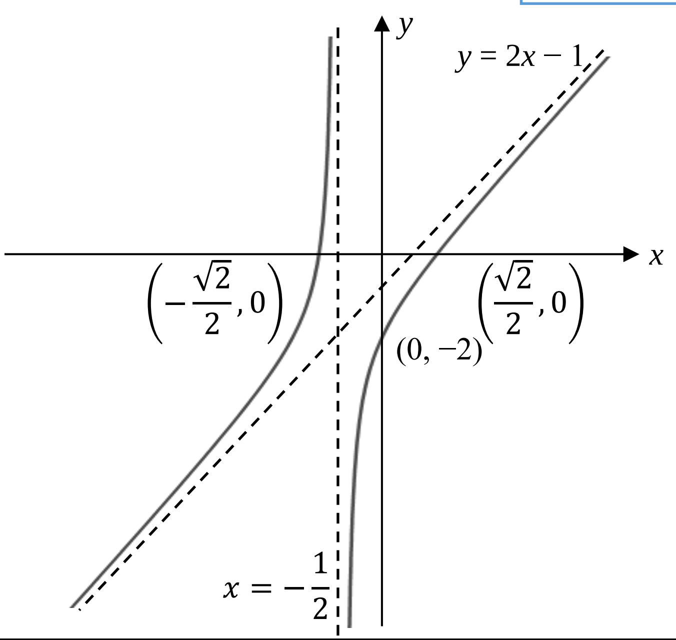
H2 Mathematics (9758)
Chapter 1 Graphing Techniques
Assignment Solutions

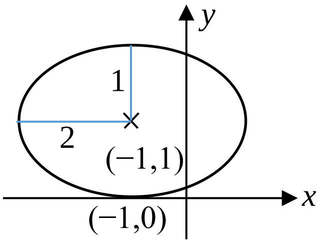
1 2017/NYJC Promo/4

The curve C has equation $y=\dfrac{4x^2-2}{2x+1}$.

Sketch C, giving the exact coordinates of all points of intersection with the axes and the equations of the asymptotes. [3]

Q1	Solution
	$y=\dfrac{4x^2-2}{2x+1}=2x-1-\dfrac{1}{2x+1}$ **Always make** the rational function **into proper fraction** using long division or juggling first, to correctly identify the asymptotes.  Please read the requirements of question carefully. Question stated **“exact coordinates”**, hence you **should not write** the x-intercepts as $(0.707,0)$ and $(-0.707,0)$.

2 Sketch the graph of $x^2+4y^2+2x-8y+1=0$. State the line(s) of symmetry.

Q2	Solution
	$x^2+4y^2+2x-8y+1=0$ $(x+1)^2-1^2+4\left[y^2-2y\right]+1=0$ $(x+1)^2+4\left[(y-1)^2-1^2\right]=0$ $(x+1)^2+4(y-1)^2=4$ $\frac{(x+1)^2}{4}+(y-1)^2=1$ $\Rightarrow \frac{(x+1)^2}{2^2}+\frac{(y-1)^2}{1^2}=1$ The lines of symmetry are $x=-1$ and $y=1$. **Read the question!** Question asks to state the lines of symmetry, so you have to state it clearly  **Note:** you do not need to label the intercepts if the centre is not on the axis. However, in this case, the *x*-intercept can be easily identified hence you should label it Note: It is an ellipse, centre at $(-1,1)$ with horizontal **semi**-major axis of length 2 units and vertical **semi**-minor axis of length 1 unit.

For ellipse, you will need to label the centre and radius clearly.

3 2018/MJC Promo/3 (Modified)

The curve C has equation $y=\dfrac{x^2+ax+10}{x-b}$. The asymptotes of C are $x=3$ and $y=x-2$.

