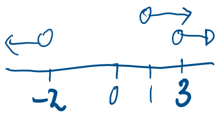
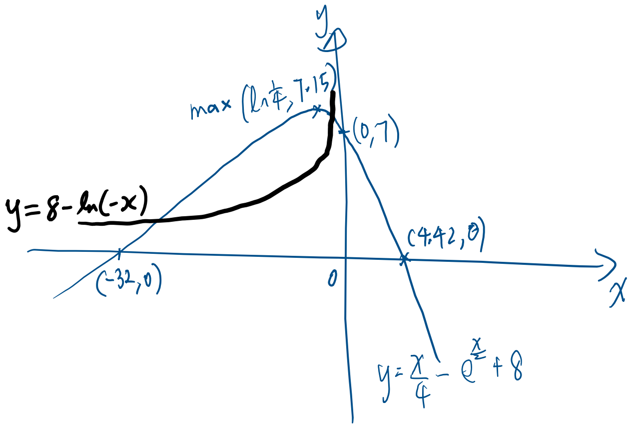
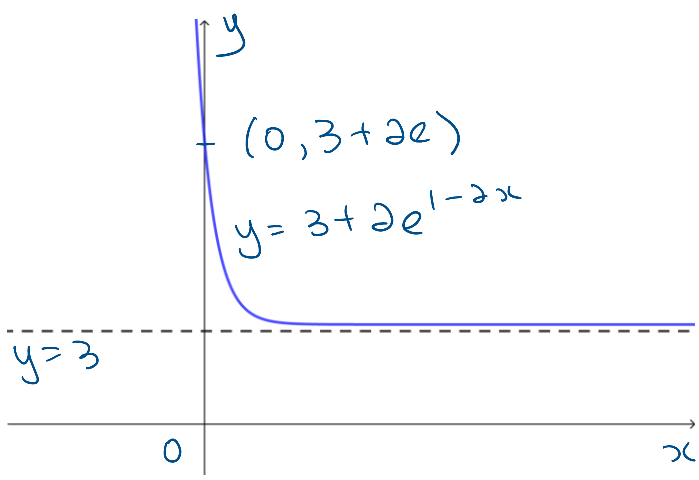
Section A: Pure Mathematics [40 marks]

1	Solution [4] Inequalities	
	For $\left(1-k\right)x^2-\sqrt{24}x-k\le 0$, Discriminant < 0 $1-k<0$ & $\left(-\sqrt{24}\right)^2-4\left(1-k\right)\left(-k\right)<0$ $k>1$ & $24+4k-4k^2<0$ $k>1$ & $k^2-k-6>0$ $k>1$ & $(k-3)(k+2)>0$ $k>1$ & $k<-2$ or $k>3$  taking intersection, $k>3$	

