matrix

(n × m matrix) × (m × p matrix) = (n × p matrix)

✓ (2×3 matrix) × (3 × 1 matrix) = (2 × 1 matrix)

✗ (3×2 matrix) × (4×3 matrix)

MULTIPLY i^{th} ROW OF 1^{st} MATRIX WITH j^{th} COLUMN OF 2^{nd} MATRIX

$$\begin{pmatrix} a_{11} & a_{12} \dots a_{1m} \\ \vdots & \vdots & \vdots \\ a_{n1} & a_{n2} \dots a_{nm} \end{pmatrix}\begin{pmatrix} b_{11} & b_{12} \dots b_{1p} \\ \vdots & \vdots & \vdots \\ b_{m1} & b_{m2} \dots b_{mp} \end{pmatrix} = \begin{pmatrix} c_{11} & c_{12} \dots c_{1p} \\ \vdots & \vdots & \vdots \\ c_{n1} & c_{n2} \dots c_{np} \end{pmatrix}$$

where c_{ij} = multiply i^{th} row of 1^{st} matrix with j^{th} column of 2^{nd} matrix

$$= a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \dots + a_{im}b_{mj}$$

i^{th} row $\begin{pmatrix} a & b & \dots & c \end{pmatrix}\begin{pmatrix} d \\ e \\ \vdots \\ t \end{pmatrix} = (ad + be + \dots + cf)$

j^{th} column

NOTE: $AB \neq BA$

SETS & NOTATIONS

Sets:

- Well-defined collection of objects → Elements
- denoted by CAPITAL LETTERS
- elements placed between {curly brackets}

Example: A is the set of days in a week

A = {Mon, Tues, Wed, Thurs, Fri, Sat, Sun} → Elements in Set A

Example: N is the set of all positive odd numbers below 10

$$N = \{x : x > 0, x < 10\}$$

$$= \{1, 3, 5, 7, 9\}$$

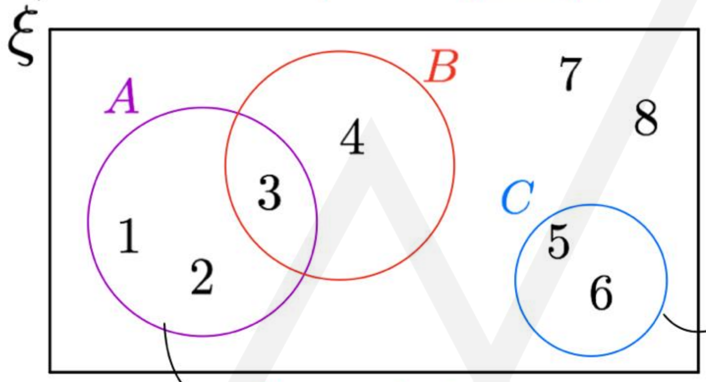
Notations	Examples
$x \in A$ x is an element of set A x belongs in set A	A = {x:x is even and $1 < x < 10$} = {2,4,6,8} $2 \in A,\ 4 \in A,\ 6 \in A,\ 8 \in A$
$x \notin A$ x is not an element of set A x does not belong in set A	$1 \notin A$, since $1 < x < 10$ $5 \notin A$, since x is even
$n(A)$ number of elements in set A	$n(A) = 4$
Finite set elements in a set are countable	A = {x:x is even and $1 < x < 10$} = {2,4,6,8} → 4 items in a set $n(A) = 4$ → finite
Infinite set elements in a set are infinite	B = {x:x are positive integers} = {1, 2, 3, 4, 5, 6...} → goes to infinity $n(B) = \infty$ ← infinite

