Binomial Distribution: Check your Understanding

Section 1: Assumptions

1. **ACJC Prelim 8865/2018/Q7part**
   A candy factory manufactured a large amount of pastilles daily and their candies are randomly packed in boxes of 20. The probability of selecting an orange-flavoured pastille to be packed into a box is 0.25. The random variable X is the number of orange-flavoured pastilles in a box of 20 pastilles.
   State an assumptions that is needed for X to be modelled by a binomial distribution. [2]

   Soln:
   A pastille being orange-flavour is independent of any other pastille being orange flavor.
   Or
   The flavour of each pastille must be independent of the flavour of any other pastille.

2. **AJC Prelim 8865/2018/Q7part**
   On average 7% of a certain brand of kitchen lights are faulty. The lights are sold in boxes of 12. State, in context, two assumptions needed for the number of faulty lights in a box to be well modelled by a binomial distribution. [2]

   Soln:
   A kitchen light being faulty is independent of any other kitchen light being faulty.
   The probability of a light being faulty is constant at 0.07 for every lights in a box.

3. **CJC Prelim 8865/2018/Q6part**
   As part of Singapore's aim to be a Smart Nation in 10 years, hawker centres are encouraged to go cashless. An initial trial of cashless payment in hawker centres shows that 1 in 5 customers uses cashless payment. A random sample of 12 customers is selected and the number of customers who use cashless payment is denoted by the random variable C. State, in context, an assumption needed for C to be well modelled by a binomial distribution. [1]

   Soln:
   A customer who uses cashless payment is independent of any other customers who use cashless payment.
   The probability of customers using cashless payment is constant at 0.2 for every customers.

Section 2: Find Probability

1. **YJC Promo 8865/2018/Q6**
   A health agency launches a '2+2' campaign to encourage people to eat two servings of fruits and two servings of vegetables per day. In a particular school, 20% of students eat at least 2+2 fruits and vegetables per day. 18 students in the school are selected at random and the number of students who eat at least 2+2 fruits and vegetables per day is denoted by X.
   (i) Find the probability that more than 3 of them eat at least '2+2' fruits and vegetables per day, [2]

(ii) Find the probability that between 2 and 7 of them eat at least '2+2' fruits and vegetables per day, [2]

Answer: (i) 0.499, (ii) 0.677

Soln:

(i)	$X \sim \mathrm{B}(18, 0.2)$ $P(X > 3) = 1 - P(X \leq 3)$ $= 0.499$
(ii)	$\mathrm{P}(2 < X < 7) = \mathrm{P}(X \leq 6) - \mathrm{P}(X \leq 2)$ $= 0.9481290011 - 0.271348775$ $= 0.677 \text{ (3 s.f.)}$

2. **RI Prelim 8865/2018/Q8part**

A dental clinic sees 50 patients each day for a total of 6 days each week, and is closed on Sunday. On average, 30 % of the patients are eligible for a dental subsidy after treatment, and the eligibility of a patient for dental subsidy is independent of another patient.

(i) Find the probability that, on a randomly chosen day, at least 20 patients are eligible for the dental subsidy. [2]

(ii) Find the probability that, in a randomly chosen week, at least 85 and at most 95 patients are eligible for the dental subsidy. [3]

Answer: (i) 0.0848, (ii) 0.512

Soln:

(i)	Let X be the random variable denoting the number of patients that are eligible for the dental subsidy out of 50 patients, $X \sim \mathrm{B}(50, 0.3)$ $\mathrm{P}(X \geq 20) = 1 - \mathrm{P}(X < 20) = 1 - \mathrm{P}(X \leq 19) = 0.0848$ (3 sf)
(ii)	Let Y be the random variable denoting the number of patients that are eligible for the dental subsidy out of 300 patients $Y \sim \mathrm{B}(300, 0.3)$ $\mathrm{P}(85 \leq Y \leq 95) = \mathrm{P}(Y \leq 95) - \mathrm{P}(Y \leq 84) = 0.512$ (3 sf)

3. **MJC Promo 8865/2018/Q4**

In a factory that produces a large number of sweets per day, a proportion, 0.15, of the sweets produced is red. A random sample of 10 sweets is taken on a particular day. The random variable X denotes the number of red sweets in the sample.

(i) Find the probability that there are at least 2 red sweets in the sample. [2]

(ii) Find the probability that the sample has at most 5 red sweets given that it has at least 2 red sweets. [3]

Answer: (i) 0.456, (ii) 0.997

Soln:

(i)	Let X be the random variable denoting the number of red sweets out of 10 sweets.

	$X \sim B(10, 0.15)$ $\text{P}(X \geq 2)$ $= 1 - \text{P}(X < 2)$ $= 1 - \text{P}(X \leq 1)$ $= 0.45570$ $= 0.456 \text{ (3 s.f)}$
(ii)	$\text{P}(X \leq 5 \mid X \geq 2)$ $= \dfrac{\text{P}(X \leq 5 \cap X \geq 2)}{\text{P}(X \geq 2)}$ $= \dfrac{\text{P}(2 \leq X \leq 5)}{\text{P}(X \geq 2)}$ $= \dfrac{\text{P}(X \leq 5) - \text{P}(X \leq 1)}{0.45570}$ $= \dfrac{0.99862 - 0.54430}{0.45570}$ $= 0.99695$ $= 0.997 \text{ (3 s.f)}$

4. **SAJC Prelim 8865/2018/Q8**

Tom owns a cheese tart specialty shop. On a daily basis, he prepares enough ingredients to bake exactly 500 cheese tarts a day. On average, Tom sells 80% of his cheese tarts per day. Find the probability that Tom sells between 300 and 420 cheese tarts inclusive given that he sells at least 380 cheese tarts in any randomly chosen day. [3]

