Example 1

According to Newton's Law of Gravitation, the force *F* between two point masses *M* and *m* separated by the distance r is given by the formula $F = \frac{GMm}{r^2}$

where *G* is the universal gravitational constant. Obtain the SI base units for *G*.

$$G = \frac{Fr^2}{Mm} \quad \text{and} \quad F = ma$$

$$\text{Unit of } G = \text{Unit of } \left(\frac{mar^2}{Mm} \right) = \frac{\text{m s}^{-2} \text{ m}^2}{\text{kg}} = \text{m}^3 \text{ kg}^{-1} \text{ s}^{-2}$$

Example 2

Bernoulli's equation, which applies to fluid flow states that

$$P + h\rho g + \tfrac{1}{2}\rho v^2 = k$$

where P is pressure, h is height, ρ is density, g is acceleration due to gravity, v is velocity and k is a constant.

Show the left hand side of the equation is homogeneous and state the SI unit for k.

Analysing each term on the LHS of the equation:

$$Units\ of\ P = Units\ of\ \frac{F}{A} = Units\ of\ \frac{ma}{A} = \frac{\mathrm{kg\ m\ s^{-2}}}{\mathrm{m^2}} = \boxed{\mathrm{kg\ m^{-1}\ s^{-2}}}$$

$$Units\ of\ h\rho g = (Units\ of\ h)(Units\ of\ \frac{M}{V})(Units\ of\ g)$$

$$= \mathrm{m} \cdot \mathrm{kg\ m^{-3}} \cdot \mathrm{m\ s^{-2}} = \boxed{\mathrm{kg\ m^{-1} s^{-2}}}$$

$$Units\ of\ \frac{1}{2}\rho v^2 = (1)\cdot \mathrm{kg\ m^{-3}} \cdot (\mathrm{m\ s^{-1}})^2 = \boxed{\mathrm{kg \cdot m^{-1} \cdot s^{-2}}}$$

Homogeneous!

Unit for k = kg m^{-1} s^{-1}

Example 3

Consider the period *T* of a simple pendulum. The possible factors which may affect it are its length *l*, its mass *m* and the acceleration due to gravity *g*. Use unit analysis to arrive at a plausible relationship between *T* and these quantities.

Suppose the relationship between T, l, m and g is given by :

$$T = kl^x m^y g^z$$

where k is a dimensionless quantity and x, y, and z are constants.

A plausible relationship must be homogeneous.

Units on the LHS : s

Units on the RHS: $(1)(\mathrm{m})^x(\mathrm{kg})^y(\mathrm{m\,s^{-2}})^z = \mathrm{m}^{x+z}\cdot\mathrm{kg}^y\cdot\mathrm{s}^{-2z}$

Comparing the indices :

$$x + z = 0,\ y = 0,\ -2z = 1$$

$$\Rightarrow x = \frac{1}{2},\ \ y = 0,\ \ z = -\frac{1}{2} \qquad \Rightarrow T = k\sqrt{(l/g)}$$

Example 4

Which estimate is realistic?

A. The kinetic energy of a bus on an expressway is 30000 J.
  - *Assuming the bus travels at 60 km h^{-1}, its KE is*

$$\frac{1}{2}mv^2 = (\frac{1}{2})(10000)(\frac{60000}{3600})^2 = 1.4 \times 10^6 \text{ J}$$

B. The power of a domestic light is 300 W
  - *A typical domestic light bulb is between 5 W and 50 W.*

C. The temperature of a hot oven is 300 K.
  - *300 K is just only about 27 ºC!*

D. The volume of air in a tyre is 0.03 m^3.
  - *Assuming an outer diameter of 30 cm and inner diameter of 20 cm, and a width of 15 cm,*

$$V = [\pi(0.30)^2 - \pi(0.20)^2](0.15) = 0.024 \text{ m}^3$$

Example 5

A student makes measurements from which he calculates the speed of sound to be 327.66 m s^{-1}. He estimates that the percentage uncertainty is 3%. Round off the speed to an appropriate number of significant figures.

$$Given \quad \frac{\Delta v}{v} = 3\%$$

$$\Delta v = 0.03 \times (327.66) = 9.82 = 10 \text{ m s}^{-1} (1 \text{sf})$$

$$\therefore \text{v} = (330 \pm 10) \text{ m s}^{-1}$$

Note that v is expressed to the same place value as Δv.
Hence in this case, the speed is expressed to the tens place (which happens to be 2 sf).

Example 6

The measurements of the dimensions of a particular piece of rectangular cardboard are (18.5 ± 0.5) mm and (12.5 ± 0.5) mm.

Determine the area of the cardboard with its associated uncertainty.

Area, A = L X B = 18.5 x 12.5 = 231.25 mm^2

Fractional uncertainty of the area,

$$\frac{\Delta A}{A} = \frac{\Delta L}{L} + \frac{\Delta B}{B} = \frac{0.5}{18.5} + \frac{0.5}{12.5} = 0.0670$$

$$\Delta A = (0.0670)(231.25) = 20 \text{ mm}^2 \text{ (to 1 s.f.)}$$

$\therefore A$ = (230 $\pm$ 20) mm^2

= (2.3 $\pm$ 0.2) x 10^2 mm^2 ("A" same place value as ΔA)

When we write 20 mm^2, it is unclear if this is a 1 or 2 s.f. value. Writing the answer in standard form removes this ambiguity, even though both are acceptable.

Example 7

The radius of a circle is r = (3.0 ± 0.2) cm. Find the circumference with its associated uncertainty.

Circumference, $C = 2\pi r = 2\pi\,(3.0) = 18.8$ cm

Absolute uncertainty of the circumference,

$\Delta C = 2\pi\Delta r = 2\pi\,(0.2) = 1.26$ cm = 1 cm (1 s.f.)

$\therefore C = (19 \pm 1)$ cm   ("C "same place value as ΔC)

Example 8

Given a sphere of radius R = (18.5 ± 0.5) mm, find the volume of the sphere with its associated uncertainty.

$$Volume\; V = \tfrac{4}{3}\pi R^3 = \tfrac{4}{3}\pi\,(18.5^3) = 2.652\times 10^4\ mm^3$$

$$\frac{\Delta V}{V} = 3\frac{\Delta R}{R} = 0 + 3\left(\frac{0.5}{18.5}\right) = 0.0811$$

$$\Delta V = 0.0811\times(2.652\times10^4) = 0.2\times10^4\ \text{mm}^3$$

$$\therefore V = (2.7\pm0.2)\times10^4\ \text{mm}^3$$

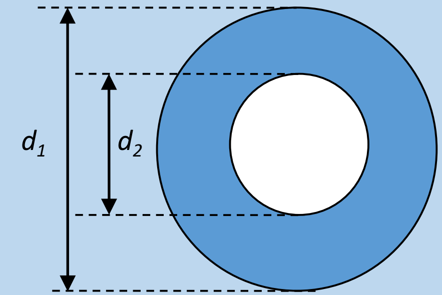
In general, if a measured number is so large or small that it calls for scientific notation, then it is simpler and clearer to express the number and its uncertainty in the same form, i.e. both are expressed to x 10^4 mm^3.