Notations	Examples
$\emptyset$ **Empty set/null set** no elements in set also denoted by { } **Note:** It is wrong to write empty set = {$\emptyset$}	A = {x: x is positive and $x < 0$ } = {} or $\emptyset$ $n(A) = 0$
ξ **Universal set** All elements relevant to solution of problem	
$A \subseteq B$ **A is a subset of B** Every element of A is in B, and $n(A) \leq n(B)$ Whatever A has, B also has and may have more	$A = \{1,2,3,4\}$, $B = \{1,2,3,4\}$ $C = \{1,2,3,4,5\}$ All elements in A are in B and C. So, $A \subseteq B$ and $A \subseteq C$
$A \nsubseteq B$ **A is not a subset of B** A consists of elements that are not in B	$A = \{5,6,7,8\}$, $B = \{1,2,3,4\}$ $C = \{1,6,7,8\}$ $D = \{1,2,3,4,5\}$ $A \nsubseteq B$, since $\{5,6,7,8\} \notin B$ $C \nsubseteq B$, since $\{6,7,8\} \notin B$ $D \nsubseteq B$, since $5 \notin B$ and $n(D) > n(B)$
$A = B$ **Sets A and B are equal** Set A and B have exactly the same elements If $A \subseteq B$ and $B \subseteq A$, then $A = B$ If $A = B$, then $A \subseteq B$ and $B \subseteq A$	$A = \{1,2,3,4\}$, $B = \{1,2,3,4\}$ $A \subseteq B$ and $B \subseteq A$ Therefore, $A = B$
$A \subset B$ **A is a proper subset of B** Condition 1 : Every element of A is in B Condition 2 : There is at least 1 element in B that is not in A, i.e. $n(A) < n(B)$	$A = \{1,2,3\}, B = \{1,2,3,4\}, D = \{3,4\}$ $A \subset B$ → since $\{1,2,3\} \in B$, and $n(A) < n(B)$ as $\{4\} \notin A$ $D \subset B$ → since $\{3,4\} \in B$, and $n(D) < n(B)$ as $\{1,2\} \notin D$
$A \not\subset B$ **A is not a proper subset of B** There are elements in A that are not in B Every element in B is in A	$A = \{1,2,3,4\}$, $B = \{1,2,3,4\}$ $D = \{1,6,7\}$ $A \not\subset B$ → since all elements in B are in A $D \not\subset B$ → since $\{6,7\} \notin B$

Venn Diagram

A venn diagram is a visual diagram to show relationship between sets. You simply got to classify the information in their appropriate categories.

universal set represented by rectangle



$\xi = \{1,2,3,4,5,6,7,8\}$

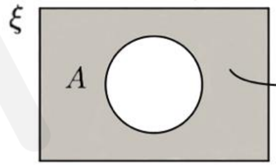
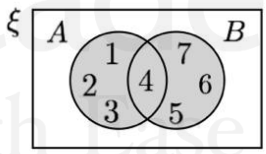
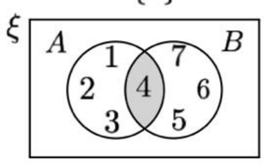
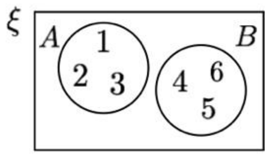
$A = \{1,2,3\}$

$B = \{3,4\}$

$C = \{5,6\}$

set represented by circle must be labelled

Elements placed in a set

Notations	Examples
A' **Complement of A** Set of elements in ξ, but not in A $A' = \{x : x \in \xi, x \notin A\}$	$A = \{1,2,3\}$, $\xi = \{1,2,3,4,5\}$ $A' = \{4,5\}$ → All other elements in ξ but not in A  Shaded region is A' shade to indicate region of interest
$A \cup B$ **Union of sets A and B** Set that contains **ALL** elements in set A or set B or both sets Common mistakes: no repetition of elements required $A \cup B \neq \{1,2,3,4,4,5,6,7\}$	$A = \{1,2,3,4\}$, $B = \{4,5,6,7\}$ $A \cup B = \{1,2,3,4,5,6,7\}$ 
$A \cap B$ **Intersection of A and B** Set that contains elements that are common to **BOTH** sets A and B	$A = \{1,2,3,4\}$, $B = \{4,5,6,7\}$ $A \cap B = \{4\}$ 
Disjoin set 2 sets with no common element $A \cap B = \emptyset$ A and B shares no connection	$A = \{1,2,3\}$, $B = \{4,5,6\}$ $A \cap B = \emptyset$  no common elements

PROBABILITY

Probability is the measure of chance or like hood of an event is happening.

