Chapter 12 SUPERPOSITION

H2 PHYSICS 9749

Syllabus Content

- Principle of superposition
- Stationary waves
- Diffraction
- Two-source interference
- Single-slit and multiple-slit diffraction

Learning Outcomes

(a) explain and use the principle of superposition in simple applications.

(b) show an understanding of the terms interference, coherence, phase difference and path difference.

(c) show an understanding of experiments which demonstrate stationary waves using microwaves, stretched strings and air columns.

(d) explain the formation of a stationary wave using a graphical method, and identify nodes and antinodes.

(e) explain the meaning of the term diffraction.

(f) show an understanding of experiments which demonstrate diffraction including the diffraction of water waves in a ripple tank with both a wide gap and a narrow gap.

(g) show an understanding of experiments which demonstrate two-source interference using water, light and microwaves.

(h) show an understanding of the conditions required for two-source interference fringes to be observed.

(i) recall and solve problems using the equation $\lambda = ax/D$ for double-slit interference using light.

new in H2 syllabus

(j) * recall and use the equation $\sin\theta = \lambda/b$ to locate the position of the first minima for single slit diffraction.

(k) * recall and use the Rayleigh criterion $\theta \approx \lambda/b$ for the resolving power of a single aperture.

(l) * recall and use the equation $d \sin\theta = n\lambda$ to locate the positions of the principal maxima produced by a diffraction grating.

(m) * describe the use of a diffraction grating to determine the wavelength of light. (The structure and use of the spectrometer are not required.)

* = not in H1 syllabus

12.1 Introduction

Principle of Superposition

A wave is a disturbance that travels through a medium or vacuum. For mechanical waves, like sound and water waves, the disturbance refers to the displacement of the particles of the media from their equilibrium position. For electromagnetic waves, the disturbance refers to the varying electric and magnetic field.

What happens when two waves of the same kind (e.g. sound waves from two sources) meet at a point in space? The answer to this question is provided by the principle of superposition.

Definition

vector addition of waves

> The principle of superposition states that when two or more waves of the same kind meet at a point in space, the resultant displacement at that point is equal to the vector sum of the displacements of the individual waves at that point.

This is illustrated below:

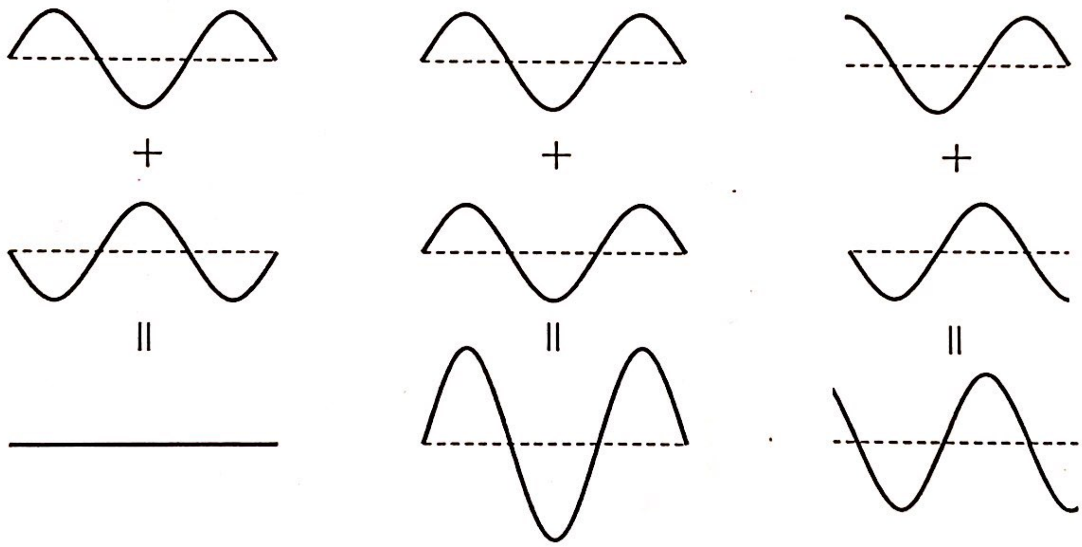


Fig. 1. When two waves arrive at a point at the same time, the resultant displacement is the vector sum of the displacements due to each wave.

12.2 Stationary Waves

What is a Stationary Wave?

Definition

A stationary wave results from the superposition of two progressive waves of the same frequency, amplitude and speed, travelling along the same line but in opposite directions.

diff particles have diff amplitude

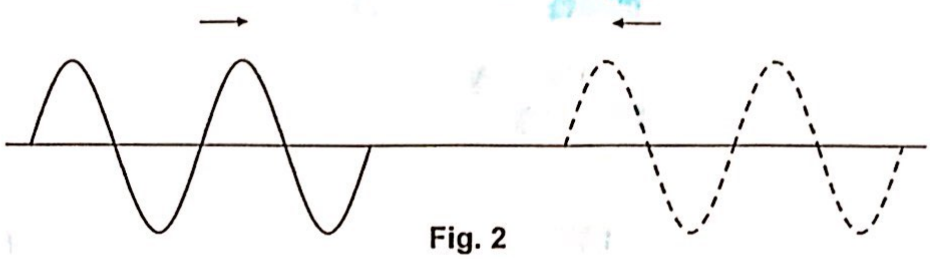


Fig. 2

The superposition of the two waves is observed along the line of propagation. The principle of superposition applies at each point and at a particular instant of time we may get a resultant waveform as shown in Fig. 3:

At a particular instant of time:

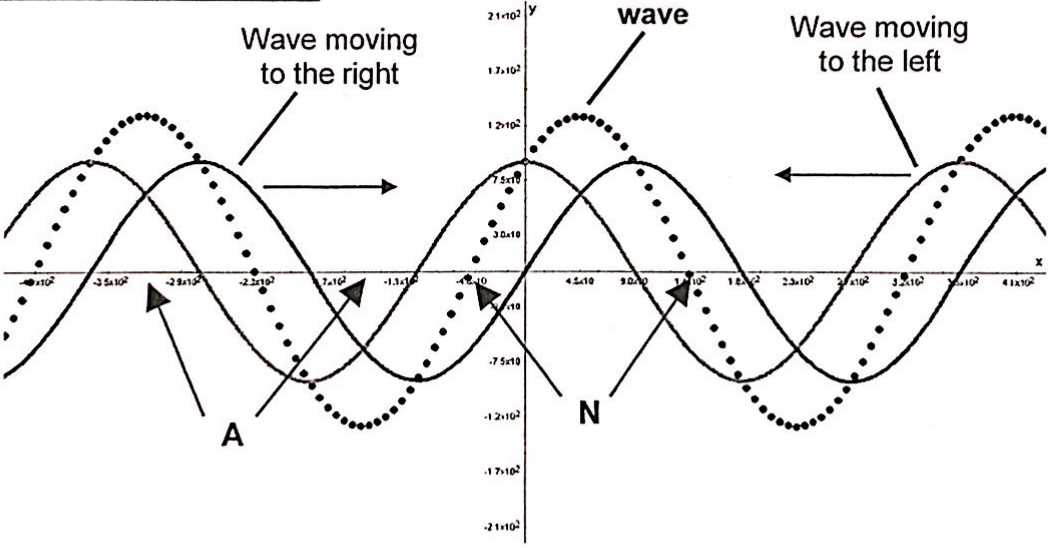


Fig. 3. Superposition of two waves displaced by ¼λ apart. A mathematical treatment can be found in Appendix A.

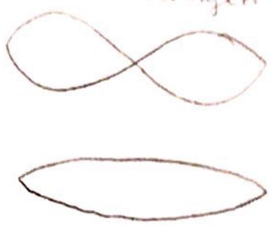
Properties of a Stationary Wave

The resultant waveform has the following features:

profile doesn't move

1. The wave profile does not propagate. As such, the resultant wave is known as a **stationary wave** or **standing wave**.
2. Every particle of the wave merely oscillates (except at the nodes) about their respective equilibrium positions with the **same frequency**, but **different amplitudes**. The frequency is the same as that of the two component waves.

envelopes stationary/unchanged



3. An **antinode** is a point in a standing wave where the amplitude is the maximum. Particles at the antinodes vibrate with the greatest amplitude. These are labelled 'A' on Fig. 3. The amplitude of oscillation at the antinodes is double that of the component waves.
4. A **node** is a point in a standing wave where the amplitude is zero. They are labelled 'N' on Fig. 3.
5. Within two consecutive nodes, every particle oscillates **in phase**, i.e., they reach their respective maxima, minima and equilibrium positions at the same instant. Note that these particles do not have the same amplitude.
6. Distance between two adjacent nodes (or antinodes) is ½λ.
7. Particles in neighbouring segments vibrate 180° (or π rad) out of phase with each other.

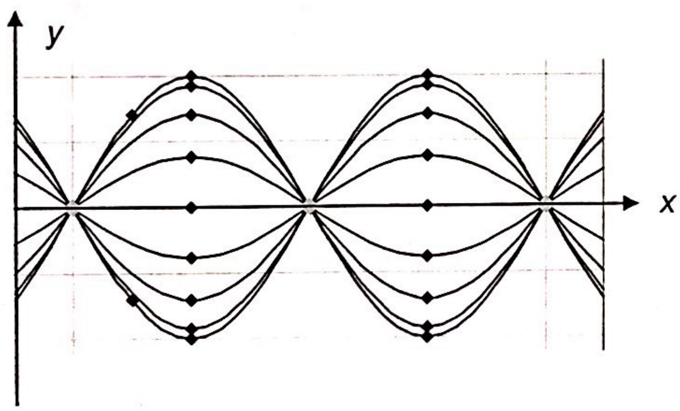


Fig. 4a: The waveform of a stationary wave at equal time interval of $T/16$ (on displacement-distance axes).

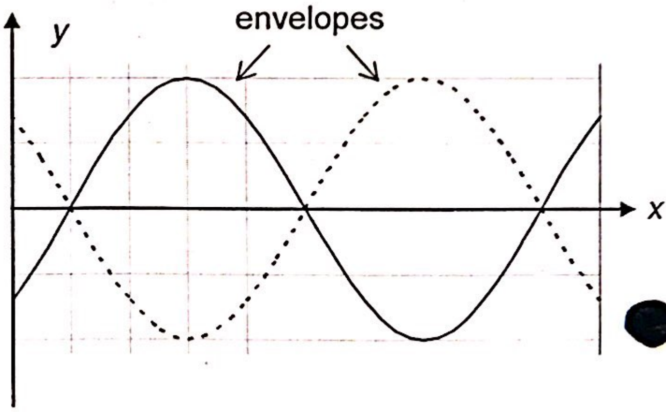


Fig. 4b: A **graphical representation** of a stationary wave (where the maximum displacement is drawn on displacement-distance axes)

Note: An **envelope** of a rapidly varying signal is a curve outlining its amplitudes.

Comparing Stationary Waves and Progressive Waves

Important Note

The table below compares the properties of a progressive wave with those of a stationary wave.

*

Property	Progressive Wave	Stationary Wave
Waveform	Propagates with the velocity of the wave	Does not propagate
Energy	Transports energy	Does not transport energy
Amplitude	Every point oscillates with the same amplitude.	Amplitude varies from 0 at the nodes to the maximum at the antinodes.
Phase	All particles within one wavelength have different phases.	All particles between two adjacent nodes have the same phase. Particles in adjacent segments have a phase difference of π rad.
Frequency	All points vibrate in s.h.m. with the frequency of the wave.	Except for the nodes which are at rest, all points vibrate in s.h.m. with the same frequency as the progressive wave that gives rise to it.
Wavelength	Is the distance between adjacent points which have the same phase.	Is equal to twice the distance between a pair of adjacent nodes or antinodes.