Example 9

In an experiment, the external diameter d_1 and internal diameter d_2 of a hollow tube are found to be (64 ± 2) mm and (47 ± 1) mm respectively. Calculate the thickness of the tube and the associated uncertainty. What is the corresponding percentage error?

Thickness $t = \dfrac{d_1 - d_2}{2} = \dfrac{64 - 47}{2} = 8.5$ mm

Uncertainty in t, $\Delta t = \dfrac{1}{2}\Delta d_1 + \dfrac{1}{2}\Delta d_2 = 2$ mm (1 s.f.)



Thus $t = (9 \pm 2)$ mm

Percentage uncertainty in $t = \dfrac{\Delta t}{t} \times 100\% = \dfrac{2}{9} \times 100\% = 22\%$

Example 10

The period of oscillation of a simple pendulum is $T = 2\pi\sqrt{\frac{l}{g}}$

A student conducts an experiment to find the acceleration of free fall, *g*. He measures the length of the pendulum, *l* = 0.23 ± 0.01 m and the time for 20 oscillations, t = 19.24 ± 0.01 s. Find g and its associated uncertainty.

Make *g* the subject of the equation* (this step is a must!)

$$g = \frac{4\pi^2 l}{T^2} = \frac{4\pi^2(0.23)}{(19.24 \div 20)^2} = 9.81\,\text{m s}^{-1}$$

$T = t/20$, so $\Delta T / T = \Delta t / t$

$$\frac{\Delta g}{g} = \frac{\Delta l}{l} + 2\frac{\Delta T}{T} = \frac{0.01}{0.23} + 2\frac{0.01}{19.24} = 0.0445$$

$$\Delta g = 0.0445 \times 9.81 = 0.4$$

$$g = (9.8 \pm 0.4)\,\text{m s}^{-2}$$

Example 11

Consider $\boldsymbol{S = x \cos\theta}$ for x = (2.0 ± 0.2) cm, $\boldsymbol{\theta}$ = (53 ± 2)°.

Find *S* with its uncertainty.

Now, *S* = (2.0 cm) cos 53° = 1.204 cm

To get the largest possible value of *S*,

x needs to be large and θ should be as close to 0° as possible.

$\Rightarrow \quad S_{max} = 2.2 \cos 51° = 1.385$ cm

Similarly, for smallest possible value of *S*, *x* is small and θ needs to be closer to 90°

$\Rightarrow \quad S_{min} = 1.8 \cos 55° = 1.032$ cm

$$\Delta S = \frac{S_{max} - S_{min}}{2} = \frac{1.385 - 1.032}{2} = 0.2 \text{ cm (to 1 s.f.)}$$

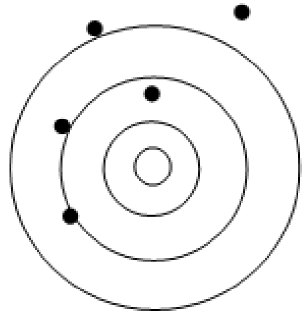
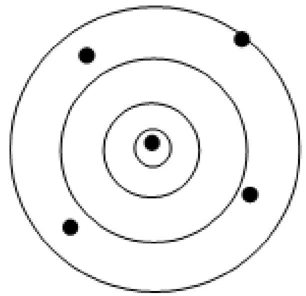
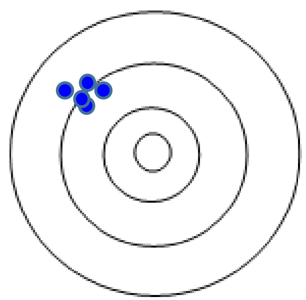
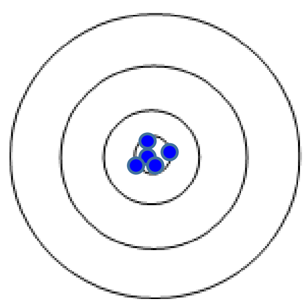
S = 1.2 ± 0.2 cm (*S* is corrected to the same place value as Δ*S* expressed as 1 s.f.)

Remarks: *S* has the same unit as *x* as cos θ is a dimensionless quantity

Example 12

Precise:
Closeness of a set of measurements

Accurate:
Closeness of the "average" of a set of measurements to the true value

Target	Precise	Accurate
	✗	✗
	✗	✓
	✓	✗
	✓	✓

Dots are far apart from each other and far away from the bull's eye.

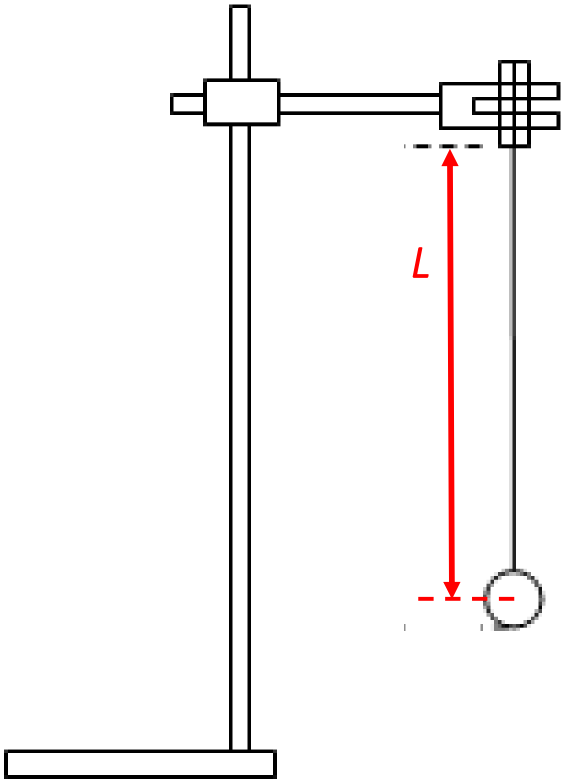
Dots are far apart from each other but their "average" position is quite near the bull's eye.

Example 13

The period of one oscillation of the pendulum T is related to the length of the pendulum L according to the equation $T = 2\pi\sqrt{\frac{L}{g}}$

Where g = 9.81 m s^{-2}. A diagram of the experiment is shown on the right. T is measured for one oscillation with a stop-watch and L is measured with a metre ruler held in hand.