Principal of Probability:

1. $0 \leq P(A) \leq 1$

   $P(A) = 0$: Event A impossible to occur

   $P(A) = 1$: Event A will surely occur

2. $P(A') =$ Probability that A will not happen $= 1 - P(A)$

3. 2 events A, B are mutually exclusive.

   ⇒ If A happens, B cannot happen, vice versa

   ⇒ A and B cannot happened at the same time

   ⇒ P(A or B) $= P(A) + P(B)$

   Example:

   P(rolling 1 or 2 on a six-sided die) = P(rolling 1) + P(rolling 2)

   $= \frac{1}{6} + \frac{1}{6}$

   $= \frac{1}{3}$

4. 2 events are independent

   ⇒ A happening does not affect B happening

   ⇒ When A happens, B can happen or not happen

   ⇒ P(A and B) = $P(A) \times P(B)$

   Example:

   P (rolling 1 on 2 die) = P(rolling 1 on 1st die) x P(rolling 1 on 2nd die)

   $= \frac{1}{6} \times \frac{1}{6}$

   $= \frac{1}{36}$

5. Sum of all probabilities = 1

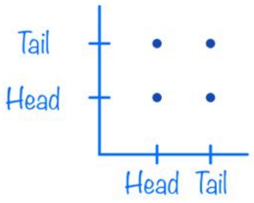
   Example:

   If A, B, C are the 3 outcome of a draw, then $P(A) + P(B) + P(C) = 1$.

Probability Diagrams

- It may help us to see the different outcomes and probabilities easier by representing them using probability diagrams.
- They are 2 types of probability diagrams: **Possibility Diagram** And **Tree Diagram**

<Possibility Diagram>



usually for: combination of a outcomes independent events

Shows: Different outcomes for 2 tosses of the coin.

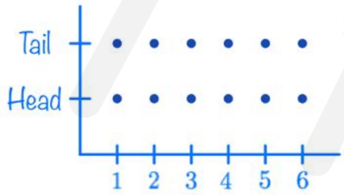
Total number of outcomes = 4

$\Rightarrow$ (Head, Head), (Head, Tail), (Tail, Head), (Tail, Tail)

P (Head, Head) = P (Head, Tail) = P (Tail, Head) = P(tail, tail) = $\frac{1}{4}$

P (at least 1 Head)