(i) State the value of b and show that $a=-5$. [2]

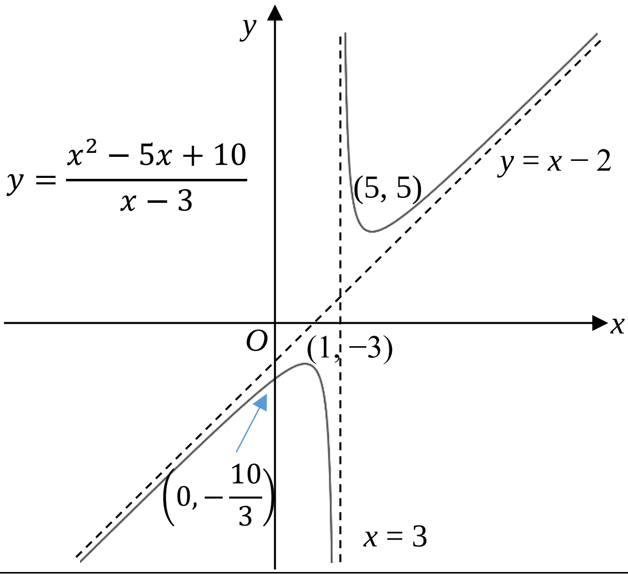
(ii) Using an algebraic method, determine the values $\dfrac{x^2+ax+10}{x-b}$ can take. [3]

(iii) Sketch C, showing its asymptotes and the coordinates of axial intercept and turning points. [3]

(iv) Deduce the range of values of c, where c is a positive real constant such that the equation $(x-1)^2+\left(\dfrac{x^2+ax+10}{x-b}\right)^2=c^2$ has at least one real root. [2]

Q3	Solution
(i)	$b=3$ $x-2+\dfrac{d}{x-3}=\dfrac{x^2+ax+10}{x-b}$ $(x-2)(x-3)+d=x^2+ax+10$ $x^2-5x+d+6=x^2+ax+10$ By comparing coefficient, $a=-5$ **Learning point 1:** To find vertical asymptote, let denominator $x-b=0 \Rightarrow x=b$
(ii)	$y=\dfrac{x^2-5x+10}{x-3}$. Let $y=k, k=\dfrac{x^2-5x+10}{x-3}$. $x^2-(k+5)x+10+3k=0$ **Learning point 2:** To determine the values a rational function can take, introduce k to be these values and ensure the quadratic equation formed has real roots For quadratic equation to have real roots, discriminant ≥ 0. $(-(k+5))^2-4(1)(10+3k)\geq 0$ $k^2+10k+25-40-12k\geq 0$ $k^2-2k-15\geq 0$ $(k-5)(k+3)\geq 0$ $k\leq -3$ or $k\geq 5$ You can still use the condition discriminant < 0, but you will need to take the complement of the answer to response to the question. Therefore, $\dfrac{x^2-5x+10}{x-3}$ can take values less than or equal to -3 or values more than or equal to 5. Please answer the question directly.

(iii)



(iv)

$$(x-1)^2 + \left(\frac{x^2 + ax + 10}{x - b}\right)^2 = c^2$$

$$\Rightarrow (x-1)^2 + y^2 = c^2$$

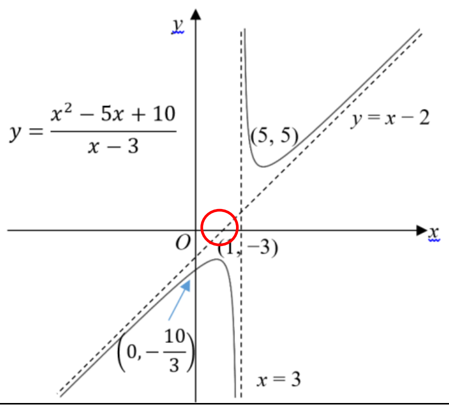
Add the graph $(x-1)^2 + y^2 = c^2$ into the previous part,

From the graph, $c \geq 3$

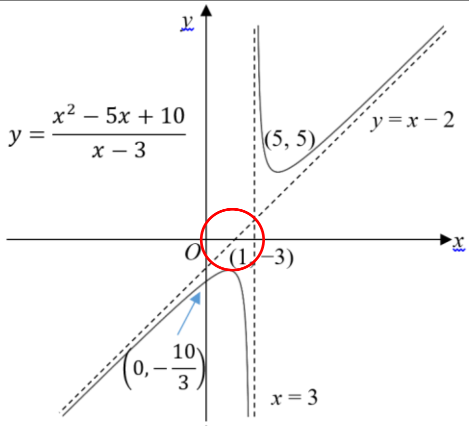
> Learning point 3:
>
> For $(x-1)^2 + \left(\frac{x^2 + ax + 10}{x - b}\right)^2 = c^2$ to have at least one real root is equivalent to having at least one point of intersection between $y = \frac{x^2 - 5x + 10}{x - 3}$ and $\left(x-1\right)^2 + y^2 = c^2$.