L/cm	T/s (1st reading)	T/s (2nd reading)
22.0	1.16	0.98
23.5	1.07	1.23
25.1	1.15	1.19
25.6	1.30	1.10
27.3	1.42	1.20
28.1	1.38	1.25



Example 13

Using the given information,

a) identify a random error in this experiment and explain the source of the error

b) suggest an appropriate method to reduce the random error you have identified

L/cm	T/s (1st reading)	T/s (2nd reading)
22.0	1.16	0.98
23.5	1.07	1.23
25.1	1.15	1.19
25.6	1.30	1.10
27.3	1.42	1.20
28.1	1.38	1.25

a) One possible random error in the measurement of T is the human judgment of pendulum starting and completing one oscillation hence introducing uncertainty in the measurement of period of oscillation.

b) Time should be taken for a large number of oscillations. If N oscillations were taken, the period is calculated as $T = t/N$. So if the uncertainty due to the random error is Δt, the uncertainty in the period is $\Delta T = \Delta t/N$. We can see that hence, the uncertainty in T will decrease with greater N.

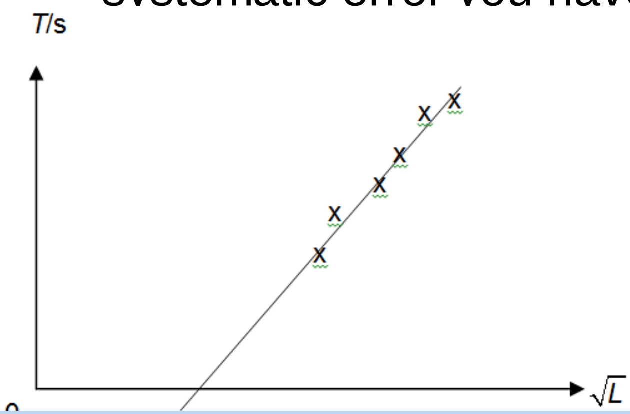
Example 14

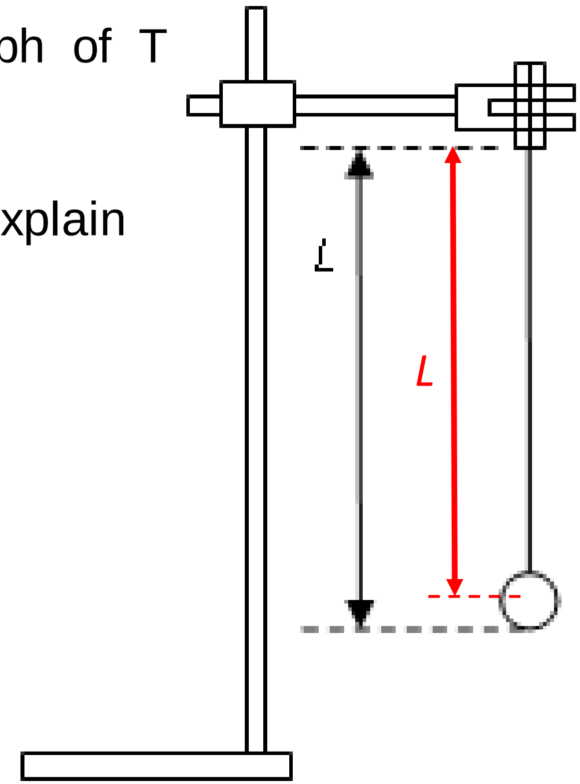
Refer to Example 13 for the same setup. The graph of T against $\sqrt{L}$ is shown:

Using the given information,

a) Identify a systematic error in this experiment and explain the source of the error.

b) Suggest an appropriate method to eliminate the systematic error you have identified.





a) ***error:*** The graph of T against $\sqrt{L}$ should pass through the origin. For every L, the period T appears to be shorter than it should be (or for every T, the length L appears to be longer than it should be) . The length of the pendulum could be measured wrongly.

source of error: The length of the pendulum could have been measured from the point the string was clamped to the bottom (instead of the middle) of the pendulum bob.

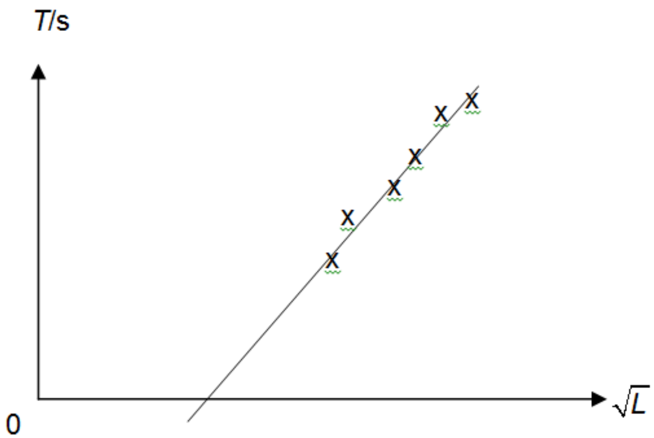
Example 14

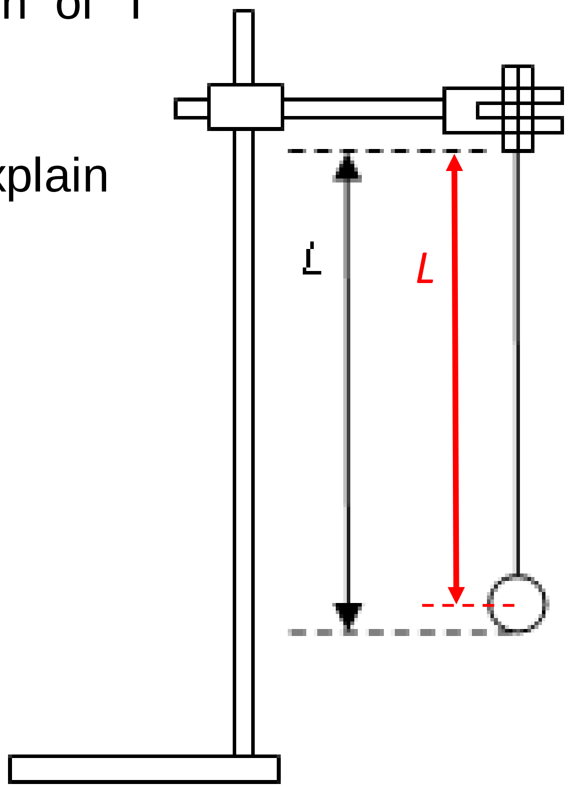
Refer to Example 13 for the same setup. The graph of T against $\sqrt{L}$ is shown:

Using the given information,

a) Identify a systematic error in this experiment and explain the source of the error.

b) Suggest an appropriate method to eliminate the systematic error you have identified.