= P (Head, Tail) + P (Tail, Head) + PC Head, Head) = $\frac{3}{4}$



Shows: Different outcomes for tossing 1 coin and then rolling 1 six-sided die

P (Obtaining tail) = $\frac{1}{2}$

P (Head, 3) = $\frac{1}{12}$

2 die roll

coin show Head ← → die roll is 3

Shows: The different outcomes for the sum of a four-sided die

+	1	2	3	4
1	2	3	4	5
2	3	4	5	6
3	4	5	6	7
4	5	6	7	8

Number of occurrence when sum is 5 = 4

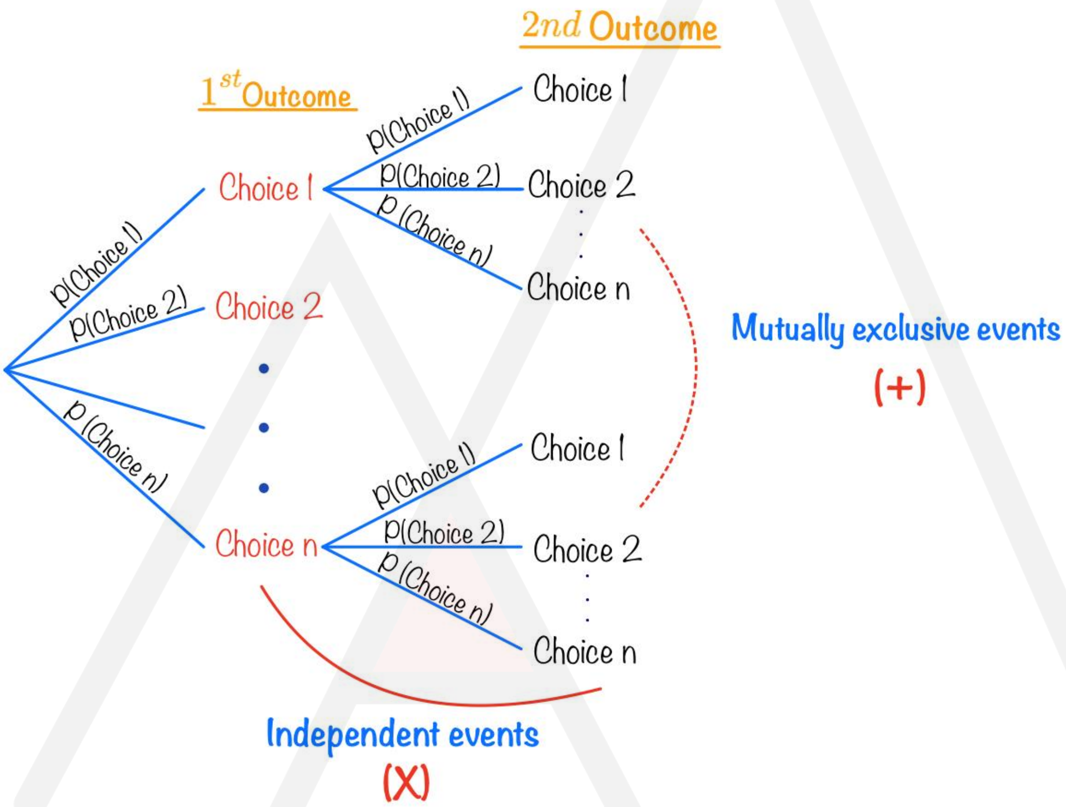
Total number of possible outcome = 16

$P(\text{Sum} = 5) = \frac{4}{16}$

$= \frac{1}{4}$

<Tree Diagram>

usually for: 2 or more outcomes. Multiple number of draws.



P (1st Outcome : Choice 1, 2nd Outcome: Choice 2)

= P (Choice 1) + P (Choice 2)

P (both Choice 1) + P (both Choice 2)+ ... + P (both choice n)

= P (Choice 1) x P (Choice 1) + P (Choice 2) x P (Choice 2) + ... + P(Choice n) x P(Choice n)

DATA HANDLING

Data can be organised in 3 different ways:

1 Frequency table:

Data classified according to the number of occurrences.

Fixed values

For ungrouped data

Example

Data collected on the number of hand phones owned by students:

1	1	3	2	2	1	1	1	1
4	2	5	1	2	1	1	2	2

Frequency table:

# HP	Tally
0	0
1	9
2	6
3	1
4	1
5	1

Class intervals

Range of values for grouped data

Example

Data collected on the marks scored by students in a test:

55	87	82	96	38	89	77
64	21	92	59	43	75	60

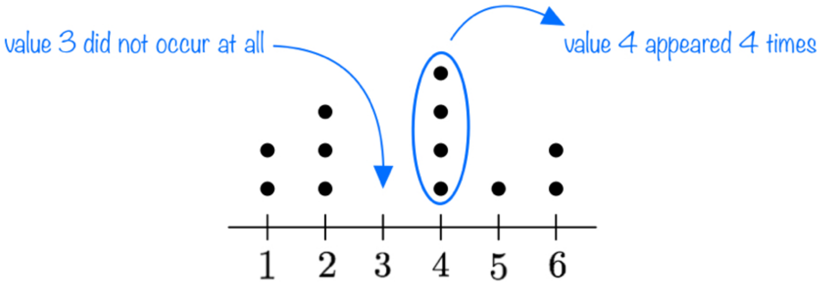
Frequency table:

Marks	Tally
81 – 100	5
61 – 80	3
41 – 60	4
21 – 40	2
0 – 20	0

This is the frequency distribution table showing how often a value / an interval of value occurs.