Explanation

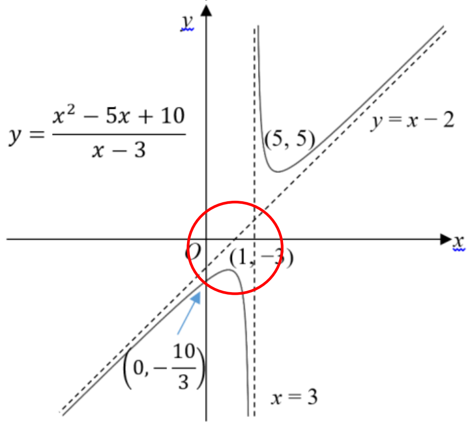
We are finding the range of values of c such that there is at least one point of intersection between $y = \frac{x^2 - 5x + 10}{x - 3}$ and a circle with centre $(1,0)$ and radius c (since $c > 0$ from the question).



> When $0 < c < 3$, no point of intersection between $y = \frac{x^2 - 5x + 10}{x - 3}$ and $\left(x-1\right)^2 + y^2 = c^2$



When $c = 3$, one point of intersection between $y = \dfrac{x^2 - 5x + 10}{x - 3}$ and $(x-1)^2 + y^2 = c^2$



When $c > 3$, more than one point of intersection between $y = \dfrac{x^2 - 5x + 10}{x - 3}$ and $(x-1)^2 + y^2 = c^2$

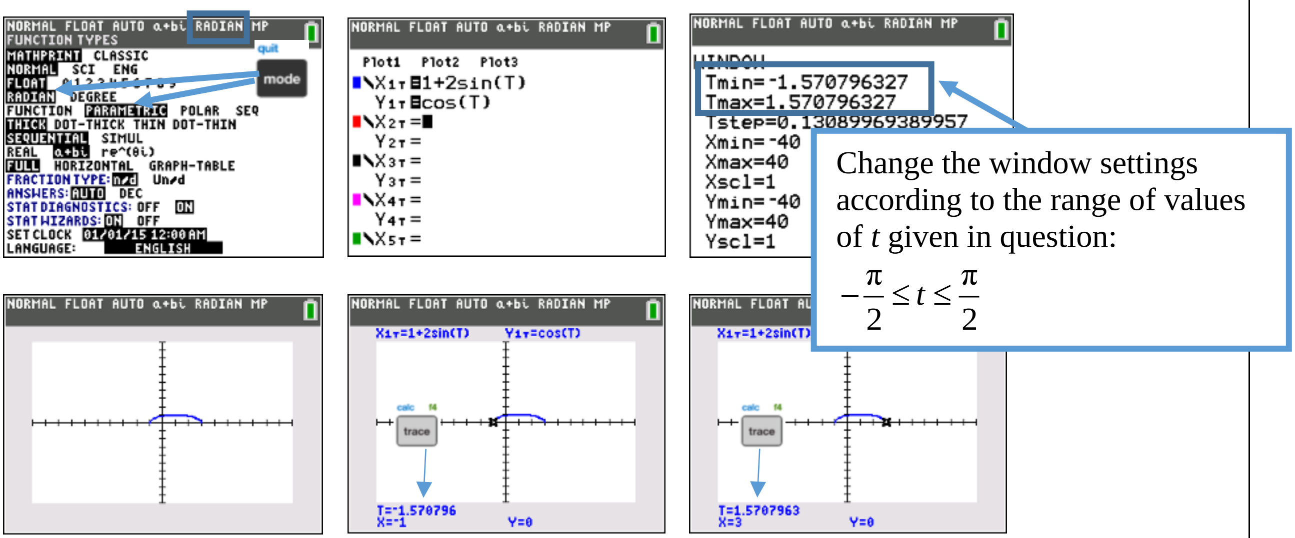
4 **2020/RVHS/JC2 MYE/Q4a(i)**

The curve C has parametric equations

$$x = 1 + 2\sin t,\ \ y = \cos t,\ \ \text{for } -\frac{\pi}{2} \le t \le \frac{\pi}{2}.$$

