

Temasek Junior College (IP) 2024 IP3 Intermediate & Advanced Mathematics June Holiday Homework

Instructions to all students

- 1. Complete all working on foolscap paper. Present all working clearly.
- 2. After completing all questions, self-mark using the answers provided on the last page.
- 3. For all incorrect answers, you are to <u>re-attempt the questions by doing the necessary</u> <u>corrections</u>.
- 5. If you require clarification for any errors made, please take the initiative to consult your Mathematics tutor via email or when you are back in College.
- 6. Submit your solutions and corrections to your tutor for checking during the first Mathematics lesson in Term 3 2024.

Intermediate Mathematics

1. Kesamet Bank offers two types of deposit accounts, the SimSaver account and the ComSavers account.

The SimSavers account offers a simple interest at 5% per annum and the ComSavers account offers an interest at 4.8% per annum, compounded half-yearly.

Jay opened and deposited \$10 000 into a ComSavers account.

(i) Find, to the nearest dollar, the amount of money Jay will have at the end of 4 years.

Tee opened and deposited \$10 000 into a SimSavers account.

(ii) Find, to the nearest dollar, the amount of money Tee will have at the end of 4 years.

At the end of 4 years, Tee intends to transfer all the money in her SimSaver account to the ComSaver account for another year.

- (iii) Explain, with supporting calculations, whether it is worthwhile for Tee to do so as compared to leaving her money in her SimSaver account. Show all working clearly.
- 2. Sketch the following graphs.
 - (a) $y = 3(x-2)^2 3$
 - (b) $y = -x^2 3$
 - (c) $y = x^2 + 6x + 13$

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- 3. (i) A quadratic graph has a minimum point at (3, -2) and cuts the y-axis at (0, 16). Find the equation of the curve.
 - (ii) Another quadratic graph cuts the *x*-axis at (-4, 0) and (6, 0) and passes through $\left(3, \frac{21}{2}\right)$. Find the equation of the curve.

4. Show that the expression
$$-x^2 + \frac{2}{5}x - \frac{2}{25}$$
 is always negative.

Hence, explain the implications of your results on the number of possible solutions of x for the equation $-x^2 + \frac{2}{5}x - \frac{2}{25} = 0$.

- 5. In 2018, the price of water was \$x per litre and a factory spent \$100 on its water bill in January.
 - (i) Express, in terms of *x*, the volume of water used in January.

In February, the price of water was reduced by \$0.14 per litre but the factory continued to spend \$100 on its water bill.

- (ii) Express, in terms of *x*, the volume of water used in February.
- (iii) If the difference in the volumes of water used in both months is 6 litres, formulate an equation in x and show that it simplifies to $6x^2 0.84x 14 = 0$.
- (iv) Hence, find the price of water per litre in February, showing your working clearly.

In 2019, a water conservation tax of a% was introduced such that the price of water is now \$1.97 per litre.

(v) Find the value of *a*, giving your answer correct to the nearest whole number.

- 6. Morph made several different objects from modelling clay. He used 500 cm3 of clay for each object.
 - (a) He made a square-based cuboid of height 2 cm. Calculate the exact length of a side of the square base.
 - (b) He made a pyramid with a base area of 150 cm^2 . Calculate the height of the pyramid.
 - (c) He made a sphere. Calculate the radius of the sphere, correct to 2 decimal places.
 - (d) He made a cone. Refer to the figure on the right. Then he cut through the cone, parallel to its base, to obtain a small cone and a frustum. The height of the small cone was two-fifths of the height of the full cone. Use a property of the volumes of similar objects to calculate the volume of clay in the small cone.



- 7. In the triangle *ABC*, angle $ABC = 90^{\circ}$ and *BC* is produced to *D*.
 - (a) Write down the value of $\cos \angle ACD$.
 - (b) Calculate the perpendicular distance from B to AC.



[Nov 2007/EM/Paper 1/Qn 12]

8. Three points, *A*, *B* and *C*, lie on a horizontal field. Angle $BAC = 75^{\circ}$ and the bearing of *C* from *A* is 217° . AB = 72 m and AC = 60 m.



- (a) Calculate
 - (i) the bearing of B from A,
 - (ii) *BC*,
 - (iii) angle ABC,
 - (iv) the bearing of C from B.
- (b) A girl standing at *B* is flying a kite. The kite, *K*, is vertically above *A*. The string, *BK*, attached to the kite is at 24° to the horizontal. Calculate the angle of elevation of the kite when viewed from *C*.

[Nov 2005/EM/Paper 2/Qn 8]

9. The cross-section of a tunnel, shaded in the figure below, is a major segment of a circle, centre *O* and radius 8 m. The total perimeter of the major segment *POQR* is 44 m.



Calculate

- (i) the magnitude, in radians, of reflex angle *POQ*,
- (ii) the area of triangle POQ,
- (iii) the total area of the cross-section of the tunnel.

[Nov 2010/EM/Paper 2/Qn 8(a)]

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- 10. PQRS is a parallelogram. P is (-4, 0), Q is (1, 0) and R is (9, 4).
 - (a) Find the coordinates of *S*.
 - (b) Find the coordinates of the midpoint of *PR*.
 - (c) Find the equation of the line *RS*.
 - (d) Find the equation of the line *QR*.
 - (e) Calculate the area of the parallelogram *PQRS*.