Answer: 0.990

Soln:

	Let A be the random variable denoting the number of cheese tarts that Tom sells out of 500 tarts. $A \sim B(500, 0.8)$ Required Probability $= \text{P}(300 \leq A \leq 420 \mid A \geq 380)$ $= \dfrac{\text{P}(300 \leq A \leq 420 \cap A \geq 380)}{\text{P}(A \geq 380)}$ $= \dfrac{\text{P}(380 \leq A \leq 420)}{\text{P}(A \geq 380)}$ $= \dfrac{\text{P}(A \leq 420) - \text{P}(A \leq 379)}{1 - \text{P}(A \leq 379)}$ $= \dfrac{0.99049 - 0.012256}{1 - 0.012256}$ $= 0.990$

Section 3: Find unknow *p*

1. JJC Prelim 8865/2018/Q8part

A market research was conducted in a town where a large number of households were asked if they subscribe to fibre broadband internet services. The probability that a household subscribes to fibre broadband internet services is found to be p. A random sample of 30 households from a particular block of flats in the town was surveyed. Given that the probability that no household subscribes to fibre broadband internet services in the sample is 0.05, determine the value of p. [2]

Answer: 0.0950

Soln:

	Let X be the random variable denoting the number of households surveyed subscribe to fibre broadband internet services out of 30. Then $X \sim \mathrm{B}(30, p)$ Given $\mathrm{P}(X=0)=0.05$ $^{30}C_0 p^0 (1-p)^{30} = 0.05$ $(1-p)^{30} = 0.05$ $1-p=(0.05)^{\frac{1}{30}}$ $p=1-(0.05)^{\frac{1}{30}}$ $\therefore p = 0.095034 = 0.0950$ (3 s.f.)

2. MI Prelim 8865/2018/Q7

A company supplies a particular type of mechanical seals, called gaskets, to an automotive manufacturer. They are supplied in batches of 100. On average, the proportion of rejected gaskets is p. The probability that there are at least two gaskets rejected in a randomly chosen batch is 0.02. Write down an equation involving p and hence find the value of p. [3]

Answer: 0.00216

Soln:

	Let X be the random variable denoting the number of gaskets that are rejected, out of 100. $X \sim \mathrm{B}(100, p)$ $\mathrm{P}(X \geq 2) = 1-\mathrm{P}(X \leq 1) = 0.02$ $1-\mathrm{P}(X=0)-\mathrm{P}(X=1)=0.02$ $1-\binom{100}{0}p^0(1-p)^{100}-\binom{100}{1}p^1(1-p)^{99}-0.02=0$ $0.98-(1-p)^{100}-100p(1-p)^{99}=0$ Using GC, $p=0.00216$ (3 sf)

3. **VJC Prelim 8865/2018/Q8**

On average $100p\%$ of a certain company's pea seeds germinate. The pea seeds are sold in trays of 24. The probability that 15 or 16 pea seeds germinate in a tray is 0.086550 correct to 6 decimal places. Find the value of p to a suitable degree of accuracy, given that $p > 0.5$. [3]

Answer: 0.7942

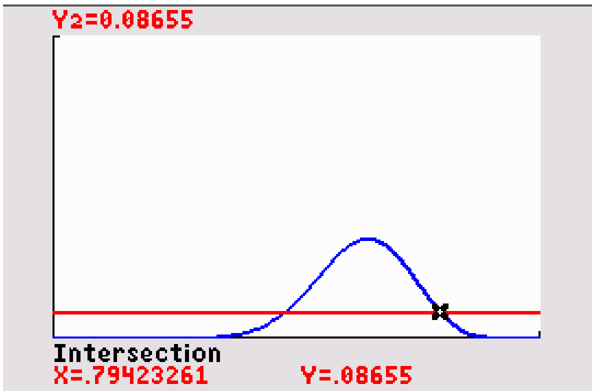
Soln:

Let X be the random variable denoting the number of seeds that germinate out of a tray of 24.

$$X \sim \mathrm{B}(24, p)$$

$$\mathrm{P}(X = 15) + \mathrm{P}(X = 16) = 0.086550$$

$$\left({}^{24}C_{15}\right)(p)^{15}(1-p)^{9} + \left({}^{24}C_{16}\right)(p)^{16}(1-p)^{8} = 0.086550$$



By GC, $p = 0.794233 = 0.7942$ or $p = 0.476512$ (rej $\because p > 0.5$)

4. **HCI Prelim 2008**

The probability of obtaining a 'head' from a biased coin is p. The coin is tossed 8 times and the number of 'tails' obtained is denoted by T. Find p, given that $Var(T) = \{\mathrm{E}(T)\}^2$.

[4]

Answer: $p = \dfrac{8}{9}$

Soln:

$T \sim \mathrm{B}(8, 1-p)$

$\mathrm{Var}(T) = \{E(T)\}^2 \Rightarrow 8(1-p)p = [8(1-p)]^2$

$\Rightarrow p = 8(1-p)$

$\Rightarrow p = \dfrac{8}{9}$

<u>Section 4: Find unknown *n*</u>

1. PJC Promo 8865/2018/Q7

On average, 75% of the customers visiting the camera section of a large electronics store make a purchase. Find the least possible number, *n*, of customers such that the probability of all the *n* customers making a purchase is less than 5%. [3]

Answer: least *n* = 11

Soln:

Let X be the random variable denoting the number of customers making a purchase out of n customers.
$X \sim \mathrm{B}(n, 0.75)$
$\mathrm{P}(X = n) < 0.05$

From GC,
$n = 10$, $\mathrm{P}(X = 10) = 0.05631 > 0.05$
$n = 11$, $\mathrm{P}(X = 11) = 0.04224 < 0.05$
$n = 12$, $\mathrm{P}(X = 12) = 0.03168 < 0.05$
So least $n = 11$