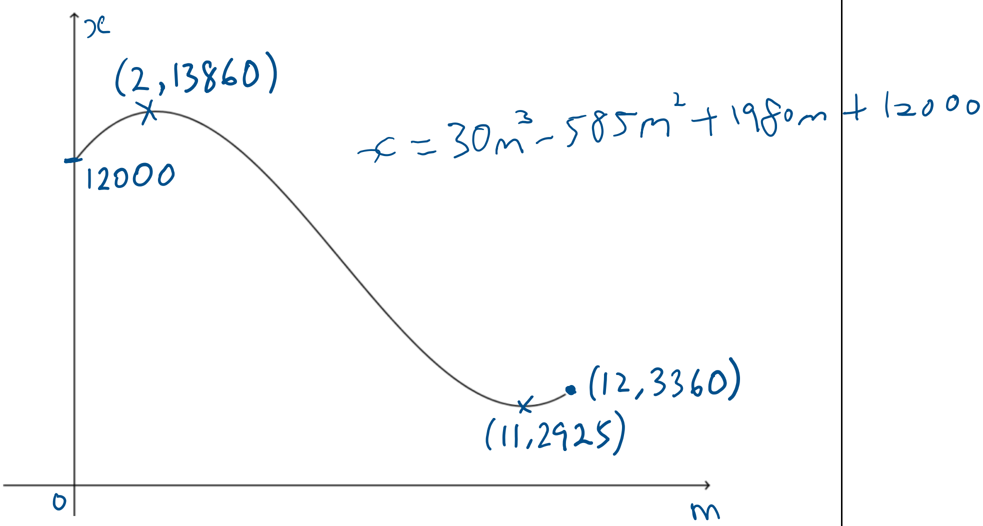
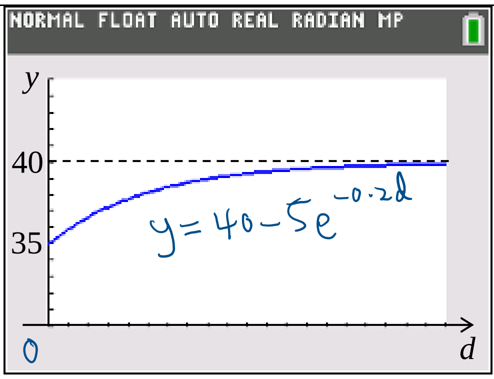
2	Solution [7] Differentiation & Integration Techniques	
(i)	Let $y=\left(1-2x\right)^{\frac{3}{2}}$ $\dfrac{dy}{dx}=\dfrac{3}{2}\left(-2\right)\left(1-2x\right)^{\frac{1}{2}}=-3\left(1-2x\right)^{\frac{1}{2}}$	
(ii)	$\displaystyle\int\frac{1}{\sqrt{1-2x}}\,dx=\int\left(1-2x\right)^{-\frac{1}{2}}\,dx$ $=\dfrac{\left(1-2x\right)^{\frac{1}{2}}}{\frac{1}{2}\left(-2\right)}+c$ $=-\sqrt{1-2x}+c$	
(iii)	$6x-5=-3\left(1-2x\right)-2$ $\displaystyle\int\frac{6x-5}{\sqrt{1-2x}}\,dx=\int\frac{-3\left(1-2x\right)-2}{\sqrt{1-2x}}\,dx$ $\displaystyle=\int-3\sqrt{1-2x}-\frac{2}{\sqrt{1-2x}}\,dx$ $\displaystyle=\int-3\sqrt{1-2x}\,dx-2\int\frac{1}{\sqrt{1-2x}}\,dx$ $=\left(1-2x\right)^{\frac{3}{2}}-2\left(-\sqrt{1-2x}\right)+c$ $=\left(1-2x\right)^{\frac{3}{2}}+2\sqrt{1-2x}+c$	

3	Solution [8] Solving Equations using GC	
(i)	$y=\frac{x}{4}-\mathrm{e}^{\frac{x}{2}}+8 \Rightarrow \frac{dy}{dx}=\frac{1}{4}-\frac{1}{2}\mathrm{e}^{\frac{x}{2}}$ for stationary points, let $\frac{dy}{dx}=0$ $\frac{1}{4}-\frac{1}{2}\mathrm{e}^{\frac{x}{2}}=0$ $\mathrm{e}^{\frac{x}{2}}=\frac{1}{2}$ $\frac{x}{2}=\ln(\frac{1}{2})$ $x=\ln\left(\frac{1}{4}\right)$ or $-\ln 4$	
(ii)		
(iii)	$\frac{x}{4}-\mathrm{e}^{\frac{x}{2}}=-\ln(-x)$ $\frac{x}{4}-\mathrm{e}^{\frac{x}{2}}+8=8-\ln(-x)$ Sketch $y=8-\ln(-x)$ from GC, $x=-8.51$ or -2.48	

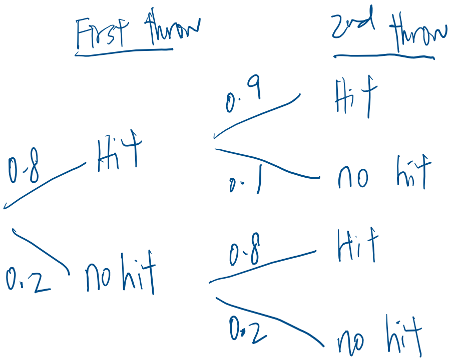
4	Solution [8] Graphing + Appln of Differentiation & Integration	
(i)		

(ii)	$y = 3 + 2e^{1-2x}$ $\dfrac{dy}{dx} = 2(-2)e^{1-2x} = -4e^{1-2x}$ When $x = 2$, $y = 3 + 2e^{1-2(2)} = 3 + 2e^{-3}$ $\dfrac{dy}{dx} = -4e^{1-2(2)} = -4e^{-3}$ Equation of tangent at $x = 2$: $y - (3 + 2e^{-3}) = -4e^{-3}(x - 2)$ $y = -4e^{-3}x + 8e^{-3} + 3 + 2e^{-3}$ $= -4e^{-3}x + (3 + 10e^{-3})$	
(iii)	Required area $= \int_0^2 (3 + 2e^{1-2x}) - (-4e^{-3}x + 3 + 10e^{-3})\, dx$ $= 2.07105$ ≈ 2.07 units^2	