[Nov 2005/EM/Paper 1/Qn 21]

11. The diagram below shows a parallelogram *ABCD* with *AD* produced to *E*. *F* is the point of intersection of *CD* and *BE*. Angle $BAD = 68^{\circ}$ and angle $BFD = 97^{\circ}$.



- (a) Find angle *ABF*.
- (b) Prove that triangles *BCF* and *EDF* are similar.
- (c) State another triangle that is similar to *BCF* and *EDF*.
- (d) The ratio AD : DE = 3 : 2.
 - (i) Find the ratio AB : CF.
 - (ii) Given that the area of triangle $EDF = 9.72 \text{ cm}^2$, find the total area of the shape *ABCFE*.

[Nov 2014/EM/Paper 2/Qn 4]

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Advanced Mathematics

1. Solve the following equations.

(a)
$$2x + \sqrt{3 - 4x} = 0$$

(b)
$$2(4^x) + 4^{x+2} = 9\left(4^{-\frac{1}{2}}\right)$$

(c)
$$4^{x-1} + 16^x = 66$$

2. Solve the following simultaneous equations.

$$8(4^{y}) = 2^{2x-1}$$
$$3^{y}\sqrt{3^{x}} = 81$$

- 3. Without the use of a calculator, find the value of k for which $\frac{1}{\sqrt{2}} \left(\frac{\sqrt{12}\sqrt{8}}{3} \frac{2}{\sqrt{6}} + \frac{3\sqrt{216}}{2} \right) = k\sqrt{3}.$
- 4. A triangle ABC in which AB = AC has an area of 46 cm2. Given that its base BC is $(8\sqrt{3}-2\sqrt{2})$ cm, find the exact height and perimeter of the triangle.
- 5. (a) Find the smallest value of the integer *a* for which $ax^2 + 5x + 2$ is positive for all values of *x*. [Nov 2008/AM/Paper 1/Qn 10(a)]
 - (b) Find the set of values of *m* for which the curve $y = 2x x^2$ and the line y = mx + 1do not intersect. [Nov 2012/AM/Paper 2/Qn 8(i)]

6. Express
$$\frac{2x^3 + 5x^2 - 9x - 11}{2x^2 - x - 6}$$
 in partial fractions.

[modified Nov 2009/AM/Paper 2/Qn 2(i)]

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- 7. The function $f(x) = x^3 6x^2 + ax + b$, where a and b are constants, is exactly divisible by x-3 and leaves a remainder of -55 when divided by x+2.
 - (i) Find the value of *a* and of *b*.
 - (ii) Solve the equation f(x) = 0 [Nov 2005/AM/Paper 2/Qn 9]
- 8. (i) Given that $\log_8 x^3 = \log_4 u$, express *u* in terms of *x*.
 - (ii) Find the value of x for which $\log_4(x^2+5x) \log_8 x^3 = \frac{1}{\log_3 4}$.

[Nov 2012/AM/Paper 2/Qn 6(a)]

Intermediate Mathematics Answer Keys

- 1. (i) \$12 089 (ii) \$12 000 (iii) She should switch to ComSaver.
- 2. Use desmos (or any graphing software) to check your sketches.

3. (i)
$$y = 2(x-3)^2 - 2$$
 (ii) $y = -\frac{1}{2}(x+4)(x-6)$

4. There will be no solutions for *x*.

5. (i)
$$\frac{100}{x}$$
 litre (ii) $\frac{100}{x-0.14}$ litre (iv) \$1.46 (2 d.p.)
(v) 35
6. (a) $5\sqrt{10}$ cm (b) 10 cm (c) 4.92 cm (2 d.p.) (d) 32 cm³

- 7. (a) $-\frac{4}{5}$ (b) 4.8 units
- 8. (a)(i) 292° (ii) 80.9 m (3 s.f.) (iii) 45.8° (1 d.p.) (iv) 157.8°
 (b) 28.1° (1 d.p.)
- 9. (i) 3.5 radians (ii) 11.2 m^2 (3 s.f.) (iii) 123 m^2 (3 s.f.)

10. (a)
$$(4, 4)$$
 (b) $\left(\frac{5}{2}, 2\right)$ (c) $y = 4$ (d) $y = \frac{1}{2}x - \frac{1}{2}$ (e) 20 units²
11. (a) 83° (c) triangle *EAB* (d)(i) 5 : 3 (ii) 82.62 cm²

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Advanced Mathematics Answer Keys

1.	(a) $x = -\frac{3}{2}$ (b) $x = -1$ (c) $x = \frac{3}{2}$
2.	x = 4, y = 2
3.	k = 10
4.	Height = $\left(4\sqrt{3} + \sqrt{2}\right)$ cm, Perimeter = $\left(20 + 8\sqrt{3} - 2\sqrt{2}\right)$ cm
5.	(a) 4 (b) $0 < m < 4$
6.	$x+3+\frac{1}{x-2}-\frac{2}{2x+3}$
7.	(i) $a = 10, b = -3$ (ii) $x = 3$ or 2.62 or 0.382 (3 s.f.)
8.	(i) $u = x^2$ (ii) $x = \frac{5}{2}$