Example 1 (J90/P1/13)

Progressive waves of frequency 300 Hz are superposed to produce a system of stationary waves in which adjacent nodes are 1.5 m apart. What is the speed of the progressive waves?

Solution

$v = f\lambda$

$= (300)(1.5)(2)$

$= 900\ \text{m s}^{-1}$

12.3 Stationary Waves in Strings and Pipes

Stationary Waves in Stretched Strings

Stationary waves of various frequencies can be set up in a stretched string by plucking it at different points along the wire. Several modes of vibration of the string are possible.

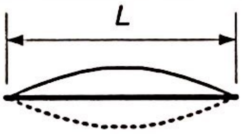
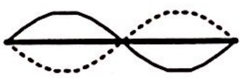
The frequency of sound produced is equal to the frequency of the stationary transverse wave set up in the string, or a combination of various harmonics.

The stationary wave with the lowest frequency is said to be vibrating with the **fundamental frequency**. Higher modes of vibration are known as the **overtones.**

The following steps can be used to determine the frequency of the different modes of vibration of a stationary wave in general.

Step 1 Select the **mode of vibration** of the stationary wave
Step 2 Draw the **graphical representation** of the stationary wave
Step 3 Derive the **wavelength** λ of the stationary wave in terms of length L of the string
Step 4 Using $f = v/\lambda$ where v is the speed of the progressive waves in the string or medium, obtain the **frequency** f of the stationary wave in terms of v and L.

Stationary Waves on a String

Modes of Vibration	Graphical Representation	Wavelength	Frequency	Also known as...
Fundamental mode/frequency		$L = 1\left(\frac{\lambda_1}{2}\right)$ $\Rightarrow \lambda_1 = 2L$	$f_1 = \frac{v}{2L}$	1st harmonic
1st overtone		$L = 2\left(\frac{\lambda_2}{2}\right)$ $\Rightarrow \lambda_2 = L$	$f_2 = 2\left(\frac{v}{2L}\right)$	2nd harmonic
2nd overtone		$L = 3\left(\frac{\lambda_3}{2}\right)$ $\Rightarrow \lambda_3 = \frac{2L}{3}$	$f_3 = 3\left(\frac{v}{2L}\right)$	3rd harmonic
$(n-1)^{th}$ overtone		$L = n\left(\frac{\lambda_n}{2}\right)$ $\Rightarrow \lambda_n = \frac{2L}{n}$	$f_n = n\left(\frac{v}{2L}\right)$	n^{th} harmonic

freq x2

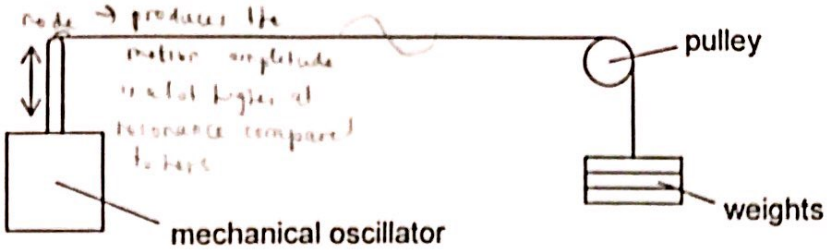
Fig. 5

Note:

- At the fixed ends, there must be nodes (since the string cannot vibrate).
- In reality, multiple harmonics give the timbre or characteristics of an instrument.

Example 2

(N95/P3/3(b))
In order to investigate stationary waves on a stretched string, a student sets up the apparatus shown below.



(i) Explain why it is necessary to adjust either the length of the string or the frequency of the oscillator in order to obtain observable stationary waves on the string.

(ii) What is meant by a node? Explain why a node must exist at the pulley.

Solution

(i) Since the tension of the string is constant, the velocity of the wave on the string is fixed. Stationary waves will only be formed if the length of the string is equal to certain integral multiples of half-wavelength of the wave (i.e. when resonance occurs). Hence, it is necessary to adjust the length of the string to fit multiples of half-wavelength, or adjust the frequency (and thus wavelength) to fit the length of the string.

(ii) A node is a point on the stationary wave where the particle is always at rest. A node must exist at the pulley because the pulley, and hence the string, is fixed in position.

Note:

- The end of the string which is attached to the oscillator is also considered as a node because its amplitude of oscillations is considered small compared to that of the antinode.
- The speed v of a wave in a string is given by $v = \sqrt{T/\mu}$, where T = tension, μ = mass per unit length.

$\uparrow T$ $\uparrow$ wavelength

Example 3

(J83/P2/12)

A taut wire is clamped at two points 1.0 m apart. It is plucked near one end. Which are the three longest wavelengths present on the vibrating wire?

Solution

Fundamental: 2.0 $1\left(\frac{\lambda}{2}\right) = 1.0 \Rightarrow \lambda = 2.0\text{ m}$

2nd harmonic: 1.0 $2\left(\frac{\lambda}{2}\right) = 1.0 \Rightarrow \lambda = 1.0\text{ m}$

3rd harmonic: 0.66 $3\left(\frac{\lambda}{2}\right) = 1.0 \Rightarrow \lambda = 0.67\text{ m}$

Stationary Waves in Air Columns or Pipes

It is also possible to set up stationary sound waves in air columns or pipes. A pipe is termed *closed* if *one* end of the pipe is closed while the other is open, and termed *opened* if *both* ends of the pipe are open.

closed end: node will be formed here (incoming wave always cancels out with ___ to form a node)

← open end antinode formed here
↳ air particles free to move at open end
↳ pressure at open end is close to atmospheric pressure

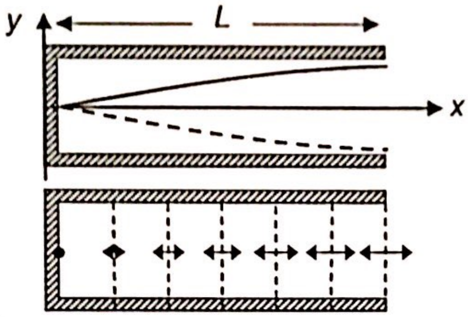
Fig. 6a closed pipe

Fig. 6b open pipe

Closed Pipes

When a sound wave is sent into a closed pipe, the wave propagates to the end of the pipe and is reflected. The reflected wave superposes with the incident wave and a stationary wave is formed. A *displacement node* is formed at the closed end of the pipe while a *displacement antinode* is formed at the open end:

harmonic in terms of fundamental
overtone ⇒ above the fundamental

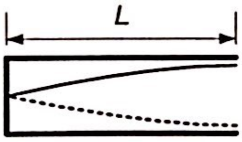
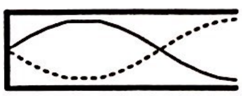
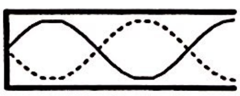


Graphical representation of a stationary wave in closed pipe – it shows the amplitude of vibration of the air molecules, but it does not mean that sound wave is a transverse wave!

Arrows indicate the amplitude of vibration of air molecules of the stationary wave.

Fig. 7

Stationary Waves in Closed Tubes (no even harmonics)

Modes of Vibration	Graphical Representation	Wavelength	Frequency	Also known as...
quarter wavelength Fundamental frequency		$L = 1\left(\frac{\lambda_1}{4}\right)$ $\Rightarrow \lambda_1 = 4L$	$f_1 = \frac{v}{4L}$	1st harmonic
1st overtone		$L = 3\left(\frac{\lambda_3}{4}\right)$ $\Rightarrow \lambda_3 = \frac{4L}{3}$	$f_3 = 3\left(\frac{v}{4L}\right)$	3rd harmonic
2nd overtone		$L = 5\left(\frac{\lambda_5}{4}\right)$ $\Rightarrow \lambda_5 = \frac{4L}{5}$	$f_5 = 5\left(\frac{v}{4L}\right)$	5th harmonic
$(n-1)^{th}$ overtone		$L = (2n-1)\left(\frac{\lambda_{2n-1}}{4}\right)$ $\Rightarrow \lambda_{2n-1} = \frac{4L}{(2n-1)}$	$f_{2n-1} = (2n-1)\left(\frac{v}{4L}\right)$	$(2n-1)^{th}$ harmonic

low intensity
amplitude should ↓

displacement / distance graph

Fig. 8

Open Pipes

When a sound wave is sent into an open pipe, a stationary wave is formed as shown in Fig. 9. As the air molecules are able to move freely at both ends, displacement antinodes are formed at both ends.

Note:
As a sound wave travels down an open pipe and reaches the other end, part of the wave is reflected. The reflected and incident waves superpose to produce a **pressure node at the open end** so that the pressure there is atmospheric. Hence, a **displacement antinode occurs at the open end.**

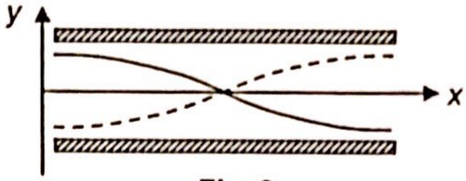
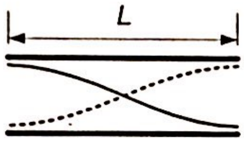
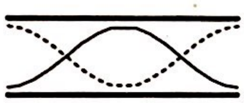
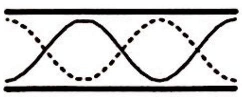
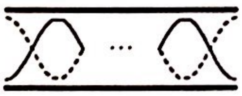


Fig. 9

Stationary Waves in Open Tubes

resonance condition

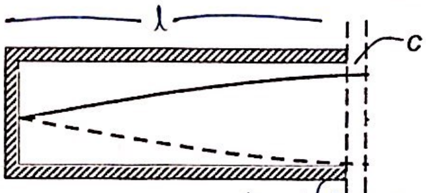
Modes of Vibration	Graphical Representation	Wavelength	Frequency	Also known as...
Fundamental frequency		$L = 1\left(\frac{\lambda_1}{2}\right)$ $\Rightarrow \lambda_1 = 2L$	$f_1 = \frac{v}{2L}$	1st harmonic
1st overtone		$L = 2\left(\frac{\lambda_2}{2}\right)$ $\Rightarrow \lambda_2 = L$	$f_2 = 2\left(\frac{v}{2L}\right)$	2nd harmonic
2nd overtone		$L = 3\left(\frac{\lambda_3}{2}\right)$ $\Rightarrow \lambda_3 = \frac{2L}{3}$	$f_3 = 3\left(\frac{v}{2L}\right)$	3rd harmonic
$(n-1)^{th}$ overtone		$L = n\left(\frac{\lambda_n}{2}\right)$ $\Rightarrow \lambda_n = \frac{2L}{n}$	$f_n = n\left(\frac{v}{2L}\right)$	n^{th} harmonic

gets softer
amplitude should decrease.

Fig. 10

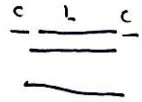
End Correction

The displacement antinode at the open ends of the pipes are actually located slightly outside the pipe as shown below. This results in a small *end correction* to be included in the calculation of the wavelength.



$\lambda/4 = (L+c)$
$f = v/[4(l+c)]$
mode: fundamental

both ends open



$\lambda/2 = (L+2c)$
$f = v/[2(L+2c)]$

Example 4

(N76/P2/3)
An organ pipe, 0.33 m long, is open at one end and closed at the other. The speed of sound in air is 330 m s^{-1}. Assuming that end corrections are negligible, calculate

(a) the frequencies of the fundamental and the first overtone,

(b) the length of a pipe which is open at both ends and which has a fundamental frequency equal to the difference of those calculated in (a).