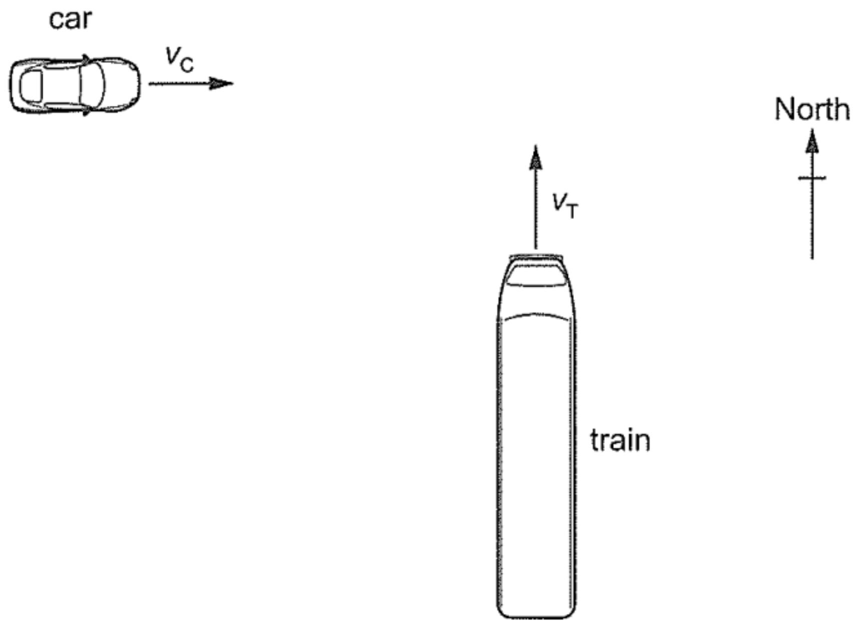


b) In the experimental setup diagram, *L* should be measured only up to the centre of mass of the bob, not to the edge of the bob.

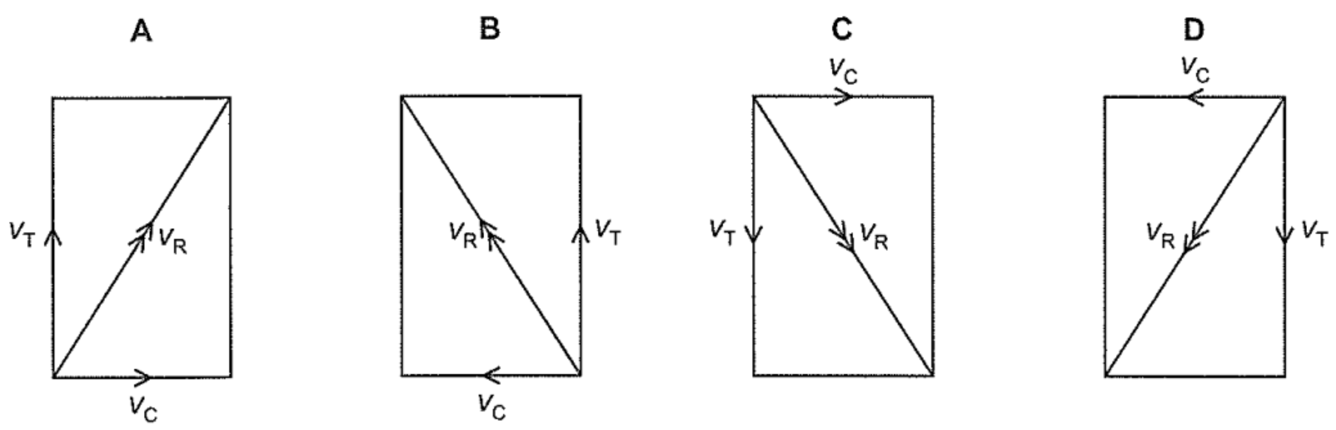
Also, the ruler for measuring *L* should be clamped vertical with a retort stand. A spirit level should be used to ensure the ruler is vertical as so to measure the correct length.

Example 15

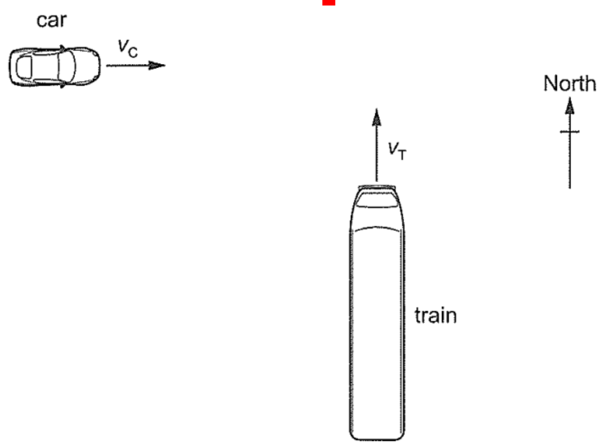
A passenger in a train travelling due north at speed v_T sees a car travelling due east at speed v_C.



Which diagram shows the velocity v_R of the car relative to the passenger in the train?



Example 15



Method 1:

The person in the train perceives himself as stationary and everything else that are stationary around him such as buildings, lampposts etc. moving towards South. The car is moving towards east, not stationary. Hence, the person in the train perceives the car as moving towards South-East.

Method 2:

In general, Relative velocity = velocity of the body A – velocity of the body B

The equation is: $v_{AB} = v_A - v_B$

where:

v_{AB}: relative velocity of the body A respect to body B

v_A: velocity of the body A

v_B: velocity of the body B

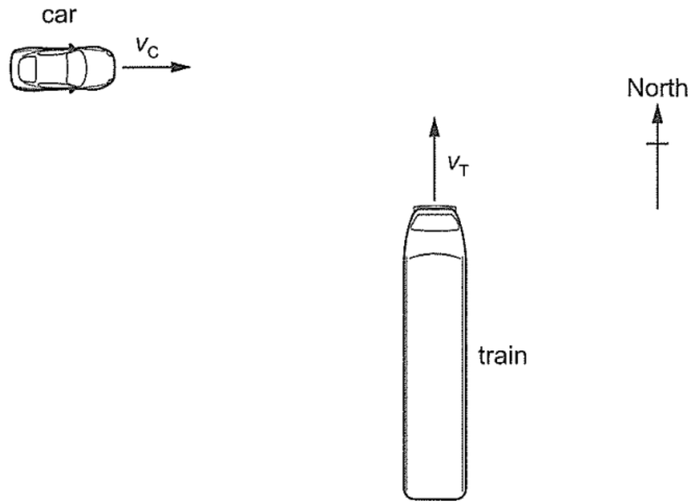
In this case, the car and train are moving relative to each other

The relative velocity is the velocity that the car would appear to an observer on the train and vice versa. Mathematically speaking the relative velocity is the vector difference between the velocities of car and the train.