5	Solution [13] Graphing + Application of Differentiation	
(i)	$x = 30m^3 - 585m^2 + 1980m + 12000$ $\dfrac{dx}{dm} = 90m^2 - 1170m + 1980$ At stationary point, $\dfrac{dx}{dm} = 0$ $90m^2 - 1170m + 1980 = 0$ $m = \dfrac{1170 \pm \sqrt{(1170)^2 - 4(90)(1980)}}{2(90)}$ $= \dfrac{1170 \pm 810}{2(90)}$ $= 2$ or 11 When $m = 2$, $x = 13860$ When $m = 11$, $x = 2925$ $\dfrac{dx}{dm} = 90m^2 - 1170m + 1980$ $\dfrac{d^2x}{dm^2} = 180m - 1170$	

	When $m=2$, $\frac{d^2x}{dm^2}=-810(<0)$ $(2,13860)$ is a maximum point When $m=11$, $\frac{d^2x}{dm^2}=810(>0)$ $(11,2925)$ is a minimum point	
(ii)	 	
(iii)	Required area $=\int_0^{12}\left(30m^3-585m^2+1980m+12000\right)\,\mathrm{d}m$ $=105120$ The **total production of fishballs** by Todo Fishball Company for the fiscal year 2021 is **105120 kg**.	
(iv)	When $d=0$, $y=40-5=35$ Therefore, an employee produces **35 kg** of fishball immediately after the training programme.	
(v)	 The model is **suitable** as the employee shows gradual improvement in the efficiency over time after the training and the improvement tapers off which is realistic.	

Section B: Statistics [60 marks]

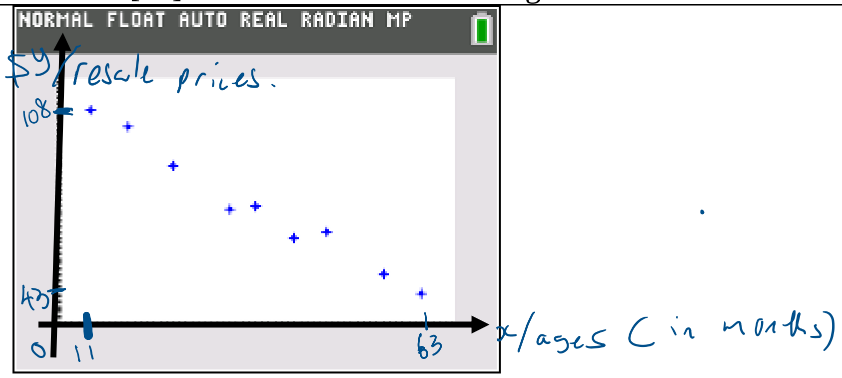
6	Solution [6] Probability	
(i)		
(ii)	P(hit bull's-eye in his 2^{nd} throw) = 0.8 x 0.9 + 0.2 x 0.8 = 0.88	
(iii)	P(hit bull's-eye on 1^{st} throw | hits bull's-eye on 2^{nd} throw) $=\dfrac{0.8\times 0.9}{0.88}$ $=0.818$ (or $\dfrac{9}{11}$)	

