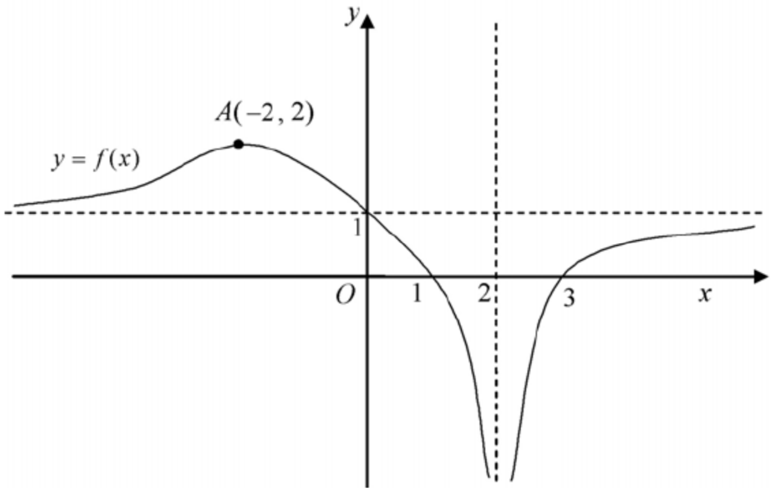
Additional Practice Questions Solutions

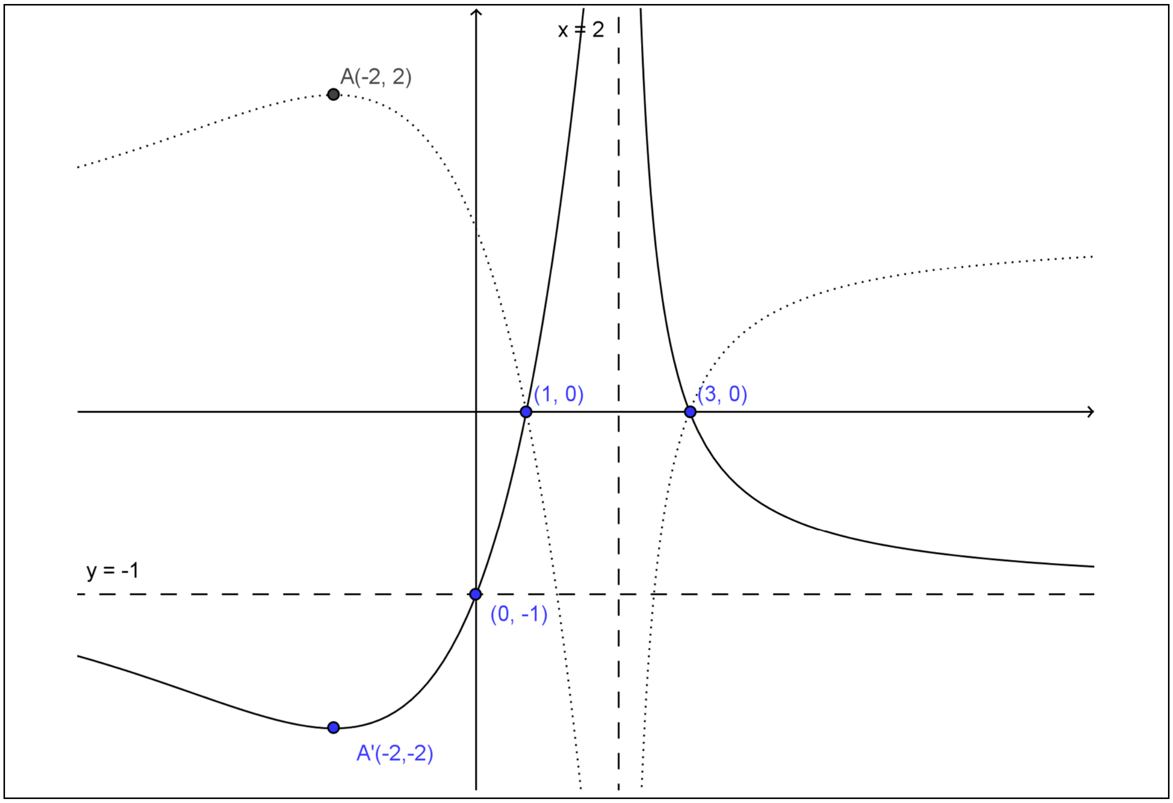
1. **NYJC/2006/Promo/Q6**



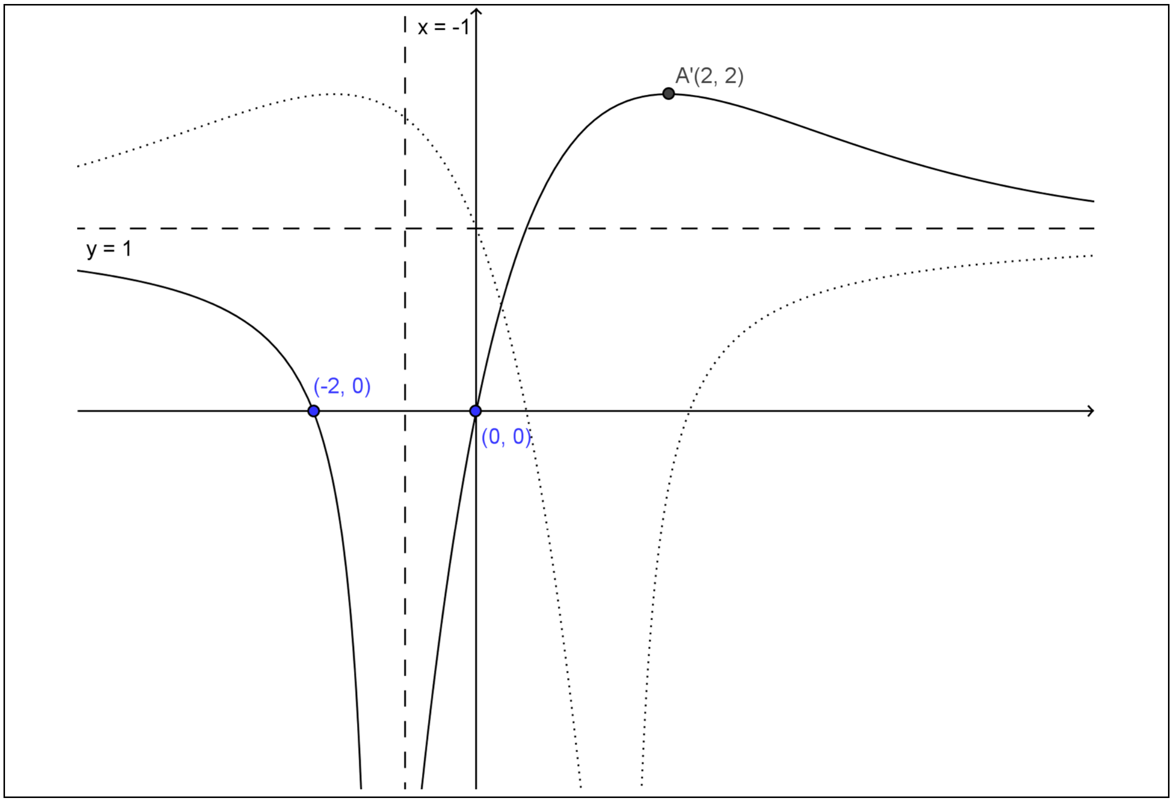
The diagram shows a sketch of $y = \mathrm{f}(x)$.