OR by definition

$$\binom{n}{n}(0.75)^n(0.25)^0 < 0.05$$

$$n\lg(0.75) < \lg 0.05$$

$$n > \frac{\lg 0.05}{\lg 0.75} = 10.41$$

$$n = 11, 12, 13, \ldots$$

2. YJC Promo 8865/2018/Q6

A health agency launches a '2+2' campaign to encourage people to eat two servings of fruits and two servings of vegetables per day. In a particular school, 20% of students eat at least 2+2 fruits and vegetables per day. A random sample of *n* students is chosen from the school. Find the greatest value of *n* such that the probability of having at least 4 students who eat at least 2+2 fruits and vegetables per day is less than 0.2. [3]

Answer: 11

Soln:

Let W be the random variable denoting the number of students who eat at least 2+2 fruits and vegetables per day out of n students

$$W \sim \mathrm{B}(n, 0.2)$$

$$\mathrm{P}(W \geq 4) < 0.2$$

$$1 - \mathrm{P}(W \leq 3) < 0.2$$

From GC,

when $n = 11$, $1 - \mathrm{P}(W \leq 3) = 0.1611 < 0.2$

when $n = 12$, $1 - \mathrm{P}(W \leq 3) = 0.2054 > 0.2$

$\therefore$ the greatest value of $n = 11$

3. **YJC Midyear 2007**

In a large city, one person in five is left-handed. A certain number of people are to be randomly selected such that the probability that there are at least one left-handed people among them, is to be greater than 0.95. Find the smallest number of people to be chosen. [5]

Answer: 14

Soln:

	Let W be the random variable denoting the number of left-handed people in a sample of n people $W \sim \mathrm{B}(n, 0.2)$ $\mathrm{P}(W \geq 1) > 0.95$ $1 - \mathrm{P}(W = 0) > 0.95 \Rightarrow \mathrm{P}(W = 0) < 0.05$ $(0.8)^n < 0.05$ $n > \dfrac{\ln 0.05}{\ln 0.8} \approx 13.425 \Rightarrow n \geq 14$ Least $n = 14$

4. **TJC Prelim 2007**

Given that 3% of the cherries in a basket are rotten. Using a binomial model, find the largest number of cherries which could be drawn so that the probability that there are no rotten cherries in the sample is greater than 0.7. [4]

Comment on the use of the binomial model in this situation. [1]

Answer:11

Soln:

	Let X be the random variable denoting the number of rotten cherries out of N cherries drawn. $X \sim \mathrm{B}(N, 0.03)$ $\mathrm{P}(X = 0) > 0.7$ $\Rightarrow\ 0.97^N > 0.7$ $\Rightarrow\ N \ln 0.97 > \ln 0.7$ $\Rightarrow\ N < 11.7$ The largest number of cherries that could be drawn = 11 If the number of cherries in the basket is not large enough, the probability of drawing a rotten cherry cannot be regarded as a constant.

5. **YJC Prelim 2007**

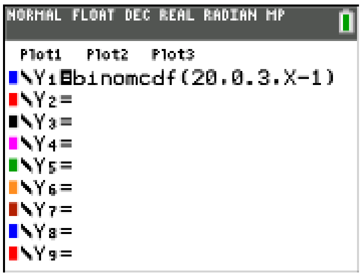
A binomial variable, X, has a mean 6 and variance 4.2. Find also the least integer r such that $P(X < r) > 0.85$. [3]

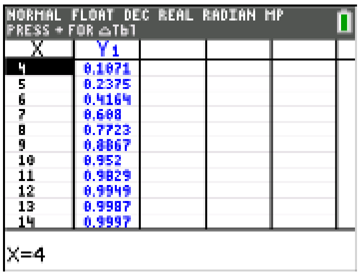
Answer: 9

Soln:

Let $X \sim B(n, p)$
Mean = $np = 6$
Variance = $np(1-p) = 4.2$
Solving, $n = 20,\ p = 0.3$

$P(X < r) = P(X \le r-1)$





Using GC,

r	$P(X < r) = P(X \le r-1)$
8	0.608
9	0.887
10	0.9527

Least integer $r = 9$

Section 5: Use of mean and variance

5. **TPJC Promo 2008**

In a binomial probability distribution, there are n trials and the probability of success for each trial is p. If the mean number of successes is 25 and the variance is 18.75, find the value of n and of p. [3]

Answer: 0.25, 100

Soln:

$X \sim B(n, p)$

$np = 25$ --------- (1)
$np(1-p) = 18.75$ -------- (2)
$1 - p = 0.75$
$p = 0.25$
$n = 100$

6. CJC Midyear 2007

The binomial random variable X is such that $E(X)=4$ and $Var(X)=2.4$. Find the probability that X assumes a value more than 4 but less than 8. [6]

Answer: 0.355

Soln:

	Let $X \sim B(n,p)$ $np=4$ $np(1-p)=2.4$ $1-p=\frac{2.4}{4}=0.6$ $\therefore\ p=1-0.6=0.4$ $n(0.4)=4 \quad \Rightarrow \quad n=10$ $\therefore X \sim B(10\ ,\ 0.4)$ $P(4<X<8)$ $=\mathrm{P}(X\le 7)-\mathrm{P}(X\le 4)$ $= 0.355$

Section 6: Find mode

1. ACJC Prelim 2008

A firm receives a large number of job applications from graduates each year. On average 20% of applications are successful. A researcher in the human resource department of the firm selects a random sample of 22 graduate applicants. Find the modal number of successful applicants in the sample. [2]

Answer: 4

Soln:

	Let X be the random variable denoting the number of successful job applicants out of 22 applicants. $X \sim B(22, 0.2)$ P(X = 3) = 0.17755 P(X = 4) = 0.21084 P(X = 5) = 0.18976 4 applicants is most likely

2. DHS Prelim 2008

The probability that a certain type of cactus seed will germinate is p. In a long-term study, 1500 of these seeds were planted, of which 600 germinated.