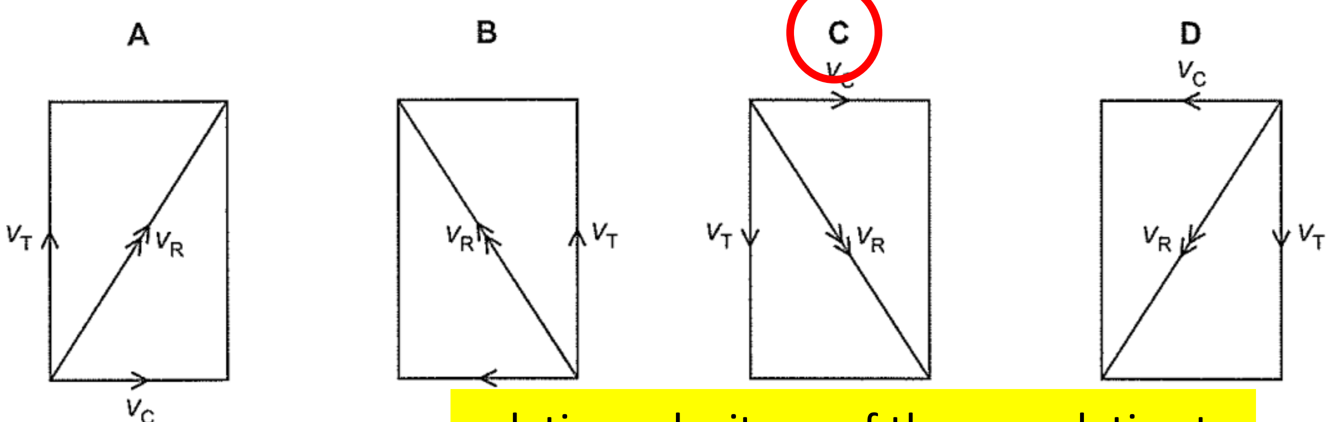
Hence the relative velocity v_R of the car relative to the passenger in the train = v_C - v_T

Example 15

A passenger in a train travelling due north at speed v_T sees a car travelling due east at speed v_C.



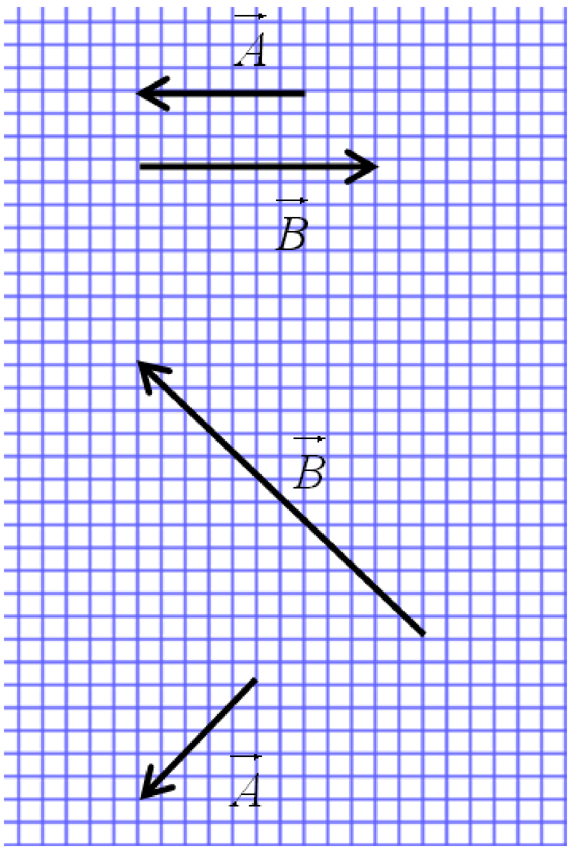
Which diagram shows the velocity v_R of the car relative to the passenger in the train?



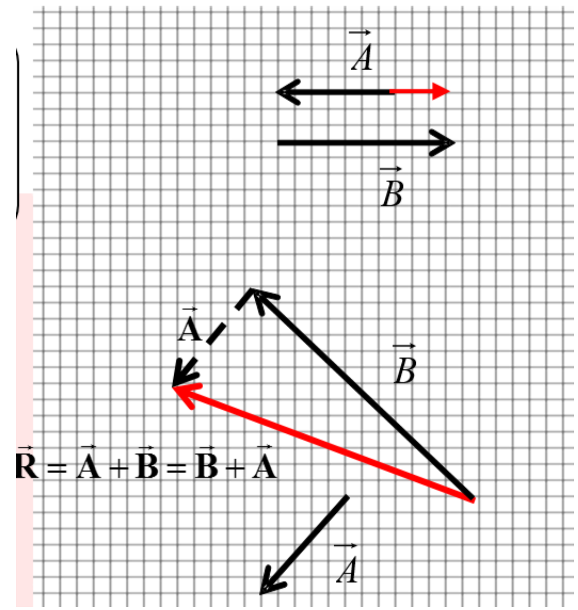
relative velocity v_R of the car relative to the passenger in the train = v_C - v_T

Example 16a

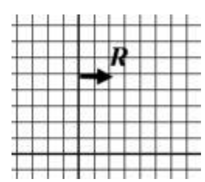
In the figure below, for each pair of vectors $\vec{A}$ and $\vec{B}$, draw the resultant vector $\vec{R}$ where $\vec{R} = \vec{A} + \vec{B}$.

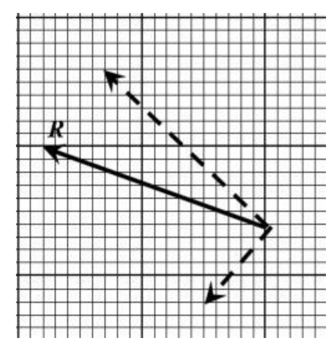


Using the triangle method



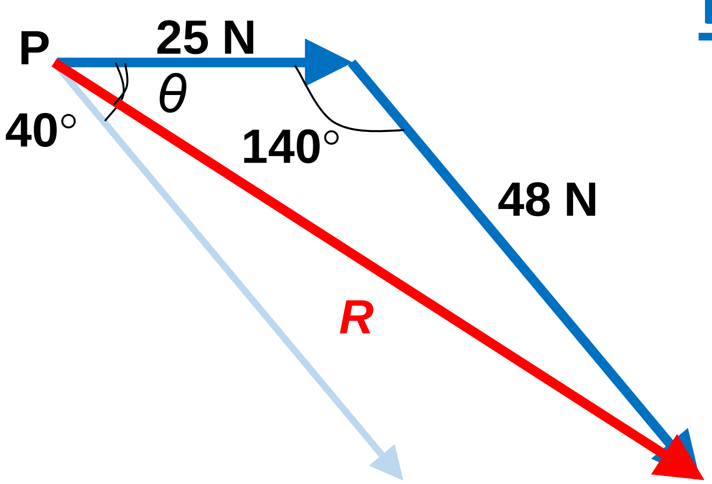
Solution:





Example 16b

Two forces act at a point P as shown below. Determine (magnitude and direction of) the resultant of these two forces.