On separate diagrams, sketch the graphs of

i) $y = -\mathrm{f}(x)$, [2]

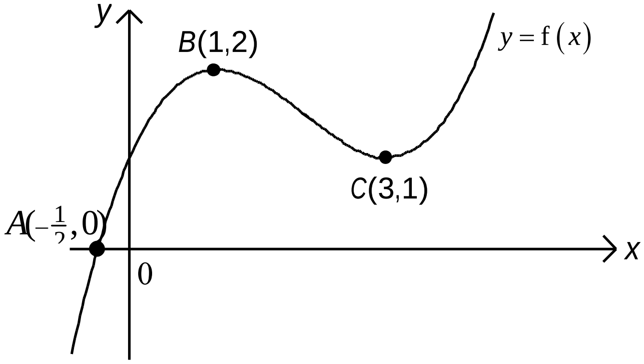


ii) $y = \mathrm{f}(1-x)$, [3]



2. **RJC/2006/Promo/Q2**

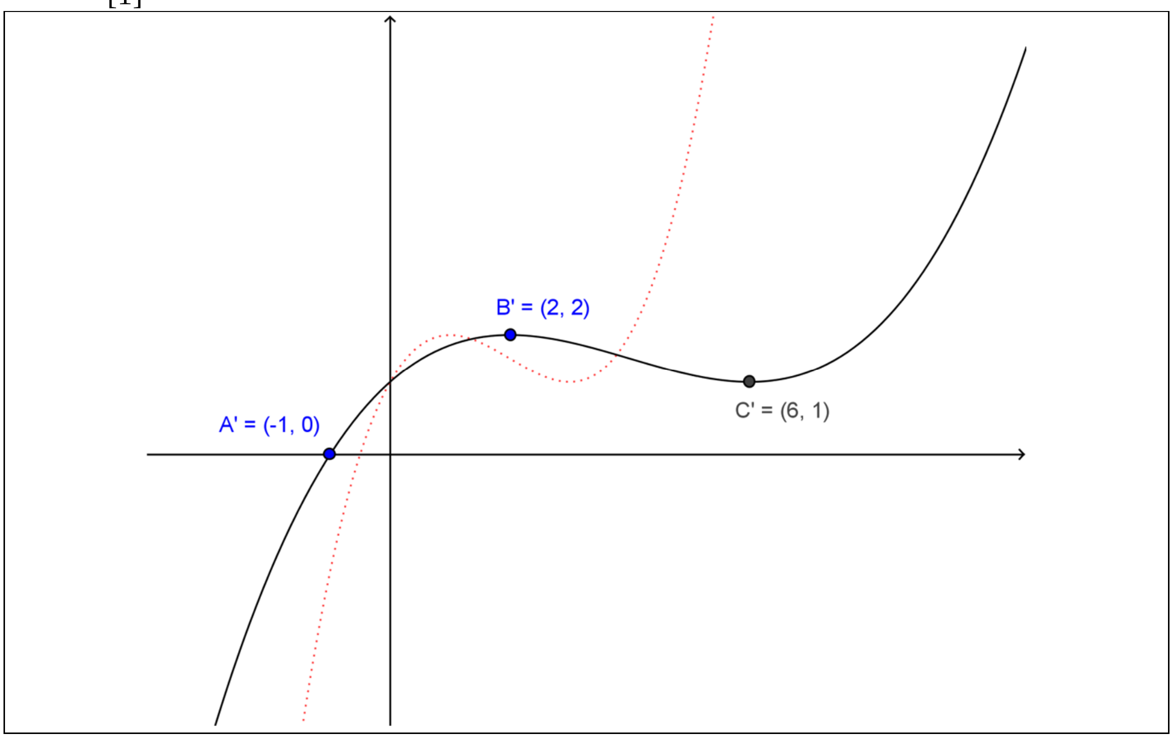
The diagram below shows the graph of $y = \mathrm{f}(x)$. The points A, B and C have coordinates $(-\frac{1}{2}, 0)$, (1, 2) and (3, 1) respectively.



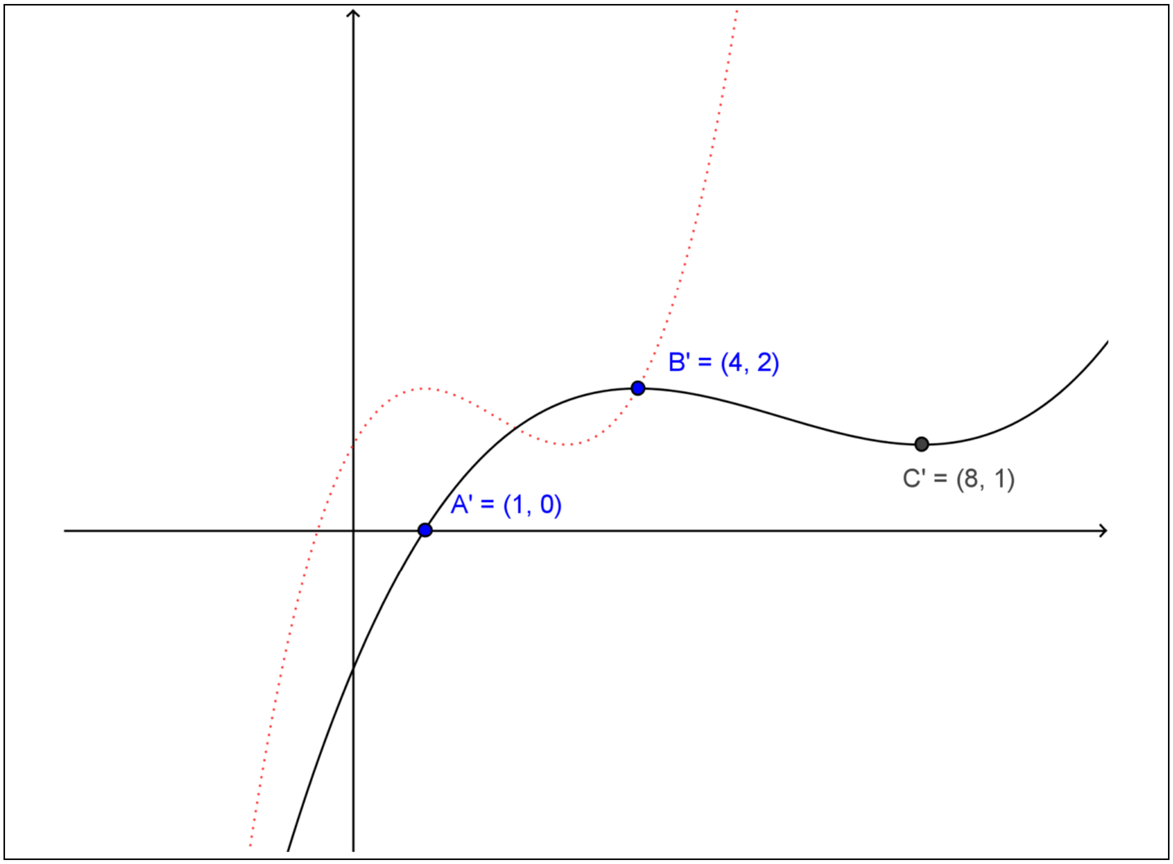
Sketch on separate diagrams, the graphs of