(i) Write down an estimate of p. [1]

Suzy plants 24 such seeds at the beginning of the growing season in three batches, each batch consisting of 8 seeds.

(ii) Assuming that the seeds germinate independently, use the value of p found in (i) to find the most probable number of seeds that will germinate in a batch of 8, [2]

Answer: (i) $\frac{2}{5}$, (ii) 3

Soln:

(i)	$p \approx \frac{600}{1500} = \frac{2}{5}$
(ii)	Let Y be the random variable denoting the number of seeds that will germinate in a batch of 8. $Y \sim \text{B}(8, \frac{2}{5})$ $\text{P}(Y = 2) = 0.209$ $\text{P}(Y = 3) = \underline{0.279}$ $\text{P}(Y = 4) = 0.232$ $\therefore$ Most probable no. that will germinate in a batch of eight is 3.

Section 7: 2-tier problems

1. EJC Prelim 8865/2018/Q10

A shop sells balloons in packs of 50 and, on average, 4% of the balloons are faulty.

(i) Find the probability that a pack of 50 balloons contains more than 2 faulty balloons. [2]

The balloons are sold in cartons and each carton contains 20 packs. In a randomly selected carton,

(ii) find the probability that less than 25% of the packs have more than 2 faulty balloons, [3]

(iii) find the probability of having at least 3 packs but fewer than 10 packs that have more than 2 faulty balloons. [2]

Answers: (i) 0.323 (ii) 0.175 (iii) 0.902

Soln:

(i)	Let X be the random variable denoting the number of of faulty balloons out of 50 balloons. $X \sim \text{B}(50, 0.04)$ $\text{P}(X > 2) = 1 - \text{P}(X \le 2) = 0.32329 = 0.323$ (to 3 s.f.)
(ii)	Let Y be the random variable denoting the number of packs with more than 2 faulty balloons out of 20 balloons. $Y \sim \text{B}(20, 0.32329)$ $\text{P}(Y < 0.25(20)) = \text{P}(Y < 5) = \text{P}(Y \le 4) = 0.17464 = 0.175$
(iii)	$\text{P}(3 \le Y < 10) = \text{P}(Y \le 9) - \text{P}(Y \le 2) = 0.90161 = 0.902$

2. H2 TPJC Prelim 2007

A factory produces chocolate which are packed into boxes of 20 and delivered to shops for sale. A chocolate will not meet the minimum criteria for packing for sale if it weighs less than 20 grams. On average, 2 % of the chocolate produced did not meet the minimum criteria.

(i) Find the probability that a randomly chosen box contain at least 1 chocolate that does not meet the minimum criteria. [2]

(ii) Find the probability that out of 4 randomly chosen boxes of chocolate, there are exactly 2 boxes with at least 1 chocolate that does not meet the minimum criteria. [2]

Answer: 0.3324; 0.295

Soln:

(i)	Let X be the random variable denoting the number of chocolates out of 20 that weighs less than 20 grams $X \sim B(20, 0.02)$ $P(X \geq 1) = 1 - P(X=0) = 0.3324$
(ii)	Let Y be the random variable denoting the number of boxes out of 4 with at least 1 chocolate that weighs less than 20 grams. $Y \sim B(4, 0.332)$ $P(X = 2) = 0.295$

3. H2 NYJC Midyear 2008

Six babies are born in a maternity ward per day. If the probability of having a girl is 0.48, calculate, find the probability that, in a week, there are at least 2 days on which exactly 3 boys are born. [2]

Answer: 0.693

Soln:

	Let X be the random variable denoting the number of babies, out of 6, that are girls. $X \sim B(6, 0.48)$ $P(X = 3) = 0.311$ Let W be the random variable denoting the number of days, out of 7, with exactly 3 boys born. $W \sim B(7, 0.311)$ $P(W \geq 2) = 1 - P(W \leq 1) = 0.693$

Section 8: Comparison problems

1. AJC Prelim 8865/2018/Q7

On average 7% of a certain brand of kitchen lights are faulty. The lights are sold in boxes of 12.

(i) Find the probability that a box of 12 of these kitchen lights contains at least 1 faulty light. [1]

The boxes are packed into cartons. Each carton contains 20 boxes.

(ii) Find the probability that each box in one randomly selected carton contains at least one faulty light. [1]

(iii) Find the probability that there are at least 20 faulty lights in a randomly selected carton. [2]

Explain why the answer to part **(iii)** is greater than the answer to part **(ii)**. [1]

Answers: (i) 0.581**, (ii)** 0.0000195**, (iii) 0.241**

Soln:

(i) Let X be the random variable denoting the number of kitchen lights that are faulty in a box of 12.

$X \sim \mathrm{B}(12, 0.07)$

$\mathrm{P}(X \geq 1) = 1 - \mathrm{P}(X = 0) = 0.58140 = 0.581$

(ii) Let Y be the random variable denoting the number of boxes in a carton with at least 1 faulty light

$Y \sim \mathrm{B}(20, 0.58140)$

$\mathrm{P}(Y = 20) = 0.0000195$

(iii) Let T be the random variable denoting the number of faulty lights in a carton

$T \sim \mathrm{B}(240, 0.07)$

$\mathrm{P}(T \geq 20) = 1 - \mathrm{P}(T \leq 19) = 0.241$

The event that there are at least 20 faulty lights in a carton can happen if one box contains at least 1 faulty light or it can happen in many other ways, e.g. 18 boxes with at least 1 faulty light each and one box with at least 2 faulty lights and one box with none. Hence, this explains why the answer to part **(iii)** is greater than the answer to part **(ii)**.