2 Dot diagram:

Number of dots represent number of times particular value occurred



3 Stem and leaf diagram:

Stem represents number in the tens place

Leaf represents number in the ones place

Stem	Leaf
1	
2	
3	4 8
4	2 9 9
5	
6	5 5 7 8
7	4 7 8 9 9 9
8	1 2 3 3 4 5 6 6 7 8
9	1 1 1 2
10	

1 student got 74 marks
1 student got 77 marks
1 student got 78 marks
3 student got 79 marks

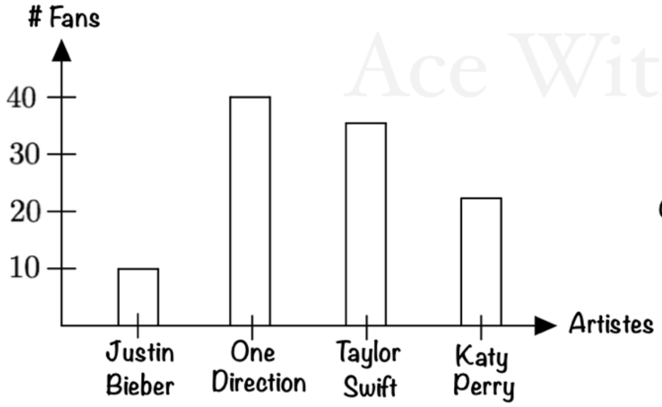
Presenting Data

Once data is organised, it has to be presented for further analysis. There are 6 ways to present data :

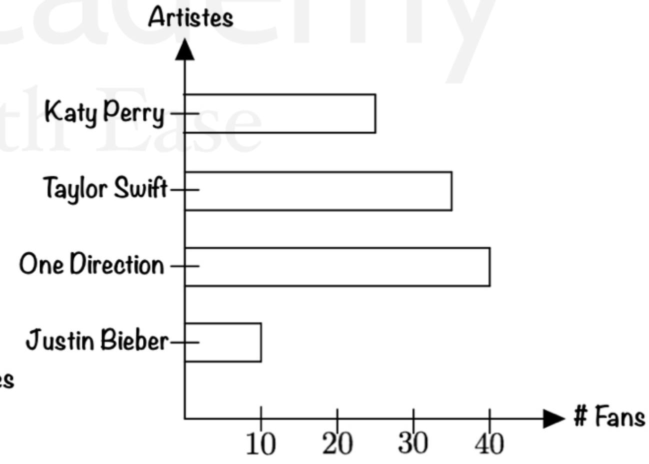
1 Table:

Artistes	Justin Bieber	One Direction	Taylor Swift	Katy Perry
# Fans	10	40	35	25

2 Bar graph:



- Vertical -



- Horizontal -

Properties:

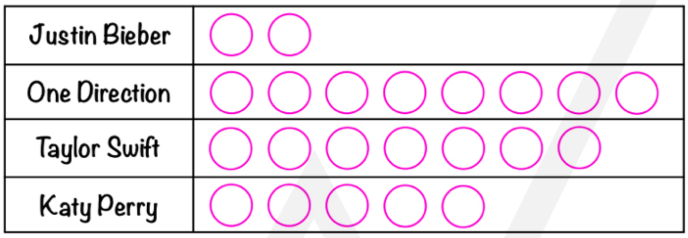
- Bars of equal width
- There are gaps between the bars
- Spaces between bars are of equal width

3 Pictogram:

Using pictures / diagrams

If frequency is high, let 1 picture denote fixed number of occurrences

If frequency is low, let 1 picture denote 1 occurrence

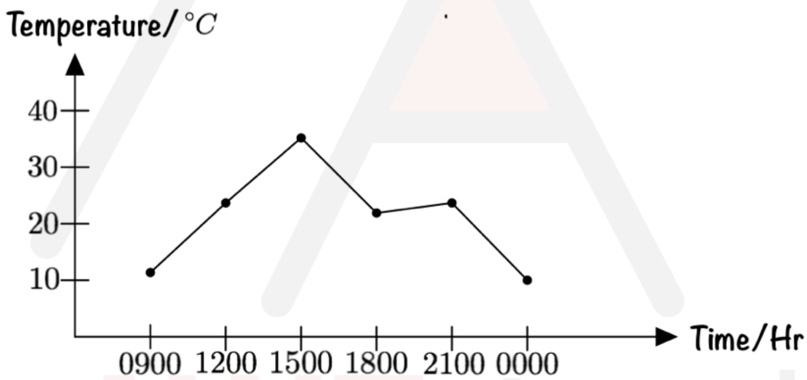


where each ○ represent 5 people

4 Line graph:

Shows trend/changes over period of time

Values between 2 readings may not have any meanings



Steps:

1. Plot value
2. Draw line connecting adjacent points

5 Pie chart:

Frequency represented by angle of sector

Angle of sector = proportion of category

$$= \frac{\text{number of occurrences for category}}{\text{total number of occurrences}} \times 360°$$

Steps:

1. For each category find angle
2. Draw pie chart
3. Label each sector

Total number of fans = $10 + 40 + 35 + 25 = 110$

Justin Bieber : $\frac{10}{110} \times 360° = 32.73°$ (to 2 d. p.)