(i) $y = \mathrm{f}\left(\frac{x}{2}\right)$,

[1]



(ii) $y = \mathrm{f}\left(\dfrac{x}{2} - 1\right)$, [2]

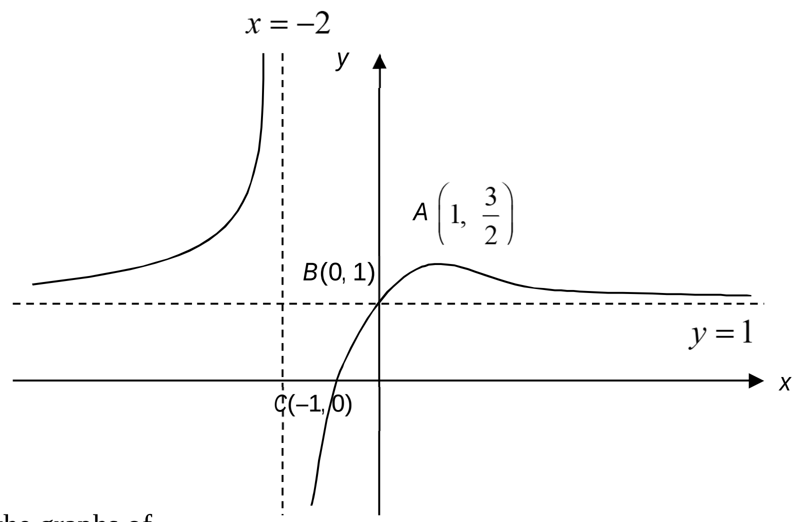


State, in each case, the coordinates of the points corresponding to *A*, *B* and *C*, where appropriate.

3. **HCI/2006/Promo/Q6**

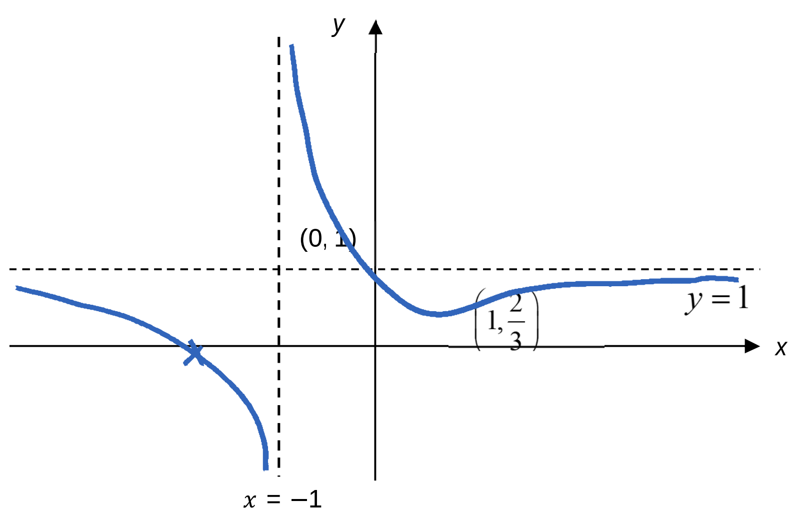
The diagram below shows the graph of $y = \text{f}\left(1-x\right)$. The curve has a maximum point at $A\left(1,\ \frac{3}{2}\right)$, and axes intercepts at $B(0, 1)$ and $C(-1, 0)$.

The lines $x = -2$ and $y = 1$ are the asymptotes.



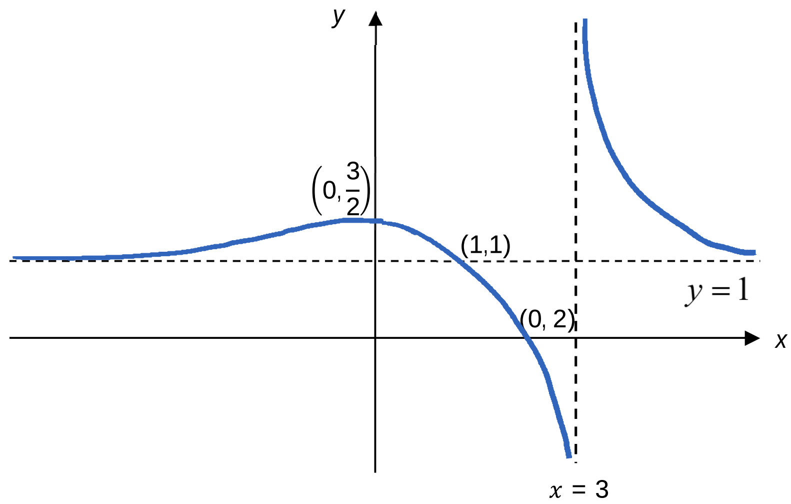
Sketch the graphs of

(a) $y = \dfrac{1}{\text{f}\left(1-x\right)}$ [3]



(b) $y = \mathrm{f}(x)$, [3]

$y = \mathrm{f}(1 - x) \rightarrow \mathrm{f}(1 + x) \rightarrow \mathrm{f}(x)$



4. **SAJC/2006/Promo/Q9 (modified)**

a) The curve C has equation $y = \dfrac{x^3 + a}{x^2}$ where a is a constant. Given that C has a stationary point at $x = 2$, find the value of a and determine the nature of the stationary point. Write down the equation of the asymptotes of C. [4]