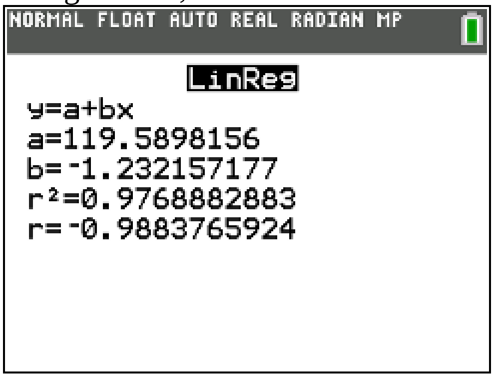
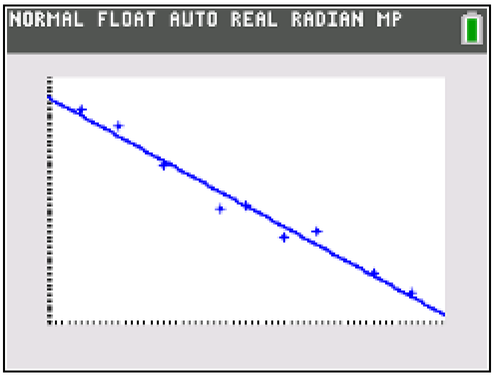
7	Solution [6] Permutations and Combinations	
(i)	Required number of 7-letter code-words = 5^7 = 78125 _ _ _ _ _ _ _ *(each of blank _ can be filled by any of the 5 letters AUXYZ)*	
(ii)	Required number of 7-letter code-words = 4^6 = 4096 E.g. *AUX**<u>Z</u>**AUX* or *AUX**<u>Z</u>**UXY* _ _ _ **<u>*Z*</u>** _ _ _ *(each of blank can be filled by any 4 of the letters A, U, X, Y)*	
(iii)	*[Note: The first 4 letters A, U, X, Y are fixed. There is only one way to do/fix that.]* For *** , choose 1 letter from 5 to be identical: $\begin{pmatrix}5\\1\end{pmatrix}$, then from remaining 4 choose 1 to be the different letter: $\begin{pmatrix}4\\1\end{pmatrix}$ (e.g. *AAU*, *UUX*, *UXX* etc) $\therefore$ No. of ways = $\begin{pmatrix}5\\1\end{pmatrix}\times\begin{pmatrix}4\\1\end{pmatrix}=20$ **<u>OR</u>** $\begin{pmatrix}5\\2\end{pmatrix}\times\begin{pmatrix}2\\1\end{pmatrix}=20$	

	(i.e. choose 2 letters from 5, then from the 2 chosen decide which 1 to be the identical.)	
(iv)	**Case 1**: all 3* identical letters (*AAA*, *UUU*, *XXX*, *YYY* or *ZZZ*) No. of ways = 5 **Case 2**: 2* identical, 1 different letter No. of ways = 20 (from part (iii)) **Case 3**: all 3* different (*AUX*, *UXZ* etc) No. of ways = $\binom{5}{3}$ = 10 Required number of codewords = 5+20+10 = 35	

8	Solution [9] Binomial distribution	
(i)	Each student has the same probability of 0.08 of being left-handed. **<u>OR</u>** The event that a student is left-handed is independent of another student.	
(ii)	Expected number = 30 x 50 x 0.92 = 1380 students	
(iii)	Let X be the number of left-handed students in a class of 30. $X \sim \mathrm{B}(30, 0.08)$ Probability $= P(X \geq 4)$ $= 1 - P(X \leq 3)$ $= 1 - 0.784206$ $= 0.215794$ $= 0.216 \quad (3 \text{ s.f.})$	
(iv)	Let W be the number of left-handed students in a lecture theatre of 250 students, and c the number of chairs in the lecture theatre for left-handed students. Then $W \sim \mathrm{B}(250, 0.08)$ Want c such that $P(X \leq c) \geq 0.9$. Using GC, $P(X \leq 25) = 0.8971$ $P(X \leq 26) = 0.9306$ $P(X \leq 27) = 0.9547$ Hence, a minimum number of 26 such chairs are needed in each lecture theatre in order to be 90% certain of meeting the needs of the left-handed students.	

9	Solution [7] Probability	
	A and B **not** mutually exclusive $\Rightarrow x+y>0$	
	$P(A\mid S)=P(A)$ $\dfrac{5+x}{25+x}=\dfrac{15+x+y}{55+x+y}$ $(5+x)(55+x+y)=(15+x+y)(25+x)$ $275+60x+x^2+5y+xy=375+40x+x^2+25y+xy$ $x=y+5$ $y=x-5$	
	<u>Alternatively</u> $P(A\cap S)=P(A)P(S)$ $\dfrac{5+x}{55+x+y}=\dfrac{15+x+y}{55+x+y}\times\dfrac{25+x}{55+x+y}$ $(5+x)(55+x+y)=(15+x+y)(25+x)$ $275+60x+x^2+5y+xy=375+40x+x^2+25y+xy$ $x=y+5$ $y=x-5$	
(i)	$P(B\mid S)=\dfrac{3}{8}$ $\dfrac{3}{8}=\dfrac{5+x}{25+x}$ $x=7$ $y=2$	
(ii)	$\mathrm{P}((S\cap A)\cup B')=\dfrac{30+x}{55+x+y}=\dfrac{37}{64}$	

10	Solution [10] Correlation and Linear Regression	
(i)		

(ii)	Using the GC,  $r = -0.988$ Since r is close to -1, there is a **strong negative linear** correlation between the age of a bicycle and its price.	
(iii)	$y = -1.23x + 119.59$ (**<u>to 2 d.p.</u>**) $c = 119.59$ means that the <u>resale value of a brand new bicycle of that particular model</u> is \$119.59. 	
(iv)	When $x = 72$, $y = -1.23216(72) + 119.590 = 30.87$ (OR $y = -1.23(72) + 119.590 = 31.03$) The cost of the 72 month old bicycle will be around \$30.87. This estimate is **unreliable** as $x = 72$ is outside the data range of the values of *x*.	