Solution

(a) Fundamental:

$f_1 = \frac{v}{4L}$

$= \frac{330}{0.33 \times 4}$

$= 250 \text{ Hz}$

$L = \frac{\lambda}{4} \Rightarrow \lambda = 4(0.33) = 1.32 \text{ m}$

$f_1 = \frac{v}{\lambda} = \frac{330}{1.32} = 250 \text{ Hz}$

1st overtone:

$f_2 = 3f_1$

$= 750 \text{ Hz}$

(b) $f = 750 - 250 = 500$ Hz

$\lambda = \frac{v}{f} = \frac{330}{500} = 0.66$ m

$L = \frac{1}{2}\lambda = 0.33$ m

Example 5

A source of sound of frequency 250 Hz is used with a resonance tube, closed at one end, to measure the speed of sound in air. Strong resonance is first obtained at tube lengths of 0.30 m and then 0.96 m. Find

(a) the speed of the sound, and

(b) the end correction of the tube.

Solution

(a) $3f_3 = \cancel{3}f_1$

$\frac{\cancel{3}v}{4(L_1+c)} = \frac{3v}{4(L_2+c)}$

$\frac{\cancel{3}1}{4(0.30+c)} = \frac{3}{4(0.96+c)}$

$\cancel{1}(0.96+c) = 3(0.30+c)$

$0.96 - 0.90 = 2c$

$c = 0.03 \text{ m}$

$250 = \frac{v}{4(0.30+0.03)}$

$v = 330 \text{ ms}^{-1}$

$\frac{\lambda}{4} = 0.30 + c$ — (1)

$3\left(\frac{\lambda}{4}\right) = 0.96 + c$ — (2)

Solving, $\lambda = 1.32$ m

$v = f\lambda = 250 \times 1.32$

$= 330 \text{ ms}^{-1}$

(b) From (1), $c = \frac{\lambda}{4} - 0.30 = 0.03$ m

WAVE FRONT: the position of points on a wave which are in phase

12.4 Diffraction

What is Diffraction?

Definition

Diffraction is the bending of waves after passing through an aperture or round an obstacle.

Due to the effects of diffraction, waves bend from a straight path and enter a region that would otherwise be shadowed.

For example, when you are in a room, you can hear someone along the corridor through the open door of your room, even if you cannot see them. Why?

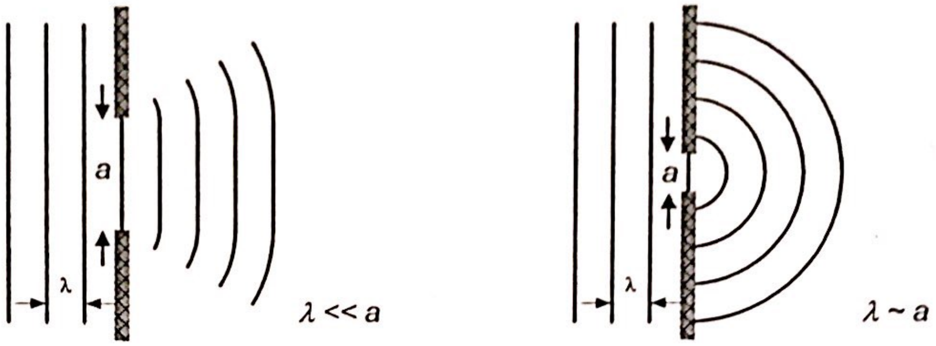


Fig. 11

The degree of diffraction depends on the relative size of the wavelength and the aperture.

Generally, **diffraction is pronounced when the wavelength of the wave is of the same order of magnitude as the width of the aperture or obstacle**. (Diffraction of waves is explained by Huygen's principle in Sect 12.9 in pg 22).

This is the reason why under normal circumstances, we do not observe any diffraction of light because the holes and apertures that we come across everyday are much larger than the wavelengths of light.

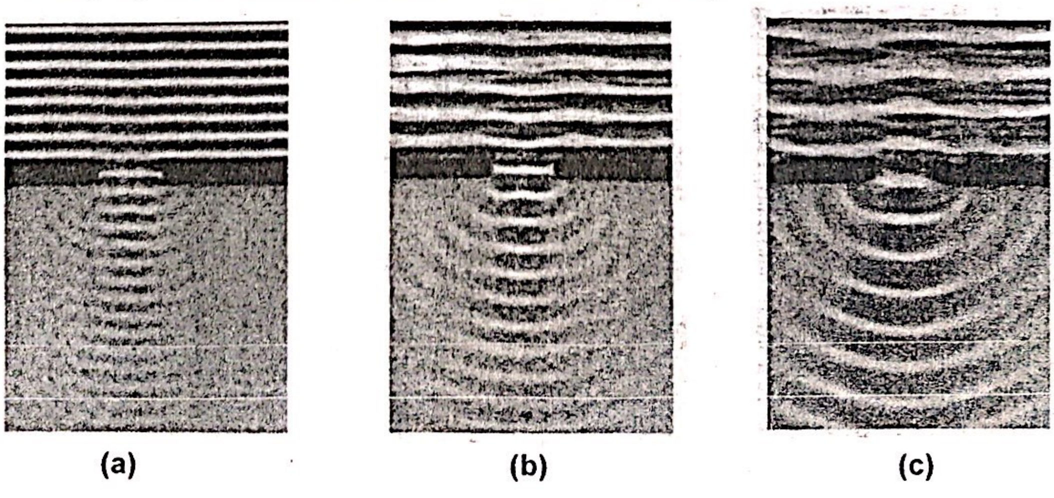


Fig. 12. Ripple tank images of water waves emerging from an opening. As the wavelength is increased from (a) to (c), the effect of diffraction becomes more pronounced.

12.5 Interference

Coherence

Definition

Waves or sources are said to be **coherent** if they have a **constant phase difference.**

This implies that coherent waves or sources must have the same frequency, but the reverse is not true. Velocities of the waves are assumed to be identical.

same f

coherent

Examples of coherent sources:

- diffracted laser beams through two slits incident by a single laser beam
- two speakers fed by same source

These are not coherent sources:

- two red lasers (same frequency, but not coherent)

two candles

- two filament lamps (not monochromatic)

special case of superposition (coherent *)

Interference

Definition

Interference is the **superposition** of two or more **coherent waves** to give a resultant wave whose resultant amplitude is given by the **principle of superposition**.

When two waves interfere, they can give rise to **constructive interference** and **destructive interference.**

Constructive Interference

Constructive interference occurs when two waves arrive at the same point with a **phase difference of zero**, i.e. a **maximum** is obtained. (Amplitude is fixed, but displacement is varying.)

<u>*A common misconception:*</u>
Constructive Interference occurs when wave crests meet wave crests or wave troughs meet wave troughs.

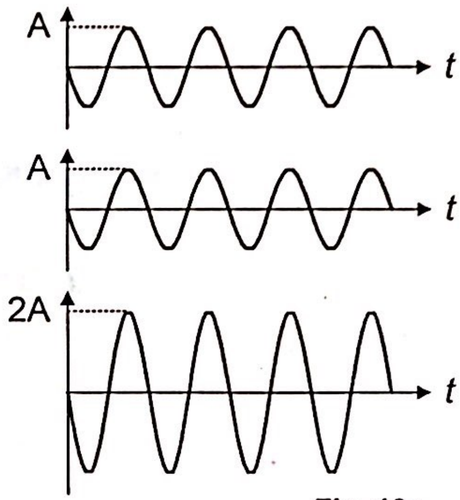


Fig. 13a

Destructive Interference

Destructive interference occurs when two waves arrive at the same point with a **phase difference of π rad**, i.e. a **minimum** is obtained. (Amplitude is zero only if both waves have the same amplitude.)

<u>*A common misconception:*</u>
Destructive interference occurs when wave crests meet wave troughs.

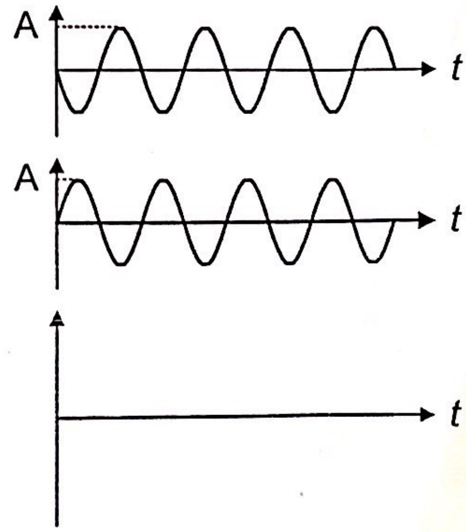


Fig. 13b

12.6 Two-source Interference

Conditions for Observable Interference

Important Note

1. The waves or sources must be **coherent** (i.e. they have the same frequency and a constant phase difference).
2. The waves must have approximately the **same amplitude** (for a better contrast). cancellat
3. The waves must **overlap** and be of the **same type** (to produce regions of constructive and destructive interference).
4. For transverse waves, they must be **unpolarised or polarised** in the same plane.

The Ripple Tank Experiment

Two ball-ended dippers S_1 and S_2, attached to a mechanical oscillator (and hence are coherent sources), send out two sets of circular wavefronts (let's say crests). These waves interfere when they overlap as shown in Fig. 14.

By the principle of superposition, **constructive interference** takes place along the **anti-nodal lines (dark lines)** when the two waves are **in phase**.

In between anti-nodal lines are the **nodal lines (dotted lines)** along which the waves arrive **exactly π rad out of phase. Destructive interference** occurs.