$y = \frac{x^3+a}{x^2} = x + ax^{-2}$

$\frac{dy}{dx} = 1 - 2ax^{-3} = 0$ when $x = 2$

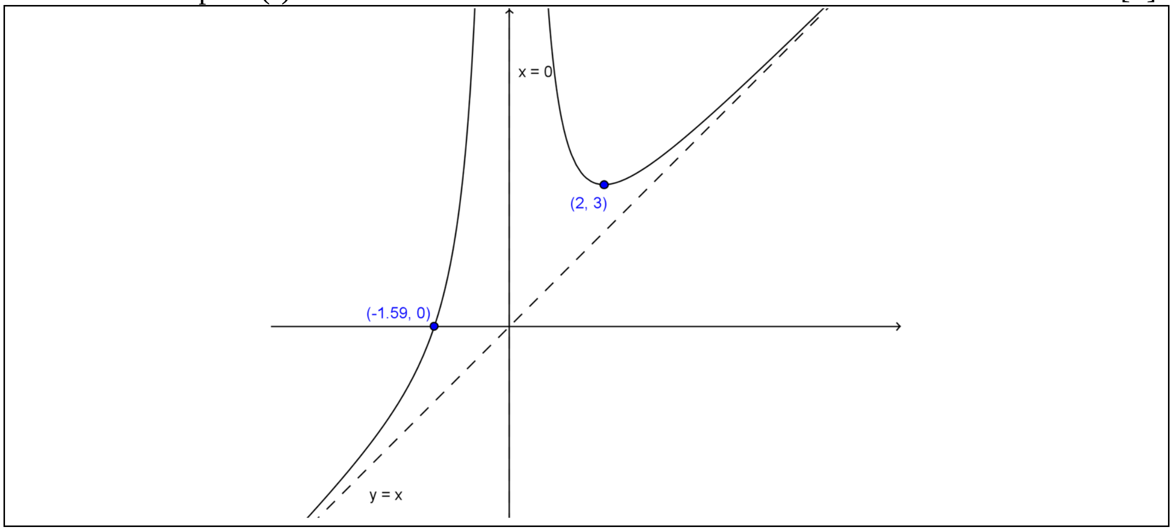
$1 - \frac{2}{8}a = 0$

$a = 4$

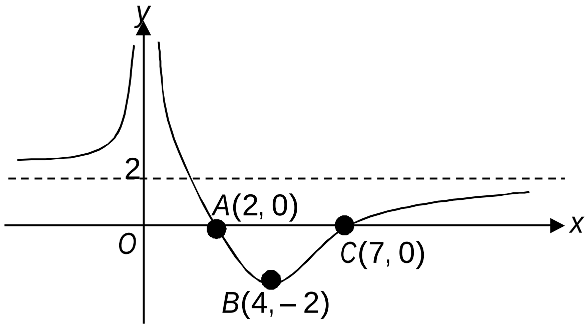
$\frac{d^2y}{dx^2} = 24x^{-4} > 0$, therefore (2, 3) is a minimum point.

Asymptotes: $x = 0$ and $y = x$

Hence sketch C, showing clearly the asymptotes and the coordinates of any point(s) of intersection with the coordinate axes. [2]

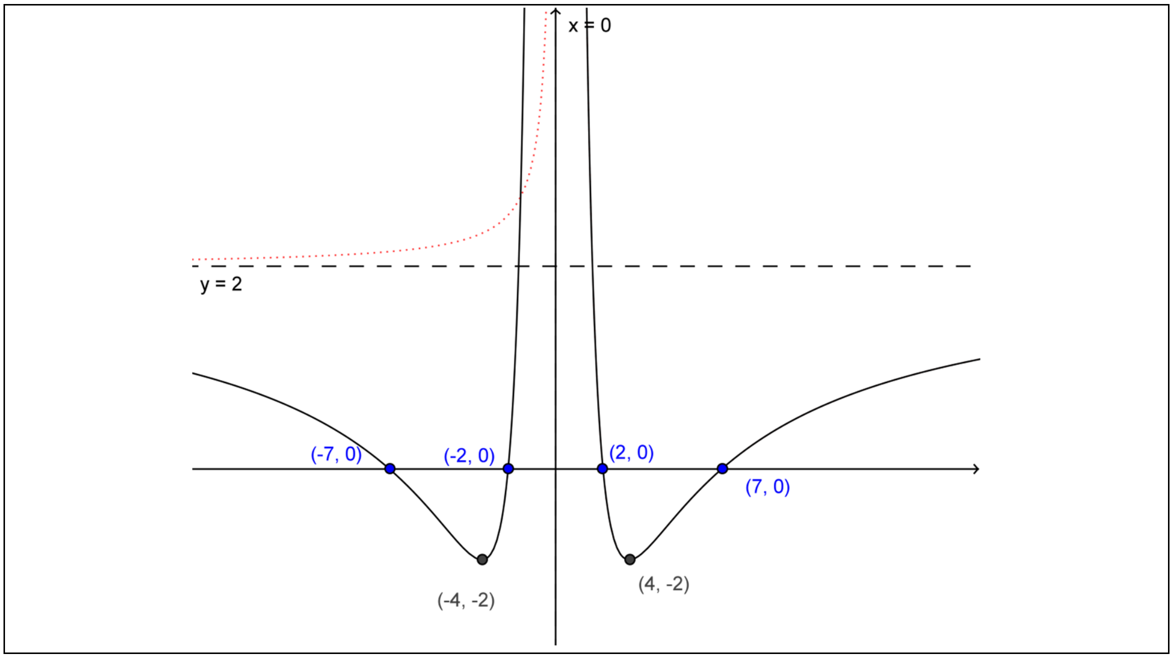


b) The given diagram shows the graph of $y = \mathrm{f}(x)$.



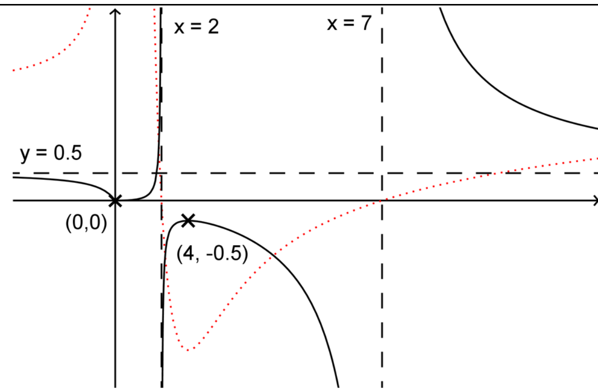
On separate diagrams, sketch the graphs of

(i) $y = \mathrm{f}\left(|x|\right)$ [1]



(ii) $y = \dfrac{1}{f(x)}$ [2]

in each case showing clearly the asymptotes and the points corresponding to *A*, *B* and *C*.



5. **MI/2006/Promo/Q8**

The equations of the curves C_1 and C_2 are given by

$C_1:\ x^2 + 3y^2 = 3$,

$C_2:\ x^2 - y^2 = 2$.