2. HCI Prelim 8865/2018/Q8

A shop sells mangoes packed in boxes of 12. It is given that 20 % of the mangoes are rotten. A buyer inspects a randomly chosen box of mangoes. Find the probability that

(a) at least 2 mangoes are rotten, [1]

(b) the 10^{th} mango inspected is the second mango that is rotten. [2]

Explain why the answer to part **(b)** is smaller than the answer to part **(a).** [1]

Answer: (a) 0.725 **(b)** 0.0472

Soln:

(a)	Let X be the random variable denoting the number of rotten mangoes out of 12 mangoes. $X \sim \mathrm{B}(12, 0.2)$ $\mathrm{P}(X \geq 2) = 1 - \mathrm{P}(X \leq 1) = 0.725$
(b)	Let Y be the random variable denoting the number of rotten mangoes out of 9 mangoes $Y \sim \mathrm{B}(9, 0.2)$ Required probability $= 0.2 \times \mathrm{P}(Y = 1)$ $= 0.0604$(to 3 s.f)
	Answer to part **(b)** is smaller than answer to **(a)** because event in **(b)** is a proper subset of event in **(a)**. Event in **(a)** includes cases where 2 rotten mango can be any 2 of the 12 whereas for event (b), the second rotten mango must be the 10^{th} mango.

3. VJC Prelim 8865/2018/Q8

On average 79.42% of a certain company's pea seeds germinate. The pea seeds are sold in trays of 24. The trays are packed into cartons. Each carton contains 8 trays.

(i) Find the probability that each tray in one randomly selected carton contains at least twenty pea seeds that germinated. [3]

(ii) Find the probability that there are at least 160 pea seeds that germinated in a randomly selected carton. [2]

(iii) Explain why the answer to part **(iii)** is greater than the answer to part **(ii)**. [1]

Answer: (i) 0.00121 (ii) 0.103

Soln:

Let X be the random variable denoting the number of seeds that germinate out of a tray of 24.

$$X \sim \mathrm{B}(24, 0.7942)$$

(i)

$$\left[\mathrm{P}(X \geq 20)\right]^8 = \left[1 - \mathrm{P}(X \leq 19)\right]^8$$
$$= 0.00121$$

(ii) Let Y be the random variable denoting the number of pea seeds that germinate out of 192 pea seeds in a carton.

$$Y \sim \mathrm{B}(192, 0.7942)$$

$$\mathrm{P}(Y \geq 160) = 1 - \mathrm{P}(Y \leq 159) = 0.103$$

(iii) Answer in part (ii) is greater than the answer in (i) because the event 'each of the 8 trays in a carton contains at least 20 pea seeds that germinated' is a subset of the event 'a carton contains at least 160 pea seeds that germinated'.

Section 9: Questions similar to Practice Questions 3(ii)

1. IJC Prelim 8865/2018/Q9

A company produces a large number of graphing calculators and on average, 5% of the graphing calculators are faulty. The graphing calculators are being packed into boxes of 30.

(i) Find the probability that there are more than 3 faulty graphing calculators in a randomly chosen box. [2]

(ii) 10 boxes are randomly selected. Find the probability that the 10^{th} box is the second box that contains more than 3 faulty graphing calculators. [3]

Answer: (i) 0.0608, (ii) 0.0201

Soln:

(i)	Let X be the random variable denoting the number of faulty calculators out of 30. $X \sim \mathrm{B}(30, 0.05)$ $\mathrm{P}(X > 3) = 1 - \mathrm{P}(X \leq 3)$ $= 0.060772$ $= 0.0608 \ (3\,\text{s.f})$
(ii)	Let Y be the random variable denoting the number of of boxes that contains more than 3 faulty calculators out of 9. $Y \sim \mathrm{B}(9, 0.060772)$ $\mathrm{P}(Y = 1) = 0.331218$ Prob required = 0.331218 (0.060772) = 0.0201 (3 s.f) **<u>Mtd 2</u>** Probability required $= \binom{9}{1}(0.060772)(1-0.060772)^8 \times 0.060772$ $= 0.0201$

2. ACJC Prelim 8865/2018/Q7part

A candy factory manufactured a large amount of pastilles daily and their candies are randomly packed in boxes of 20. The probability of selecting an orange-flavoured pastille to be packed into a box is 0.25.

(i) Find the probability that a box of pastilles contains at most 8 orange-flavoured pastilles. [1]

Jimmy likes the pastilles very much and would buy a box of pastilles each day from Monday to Friday. For a randomly selected week from Monday to Friday, find the probability that

(ii) the box John gets on Friday is the third box of pastilles that contain at most 8 orange-flavoured pastilles. [2]

Answer: (i) 0.959, (ii) 0.00887

Soln:

(i) (ii)	Let X be the random variable denoting the number of orange-flavoured pastilles in a box of 20 pastilles. $X \sim B(20, 0.25)$ $P(X \le 8) = 0.959$ Let W be the random variable denoting the number of boxes of pastilles that contain at most 8 orange-flavoured pastilles out of 4 boxes. $W \sim B(4, 0.959)$ Let A = {2 boxes out of 4 boxes John gets from Monday to Thursday contain at most 8 orange-flavoured pastilles} B = {the box John gets on Friday contains at most 8 orange-flavoured pastilles} $P(A \cap B)$ $= P(A).P(B)$ $= P(W = 2)(0.959)$ $= 0.00887$ (3 sig. fig.)

Section 10: Common questions

1. **MJC Prelim 8865/2018/Q9**

A machine produces n toys a day. Over a long time, it is found that 100p% of the toys produced by the machine is defective. The number of defective toys produced in a day by the machine is denoted by X.

(i) State, in the context of this question, two conditions needed for X to be well modelled by a binomial distribution. [2]

Assume now that X indeed follows a binomial distribution.

(ii) The mean number and variance of defective toys produced by the machine in a day is 10 and 9.75 respectively. Find the value of n and p. [3]

(iii) Given that there are at most five defective toys produced by the machine on a particular day, find the probability that the last toy produced by the machine is defective. [4]

Answer: (i) (ii) $p = 0.025$**;** $n = 400$**, (iii) 0.0109**

Soln:

(i)	The probability that a randomly chosen toy is defective is constant for all toys. A toy being defective is independent of any other toys being defective.
(ii)	$X \sim \mathrm{B}(n, p)$ $\mathrm{E}\left(X\right) = 10 \quad \Rightarrow \quad np = 10$ $\mathrm{Var}\left(X\right) = 9.75 \quad \Rightarrow \quad np\left(1-p\right) = 9.75$ $1 - p = \dfrac{9.75}{10} \quad \Rightarrow \quad p = 0.025$

	$\therefore n = \dfrac{10}{0.025} = 400$
(iii)	Let Y be the random variable denoting the number of defective toys produced out of 399 toys $Y \sim \text{B}(399, 0.025)$ Required probability $= \dfrac{\text{P}(Y \le 4) \times 0.025}{\text{P}(X \le 5)}$ = 0.0109 (3 s.f.)

2. **PJC Prelim 8865/2018/Q13**

In a large batch of light bulbs produced by a manufacturer, one in 50 light bulbs is faulty. The light bulbs are packed in boxes of 50. A box is considered "Bad" if it contains more than 2 faulty lights.

Find the probability that

(i) a randomly chosen box is "Bad", [2]

(ii) in 3 randomly chosen boxes, only 1 box is "Bad". [3]

The manufacturer wishes to conduct an inspection on the batch of light bulbs for quality control. A sample of 20 boxes is chosen at random. If there are less than 5 "Bad" boxes, then the batch of light bulbs is accepted. If there are more than 5 "Bad" boxes, then the batch of light bulbs is rejected. If there are exactly 5 "Bad" boxes, a further sample of 10 boxes is taken. If more than 1 "Bad" box is found, the batch of light bulbs will be rejected, but otherwise the batch is accepted.

(iii) Find the probability that the batch is accepted. [4]

(iv) Find the probability that a second sample is taken if that batch is rejected. [4]

(v) Explain the significance of the phrase "large batch" in the first sentence of this question. [1]

Answer: (i) 0.0784, (ii) 0.200, (iii) 0.994,(iv) 0.417

Soln:

(i) Let X be the random variable denoting the number of number of faulty light bulbs, out of 50

$X \sim \text{B}\left(50, \dfrac{1}{50}\right)$

$\text{P}(X > 2) = 1 - \text{P}(X \le 2) = 0.078428 \approx 0.0784$

(ii) $\left[\text{P}(X \le 2)\right]^2 \times \text{P}(X > 2) \times 3$

$= (0.92157)^2 (0.078428) \times 3$

$= 0.19982 \approx 0.200$

Alternative:

Let V be the random variable denoting the number of boxes containing more than 2 faulty light bulbs, out of 3

$V \sim \text{B}(3, 0.078428)$

$\text{P}(V = 1) = 0.19982 \approx 0.200$

(iii) Let Y be the random variable denoting the number of boxes containing more than 2 faulty light bulbs, out of 20

$Y \sim \mathrm{B}(20, 0.078428)$

Let W be the random variable denoting the number of boxes containing more than 2 faulty light bulbs, out of 10

$W \sim \mathrm{B}(10, 0.078428)$

Probability required

$= \mathrm{P}(Y < 5) + \mathrm{P}(Y = 5) \times \mathrm{P}(W \leq 1)$

$= \mathrm{P}(Y \leq 4) + \mathrm{P}(Y = 5) \times \mathrm{P}(W \leq 1)$

$= 0.98305 + (0.013513) \times (0.81791)$

$= 0.99410 \approx 0.994$

(iv) $\mathrm{P}(\text{second sample is taken} \mid \text{batch is rejected})$

$$= \frac{\mathrm{P}(\text{second sample is taken} \cap \text{batch is rejected})}{\mathrm{P}(\text{batch is rejected})}$$

$$= \frac{\mathrm{P}(Y = 5) \times \mathrm{P}(W > 1)}{1 - 0.99410}$$

$$= \frac{(0.013513) \times (1 - \mathrm{P}(W \leq 1))}{1 - 0.99410}$$

$$= \frac{(0.013513) \times (1 - 0.81791)}{1 - 0.99410}$$

$$= \frac{(0.013513) \times 0.18209}{1 - 0.99410}$$

$= 0.417$

(v) The phrase 'large batch' in the first sentence is required in order to assume that the conditional probability of the event that a light bulb in the batch is faulty is approximately the same at $\frac{1}{50}$ and hence the trials may be considered to be independent.