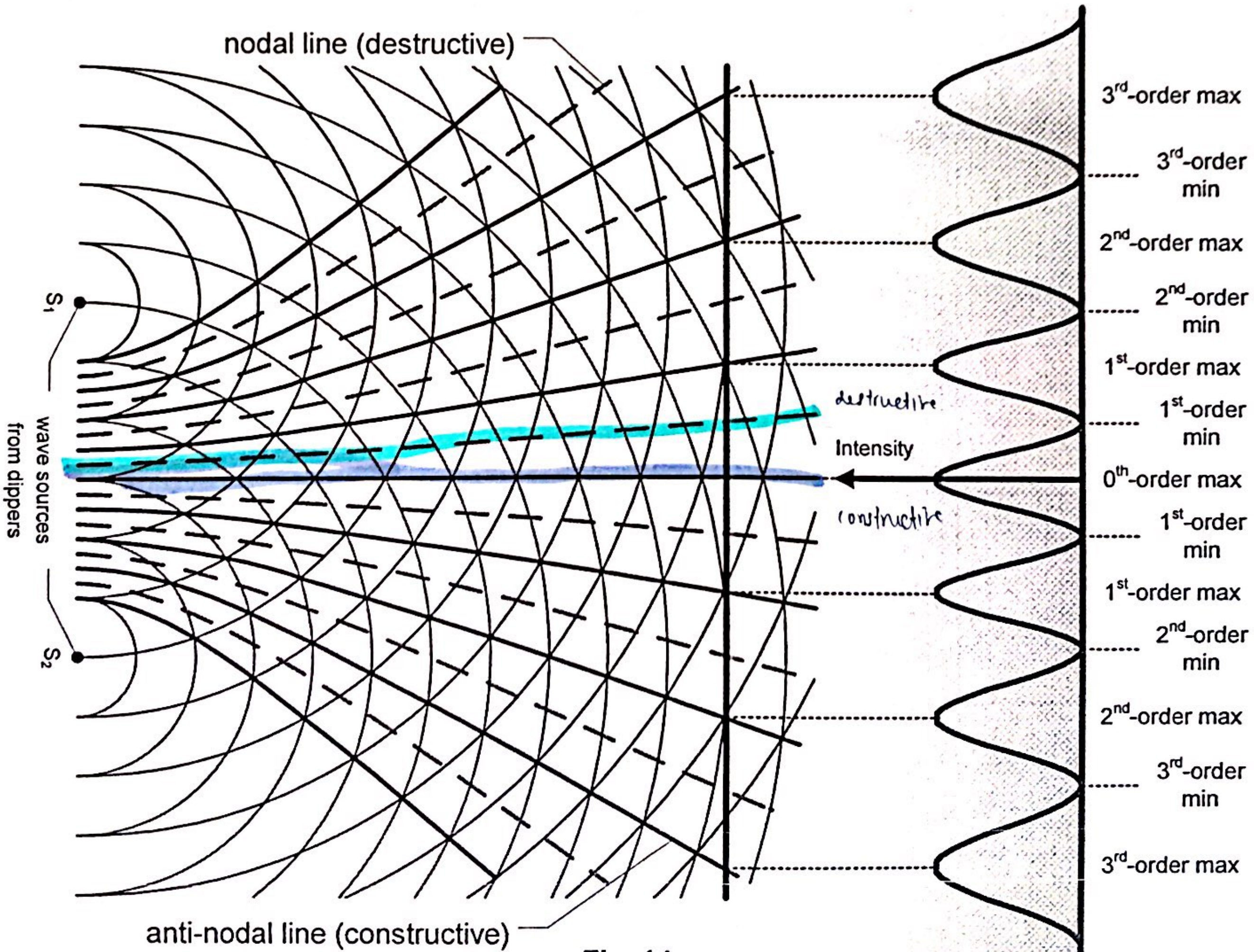


Fig. 14

any point that not intersecting:
· destructive

why doesn't light have bright/dark spots? the lights have many many frequencies

Path Difference

Definition

Path difference is the difference in the distances that each wave travels from its source to the point where the two waves meet.

We can look at path difference to analyse whether two waves meet in phase or out of phase, and hence whether constructive or destructive interference results.

Consider sources S_1 and S_2, and points P & Q where the two waves meet:

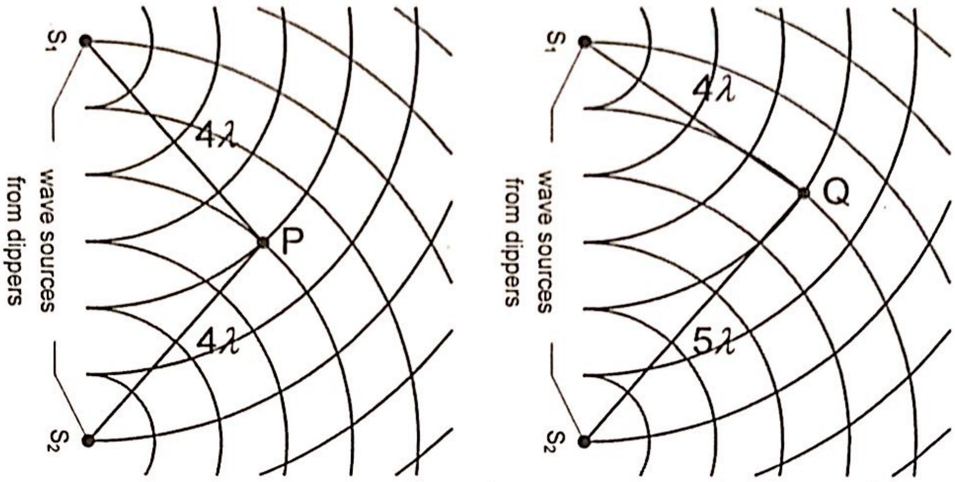


Fig. 15a **Fig. 15b**

condition: two sources must be in phase

At point P, the two waves meet in phase (i.e. constructive interference),

$$\text{Path difference, } \Delta x = S_2P - S_1P = 0$$

At point Q, the two waves also meet in phase (i.e. constructive interference),

$$\text{Path difference, } \Delta x = S_2Q - S_1Q = \lambda$$

Hence, when two sources are **in phase**, for **constructive interference** to occur,

$$\text{Path difference, } \boldsymbol{\Delta x = n\lambda} \text{ (where n = 0, 1, 2, ...)}$$

The n[th] order maximum (e.g. bright fringe or loud sound) occurs at positions where the path difference is $n\lambda$.

Now, consider points R & T where the two waves meet:

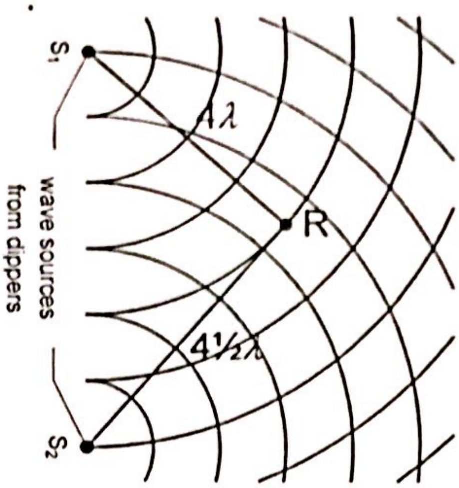


Fig. 16a

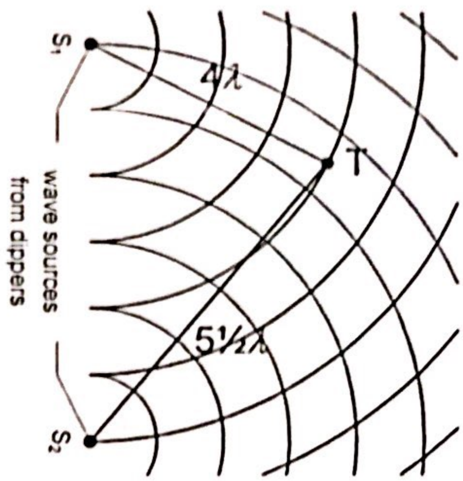


Fig. 16b

At point R, the two waves meet π out of phase (i.e. destructive interference),

$$\text{Path difference, } \Delta x = S_2R - S_1R = \tfrac{1}{2}\lambda$$

At point T, the two waves also meet π out of phase (i.e. destructive interference),

$$\text{Path difference, } \Delta x = S_2Q - S_1Q = 1\tfrac{1}{2}\lambda$$

Hence, when two sources are **in phase**, for **destructive interference** to occur,

Path difference, $\boldsymbol{\Delta x = (n + \frac{1}{2})\lambda}$ (where n = 0, 1, 2, ...)

odd number of $\frac{1}{2}$ wavelengths

Question: What happens at P, Q, R and T if the two sources are out of phase by π rad?

Answer:

Waves arrive at P & Q π rad out of phase $\Rightarrow$ destructive interference at P & Q

Waves arrive at R & T in phase $\Rightarrow$ constructive interference at R & T

Path Difference Summary

In summary, the following table can be used to determine whether constructive or destructive interference occurs at a certain point where the two waves meet.

1. Determine whether the two sources are in phase or π rad out of phase.
2. Determine the path difference in terms of λ.
3. Use the table to check whether constructive or destructive interference occurs.

Formula

	2 sources in phase	2 sources π rad out of phase
Constructive interference (maxima)	$\boldsymbol{\Delta x = n\lambda}$	$\boldsymbol{\Delta x = (n + \frac{1}{2})\lambda}$
Destructive interference (minima)	$\boldsymbol{\Delta x = (n + \frac{1}{2})\lambda}$	$\boldsymbol{\Delta x = n\lambda}$

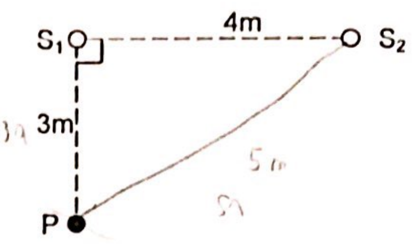
* check: don't apply formula / swap blindly

Example 6

(J80/P2/10; N88/P1/8)
Two wave generators S_1 and S_2 produce water waves of wavelength 1 m. They are placed 4 m apart in a water tank and a detector P is placed on the water surface 3 m from S_1 as shown in the diagram.

When operated alone, each generator produces a wave at P which has amplitude A.

When the generators are operating together and in phase, what is the resultant amplitude at P?



Solution

$S_2P = 5m$
constructive.

2A.

$\Delta x = S_2P - S_1P$
$= 5 - 3 = 2m$

since path difference = 2m = 2λ, constructive interference occurs at P.
resultant amplitude = A + A
= 2A

12.7 Young's Double-Slit Experiment

only refers to light

Experimental Set-Up

An experimental set-up for viewing two-source interference pattern with light is the Young's double-slit experiment shown below.

A monochromatic light source is placed behind a single slit to create a small, well-defined source of light. Light from this source is diffracted at the single slit, producing two light sources at the double slits. Because light from the two slits originate from the same single slit, they are coherent and create a sustained and observable interference pattern.

At points of constructive interference (maxima), bright fringes are observed.
At points of destructive interference (minima), dark fringes are observed.

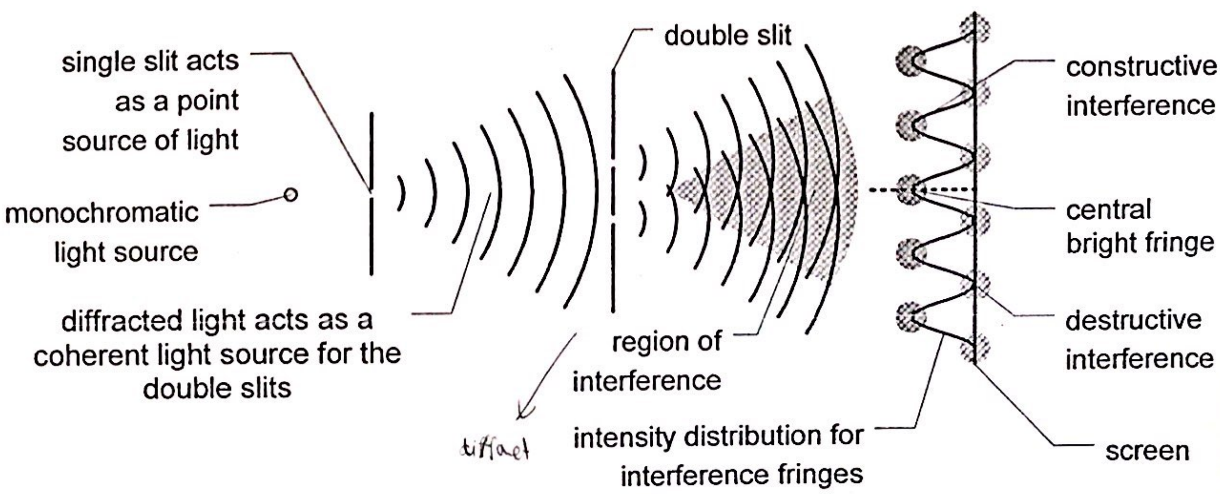


Fig. 17

Derivation of $x = \lambda D / a$

Let us now derive an equation for the fringe separation, i.e. the spacing between two successive bright (or dark) fringes.