Method 1: Trigo rules

Find magnitude by Cosine Rule,

$$R^2 = 25^2 + 48^2 - 2(25)(48)\cos 140°$$

$$R = 69\text{ N}$$

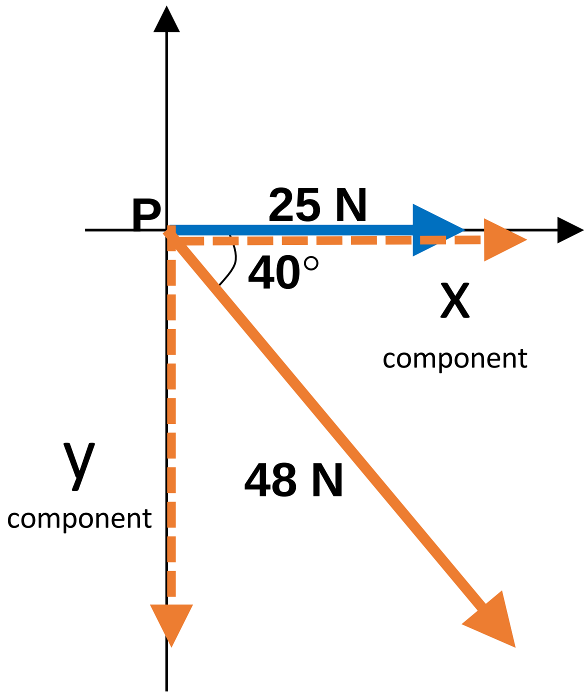
By Sine Rule,

$$\frac{\sin\theta}{48} = \frac{\sin 140°}{69}$$

$\theta = 27°$ below the horizontal

Example 16b (alternative method)

Two forces act at a point P as shown below. Determine (magnitude and direction of) the resultant of these two forces.



Method 2: Resolution of vectors

In the x-direction,

Sum of forces = 48 cos 40° + 25 = 61.8 N

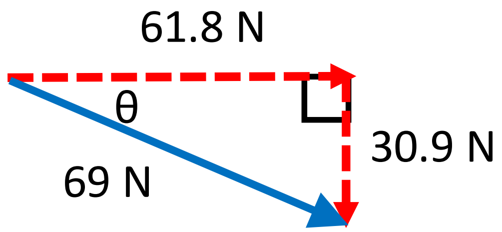
In the y-direction,

Sum of forces = 48 sin 40° = 30.9 N

$$\text{Net force} = \sqrt{61.8^2 + 30.9^2} = 69\,\text{N}$$

$$\theta = \tan^{-1}\left(\frac{30.9}{61.8}\right)$$

$$\theta = 27°$$



Example 16c

(c) An object is moving at 5.0 m s^{-1} due east. Its direction changes to due south with a speed of 7.5 m s^{-1}. Determine

(i) the change in **speed**.

(ii) the change in **velocity**.

What is the difference between change in speed and change in velocity??

Please study the suggested solution and learn the technique of calculating the change in velocity.

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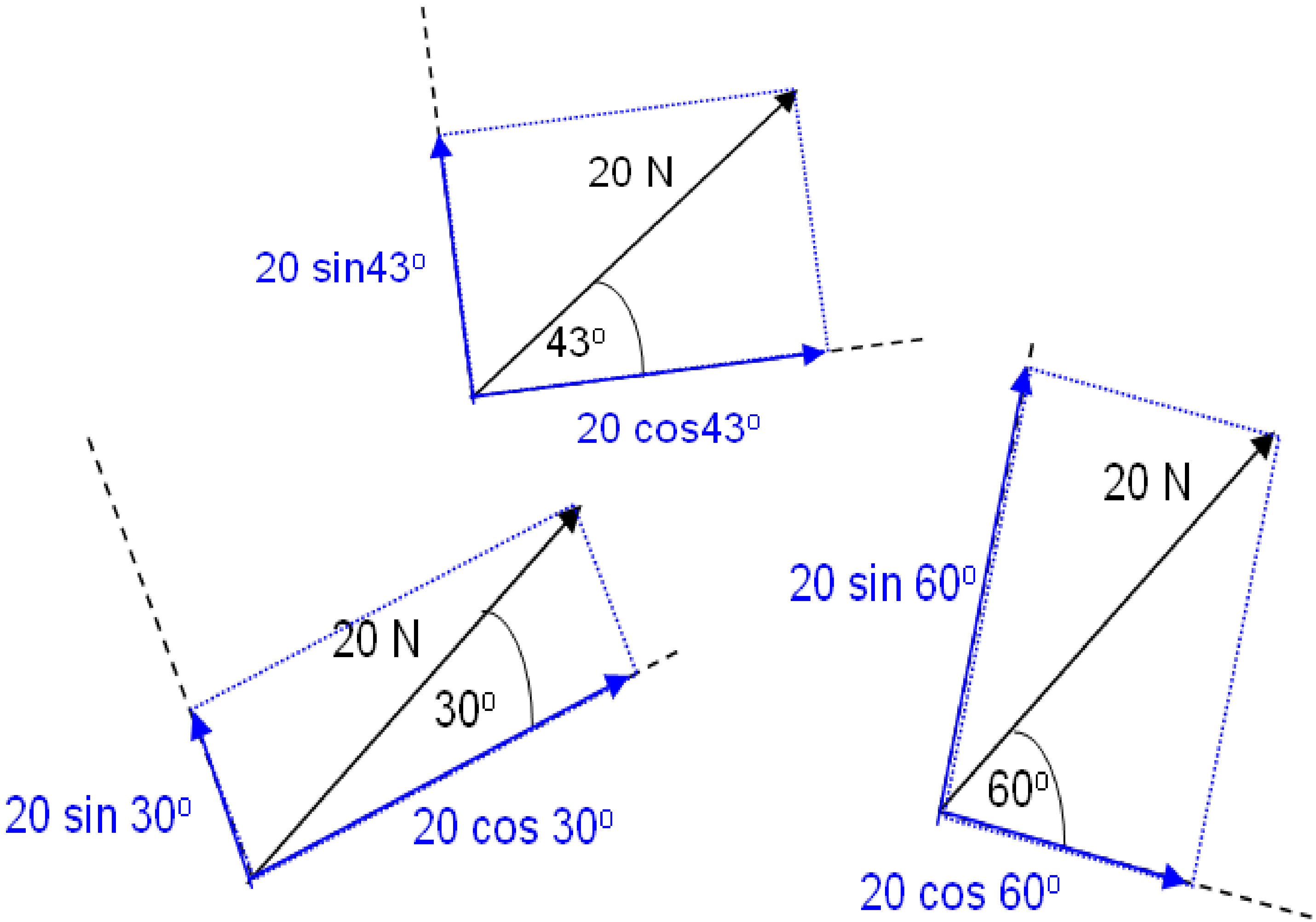
Resolution of vectors
Practice problem solutions



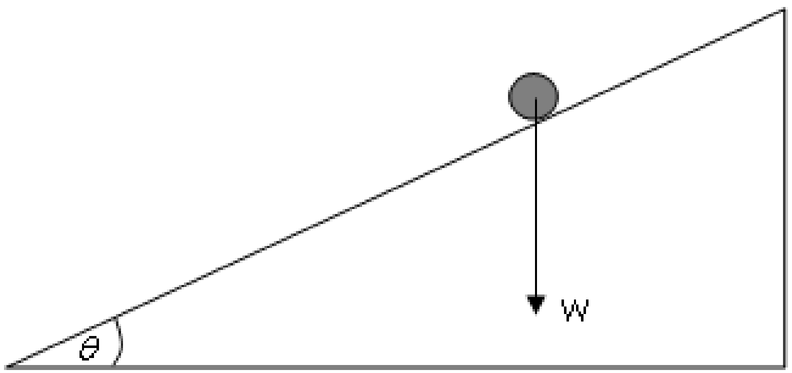
Example 17a

An object rests on the plane of an inclined slope as shown. The weight W acts vertically down. Draw components of the weight

(i) parallel to the slope, W_P

(ii) Perpendicular (normal), W_N to the slope.