(i) Give a geometrical description of the curve C_1 and state any axes of symmetry. [4]

C_1 is an ellipse, centred on (0, 0). Horizontal diameter is $2\sqrt{3}$, vertical diameter 2.

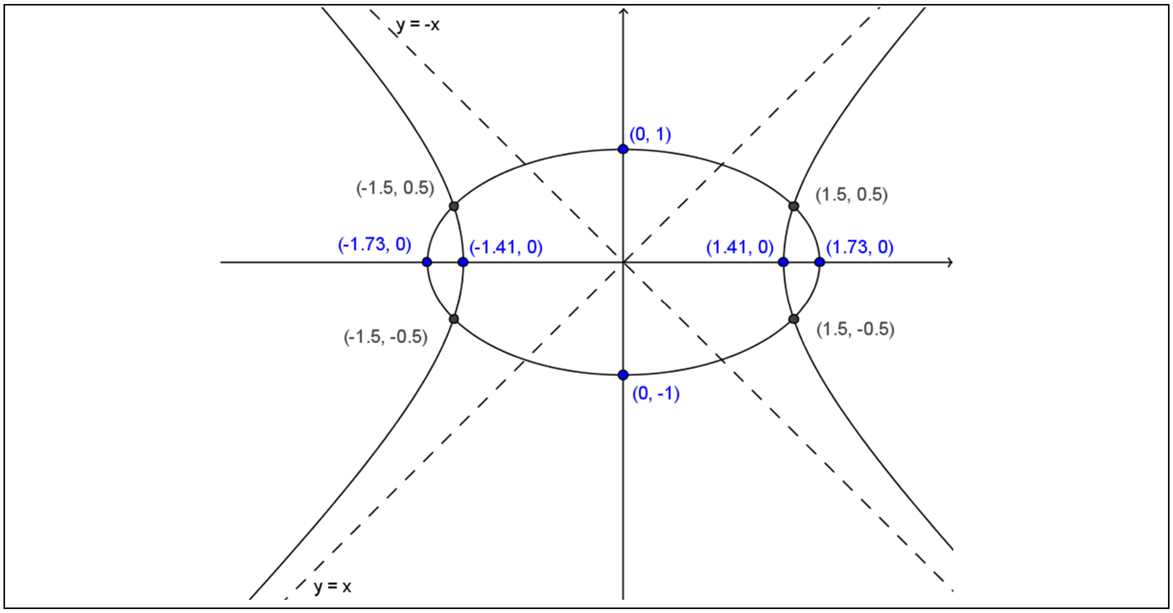
Axes of symmetry are $y = 0$ and $x = 0$.

(ii) State the equations of the asymptotes of C_2. [1]

$y = \pm x$

Sketch C_1 and C_2 on the same diagram, indicating clearly the asymptotes and state the coordinates of the points of intersection of the curves C_1 and C_2.

[5]



6. **CJC/2006/Promo/Q3**

A graph with equation $y = \text{f}(x)$ undergoes 3 successive transformations as follows:

I: A scaling parallel to x–axis with scale factor $\frac{1}{2}$

II: A translation of 3 units in the direction of the positive x–axis

III: A scaling parallel to y–axis with scale factor 2

The resulting equation is $y = 2\ln(6 - 2x)$.
Determine the equation of the graph $y = \text{f}(x)$, showing your workings clearly.

[3]

Let:

III': Scaling parallel to y-axis with scale factor ½

II': Translation of -3 units in the direction of the positive x-axis

I': Scaling parallel to x-axis with scale factor 2.

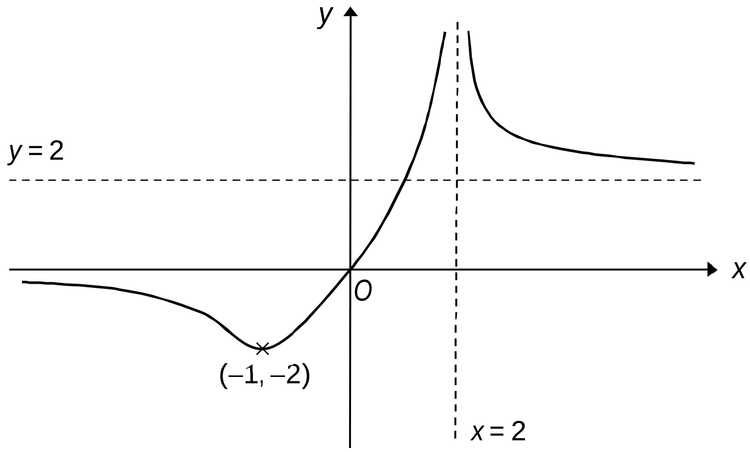
These will undo the original three transformations. Applying them successively to $y = 2\ln(6 - 2x)$:

$$2\ln(6-2x) \xrightarrow{III'} \ln(6-2x) \xrightarrow{II'} \ln\left(6-2\left(x-3\right)\right) = \ln(-2x) \xrightarrow{I'} \ln\left(-2\left(\frac{1}{2}x\right)\right) = \ln\left(-x\right)$$

Therefore the original graph was $y = \ln(-x)$

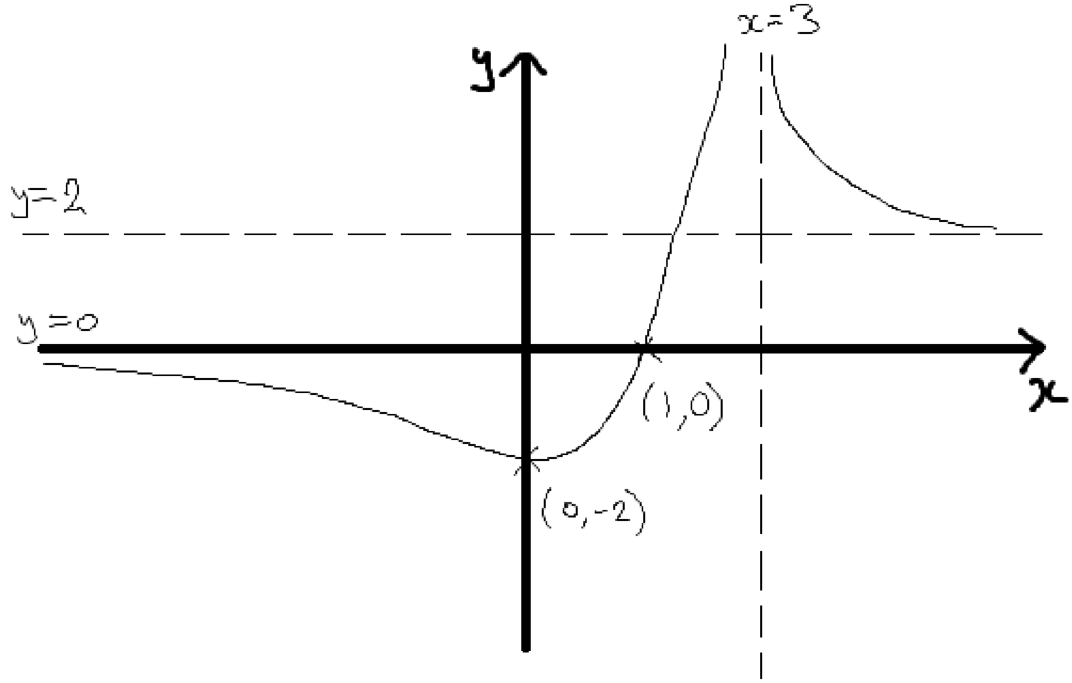
7. **VJC/2006/Promo/Q12 (modified)**

The diagram shows the graph of $y = \mathrm{f}(x)$. The curve passes through the origin and has minimum point (–1, –2). The asymptotes are $x = 2$, $y = 0$ and $y = 2$.