Notations: x_n = distance of n[th] bright fringe from central fringe,
a = slit separation, D = distance between slits and screen

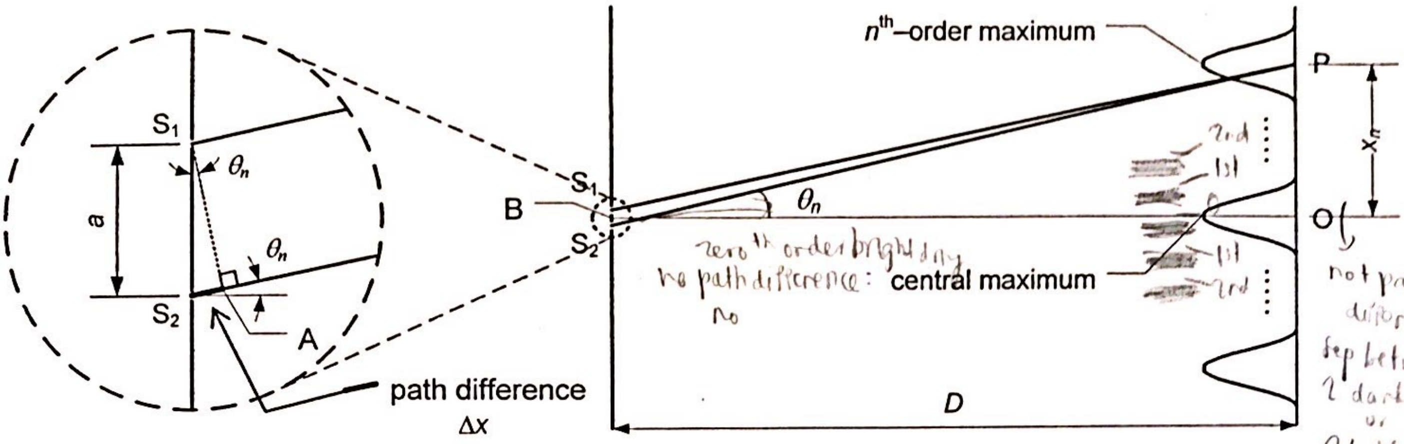


Fig. 18a

Fig. 18b

Consider position P where the n^{th} order bright fringe (maximum) lies. If $a << D$, the rays from S_1 and S_2 to P are almost parallel and $\angle A \approx 90°$.

$$\therefore \text{ path difference } \Delta x = S_2P - S_1P = S_2A \approx a \sin\theta_n \text{ (Fig. 18a)}$$

constructive interference

Since the waves passing through slits S_1 and S_2 are in phase, for constructive interference,

$$\boxed{a \sin\theta_n = n\lambda}$$

same as diffraction grating

cambridge: double slit a, multiple slit d

don't need to know how to derive

$$\sin\theta_n = \frac{n\lambda}{a} \qquad \text{where } n = 0, 1, 2, \ldots$$

From ΔOBP (Fig. 18b),

$$\tan\theta_n = \frac{x_n}{D}$$

almost parallel!

When $a >> \lambda$, using small angle approximation ($\theta_n < 6°$),

$$\sin\theta_n \approx \tan\theta_n$$ → radian not degree

$$\frac{n\lambda}{a} \approx \frac{x_n}{D}$$

Hence, constructive interference takes place at these positions,

$$x_n = \frac{n\lambda D}{a} \qquad \text{where } n = 0, 1, 2, \ldots$$

Therefore, spacing between successive bright fringes,

$$\boxed{x = x_n - x_{n-1} = \frac{n\lambda D}{a} - \frac{(n-1)\lambda D}{a} = \frac{\lambda D}{a}.}$$

π rad out of phase: interference pattern swapped.

7/12°: bright fringes further away (separation no longer constant)

Fringe Separation

Formula

$$\boxed{x = \frac{\lambda D}{a}}$$

What are the assumptions for the equation to be valid?

Intensity Distribution of Interference Fringes

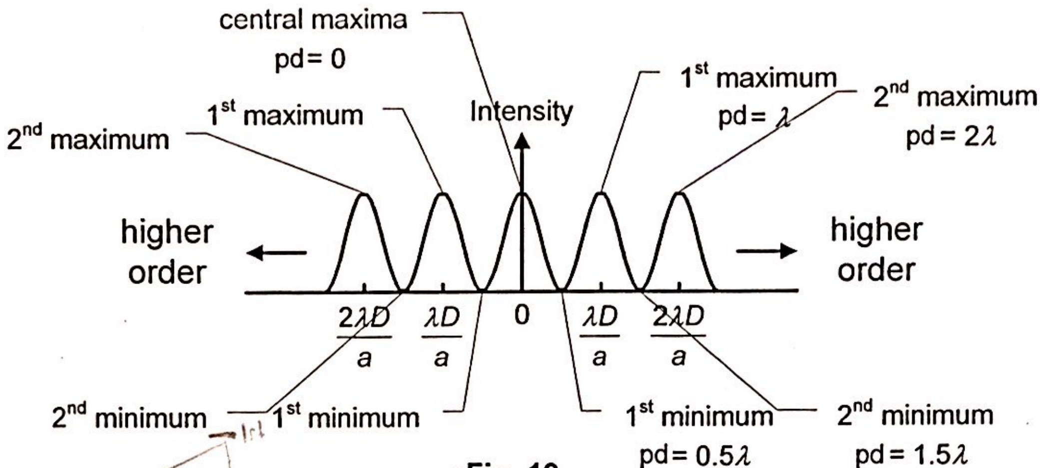
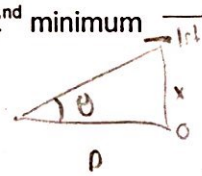


Fig. 19



use $\tan\theta = \frac{x}{D}$ if θ is not small

Take Note

Important Note

- The formula $x = \frac{\lambda D}{a}$ is applicable only if
  - $a \ll D$ (assumed that rays are parallel)
  - $a \gg \lambda$ (so that $\theta < 6°$ and $\sin\theta \approx \tan\theta$)

  usually valid in Young's Double slit, not diffraction grating
- The fringe separation x is only approximately constant for $\theta < 6°$. For larger θ, fringe separation actually increases.
- For interference of light, the typical values for slit width $w \sim 0.2$ mm, slit separation $a \sim 0.5$ mm, slit-screen distance $D \sim 1$ m, wavelength of light $\lambda \sim 500$ nm.
- The sources must be more than one wavelength apart (or the path difference is more than one wavelength) to produce an observable interference pattern.
- The single slit acts as a well-defined source of light to ensure that waves incident on the double slits are coherent.
- If the two coherent sources have a phase difference of π rad, the conditions for constructive and destructive interference would be interchanged.
- Light is emitted from sources as a series of pulses or packets of energy. These pulses last for very short durations of nanoseconds. Between each pulse, there is an abrupt change in the phase of the waves. Separate light sources, even of the same frequency, produce incoherent waves. Waves from two separate sources may be in phase at one instant, but out of phase the next. The human eye cannot cope with the rapid changes, so the pattern is not observable.

two conditions:
$\theta < 6°$ don't exceed 12°
$a \ll D$

fluorescent tube / light bulb: not coherent.
put — small hole for it to diffract.

Example 7

In a Young's double-slit experiment, the separation between the first and the fifth bright fringe is 2.5 mm when the wavelength used is 620 nm. If the distance from the slit to the screen is 0.80 m, calculate the separation of the two slits.

Solution

$x = \frac{\lambda D}{a}$ $\qquad x = \frac{2.5}{4} = 0.625\,mm$

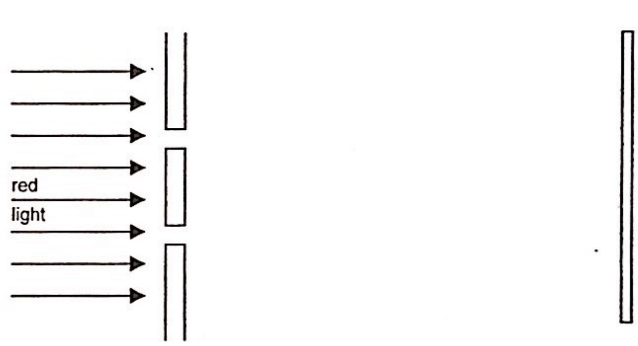
$= \frac{(620\times10^{-9})(0.80)}{a}$

$a = \frac{(620\times10^{-9})(0.8)}{(0.625\times10^{-3})} = 0.794\times10^{-3}\,m$

* distance of 5th bright fringe from central = 5 x ...
= 313 ...
angle of 5th bright fringe from central: $\tan^{-1}\left(\frac{3.13}{800}\right)$
$= \tan^{-1}(0.00...)$
$= 0.00391$ rad
(0.22°)
So it is accurate to assume that the fringes are equally spaced and apply the formula $x = \frac{\lambda D}{a}$.

Example 8

(N99/P2/3)
A student sets up the apparatus shown below to observe two-source interference fringes.



State and explain what change, if any, occurs in the separation of the fringes and in the contrast between bright and dark fringes observed on the screen, when each of the following changes is made separately.

(a) increasing the intensity of the red light incident on the double-slit.

(b) increasing the distance between the double-slit and the screen.

(c) reducing the intensity of light incident on one of the double-slit.

Solution

(a) Fringe separation remains the same based on $x=\lambda D/a$. However, contrast increases because intensity of bright fringes increases.

(b) Fringe separation increases based on $x=\lambda D/a$. However, contrast decreases because intensity of bright fringes decreases due to increased distance of screen. (waves)

(c) Fringe separation remains the same based on $x=\lambda D/a$. However, contrast decreases because the minima are now not completely dark and the maxima are not as bright as before. → no complete cancellation

12.8 Diffraction Grating

Diffraction Grating

A diffraction grating consists of a large number of parallel, equally spaced lines (or slits) of equal width. A diffraction grating typically consists of 100 lines to 1000 lines per mm.

When a narrow beam of monochromatic light is incident on a diffraction grating, sharp maxima are obtained at various angular positions θ_n as shown in Fig. 20b.

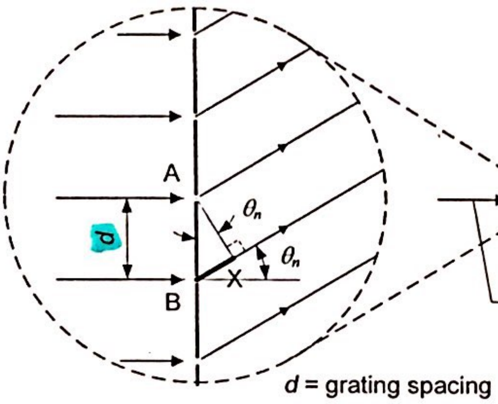


Fig. 20a

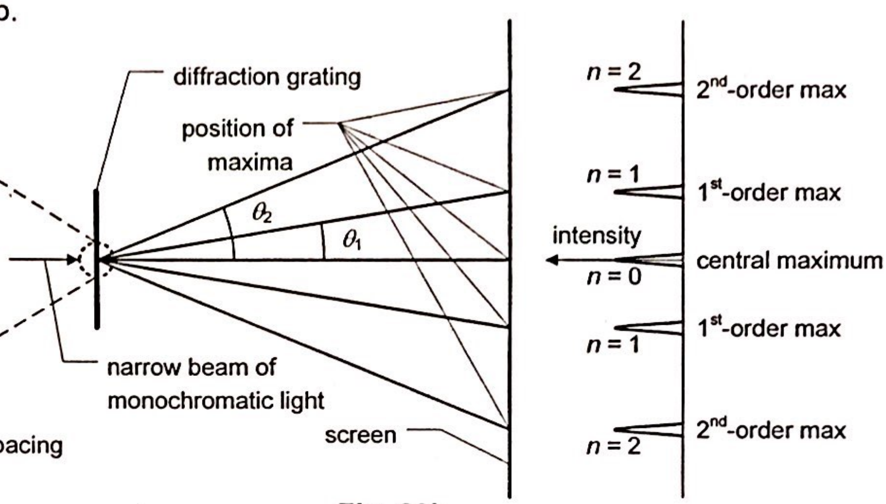


Fig. 20b

Derivation of $d \sin \theta = n\lambda$

Consider a narrow beam of monochromatic light incident on a diffraction grating of N lines per metre. At angular position θ_n, n^{th} order bright fringe (maxima) is obtained. The path difference between adjacent rays (Fig. 20a) is

$$\Delta x = BX = d \sin\theta_n \qquad \text{where } d = \text{ slit separation} = \frac{1}{N} \text{ in metre.}$$

Since the waves passing through all the slits are in phase, for constructive interference, (what is the assumption?)