Label the magnitude of the two components in terms of W and θ.

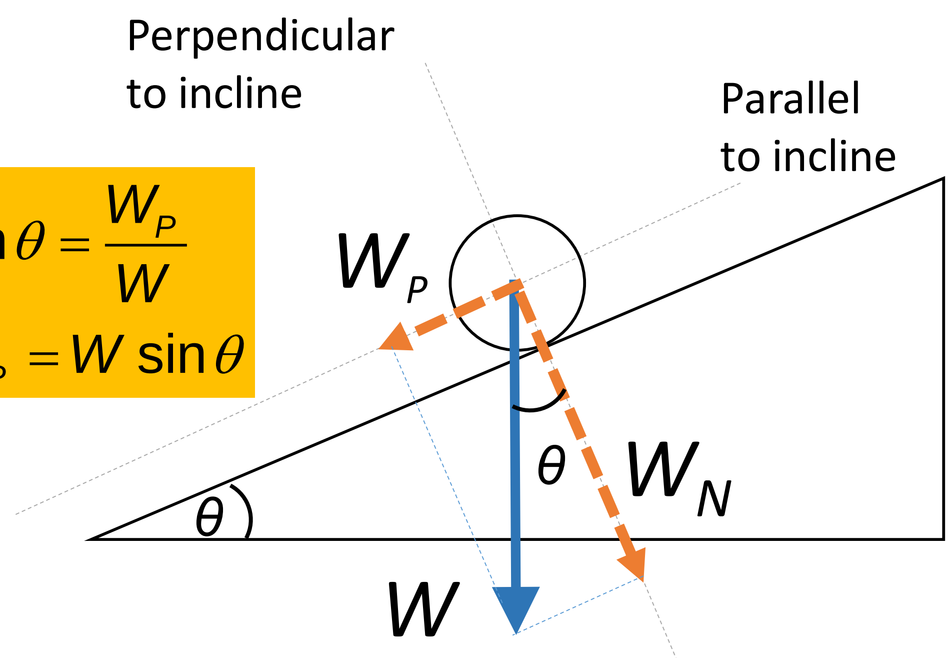


$$\sin\theta = \frac{W_P}{W}$$

$$W_P = W\sin\theta$$

$$\cos\theta = \frac{W_N}{W}$$

$$W_N = W\cos\theta$$

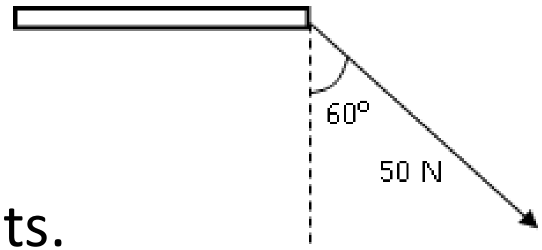


Example 17b

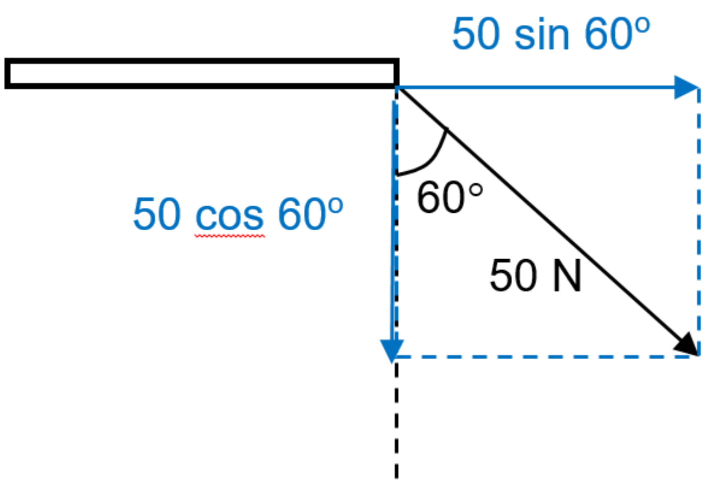
A force of 50 N acts on a horizontal plank at angle of 60 to the vertical as shown. Draw components of this force

(i) parallel to the plank,

(ii) perpendicular to the plank.



Determine the magnitude of these components.