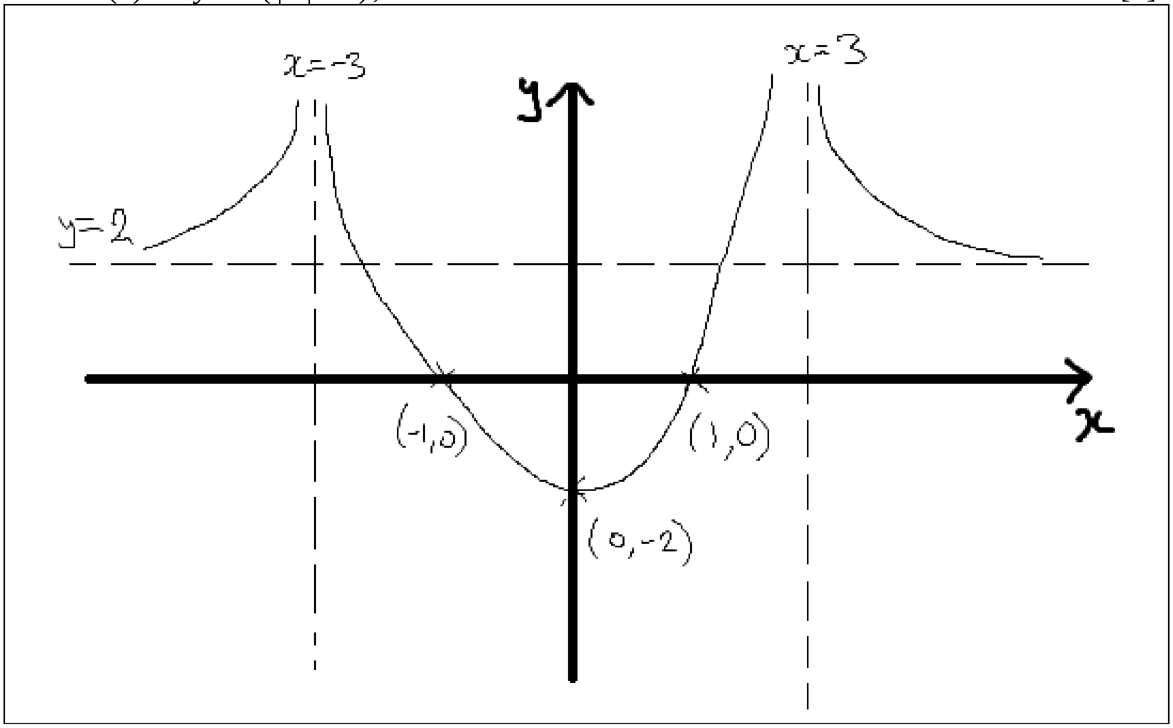


Sketch, on separate diagrams, the graphs of

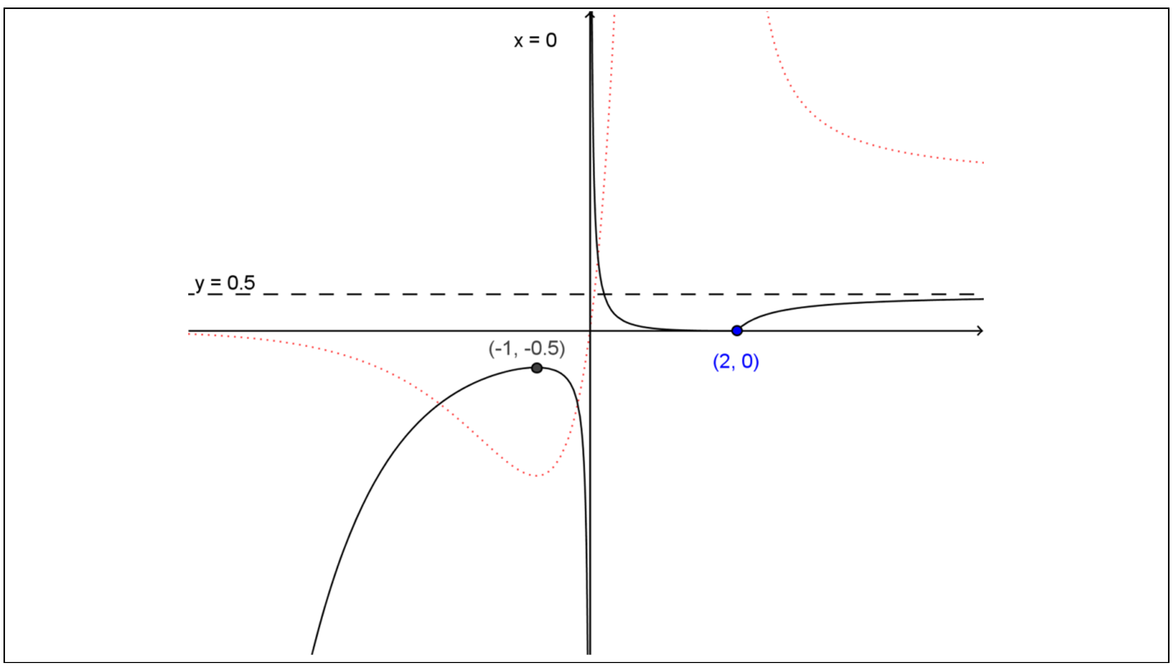
(i) $y = \mathrm{f}(x-1)$, [2]



(ii) $y = \mathrm{f}(|x|-1)$, [2]



(iii) $y = \frac{1}{\mathrm{f}(x)}$



8. ASRJC/JC1 Promo/2019/Q1

The graph of $y = \mathrm{f}(x)$ undergoes transformations in the following order:

I. Reflection in the x-axis
II. Translation in the positive x-direction by 4 units
III. Scaling parallel to the x-axis by a scale factor of $\frac{1}{3}$