Formula

$$\boxed{d \sin\theta_n = n\lambda} \qquad \text{where } n = 0, 1, 2, \ldots$$

both double slit & diffraction grating

Take Note

1. Since N is typically 100 to 1000 lines per mm, d is of the order of 10^{-6} m. Since d is very small, in fact only a few times more than the wavelength of visible light (about 0.4×10^{-6} to 0.7×10^{-6} m), the angle of diffraction of even the first order (n = 1) is quite big. Hence, $x = \lambda D/a$ may not be applicable to diffraction gratings, unless it has a small N or large d.

2. Since

$$\theta_n < 90^\circ$$
$$\sin\theta_n < 1$$
$$\frac{n\lambda}{d} < 1$$

Formula

$$\boxed{n < \frac{d}{\lambda}}$$

$d = \frac{1}{N}$ in m
no need $d = \frac{1}{N-1}$ or $\frac{1}{N+1}$

Hence, we can determine the maximum order of the bright fringes.

Example 9

Light of wavelength 656 nm is incident normally on a diffraction grating which has 400 lines per mm. Determine the angular positions of the first, second and third-order maxima.

Solution

Slit separation: (or line spacing)

$$d = \frac{1\times10^{-3}}{400} = 2.50\times10^{-6}\ \text{m}$$

For $n = 1$:

$$\sin\theta_1 = \frac{\lambda}{d} = \frac{656\times10^{-9}}{2.50\times10^{-6}} = 0.2624$$

$$\theta_1 = 15.2°$$

For $n = 2$:

$$\sin\theta_2 = \frac{2\lambda}{d} = \frac{2\times656\times10^{-9}}{2.50\times10^{-6}} = 0.5248$$

$$\theta_2 = 31.7°$$

For $n = 3$:

$$\sin\theta_3 = \frac{3\lambda}{d} = \frac{3\times656\times10^{-9}}{2.50\times10^{-6}} = 0.7872$$

$$\theta_3 = 51.9°$$

Note that $\Delta\theta$ is not constant between the orders: (increases with orders)

$$\Delta\theta_{10} = \theta_1 - \theta_0 = 15.2 - 0 = 15.2$$

$$\Delta\theta_{21} = \theta_2 - \theta_1 = 31.7 - 15.2 = 16.5$$

$$\Delta\theta_{32} = \theta_3 - \theta_2 = 51.9 - 31.7 = 20.2$$

Example 10

How many bright fringes can we observe using a diffraction grating of 600 lines per mm illuminated normally with light of wavelength 633 nm?

Solution

$$n < \frac{d}{\lambda}$$

~~$n<$~~ $d = \frac{1\times10^{-3}}{600} = 1.67\times10^{-6}\ \text{m}$

$\lambda = 633\times10^{-9}\ \text{m}$

$$n < \frac{1.67\times10^{-6}}{633\times10^{-9}}$$

$$< 2.63$$

Since n must be an integer, the highest order bright fringe observable is 2nd order.

Hence, the number of bright fringes is 5 central bright fringe and 2 on either slide.

Example 11 Describe, with the aid of a labelled diagram, the appearance of the 1st order spectra when white light having wavelengths from 380 nm (violet) to 780 nm (red) is incident normally on a diffraction grating of 500 lines per mm.

Solution Slit separation: $d = \frac{1\times10^{-3}}{500} = 2.0\times10^{-6}\ m$

For violet light: $\sin\theta_{1v} = \frac{\lambda}{d} = \frac{380\times10^{-9}}{2.0\times10^{-6}} = 0.19$

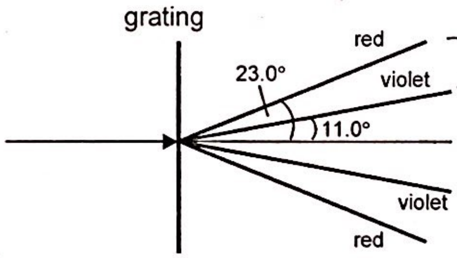
$\theta_{1v} = 11.0°$

For red light: $\sin\theta_{1R} = \frac{\lambda}{d} = \frac{780\times10^{-9}}{2.0\times10^{-6}} = 0.39$

$\theta_{1R} = 23.0°$

Note that the longer wavelength deviates the most from the central fringe. The central fringe remains white as all colours fall on it.

- red bends most
- higher order spectrum may overlap



central fringe is white as all colours fall on it

12.9 Single-Slit Diffraction

Huygen's Principle

Dutch physicist Christian Huygen, a contemporary of Newton, proposed that at any instant, all points on a wavefront could be regarded as secondary sources of wavelets. The envelope of the wavefronts produced by these secondary sources gives the new position of the original wavefront.

① why waves diffract after passing through an aperture
② why a single slit can produce an interference pattern

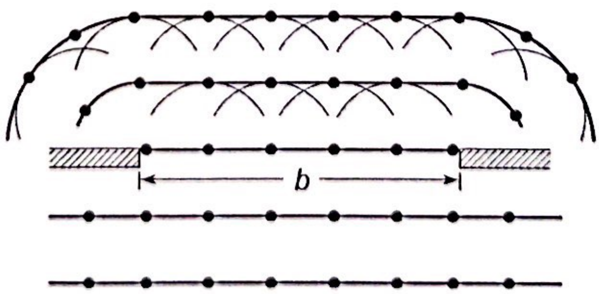


Fig. 21

If a plane wavefront is restricted by an obstacle as shown in Fig. 21, some of the wavelets making up the wavefront are removed, causing the edges of the wavefront to be curved. If the aperture is comparable to the wavelength of the incident wave, the diffraction effect becomes pronounced and the transmitted wavefront looks circular. (See Fig.11 on pg 11.)

Singe-Slit Interference Pattern

We have observed and studied two-slit and multiple-slit interference pattern. But how can a single-slit produce the interference pattern shown Fig. 22? The explanation that follows is based on Huygen's principle (it was explained in greater detail by French scientist Augustin Fresnel two centuries later).

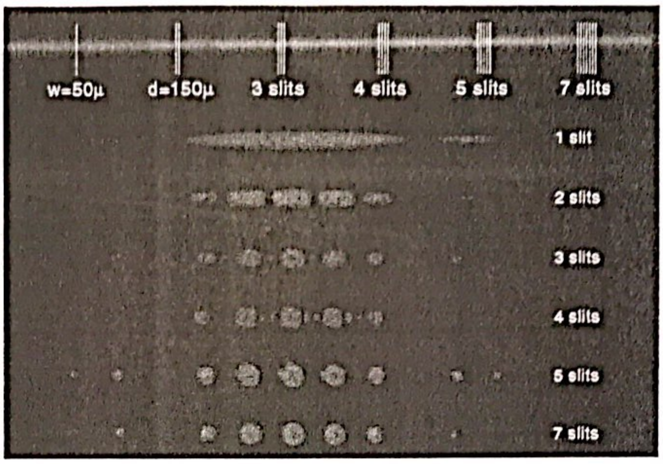


Fig. 22: slits of same width and slit separation

Single-Slit Minima

Fig. 23 shows plane wavefronts incident on a single-slit of width *b*. At the slit, we can divide the wavefront into two halves, and consider a secondary source of waves at the top and another one at the mid-point a distance *b*/2 below. Waves from these 2 points behave like sources in a two-source interference experiment.

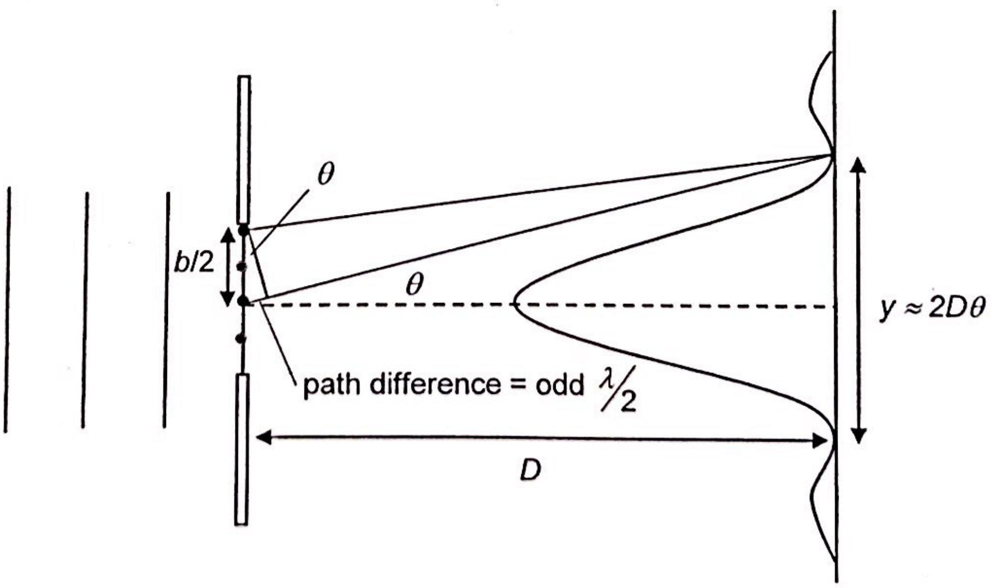


Fig. 23

To determine the positions of the dark fringes or minima, the path difference between waves from the two secondary sources is an odd number of half-wavelengths. This argument is applied to any point in the top-half of the slit and one a distance $b/2$ below it in the lower half of the slit:

$$\text{Path difference, } \frac{b}{2}\sin\theta = (2n-1)\frac{\lambda}{2}, \text{ where } n \text{ is a positive integer}$$

$$b\sin\theta = (2n-1)\lambda \qquad \text{– equation (1)}$$

$$= 1\lambda, 3\lambda, 5\lambda \ldots \text{odd number of wavelengths}$$

If we divide the wavefront into four quarters and apply the same argument to determine the positions of destructive interference, we now get:

$$\frac{b}{4}\sin\theta = (2n-1)\frac{\lambda}{2}, \text{ where } n \text{ is a positive integer}$$

$$b\sin\theta = 2(2n-1)\lambda \qquad \text{– equation (2)}$$

$$= 2\lambda, 4\lambda, 6\lambda \ldots \text{even number of wavelengths}$$

Combining equations (1) and (2), for single-slit interference pattern, the minima or destructive interference occurs at angle θ according to:

$$b\sin\theta = n\lambda, \text{ where } n \text{ is a positive integer.}$$

Hence, the angle θ of the first minima from the straight-through position for light of wavelength λ incident on a single slit of width b is

Formula

$$\boxed{\sin\theta = \frac{\lambda}{b}.} \qquad \text{– equation (3)}$$

For $\theta < 6°$, the width y of the central bright fringe on a screen a distance D away is therefore

$$\boxed{y \approx D(2\theta) \approx \frac{2D\lambda}{b},} \qquad \text{where } \theta \approx \frac{\lambda}{b}.$$

Take Note

Important Note

- Equation (3) is only valid when the distance D of the screen from the single-slit is much larger than the width b of the single-slit as the rays used in the derivation are assumed to be parallel.
- To determine the positions of the minima, the wavefront along the slit can only be divided into even parts such as 2, 4, 6, 8, etc . This is to allow an even number of wavelets to interfere destructively to produce a minimum. If there were an odd number of sections in the wavefront, complete destructive interference is not possible.
- Huygen's principle cannot be used to determine positions of constructive interference because a set of wavelets which arrive at a point in phase, may not be in phase with another set of wavelets (which are in phase among themselves).

don't divide wavefront into odd sections, need a pair to cancel each other to get destructive interference.