(i) Component of force parallel to the plank,

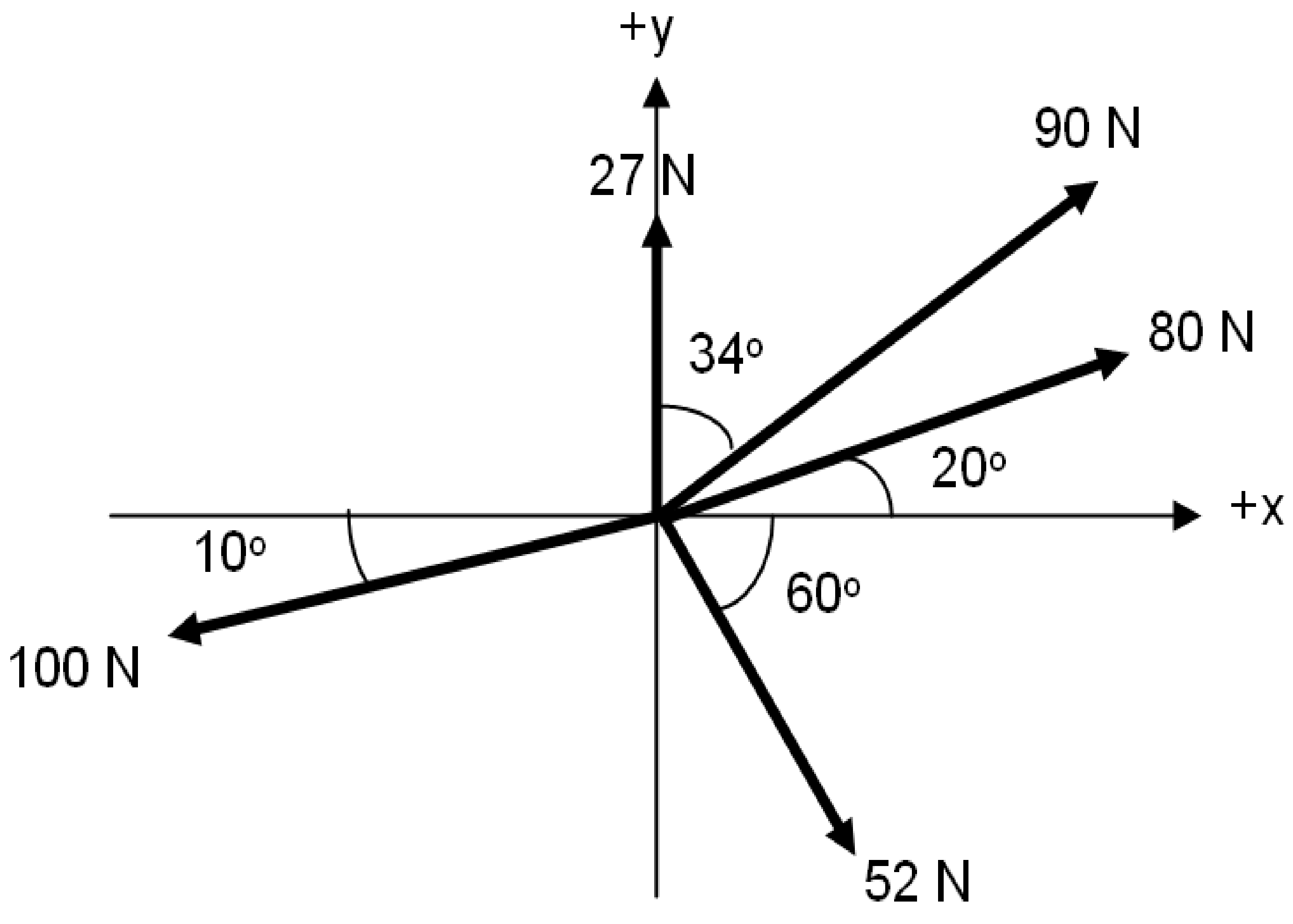
$$F_P = 50 \sin 60^o = 43.3\,N$$

(ii) Component of force perpendicular to the plank.

$$F_N = 50 \cos 60^o = 25.0\,N$$

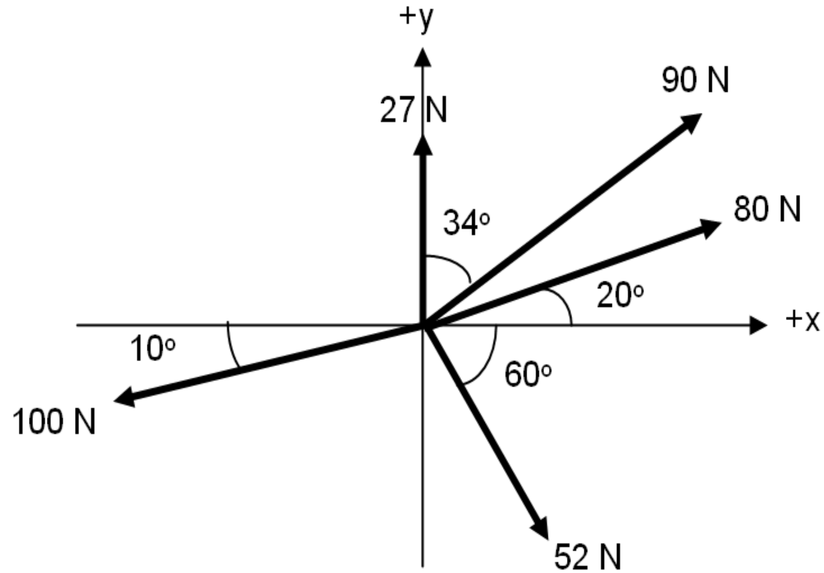
Example 18

Five forces shown act on an object. Find the resultant force.



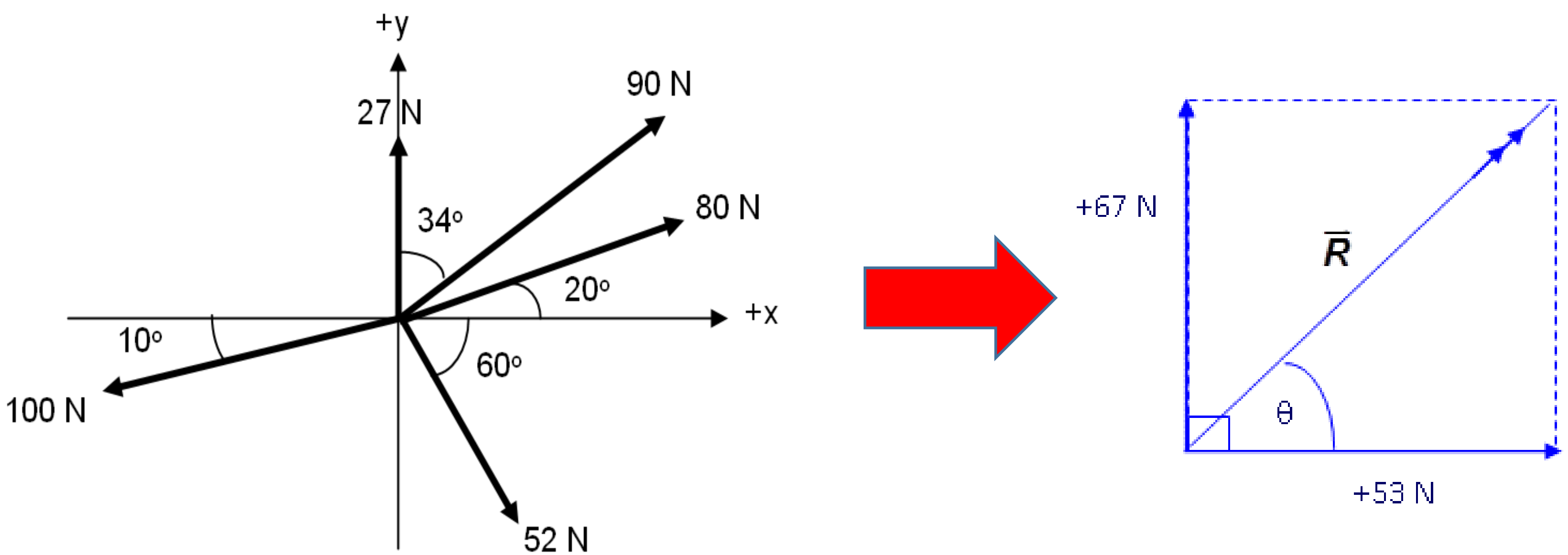
Example 18

The component method



Vector/N	x-component /N (+ →)	y-component /N (+↑)
80	80 cos20°	80 sin20°
90	90 sin34°	90 cos34°
27	0	27
100	-100 cos10°	- 100 sin10°
52	52 cos60°	- 52 sin60°
Resultant	53	67

Example 18



The magnitude of the resultant vector, $R = \sqrt{53^2 + 67^2} = 85.4\ N$

The direction of the resultant vector anti-clockwise from the positive x-direction,

$$\theta = \tan^{-1}\left(\frac{67}{53}\right) = 51.6^{o}$$