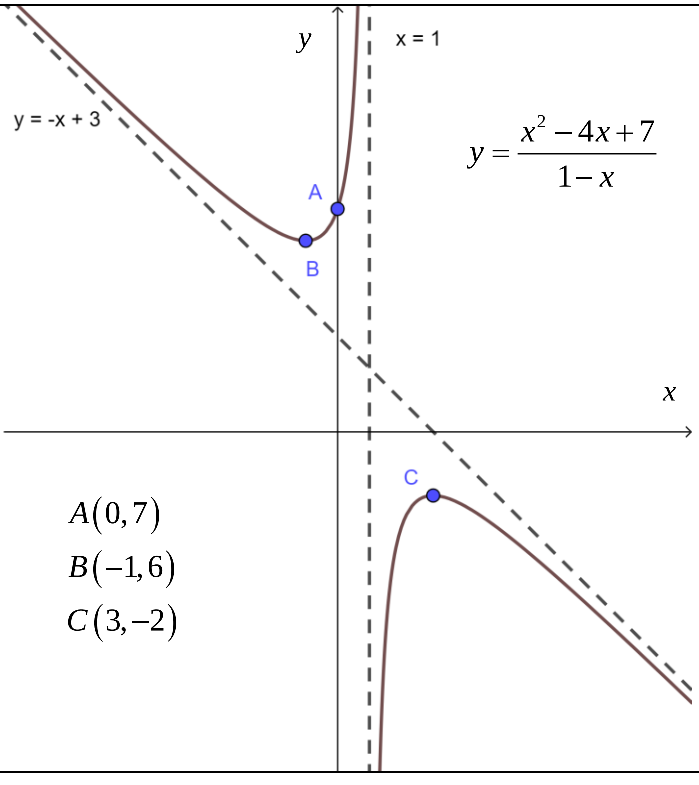
2022 H2 Maths JC1 Lecture Test 2 Solution (Student Copy)

Question 1 [6 marks] AP/GP		
(a)	Let a_n be the amount of gold received for logging in on the n^{th} day. Then $a_1 = 100,\ \ a_2 = 130,\ \ a_3 = 160, \ldots$ Which forms an arithmetic progression. Thus, $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$ $900 \le \frac{n}{2}[2(100) + (n-1)(30)]$ $0 \le 15n^2 + 85n - 900$ $n \ge 5.41$ or $n \le -11.1$ (rej) Therefore, it will take 6 days.	
(b)i.	Let b_n be the amount of XP received for completing the quest on the n^{th} time. Then $b_1 = 1400,\ \ b_2 = 1400(0.9),\ \ \mathrm{b}_3 = 1400(0.9)^2, \ldots$ Which forms a geometric progression. Thus $S_n = \frac{a(1-r^n)}{1-r}$ $= \frac{1400(1-0.9^{10})}{1-0.9}$ $= 9118.501839$ Therefore, 9120 XP (3 s.f.) would be earned	
(b)ii.	Maximum XP is the amount earned if played infinitely many times. $S_\infty = \frac{a}{1-r}$ $= \frac{1400}{1-0.9}$ $= 14\ 000$ The maximum theoretical amount of XP to be earned is 14 000 XP	

Question 2 [8 marks] MOD and Sigma Notation		
(i)	$\mathrm{f}(n+1)-\mathrm{f}(n)$ $=\ln\left(\frac{1}{n+1}\right)-\ln\left(\frac{1}{n}\right)$ $=\ln\left(\frac{1/(n+1)}{1/n}\right)$ $=\ln\left(\frac{n}{n+1}\right)$	
(ii)	$\sum_{n=1}^{N}\ln\left(\frac{n}{n+1}\right)$ $=\sum_{n=1}^{N}\mathrm{f}(n+1)-\mathrm{f}(n)$ $=\begin{bmatrix} \mathrm{f}(2)-\mathrm{f}(1) \\ +\mathrm{f}(3)-\mathrm{f}(2) \\ \ldots \\ +\mathrm{f}(N)-\mathrm{f}(N-1) \\ +\mathrm{f}(N+1)-\mathrm{f}(N) \end{bmatrix} \begin{matrix} n=1 \\ n=2 \\ \ldots \\ n=N-1 \\ n=N \end{matrix}$ $=-\mathrm{f}(1)+\mathrm{f}(N+1)$ $=-\ln(1)+\ln\left(\frac{1}{N+1}\right)$ $=-\ln(N+1)$	
(iii)	$\sum_{n=1}^{\infty}\ln\frac{n}{n+1}=\lim_{N\to\infty}\sum_{n=1}^{N}\ln\frac{n}{n+1}$ $=\lim_{N\to\infty}-\ln(N+1)$ $=-\infty$ Hence, the series diverges	
(iv)	$\sum_{n=5}^{2N}\ln\left(\frac{n}{n+1}\right)=\sum_{n=1}^{2N}\ln\left(\frac{n}{n+1}\right)-\sum_{n=1}^{4}\ln\left(\frac{n}{n+1}\right)$ $=-\ln(2N+1)+\ln 5$ $=\ln\left(\frac{5}{2N+1}\right)$	

<table>
<tr><td colspan="3">Question 3 [8 marks] Graphing Techniques</td></tr>
<tr><td>(i)</td><td>$y = \dfrac{ax^2 - 4x + 7}{b - x}$
with asymptotes $y = -x + 3$ and $x = -1$
By observation, $b = 1$, $a = 1$
$y = \dfrac{x^2 - 4x + 7}{1 - x} = -x + 3 + \dfrac{4}{1 - x}$</td><td></td></tr>
<tr><td>(ii)</td><td>Consider the points of intersection between $y = k$ and
$y = \dfrac{x^2 - 4x + 7}{1 - x}$.
$k = \dfrac{x^2 - 4x + 7}{1 - x}$
$0 = x^2 + (k - 4)x + (7 - k)$
For no points of intersection,
$(k - 4)^2 - 4(1)(7 - k) < 0$
$k^2 - 8k + 16 + 4k - 28 < 0$
$k^2 - 4k - 12 < 0$
$(k - 6)(k + 2) < 0$
$-2 < k < 6$
Therefore, there are no y values between –2 and 6.</td><td></td></tr>
<tr><td>(iii)</td><td>
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Question 4 [8 marks] Graphing Transformations		
(a)	$x^2+y^2=1$ $\downarrow$ Replace y with $\frac{y}{3}$ Scale parallel to y-axis, with factor 3 $x^2+\left(\frac{y}{3}\right)^2=1$ $\downarrow$ Replace y with $y+1$ Translate 1 unit in the negative y-direction $x^2+\frac{\left(y+1\right)^2}{9}=1$	
	<u>Alternatively:</u> $x^2+y^2=1$ $\downarrow$ Replace y with $y+\frac{1}{3}$ Translate $\frac{1}{3}$ units in the negative $y-$direction $x^2+\left(y+\frac{1}{3}\right)^2=1$ $\downarrow$ Replace y with $\frac{\mathrm{y}}{3}$ Scale parallel to y-axis, s.f. 3 $x^2+\left(\frac{y}{3}+\frac{1}{3}\right)^2$ $x^2+\frac{\left(y+1\right)^2}{9}=1$	
(b)i.	$y=\mathrm{f}\left(x\right)$ $\downarrow$ Replace x with $x-1$ Translate 1 unit in the positive $x-$direction $y=\mathrm{f}\left(x-1\right)$ $\downarrow$ Replace x with $\frac{x}{2}$ Scale parallel to x-axis, with factor 2 $y=\mathrm{f}\left(\frac{x}{2}-1\right)$	

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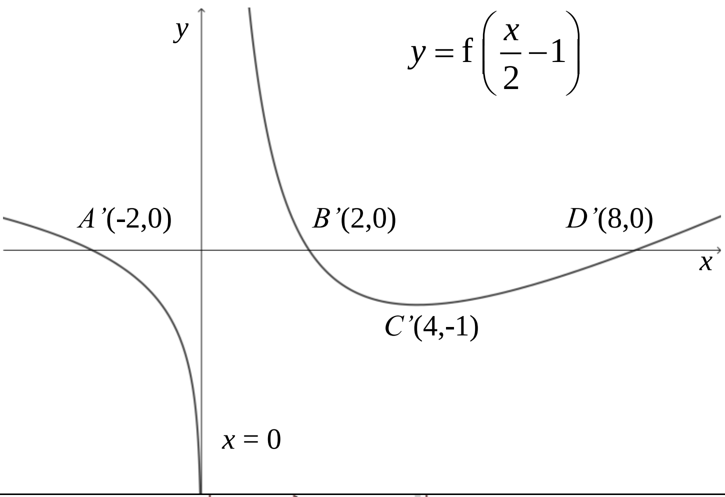
<td>
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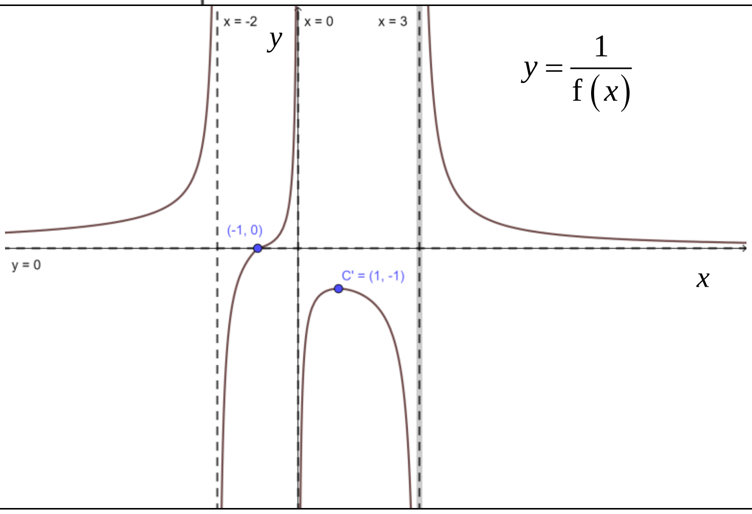
<td>(b)ii.</td>
<td>
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