Linking Single-Slit Diffraction Pattern to that of Double-Slit and Diffraction Grating

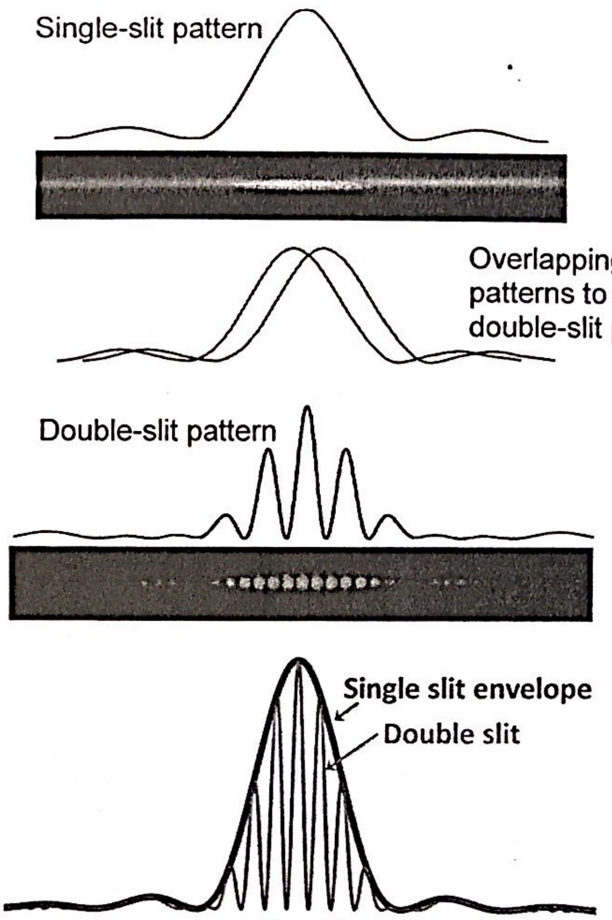


Fig. 24

Sources:

- http://hyperphysics.phy-astr.gsu.edu/hbase/phyopt/sindoub.html#c1
- http://hyperphysics.phy-astr.gsu.edu/hbase/phyopt/sinslitd.html
- http://labman.phys.utk.edu/phys250/modules/module%201/diffraction_and_interference.htm
- http://en.wikipedia.org/wiki/File:Double-slit_diffraction_pattern.png
- https://www.khanacademy.org/science/physics/light-waves/interference-of-light-waves/v/single-slit-interference

Example 12

(a) Why are the higher-order maxima dimmer in the Young's double-slit and diffraction grating interference pattern?

(b) Why are some orders missing from the Young's double-slit and diffraction grating interference pattern?

Solution

(a) This is because the intensity of the double-slit and diffraction grating interference pattern is restricted by that of the single-slit diffraction pattern.

(b) This is because the positions of the maxima of the double-slit and diffraction grating interference pattern coincide with the minima of the single-slit diffraction pattern.

In your notes, Fig. 14, 17, 18b, 19 and 20b ignored these facts!

12.10 Rayleigh Criterion

Resolving Power of A Single-Slit

When the images of objects through a lens system is distinguishable, we say that the images are well resolved. However, when the objects are too close together or too far away from the lens system, the images may overlap and become indistinguishable.

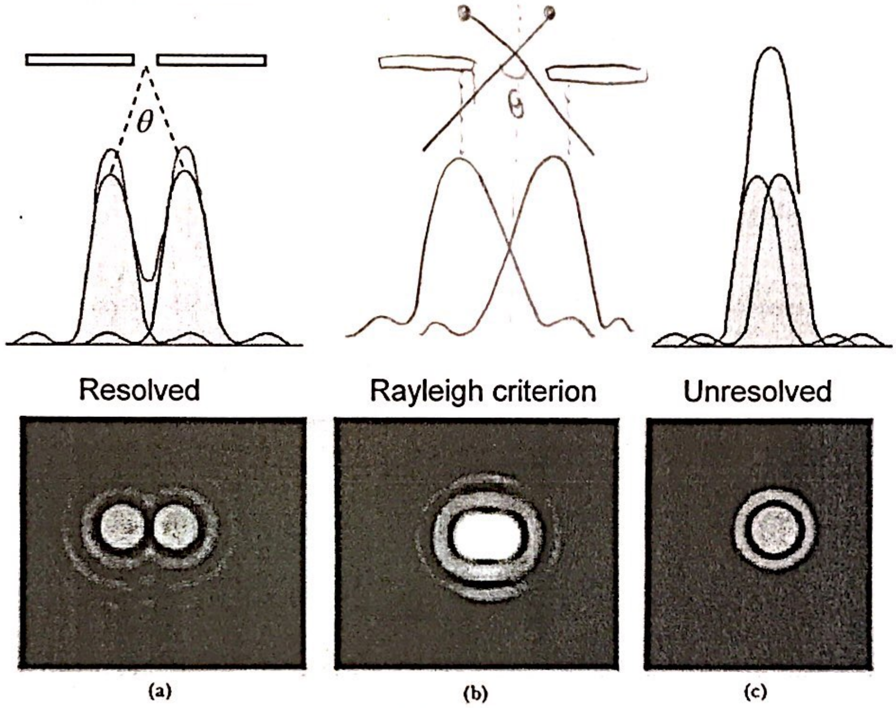


Fig. 25

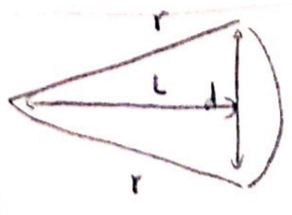
Lord Rayleigh arbitrarily set the criterion that two images are **just resolved** through a slit of width b when their angular separation θ satisfies

Formula

$$\boxed{\theta \approx \frac{\lambda}{b},}$$

where λ is the wavelength of the light radiating or reflected from the two objects. **This happens when the peak intensity of one image lies on the first minimum of the other image.** (For a circular aperture, $\boxed{\theta \approx 1.22\frac{\lambda}{b}}$.)

If the smallest resolvable separation between images is to be reduced (i.e. increasing the resolving power), shorter wavelength light might be used. For example, using UV light, rather than visible light, allows finer details to be seen.



$r = s\theta$

$d \approx s$

$d = L\theta$

$\theta \approx \frac{\lambda}{b}$

Appendix A **Stationary Waves – A Mathematical Approach (Optional)**

An analytical treatment of the production of stationary waves from the superposition of two progressive waves having the same frequency and amplitude is given below.

The two progressive waves may be represented by the equations

$$y_1 = a\sin\left(\frac{2\pi x}{\lambda} - \omega t\right) \text{ and } y_2 = a\sin\left(\frac{2\pi x}{\lambda} + \omega t\right)$$

where y_1 and y_2 are travelling toward the right and left respectively.

Using the principle of superposition of waves, the resultant wave can be represented by the equation

$$\begin{aligned} y &= y_1 + y_2 \\ &= a\sin\left(\frac{2\pi x}{\lambda} - \omega t\right) + a\sin\left(\frac{2\pi x}{\lambda} + \omega t\right) \\ &= 2a\sin\frac{2\pi x}{\lambda}\cos\omega t \\ &= \left(2a\sin\frac{2\pi x}{\lambda}\right)\cos\omega t \\ &= A\cos\omega t \end{aligned}$$

where $A = 2a\sin\frac{2\pi x}{\lambda}$ is the amplitude of the resultant stationary wave for a point at a distance x from a reference point.

At the nodes, the amplitude is always zero and

$$\frac{2\pi x}{\lambda} = 0, \pi, 2\pi, \ldots$$
$$x = 0, \frac{\lambda}{2}, \lambda, \ldots$$

At the antinodes, the amplitude is maximum and equals $2a$. This occurs when

$$\frac{2\pi x}{\lambda} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$$
$$x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \ldots$$

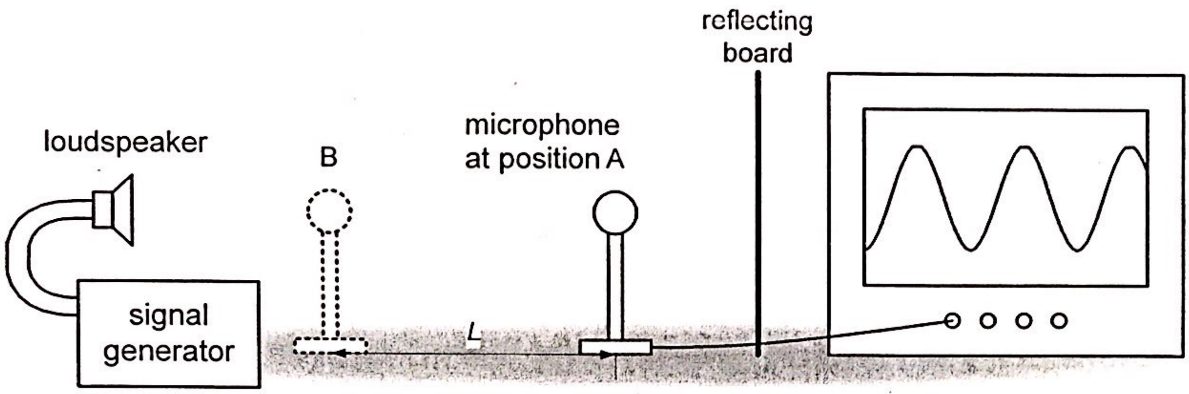
We find that the distance between two successive nodes or antinodes is always $\frac{\lambda}{2}$.

Note: $\sin P + \sin Q = 2\sin\frac{P+Q}{2}\cos\frac{P-Q}{2}$

Appendix B **Measuring Speed of Sound in Air**

The speed of sound in air can be measured either using a cathode-ray oscilloscope or a resonance tube.