11	Solution [10] Hypothesis Testing	
(i)	Unbiased estimate of the population mean, $\bar{x} = \dfrac{539}{35} = 15.4$ Unbiased estimate of the population variance,	

	$s^2=\frac{1}{34}\left(\sum x^2-\frac{\left(\sum x\right)^2}{35}\right)$ $=\frac{1}{34}\left(8647.4-\frac{539^2}{35}\right)$ $=10.2$	
(ii)	Let μ be the population mean travel time from Town A to Park B. To test H_0: $\mu = 14.5$ against H_1: $\mu > 14.5$ at 5% significance level Test statistic: Under H_0, $\bar{X} \sim N\left(14.5, \frac{s^2}{35}\right)$ approximately by Central Limit Theorm since $n=35\geq 30$ is large, $\Rightarrow Z=\frac{\bar{X}-14.5}{s/\sqrt{35}} \sim N(0,1)$ approximately. p-value = 0.0477 (3 s.f.) As p-value < 0.05, we reject H_0. There is sufficient evidence at 5% significance level that the mean travel time from Town A to Park B is greater than 14.5 minutes.	
(iii)	At 5% level of significance means there is a probability of 0.05 that the test will conclude that the mean travel time from Town A to Park B is more than 14.5 minutes when in fact it is 14.5 minutes. **<u>OR</u>** At 5% level of significance means there is a probability of 0.05 that the test will indicate to reject the claim that the mean travel time from Town A to Park B is 14.5 minutes when in fact the mean travel time from Town A to Park B is 14.5 minutes.	
(iv)	Let $\bar{T}$ be the random variable for the new sample mean travel time. Test $H_0: \mu=14.5$ Against $H_1: \mu<14.5$ Perform a 1-tail Z-test at 1% significance level. Under H_0, $\bar{T} \sim N\left(14.5, \frac{9.6}{100}\right)$ approximately by Central Limit Theorem since sample size ($n=100$) is large	

	H_0 is rejected. Hence $z_{calc} < -2.326347877$ $\dfrac{m-14.5}{\sqrt{9.6/100}} < -2.326347877$ $m < 13.77921$ i.e. $m < 13.8$ (to 3sf)	

12	Solution [12] Normal Distribution + Sampling	
(i)	Let X be the mass of a "Comforta" cushion. $X \sim N(170, 12^2)$ $P(X > 165) = 0.6615388$ ≈ 0.662	
(ii)	$P(X_{max} < 165)$ where $X_{max} = \max\{X_1, X_2, X_3\}$ $= [P(X < 165)]^3$ $= (1-0.6615388)^3$ $= 0.0388$	
(iii)	Let Y be the mass of a cushion cover. $Y \sim N(20, \sigma^2)$ $X + Y \sim N(190, 12^2 + \sigma^2)$ $P(X + Y \geq 170) > 0.9$ $P\left(Z \geq \dfrac{170-190}{\sqrt{12^2+\sigma^2}}\right) > 0.9$, $Z \sim N(0,1)$ $\dfrac{-20}{\sqrt{12^2+\sigma^2}} < -1.281551567$ $\sigma < 9.9774658$ $\sigma < 9.98$ (shown)	
(iv)	$0.05X + 0.02Y$ $\sim N(0.05(170)+0.02(20), 0.05^2(12^2)+0.02^2(8^2))$ i.e. $0.05X + 0.02Y \sim N(8.9, 0.3856)$ $P(0.05X + 0.02Y < 10)$ $= 0.962$	
	Let S be the mass of a "Serena" cushion. $s^2 = \dfrac{50}{49}(17^2)$ Since $n = 50$ is large, by Central Limit Theorem, $\bar{S} \sim N\left(250, \dfrac{s^2}{50}\right)$ approximately. $P(\bar{S} \leq 245) = 0.0198$	