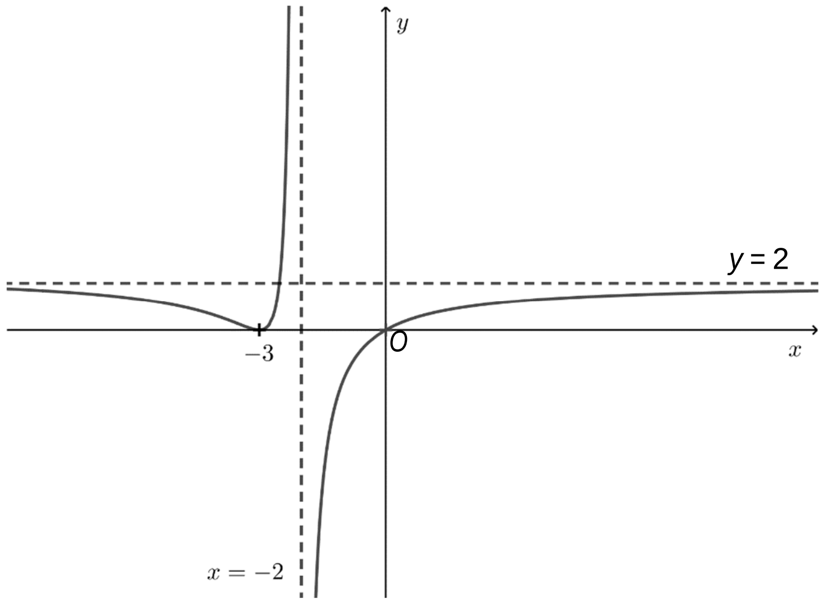
The equation of the resulting graph is $\mathrm{e}^{2y} = 7 - 3x$.

Find the equation of the original graph in the form $y = \mathrm{f}(x)$. [3]

	Method 1 Reverse III: Scaling parallel to the x-axis by a scale factor of 3. (Replace x by $\frac{x}{3}$) Equation becomes: $\mathrm{e}^{2y} = 7 - x$ Reverse II: Translation in the negative x-direction by 4 units. (Replace x by $x+4$) Equation becomes: $\mathrm{e}^{2y} = 7 - (x+4)$ $\mathrm{e}^{2y} = 3 - x$ Reverse I: Reflection in the x-axis (replace y by $-y$) Equation becomes $\mathrm{e}^{-2y} = 3 - x$ $\Rightarrow y = \mathrm{f}(x) = -\frac{1}{2}\ln(3-x)$ **Method 2** After the 3 transformations, $y = \mathrm{f}(x)$ becomes $y = -\mathrm{f}(3x-4)$ As the final curve is $y = -\frac{1}{2}\ln(7-3x)$ So $\mathrm{f}(3x-4) = \frac{1}{2}\ln(7-3x)$ Let $z = 3x - 4 \Rightarrow x = \frac{z+4}{3}$ $\mathrm{f}(z) = \frac{1}{2}\ln\left[7 - 3\left(\frac{z+4}{3}\right)\right] = \frac{1}{2}\ln(3-z)$ $y = \mathrm{f}(x) = \frac{1}{2}\ln(3-x)$ **Method 3** After the 3 transformations, $y = \mathrm{f}(x)$ becomes $y = -\mathrm{f}(3x-4)$ As the final curve is $y = -\frac{1}{2}\ln(7-3x)$ So $\mathrm{f}(3x-4) = \frac{1}{2}\ln(7-3x) = \frac{1}{2}\ln\left[-(3x-4)+3\right]$ $\mathrm{f}(x) = \frac{1}{2}\ln(3-x)$

9. RVHS/JC1 Promo/2019/Q5

(a) The diagram shows the graph of y = f(x). The curve passes through the origin, has a minimum point at $(-3, 0)$ and its equations of asymptotes are $x = -2$ and $y = 2$.

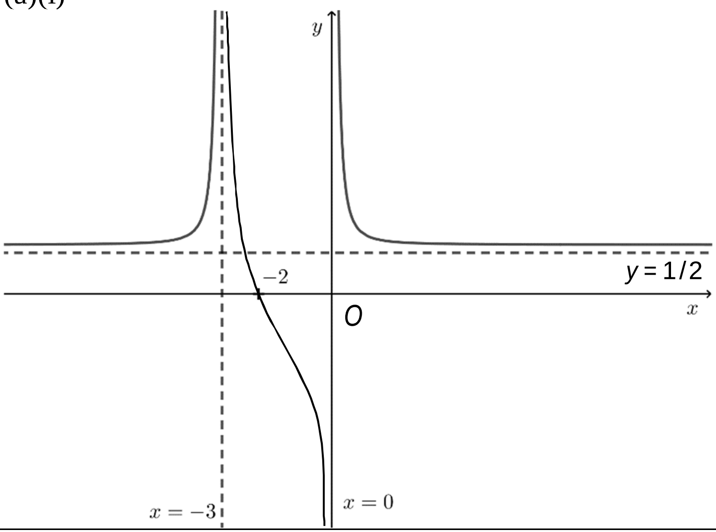
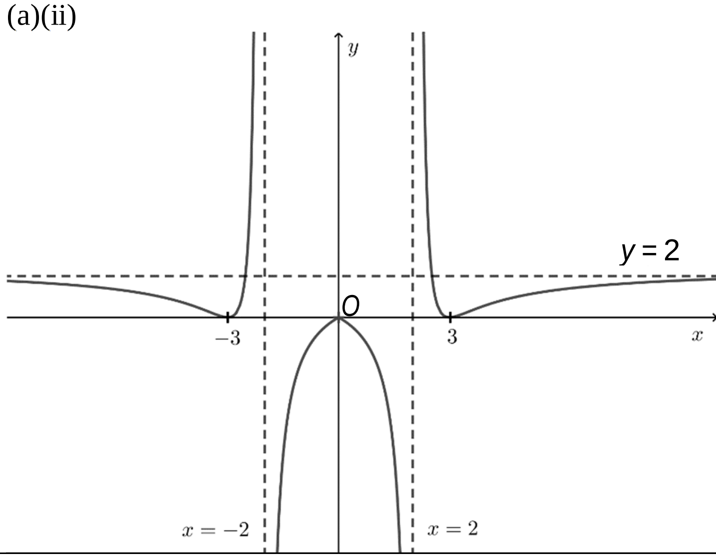


Sketch the following graphs on separate axes.