(i) Sketch C, indicating clearly the exact coordinates of the axial intercepts. [2]

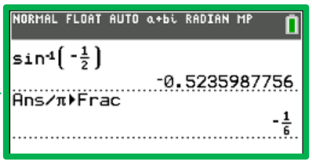
(ii) Find the cartesian equation of C. [1]

4	**Suggested Solutions**
	

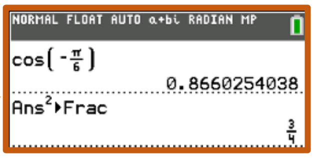
Solving for y-intercept:
When $x=0,\ 1+2\sin t=0$

$\Rightarrow \sin t=-\frac{1}{2}\Rightarrow -\frac{\pi}{6}$

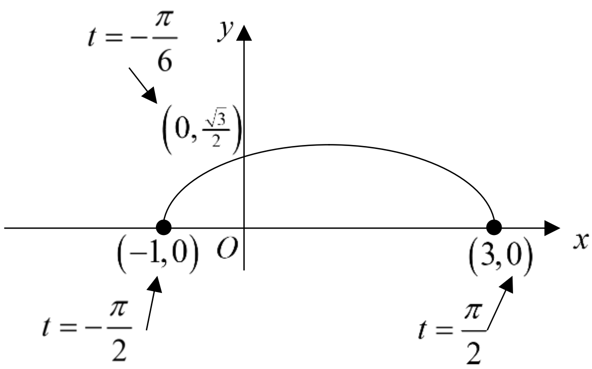
$\therefore\ y=\cos\left(-\frac{\pi}{6}\right)=\frac{\sqrt{3}}{2}$



$$\frac{\sin^{-1}\left(-\frac{1}{2}\right)}{\pi}=-\frac{1}{6}$$
$$\Rightarrow \sin^{-1}\left(-\frac{1}{2}\right)=-\frac{\pi}{6}$$



$$\left[\cos\left(-\frac{\pi}{6}\right)\right]^2=\frac{3}{4}$$
$$\cos\left(-\frac{\pi}{6}\right)=\sqrt{\frac{3}{4}}=\frac{\sqrt{3}}{2}$$



Note:
(1) Sketch the curve according to the range of values of t given in question.
(2) Label end-points which are the x-intercepts in this case
(3) Label in coordinates as indicated in question
(4) Indicate open/closed circle for end points according to range of values of t. In this case, both end-points are closed circle because $-\frac{\pi}{2}\le t\le\frac{\pi}{2}$.

A good habit to label the corresponding values of t on the graph itself for easy reference in subsequent parts of the question.

$x=1+2\sin t\Rightarrow \sin t=\frac{x-1}{2}$ --- (1)

$y=\cos t$ --- (2)

Substitute (1) and (2) into $\sin^2 t+\cos^2 t=1$,

$$\left(\frac{x-1}{2}\right)^2+y^2=1$$

$$\frac{(x-1)^2}{4}+y^2=1$$

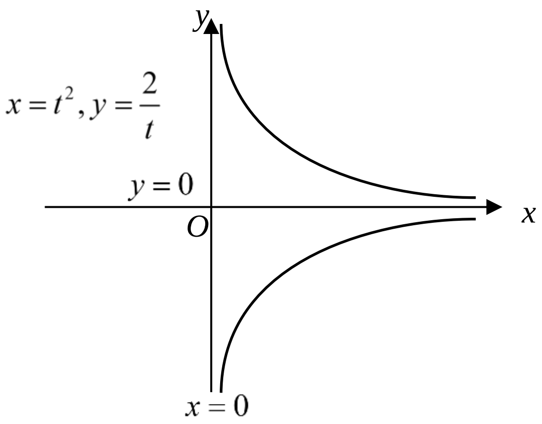
Generally, if the parametric equation has trigonometry, you will be using trigo identities like $\sin^2\theta+\cos^2\theta=1$ to convert to cartesian equation