3. **SRJC Prelim 8865/2018/Q8**

(a) The random variable X has distribution B(7, p) and P(X = 1 or 4) = 0.25. Write down an equation in terms of p and find the possible values of p. [2]

(b) A company manufactures a large number of teapots. On average, p% of the teapots are defective. The teapots are randomly packed into boxes of 15. Given that the mean number of defective teapots in a box is 0.3, show that p = 2. [1]

(i) State, in context of this question, one assumption needed to model the number of defective teapots by a binomial distribution. [1]

(ii) Find the probability that at least 90% of the teapots in a randomly chosen box are not defective. [2]

(iii) n boxes are randomly chosen. The boxes which have more than 10% of the teapots that are defective are rejected. Find the greatest value of n such that the probability of rejecting at least 3 boxes is at most 0.01. [3]

Answer: (a) p = 0.0479 or p = 0.680, (ii) 0.965, (iii) 13

Soln:

(a)	$P(X = 1) + P(X = 4) = 0.25$ ${}^7C_1 p(1-p)^6 + {}^7C_4 p^4(1-p)^3 = 0.25$ $7p\,(1-p)^6 + 35p^4(1-p)^3 = 0.25$ Using GC, $p = 0.0479$ or $p = 0.680$
(b)	Let X be the random variable denoting the number of defective teapots in a box of 15 $X \sim B(15, p)$ $E(X) = 15\left(\frac{p}{100}\right)$ $0.3 = 15\left(\frac{p}{100}\right)$ $p = 2$ (Shown)
(i)	**A** teapot being defective is independent of any other teapots being defective. The probability of a teapot being defective is 0.02 for every teapots.
(ii)	Let Y be the random variable denoting the number of of non-defective teapots out of 15 $Y \sim B(15, 1 - 0.02)$ $Y \sim B(15, 0.98)$ $P(Y \geq 0.9 \times 15)$ $= P(Y \geq 13.5)$ $= P(Y \geq 14)$ $= 1 - P(Y \leq 13)$ $= 0.96466$ $= 0.965$ **Alternative Method** $P(15 - X \geq 0.9 \times 15)$

(iii)	$= P(15 - X \geq 13.5)$ $= P(X \leq 1.5)$ $= P(X \leq 1)$ $= 0.96466$ $= 0.965$ $P(X > 0.10 \times 15)$ $= P(X > 1.5)$ $= P(X \geq 2)$ $= 1 - P(X \leq 1)$ $= 1 - 0.96466$ $= 0.03534$ Let W be the random variable denoting the number of rejected boxes out of n boxes $W \sim B(n, 0.03534)$ $P(W \geq 3)$ $= 1 - P(W \leq 2) \leq 0.01$ When $n = 12$, $1 - P(W \leq 2) = 0.0076385$ When $n = 13$, $1 - P(W \leq 2) = 0.0096713$ When $n = 14$, $1 - P(W \leq 2) = 0.011989$ Greatest value of $n = 13$

4. RVHS Prelim 8865/2018/Q9

A receptionist considers her day to be a "good" day when she gets less than 10 rude calls for the particular day. The number of "good" days in a work week of 5 days, denoted by X, is observed over a long period of time. The probability of having a "good" day is denoted by p.

(i) State two assumptions made to model X as a binomial distribution. [2]

(ii) Given that the probability of a work week with less than two "good" days is 0.888, form an equation in terms of p and solve for p, giving your answer correct to 3 decimal places. [3]

(iii) Given that the receptionist has less than two "good" days in a particular work week, find the probability that the receptionist has a "good" day on the last day of the week. [3]

(iv) A calendar year has 52 weeks and the receptionist is assumed to work 5 days per week regardless of any public holiday. Find the probability that the mean number of "good" days for a week over a calendar year is not more than 2. [3]

Answer: (ii) 0.370, (iii) 0.149, (iv) 0.839

Soln:

(i)	The probability of a day being "good" is constant for all days. A day being "good" is independent of any other days being "good" OR "Good" days occur independently
(ii)	Let X be the random variable denoting the number of "good" days in a week.

	$X \sim \mathrm{B}(5, p)$ $\mathrm{P}(X < 2) = 0.888$ $\mathrm{P}(X = 0) + \mathrm{P}(X = 1) = 0.888$ $\binom{5}{0}(1-p)^5 + \binom{5}{1}p(1-p)^4 = 0.888$ $(1-p)^5 + 5p(1-p)^4 = 0.888$ Solving, $p = 0.36980863 \approx 0.370$
(iii)	Let Y be the random variable denoting the number of "good" days for the first 4 days of a week $Y \sim \mathrm{B}(4, p)$ P("good" day on last day | less than 2 "good" days in a week) $= \dfrac{P(Y=0) \times p}{P(X<2)}$ $= \dfrac{P(Y=0) \times 0.36980863}{0.888}$ $= 0.0656831364$ ≈ 0.0657

5. **TJC Prelim 8865/2018/Q11**

A confectionary produces a large number of sweets every day. On average, 20% of the sweets are wasabi-flavoured and the rest are caramel-flavoured.

(i) A random sample of n sweets is chosen. If the probability that there are at least three wasabi-flavoured sweets in the sample is at least 0.7, find the least possible value of n. [3]

The manufacturer decides to put the sweets randomly into packets of 20.

(ii) Find the probability that such a packet contains less than 3 wasabi-flavoured sweets. [2]

(iii) A customer selects packets of 20 sweets at random from a large consignment until she finds a packet with exactly 12 caramel-flavoured sweets. Give a reason why a Binomial Distribution is not an appropriate model for the number of packets she selects in the context of the question. [1]

The packets are then packed into boxes. Each box contains 10 packets.

(iv) Find the probability that all the packets in a randomly chosen box contain at least 3 wasabi-flavoured sweets. [2]

(v) Find the probability that there are at least 30 wasabi-flavoured sweets in a randomly chosen box. [1]

(vi) Explain why the answer to **(v)** is greater than the answer to **(iv)**. [1]

Answers: (i) Least *n* is 18, (ii) 0.206 **, (iii)** 0.0995**, (iv) 0.972**

Soln:

<table>
<tr><td>(i)</td><td>Let X be the random variable denoting the number of wasabi-flavoured sweets out of n.
$X \sim \mathrm{B}(n,\ 0.2)$
$\mathrm{P}(X \geq 3) \geq 0.7$
$1-\mathrm{P}(X \leq 2) \geq 0.7$
Least n is 18.
<table><tr><td>n</td><td>$1-\mathrm{P}(X \leq 2)$</td></tr><tr><td>17</td><td>0.6904 < 0.7</td></tr><tr><td>18</td><td>0.7287 > 0.7</td></tr></table></td></tr>
<tr><td>(ii)</td><td>Let Y be the random variable denoting the number of wasabi-flavoured sweets out of 20.
$Y \sim \mathrm{B}(20,\ 0.2)$
$\mathrm{P}(Y < 3) = \mathrm{P}(Y \leq 2)$
$= 0.206$</td></tr>
<tr><td>(iii)</td><td>The number of packets selected (i.e, the number of trials) is not fixed.</td></tr>
<tr><td>(iv)</td><td>Let W be the random variable denoting the number of packets which contains at least 3 wasabi-flavoured sweets out of 10.
$W \sim \mathrm{B}(10,\ 1-0.20608)$
$W \sim \mathrm{B}(10,\ 0.79392)$
$\mathrm{P}(W = 10) = 0.0995$
Alternative method:
$(1-0.20608)^{10} = 0.0995$</td></tr>
<tr><td>(v)</td><td>Let V be the random variable denoting the number of wasabi-flavoured sweets out of 200.
$V \sim \mathrm{B}(200,\ 0.2)$
$\mathrm{P}(V \geq 30) = 1-\mathrm{P}(V \leq 29) = 0.972$</td></tr>
<tr><td></td><td>Part (iv) is a subset of part (v), for example part (v) include cases where some packets have less than 3 wasabi sweets but overall the 10 packets have at least 30 wasabi sweets.</td></tr>
</table>

6. **YJC Prelim 8865/2018/Q9**

Eggs produced at a chicken farm are packaged in boxes of six. For any egg, the probability that it is broken when it reaches the retail outlet is 0.1. A box is said to be sub-standard if it contains at least two broken eggs. The number of broken eggs in a box is the random variable X.

(i) State, in context, the assumption needed for X to be well modelled by a binomial distribution. [1]

Assume now that X has a binomial distribution.

(ii) Find the probability that a randomly selected box is sub-standard. [2]

(iii) A random sample of n boxes is taken. Find the greatest value of n such that the probability that there are more than three sub-standard boxes is less than 0.01. [3]

(iv) Ten boxes are chosen at random. Find the most likely number of boxes that are sub-standard. [2]

(v) Give a possible reason in context why the assumption made in part **(i)** may not be valid. [1]

Answers: (ii) 0.114 (iii) 8 (iv) 1

Soln:

(i) Assumption: An egg being broken is independent of any other eggs being broken.

(ii) $X \sim \text{B}(6, 0.1)$

P(a box of eggs is sub-standard) $= \text{P}(X \geq 2)$

$= 1 - \text{P}(X \leq 1)$

$= 0.114$ (3 sf)

(iii) Let W be the random variable denoting the number of boxes that are sub-standard out of n.

$W \sim \text{N}(n, 0.114265)$

Given $\text{P}(W > 3) < 0.01$

$1 - \text{P}(W \leq 3) < 0.01$

From GC,

When $n = 8$, $\text{P}(W > 3) = 0.00815 < 0.01$

When $n = 9$, $\text{P}(W > 3) = 0.01336 > 0.01$

Therefore, greatest value of number of boxes chosen is 8.

(iv) Let Y be the random variable denoting the number of boxes that are sub-standard out of 10.

$Y \sim \text{B}(10, 0.114265)$

$\text{P}(Y = 0) = 0.2972$

$\text{P}(Y = 1) = 0.3834$

$\text{P}(Y = 2) = 0.2226$

i.e. Most likely number = 1

(v) Breakages are not independent of each other. (if one egg in a box is broken, it is more likely the others will be).

7. CJC Midyear 2008

In a large consignment of toys, the proportion of defective toys is 10%.

(i) A random sample of five toys is tested. Find the probability that less than three of them are defective. [1]

(ii) A random sample of n toys is selected. Find the least value of n such that the probability that at least one of them is defective exceeds 0.9. [3]

(iii) Three samples of five toys each are taken. Find the probability that one of these three samples has exactly one defective toy and the other two samples have exactly two defective toys each. [3]

(iv) The toys are packed into boxes with each box containing five toys. M of such boxes are delivered to a wholesaler. The wholesaler returns a box if there are more than one defective toy in it. If the expected number of boxes returned by the wholesaler is less than 40, what is the largest value of M? [3]

Answer: (i) 0.991; (ii) 22; (iii) 0.00523; (iv) 491

Soln:

(i)	Let X be the random variable denoting the number of defective toys out of 5 toys. $X \sim B(5, 0.1)$ $P(X < 3) = 0.991$
(ii)	Let Y be the random variable denoting the number of defective toys out of n toys. $Y \sim B(n, 0.1)$ $P(Y \geq 1) > 0.9$ $1 - P(Y = 0) > 0.9$ ${}^{n}C_0 (0.1)^0 (0.9)^n < 0.1$ $n \lg 0.9 < \lg 0.1$ $n > \dfrac{\lg 0.1}{\lg 0.9} = 21.85$ So least value of $n = 22$
(iii)	Probability $= P(X = 1) \times [P(X = 2)]^2 \times 3 = 0.00523$ (3 s.f.)
(iv)	Expected number returned $= M \times P(X > 1)$ So, $M \times P(X > 1) < 40$ $M < \dfrac{40}{1 - P(X \leq 1)}$ $M < \dfrac{40}{1 - \text{binomcdf}(5, 0.1, 1)} = 491.03$ So largest value of M is 491