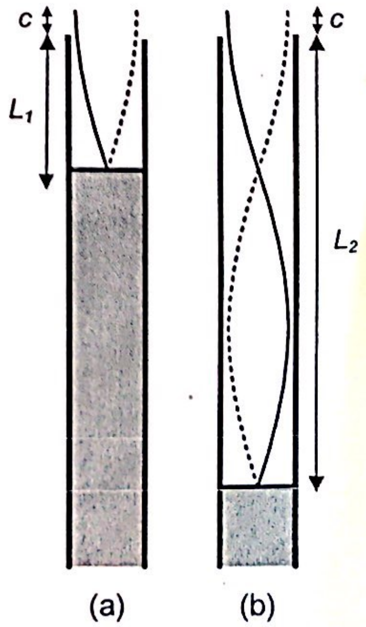
Using Cathode-Ray Oscilloscope



1. A small microphone, connected to a CRO, is positioned between a reflecting board and a loudspeaker connected to a signal generator.
2. Sound wave of constant frequency travels from the loudspeaker towards the reflecting board. Interference between the incident and reflected sound waves produces a stationary wave.
3. As the microphone is moved slowly towards the loudspeaker, the amplitude of the waveform on the CRO increases to a maximum (pressure antinode) at position A and then the next maximum at position B.
4. Since the distance L between two successive pressure antinodes (or displacement nodes) is $\frac{1}{2}\lambda$ and the frequency f can be determined from the CRO, the speed of the sound can be calculated using

$$v = f\lambda = 2fL$$

Using Resonance Tube



1. A tuning fork of frequency f is struck and held over the top of a tube filled with water.
2. The water level is gradually lowered or drained until the note is at its loudest as shown in (a). Note the length of the air column L_1. The air column is said to be resonating with the frequency of the tuning fork: $L_1 + c = \frac{1}{4}\lambda$ --- (a)
3. Repeat the above steps while lowering the water level further until a second weaker resonance is heard in (b). Note the new length L_2: $L_2 + c = \frac{3}{4}\lambda$ --- (b)
4. As shown in the diagram, equations (b) – (a):

$$\frac{1}{2}\lambda = L_2 - L_1$$

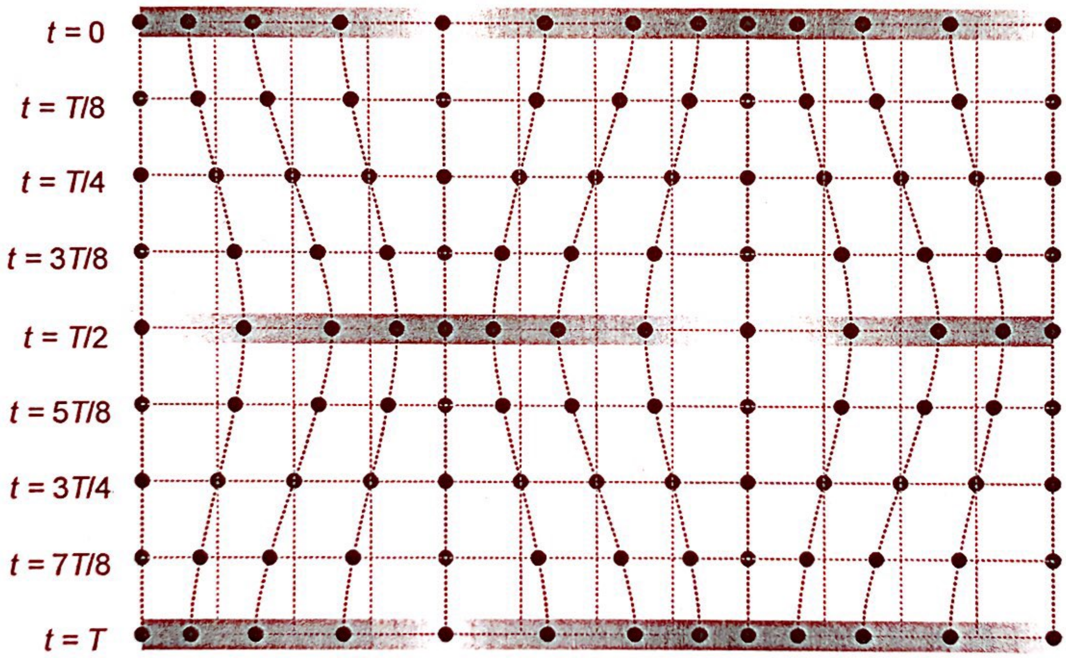
5. Since the frequency f of the tuning fork is known, speed of sound in air can be calculated using

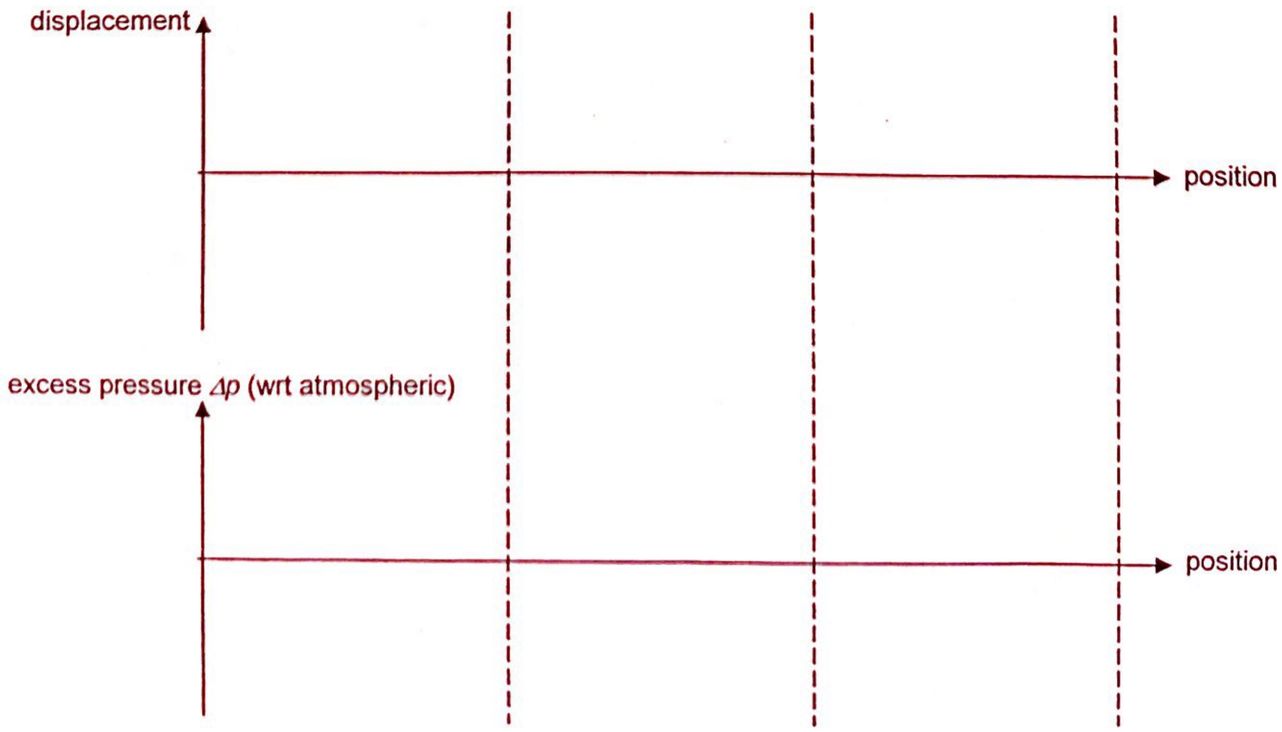
$$v = f\lambda = 2f\left(L_2 - L_1\right)$$

Appendix C Visualizing a Longitudinal Stationary Wave (Useful)

The following diagram shows the actual position of the particles in a longitudinal stationary wave. Dark regions represent areas of **maximum compression (highest pressure)** while light regions represents areas of **maximum rarefaction (lowest pressure)**.

Notice that both the maximum compressions and maximum rarefactions occur at the nodes. This means that the **displacement nodes** are also the **pressure antinodes** and vice versa.



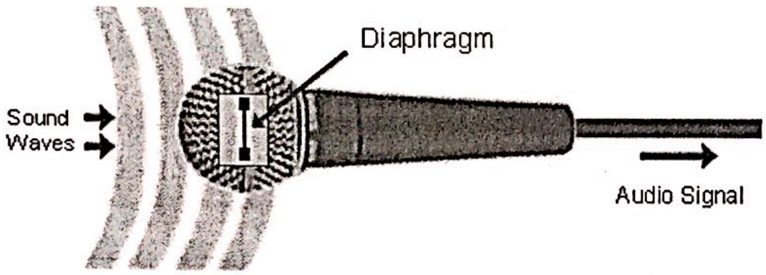


Complete the above graphs for time $t = T/2$ and $t = T$.

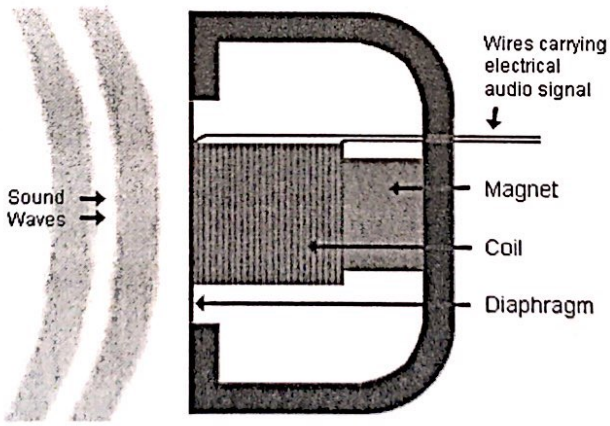
Appendix D How Do Microphones Work?

Microphones are a type of *transducer* - a device which converts energy from one form to another. Microphones convert acoustical energy (sound waves) into electrical energy (audio signal).

Different types of microphone have different ways of converting energy but they all share one thing in common: the *diaphragm*. This is a thin piece of material (such as aluminium) which vibrates when it is struck by sound waves. In a typical hand-held microphone like the one below, the diaphragm is located in the head of the microphone.

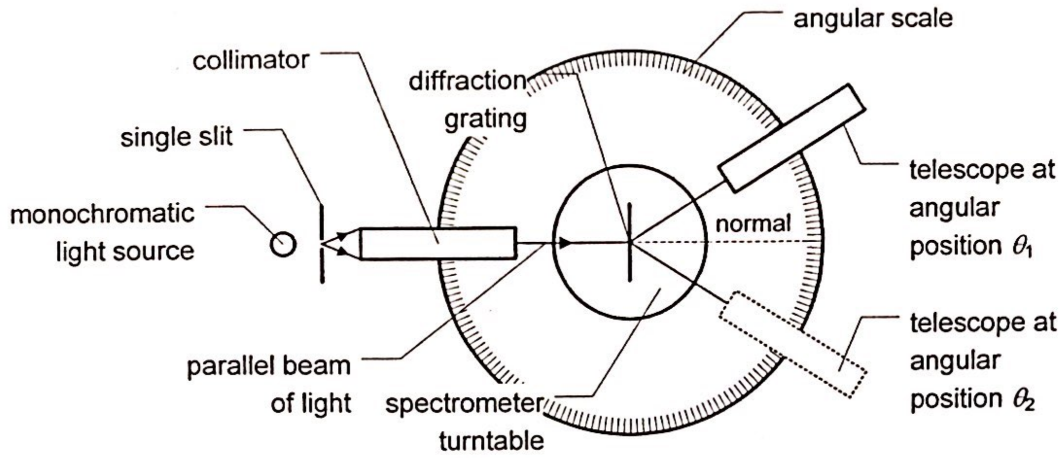


Cross-Section of Dynamic Microphone



When the diaphragm vibrates, it causes other components in the microphone to vibrate. These vibrations are converted into an emf which becomes amplified and converted to audio signal.

Appendix E Determination of Wavelength of Light using Diffraction Grating



A parallel, narrow beam of monochromatic light, after emerging from the collimator, is incident normally on the diffraction grating. As a result, bright fringes are produced at various angular positions.

The telescope is first rotated such that the n^{th} order bright fringe is at the centre of the crosswire in the telescope and its angular position θ_1 is noted. It is then positioned on the opposite side of the normal to the grating and the angular position θ_2 of the same order bright fringe is noted.

The angular position of the n^{th} order bright fringe is

$$\theta_n = \frac{1}{2}(\theta_2 - \theta_1)$$

Hence,

$$\lambda = \frac{d}{n}\sin\theta_n$$

Appendix F Simulation Applets to Aid Visualisation of Interference

Single slit pattern:
http://www.lon-capa.org/~mmp/kap27/Gary-Diffraction/app.htm
http://labman.phys.utk.edu/phys136/modules/m9/diff.htm

Double slit pattern:
http://www.lon-capa.org/~mmp/kap27/Gary-TwoSlit/app.htm
http://vsg.quasihome.com/interfer.htm
http://zonalandeducation.com/mstm/physics/waves/interference/twoSource/TwoSourceInterference1.html

Standing wave in a violin:
http://zonalandeducation.com/mstm/physics/waves/standingWaves/standingWaves1/StandingWaves1.html