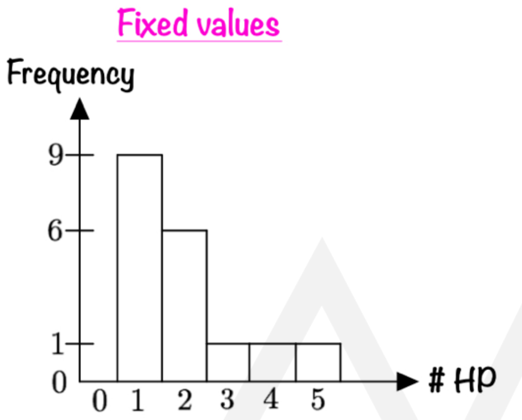
One Direction : $\frac{40}{110} \times 360° = 130.91°$ (to 2 d. p.)

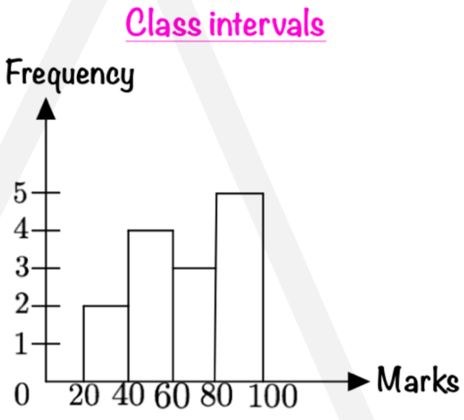
Taylor Swift : $\frac{35}{110} \times 360° = 114.55°$ (to 2 d. p.)

Katy Perry : $\frac{25}{110} \times 360° = 81.82°$ (to 2 d. p.)

6 Histogram:

Used to represent frequency distribution or grouped data with class intervals





Properties:

- Columns of equal width
- No gaps between columns
- Height of column = Frequency (number of occurrences)

Check out this table below to know when to use what.

Types	Purpose	Advantages	Disadvantages
Table	Consolidate data into categories	• Exact results	• Hard to see comparison • Boring
Bar Graph	Use of bars to show data	• Accurate • Useful in comparison among categories	Does not show proportion of one category in relation to the whole
Pictogram	Use of pictures/ diagram to show data	• Fun • Easy to present • Visual	• Inaccurate • Troublesome to draw
Line Graph	Shows data that changes with time	• Shows trends • Track changes with respect to time	No significance of values between 2 readings
Pie Chart	Compare each category to the whole	• Shows proportion	Tedious: need to convert data
Histogram	Use of columns to show data	• There is continuity • Accurate • Useful for comparisons	Does not show proportion

DATA ANALYSIS

1 Mean: Average value

$$\overline{x} = \frac{\text{sum of values}}{\text{total number of values}}$$

- For ungrouped data with set of n values $x_1, x_2 \ldots, x_n$:

$$\overline{x} = \frac{x_1 + x_2 + \ldots + x_n}{n}$$

- stem and leaf diagram

- For frequency distribution **without** class intervals:

$$\overline{x} = \frac{f_1x_1 + f_2x_2 + \ldots f_nx_n}{f_1 + f_2 + \ldots + f_n}$$

where f_i = frequency of value x_i

- dot diagram
- frequency table without class intervals
- histogram with ungrouped data

- For group data **with** class intervals

$$\overline{x}^{\cdot} = \frac{f_1x_1^{\cdot} + f_2x_2^{\cdot} + \ldots f_nx_n^{\cdot}}{f_1 + f_2 \ldots + f_n}$$

where f_i = frequency of i^{th} class interval

x_i = mid value of i^{th} class interval

- histogram with class intervals
- frequency table with class intervals

2 Median: Middle value

When distribution is arranged in ascending/descending order

- For odd numbers: Median = middle value
- For even numbers: Median = $\frac{\text{sum of 2 middle values}}{2}$

3 Mode: Value that has highest frequency, ie. occurs most frequently

- For set of values: Mode = value with highest frequency
- For class intervals: Mode = class intervals with highest frequency

When To Use What?