(i) $y = \dfrac{1}{\mathrm{f}(x)}$ [3]

(ii) $y = \mathrm{f}(-|x|)$ [2]

(b) Describe a series of transformations that transforms the graph of $x^2 + (y+1)^2 = 1$ onto the graph of $(x-1)^2 + \dfrac{y^2}{9} = 1$. [3]

(a)(i) 	
(a)(ii) 	
(b) Translate 1 unit in the positive *x*-axis Translate 1 unit in the positive *y*-axis Scale parallel to the *y*-axis by factor 3. OR Translate 1 unit in the positive *x*-axis Scale parallel to the *y*-axis by factor 3. Translate 3 units in the positive *y*-axis (Note: Transformation of *x* can be at any step)	

10. ASRJC/Promo 9758/2020/Q6 (part)

The curve C_1 whose equation is $x^2+y^2=16$ undergoes, in succession, the following transformations:

A: Translation of 4 units in the negative x-direction.
B: Translation of 2 units in the positive y-direction.
C: Scaling by factor 2 parallel to the y-axis.

(i) Find the equation of the resulting curve C_2. [2]

(ii) Sketch C_1 and C_2 on the same diagram, indicating clearly the relevant features of the two curves. You need not find the coordinates of axial intercepts. [2]

(iii) State the coordinates of the points of intersection between C_1 and C_2. [1]

Question 10

(i) $x^2+y^2=16$

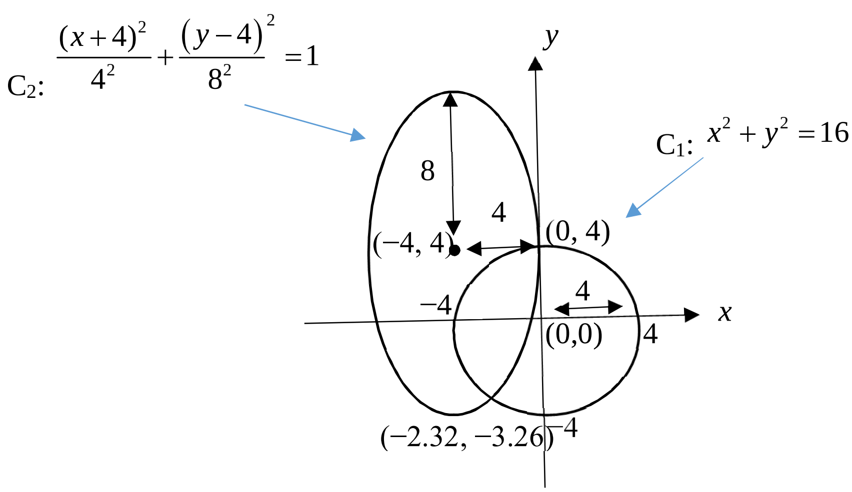
Transformation A, replace x with $x+4$: $(x+4)^2+y^2=16$

Transformation B, replace y with $y-2$: $(x+4)^2+(y-2)^2=16$

Transformation C, replace y with $\frac{y}{2}$: $(x+4)^2+\left(\frac{y}{2}-2\right)^2=16$

$$\frac{(x+4)^2}{4^2}+\frac{(y-4)^2}{8^2}=1$$

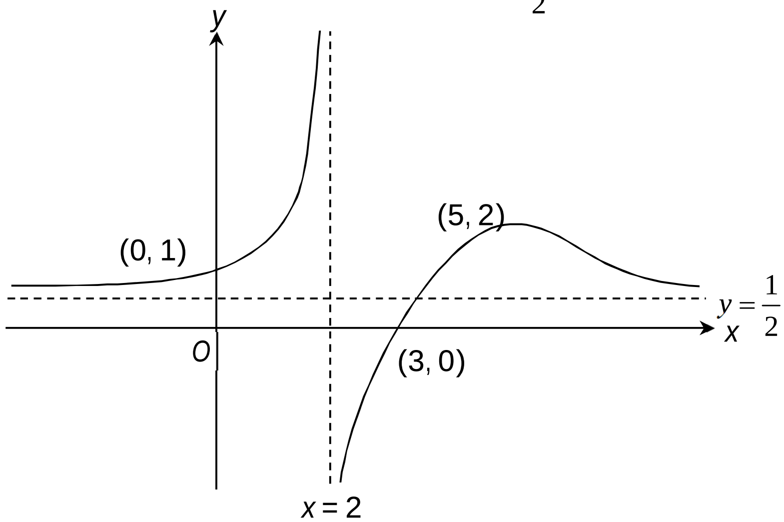
(ii)



(iii) Intersection points are (–2.32, –3.26) and (0, 4)

11. MI 2020 Promo 9758/2020/PU1/Q3

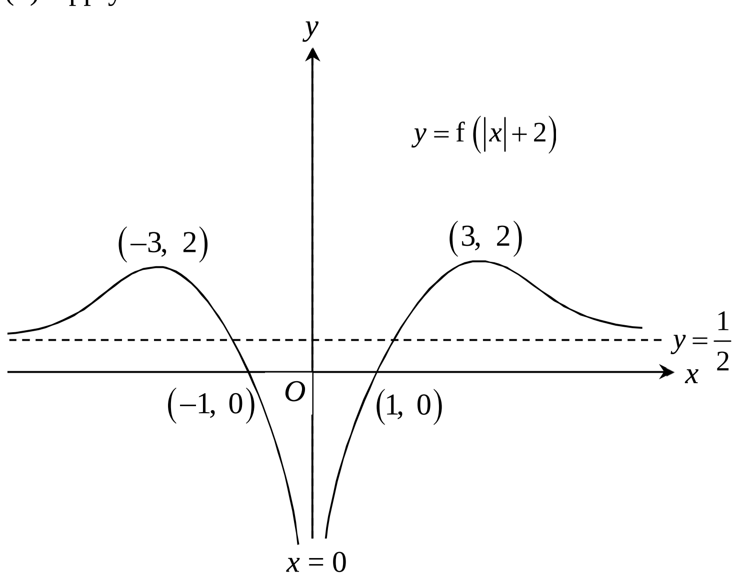
The diagram below shows the graph of $y = \mathrm{f}(x)$. The curve intersects the *x*-axis and *y*-axis at the points $(3,\ 0)$ and $(0,\ 1)$ respectively. It has a maximum point at $(5,\ 2)$. The asymptotes of the curve are the lines with equations $x = 2$ and $y = \frac{1}{2}$.



On separate diagrams, sketch the following graphs, indicating the coordinates of any stationary points and points of intersections with the axes and the equations of any asymptotes.

(i) $y = \mathrm{f}(|x| + 2)$ [3]

(ii) $y = \dfrac{1}{\mathrm{f}(x)}$ [3]

(i) **[3]**	***Thinking steps:*** $y = \mathrm{f}(x) \xrightarrow[(1)]{\text{Replace } x \text{ by } x+2} y = \mathrm{f}(x+2) \xrightarrow[(2)]{\text{Replace } x \text{ by } \|x\|} y = \mathrm{f}(\|x\|+2)$ **Sequence:** (1) Translation of 2 units in the negative *x*-direction (2) Apply modulus 

(ii) **[3]**	Sketch of $y=\dfrac{1}{\mathrm{f}(x)}$: