RI H2 Mathematics 2017 Prelim Exam Paper 2 Question

	1	Refe that	Referred to the origin O, the points A, B and C have position vectors a , b and c respectively such that								
		a = 2i + 3j - k, $b = 5i - 2j + 3k$ and $c = 4i + j - 2k$.									
		(i) Given that M is the mid-point of AC , use a vector product to find the exact area of tri ABM .									
		(ii)	(ii) Find the position vector of the point N on the line AB such that \overrightarrow{MN} is perpendicular								
			AB.	[4]							
	2	(a)	(i) Show that $\frac{1}{r-1} - \frac{2}{r} + \frac{1}{r+1} = \frac{2}{r(r-1)(r+1)}$.	[1]							
		(ii) Hence find $\sum_{r=3}^{n} \frac{4}{r(r-1)(r+1)}$.									
			(There is no need to express your answer as a single algebraic fraction).	[4]							
		(b) Amy and her brother Ben are saving money together for their family trip. In the of 2017, Amy saves \$25 and Ben saves \$2. In each subsequent week, Amy save than the amount she saved in the previous week, and Ben saves 22% more than the saved in the previous week.									
			 (i) Which is the first week in which Ben saves more than Amy in that week? (ii) They need a combined total of \$2400 for the trip. How many complete we Amy and Ben need to save before they can achieve their targeted amount? 	[2] eeks do [2]							
	3	The	function f is defined as follows.								
		$f: x \mapsto \sqrt{3} \sin x + \cos x, x \in \Box, 0 < x < \pi.$									
		(i) Write $f(x)$ as $R\sin(x+\alpha)$, where R and α are constants with exact values to be four									
		(ii) Sketch the graph of $y = f(x)$, stating the axial intercepts, and find the range of f									
		(iii) Hence, solve $f(x) \le 1$ exactly.									
		The	The function g is defined as follows:								
			$g: x \mapsto 2\cos\left(x + \frac{\pi}{6}\right), x \in \Box, -\frac{\pi}{6} \le x \le b.$								
		(iv)	Write down the largest exact value of b , for g^{-1} to exist.	[1]							
		(v)	Taking the value of <i>b</i> found in part (iv), show that the composite function $g^{-1}f$ exist and solve $g^{-1}f(x) = x$ exactly.	sts [3]							
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4	The line l_1 has equation $\frac{x}{-3} = \frac{y}{12} = \frac{z-1}{4}$ and the line l_2 has equation $\frac{x-1}{-3} = y-4 = \frac{z-1}{4}$.							
	(i) Show that l_1 and l_2 are skew lines. [3]							
	(ii) Find a cartesian equation of the plane p which is parallel to l_1 and contains l_2 . [3]							
	(iii) The point $A(0, a, 1)$ is equidistant from p and l_1 . Calculate the possible values of a exactly [6]	y.						
5	For events X and Y, it is given that $P(X Y) = \frac{1}{2}$, $P(Y X) = \frac{2}{3}$ and $P(X \cup Y) = \frac{5}{6}$.							
	Find							
	(i) $P(X)$, [3]							
	(ii) $P(X \cup Y')$. [2]							
6	The power consumption of a randomly chosen Effixion laptop has a normal distribution. The salesman at Elf Superstore claims that the average power consumption of an Effixion laptop is 100 watts. The power consumption, w watts, is measured for a random sample of 50 Effixion laptops. The results are summarised as follows.	is n						
	$\sum (w - 100) = 26 \qquad \sum (w - 100)^2 = 273$							
	Test whether this data provides evidence at the 3% level of significance, that the salesman has made an understatement. [6]							
	The power consumption of another random sample of 50 Effixion laptops is measured. It is found that the sample variance is 6.25. Using this sample only, find the set of values of \overline{w} , correct to 2 decimal places, for which the test would result in the rejection of the null hypothesis in favour of the alternative hypothesis at the 1% level of significance. [4]							
7	An unbiased cubical die has the number 1 on one face, the number 2 on two faces and th number 3 on three faces. Adrian invites Benny to play a game. In each round, Benny rolls the di twice. Adrian pays Benny a if the total score is 2 and 3 if the total score is 3. However, if th total score is 4, Benny pays Adrian 2 . No payment is made otherwise.	ne ie ne						
	(i) Find the probability that Adrian pays Benny at least 5 times in 20 rounds. [4]							
	The random variable X represents Benny's winnings in each round.							
	(ii) Given that $a = 6$, find the probability distribution of X. Hence, help Benny decide if h should accept Adrian's invitation to play the game. Justify your answer. [5]	ıe						
	(iii) Determine the value of <i>a</i> for the game to be fair. [1]							
8	(a) In Country S, each household's monthly income per capita is calculated by taking the gross household income divided by the total number of members in the household. It is assume	ss ed						

		that this amount for a randomly chosen household consisting of 3 members follows a normal distribution with mean \$2601 and standard deviation \$768.										
		 (i) The Ministry of Education offers financial aid to students from households consisting of 3 members each and with a household monthly income per capita lower than \$1800. Find the probability that a randomly chosen household with 3 members does not qualify for financial aid. 										
		(ii) It is found that there is a 50% chance that a randomly chosen household members has a gross household income between \$5000 and \$ a , where a Find the value of a , correct to the nearest dollar.										l with 3 > 5000. [3]
(b) Mr Tan is self-employed and his monthly income follows \$6000 and standard deviation \$1000 whereas Mrs Tan w amount of \$1500 a month. Their family's monthly distribution with mean μ dollars and standard deviation 6								s a norm works pa expend 550 dolla	al distrit rt-time a liture fo rs.	oution wi and earns llows a	ith mean s a fixed normal	
(i) It was found that 10% of the time they spend more than \$5900 value of μ , correct to the nearest dollar.									900 in a	month.	Find the [2]	
	(ii) Mr and Mrs Tan save the remaining amount of their income after deduct expenditure every month. Find the probability that their monthly savings ir and in September differ by more than \$1000.									ing their August [4]		
		(iii)	State an	assump	tion need	led for yo	our calcu	lation in p	part (b)(i	i).		[1]
9	(i)	i) Sketch a scatter diagram that might be expected when x and y are related approximately a given in each of the cases (A) and (B) below. In each case, your diagram should include points, approximately equally spaced with respect to x , and with all x - and y -value positive. The letters a, b, c and d represent constants.								nately as nclude 6 y-values		
		(A) y	=a+bx	² , where	<i>a</i> is posi	itive and	b is nega	tive,				
		(B) y	= c + d 1	n <i>x</i> , when	re <i>c</i> is po	ositive and	d d is neg	gative.				[2]
		The following table shows the Gross Domestic Product (GDP) per capita, x , and infan mortality rate, <i>y</i> , for a sample of 9 countries.								nd infant		
x (\$) 1375 2502 10569 2966 11539 2036 4260 14									1433	7427		
			у	115	69	18	65	17	83	44	112	27
	(ii) Draw a scatter diagram for these values, labelling the axes clearly.								[2]			
	(iii)	(iii) Calculate the product moment correlation coefficient, and explain why its value does not necessarily mean that a linear model is the best model for the relationship between x and y[2]								does not x and y. [2]		

	(iv)	State which of the two cases in part (i) is more appropriate for modelling the relationship between x and y . Calculate the product moment correlation coefficient and the equation of the appropriate regression line for this case. [3]									
	(v)	Use the regression line in part (iv) to find an estimate of the infant mortality rate for a country with GDP per capita of \$723. Comment on the reliability of your estimate. [3]									
10	(a)	It is given that the probability that 21 randomly chosen people were all born on different days of the year is 0.55631, correct to 5 decimal places.									
		Find the probability that in a random sample of 22 people, there are at least 2 people with the same date of birth. [3]									
		[You m any of t	[You may assume there are 365 days in a year and the probability that a person is born on any of the 365 days is the same.]								
	(b)	A socce	er team con	sists of 1 goal	keeper, 4 defe	nders	, 4 midfield	ers and 2 forw	vards.		
		Country	y N has a so	uad of 3 goal	keepers, 6 def	ender	s, 9 midfiel	ders and 4 for	wards.		
		(i) H	low many d	lifferent socce	r teams can be	form	ed by coun	try N?	[2]		
		One of	One of the defenders and one of the midfielders in the squad are twin brothers.								
		(ii) H b	(ii) How many different teams can be formed which include at most one of the twin brothers? [3]								
		The fol	lowing table	e shows the da	tes of birth of	the 22	2 players in	the squad of c	ountry N:		
			Jersey	Desition	Date of		Jersey	Desition	Date of		
			Number	Position	birth		Number	Position	birth		
			1	Goalkeeper	29 October		12	Midfielder	15 August		
			2	Defender	3 May		13	Defender	11 July		
		3Defender17 July14Midfielder29 March									
		4Defender15 May15Defender22 October									
		5 Defender 14 December 16 Midfielder 13 March									
		6Midfielder12 October17Forward29 November									
		7Midfielder15 May18Goalkeeper20 December									

	8	Forward	10 May		19	Midfielder	5 February
	9	Forward	1 July		20	Midfielder	1 March
	10	Midfielder	1 April		21	Forward	27 March
	11	Midfielder	29 October		22	Goalkeeper	31 October
(iii) F sa	ind the pro ame date of	bability that the that the the the the the the the the tensor of	he team forme	d by	country N d	contains no pl	ayers with the [4]