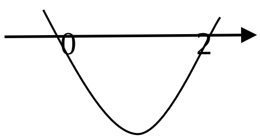
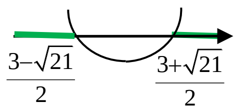
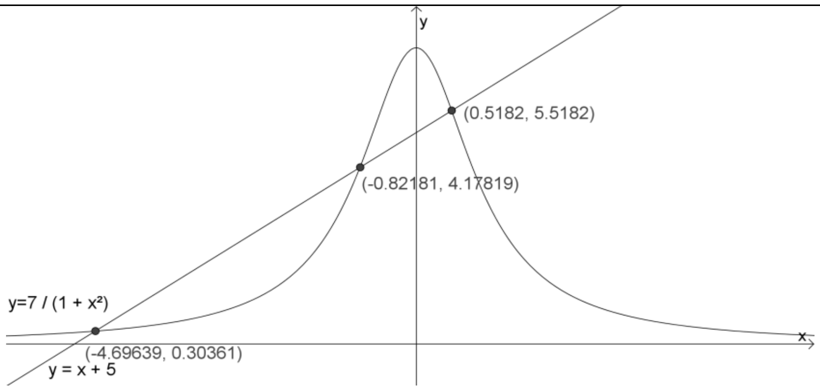
2023 JC1 H1 REVISION SET A-3
COMPLETE SOLUTIONS

Qn	EQUATIONS & INEQUALITIES
1	$6x^2+6y^2=60 \Rightarrow x^2+y^2=10$ ...(1) $12x+12y=48 \Rightarrow x+y=4$ ...(2) Substitute (2) into (1): $(4-y)^2+y^2=10 \Rightarrow 2y^2-8y+6=0$ $\Rightarrow y^2-4y+3=0 \Rightarrow (y-3)(y-1)=0$ $\Rightarrow y=3 \quad \therefore x=1$ since $y>x$
2	The equations are $c=1.2$ (1) $4a+2b+c=34.4$ (2) $9a-3b+c=-11.1$ (3) From GC, $a=2.5,\ b=11.6,\ c=1.2$
3	Let the price of 1 litre of A, B and C be a, b and c respectively. Given that $a+b+2c=9$ $b+c=3.50$ $2.5b+2c=2a \quad \Rightarrow 2a-2.5b-2c=0$ Using GC, $a=\$4,\ b=\$2,\ c=\$1.50$
4	$x^2+kx+11=3x+k \quad \Rightarrow x^2+(k-3)x+(11-k)=0$ $b^2-4\text{ac} \geq 0 \quad \Rightarrow \quad (k-3)^2-4(11-k)\geq 0$ $\Rightarrow \quad k^2-2k-35\geq 0 \quad \Rightarrow \quad k \leq -5$ or $k \geq 7$
5	$k-x=\dfrac{k}{2x} \quad \Rightarrow \quad 2kx-2x^2=k$ $\Rightarrow \quad 2x^2-2kx+k=0$ Since there is no intersection, discriminant < 0 $4k^2-4(2)(k)<0 \quad \Rightarrow \quad k(k-2)<0$  $\therefore \quad 0<k<2$
6 (a)	$5-x^2 \leq 2-3x \quad \Rightarrow \quad x^2-3x-3\geq 0$ Let $x^2-3x-3=0 \quad \Rightarrow \quad x=\dfrac{3\pm\sqrt{9-4(1)(-3)}}{2}=\dfrac{3\pm\sqrt{21}}{2}$  $\therefore \quad x \leq \dfrac{3-\sqrt{21}}{2}$ or $x \geq \dfrac{3+\sqrt{21}}{2}$
6 (b)	(i) $4x^2-24x+39 = 4\left(x^2-6x\right)+39$

Qn	EQUATIONS & INEQUALITIES
	$= 4\left[(x-3)^2 - 9\right] + 39 = 4(x-3)^2 + 3$ Since $(x-3)^2 \geq 0$ for all real values of *x*, we have $4(x-3)^2 \geq 0$ $\Rightarrow 4(x-3)^2 + 3 \geq 3 > 0$ $\therefore 4x^2 - 24x + 39 > 0$ for all real values of *x*. (ii) $\dfrac{4x^2 - 24x + 39}{(x+2)(x-1)} \leq 0$ From (i), $4x^2 - 24x + 39 > 0$ for all real values of *x*, so $(x+2)(x-1) < 0$ giving $-2 < x < 1$
7	$x^2 + 2k = 4 - kx \quad \Rightarrow \quad x^2 + kx + (2k - 4) = 0$ Discriminant = $k^2 - 4(1)(2k - 4) = (k - 4)^2 \geq 0$ Hence $x^2 + 2k = 4 - kx$ has real roots for all real values of *k*.
8	$x^2 + x + 7 \leq 2x^2 + 1 \quad \Rightarrow \quad x^2 - x - 6 \geq 0$ $\therefore (x - 3)(x + 2) \geq 0$ giving $x \leq -2$ or $x \geq 3$ (i) Replace *x* by ln *x* : $\ln x \leq -2$ or $\ln x \geq 3$ giving $0 < x \leq e^{-2}$ or $x \geq e^3$ (ii) Replace *x* by e^x : $e^x \leq -2$ (reject because $e^x > 0$) or $e^x \geq 3$ $x \geq \ln 3$
9	 Intersection points at $x = -4.70, -0.822$ or 0.518 Therefore, solution set is $\{x : x \in \mathbb{R}, -4.70 \leq x \leq -0.822 \text{ or } x \geq 0.518\}$ $\dfrac{7}{1+x^2} + x \leq 5 \Rightarrow \dfrac{7}{1+(-x)^2} \leq -x + 5$ Replace *x* by $-x$: $-4.70 \leq -x \leq -0.822 \text{ or } -x \geq 0.518$ $0.822 \leq x \leq 4.70 \text{ or } x \leq -0.518$

Qn	EQUATIONS & INEQUALITIES
10	$kx^2 + x + k > 4 - 2x$ $kx^2 + 3x + k - 4 > 0$ For quadratic curve to be positive, it must not cut the *x*-axis, i.e. no real solution, discriminant is negative ($b^2 - 4ac < 0$) and the curve must lie above the *x*-axis, so coefficient of x^2 is positive $3^2 - 4(k)(k-4) < 0$ and $k > 0$ $-4k^2 + 16k + 9 < 0$ $k > \frac{9}{2}$ or $k < -\frac{1}{2}$ Overall, $k > \frac{9}{2}$
11	$A + B + C = 280$ ------------ (1) $6.5A + 4B + 2C = 1395$ -------- (2) $2B - A = 10$ --------------- (3) Using GC, *A* = no. of adult tickets = 150 *B* = no. of student tickets = 80 *C* = no. of children tickets = 50
12	$5x + 3y + z = 593$ $2x + 7y + 5z = 829$ $6x + 4y + 2z = 778$ Using GC, $x = 72, y = 60, z = 53$ Mr Tan pays $= 3\times72 + 1\times60 + 5\times53 = 541$