Mode	Mean	Median
For: Finding most common value/popular entity	**When:** Data is evenly spread **For:** Finding the average	**When:** Data is skewed - presence of outliers (extreme values) **For:** Finding the 'typical' value Identifying where the data is centred at

Spread

To measure how spread out the data is, there are 5 tools:

1 **Range =** highest value-lowest value

Shows entire spread of the data

2 **Standard deviation:**

Shows deviation from the average

- For ungrouped data with n values $x_1, x_2, \ldots, x_n$ and $\overline{x}$ = mean:

$$S = \sqrt{\frac{(x_1-\overline{x})^2 + (x_2-\overline{x})^2 + \ldots + (x_n-\overline{x})^2}{n}}$$
$$= \sqrt{\frac{x_1^2 + x_2^2 + \ldots + x_n^2}{n} - \overline{x}^2}$$

- stem and leaf diagram

- For frequency distribution where values $x_1, x_2, \ldots, x_n$ occur with corresponding frequency $f_1, f_2, \ldots, f_n$:

$$S = \sqrt{\frac{f_1(x_1-\overline{x})^2 + f_2(x_2-\overline{x})^2 + \ldots + f_n(x_n-\overline{x})^2}{f_1 + f_2 + \ldots f_n}}$$
$$= \sqrt{\frac{f_1x_1^2 + f_2x_2^2 + \ldots + f_nx_n^2}{f_1 + f_2 + \ldots f_n} - \overline{x}^2}$$

- dot diagram
- frequency table without class intervals
- histogram with ungrouped data

- For group data where $x_i^{\cdot}$ is the midpoint of class interval:

$$S = \sqrt{\frac{f_1(x_1^{\cdot}-\overline{x})^2 + f_2(x_2^{\cdot}-\overline{x})^2 + \ldots + f_n(x_n^{\cdot}-\overline{x})^2}{f_1 + f_2 + \ldots f_n}}$$
$$= \sqrt{\frac{f_1x_1^{\cdot 2} + f_2x_2^{\cdot 2} + \ldots + f_nx_n^{\cdot 2}}{f_1 + f_2 + \ldots f_n} - \overline{x}^2}$$

- histogram with class intervals
- frequency table with class intervals

3 Quartile:

There are 3 quartiles of importance :

1. Lower : Q_1(Ist quartile)
   → 25% values below, 75% values above
2. Median : Q_2(2nd quartile)
   → 50% values below and above
3. Upper : Q_3(3rd quartile)
   → 75% values below, 25% values above

4 Interquartile range = $Q_3 - Q_1$

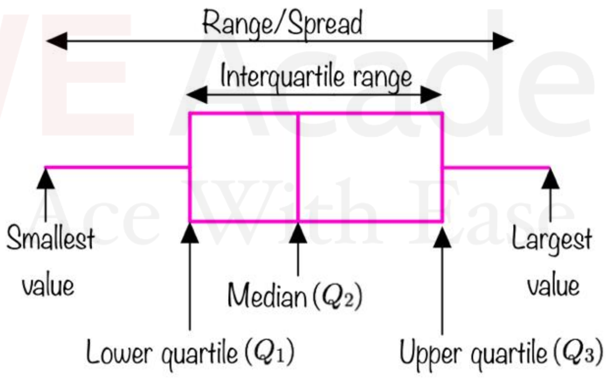
5 Percentile :

n^{th} percentile : smallest number such that n % of numbers $\leq$ to it.

Representing Analysis

1 Box-whisker plot/ box plot:

To represent the data to show its spread and central tendency, we can draw:



2 Cumulative frequency curve :

Obtained by adding previous frequencies from frequency table

Shows number of observations that lie below particular value