Note: Only convert parametric equations to cartesian equation if the question asks for it

5 The parametric equations of a curve are

$$x = t^2, y = \frac{2}{t} \qquad \text{for } t \in \mathbb{R}, t \neq 0.$$

Sketch C. [2]

5	**Suggested Solutions**
	 Since $t \in \mathbb{R}$, $t \neq 0$, it means t can be any real number except zero Change the window settings to include negative values to observe the complete graph. $\text{T}\min = -10$ $\text{T}\max = 10$ Observing the asymptotes: As $x \to \infty, t \to \pm\infty, y \to 0$ Thus, $y = 0$ is an asymptote. As $x \to 0, t \to 0, y \to \infty$ Thus, $x = 0$ is an asymptote.

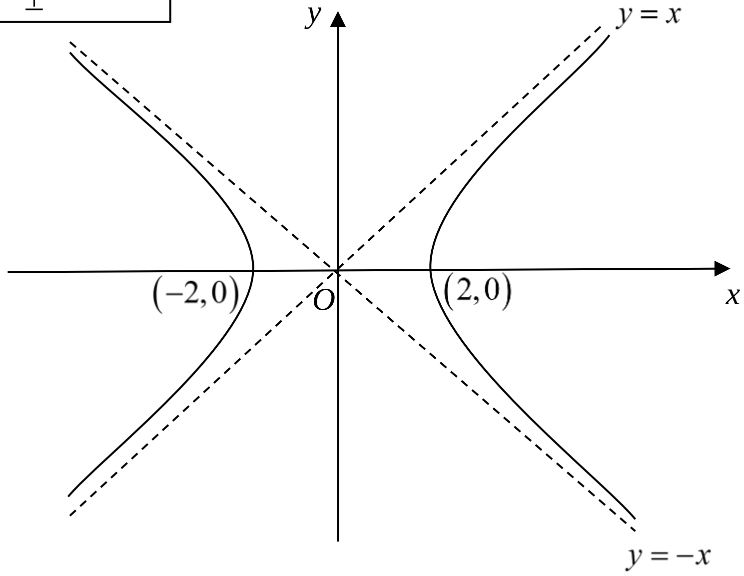
6 **2010/A-Level/P1/Q11(iii) – Challenge yourself**

A curve C has parametric equations

$$x = t + \frac{1}{t}, \qquad y = t - \frac{1}{t}.$$

Find a cartesian equation of C. Sketch C, giving the coordinates of any points where C crosses the x- and y-axes and the equations of any asymptotes. [4]

6	**Suggested Solutions**
	$x = t + \frac{1}{t} \quad ----------(1)$ $y = t - \frac{1}{t} \quad ----------(2)$ $(1)+(2): x + y = 2t \quad ----------(3)$ $(1)-(2): x - y = \frac{2}{t} \quad ----------(4)$ $(3)\times(4): (x+y)(x-y) = 4$ The cartesian equation is $x^2 - y^2 = 4$ $\frac{x^2}{2^2} - \frac{y^2}{2^2} = 1$ The idea to obtain the cartesian equation from the parametric equations is to eliminate the parameter t.

	To find the equation of the oblique asymptotes, let $\frac{x^2}{2^2}-\frac{y^2}{2^2}=0$ $y^2=x^2$ $y=\pm x$ Do not forget the "$\pm$" When $y=0$, $\frac{x^2}{2^2}=1$ $x=\pm 2$ 

7 Match the following equations with their corresponding sketches, and fill in the boxes accordingly.

1. $y=6+\dfrac{1}{x^2-4}$	D	6. $x^2+y^2-2(x-y)=0$	B
2. $\dfrac{y^2}{36}-\dfrac{x^2}{9}=1$	G	7. $y=3x-1+\dfrac{5}{x-4}$	A
3. $y-5=(x+2)^2$	F	8. $\dfrac{x^2}{25}-\dfrac{y^2}{16}=1$	I
4. $(x-3)^2+(y-4)^2=25$	E	9. $4x^2+y^2-8x-2y=4$	C
5. $\dfrac{x^2}{4}+y^2=1$	J	10. $(y-4)^2=x-2$	H