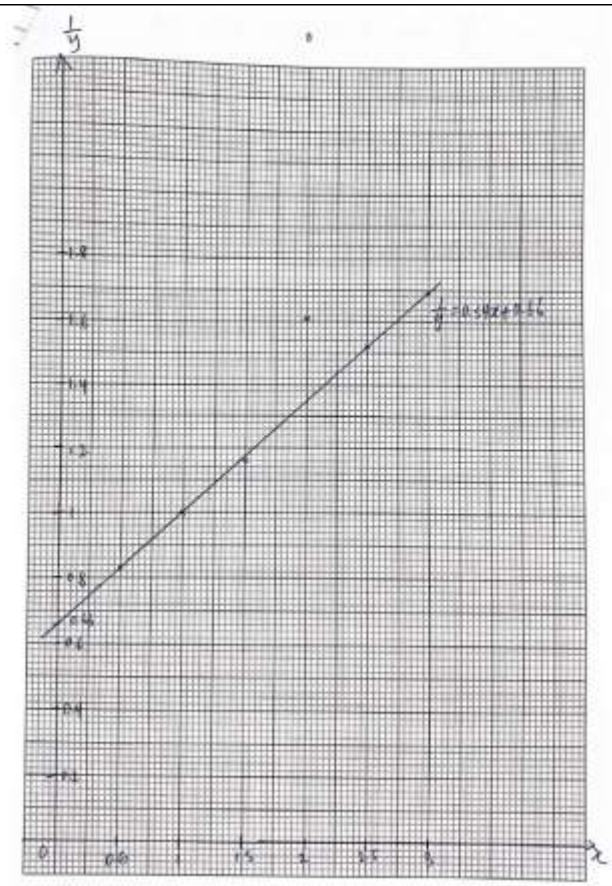
2021 KSS AM Prelim P2 (Solutions)

1ai	$2x^2+(4-6m)x+2m+2=0$ $b^2-4ac\geq 0$ $(4-6m)^2-4(2)(2m+2)\geq 0$ $16-48m+36m^2-8(2m+2)\geq 0$ $36m^2-64m\geq 0$ $m(9m-16)\geq 0$ $m\leq 0 \quad or \quad m\geq \frac{16}{9}$	
1aii	$2x^2+4x+2=6x-2$ By comparing with (i), m=1 <u>When m=1, it is not within the set of values of m for which there will be real roots</u>, <u>hence the curve will not meet the line/ the curve will not cut the line</u>.	
1b	$px^2+qx+p=-3q-3px$ $px^2+(3p+q)x+p+3q=0$ $b^2-4ac<0$ $(3p+q)^2-4p(p+3q)<0$ $5p^2-6pq+q^2<0$ $(q-5p)(q-p)<0$ $p<q<5p \quad (shown)$	
2i	$T_{r+1}=\binom{9}{r}(2x^{-1})^{9-r}(ax^2)^r=\binom{9}{r}2^{9-r}a^rx^{3r-9}$ $3r-9=-3 \quad \square \; r=2$ $512=\binom{9}{r}2^7a^2$ $a=-\frac{1}{3} \quad or \quad \frac{1}{3}\,(reject,\, a<0)$	
2ii	$\frac{1}{x^4}\,term=\left(\frac{1}{8x}\right)\left(\frac{512}{x^3}\right)+\left(\frac{x^2}{12}\right)\binom{9}{1}2^8\left(-\frac{1}{3}\right)x^{-6}=0$ Since the coefficient of the $\frac{1}{x^{-1}}$ term is 0, it does not exist in the expansion.	
3i	$\angle CBA=\angle BAC \quad (CA=CB)=\angle BCY \quad (alt\ seg\ theorem)$	

	Since $\angle CBA = \angle BCY$, <u>by the converse of alternate angles</u>, *AB*//*CY* (proven).	
3ii	$\angle CAY = \angle XBC \quad (\angle \text{ in same seg})$ $\angle CYA = \angle BAY \quad (AB\,//\,CY, alt\ \angle) = \angle BCX \quad (\angle \text{ in same seg})$ Since two pairs of corresponding angles are equal, ΔACY and ΔBXC are similar.	
4i		
4ii	$m = \dfrac{0.66-1}{0-1} = 0.34$ $\dfrac{1}{y} = 0.34x + 0.66$ Accept $0.32 \le m \le 0.37$ Accept $0.62 \le y-\text{int}ercept \le 0.68$	
4iii	$\dfrac{1}{y} = 1.35 \quad Accept\ 1.33 \le \dfrac{1}{y} \le 1.38$ $y = 0.74 \quad Accept\ \ 0.72 \le y \le 0.75$ Accept $1.33 \le \dfrac{1}{y}\ wrong\ value \le 1.38$	

5i	$ED = 2\sin\theta, \quad AB = 2\cos\theta$ $AF = 5\sin\theta, \quad BC = 5\cos\theta$ $P = 5\sin\theta + 2\cos\theta + 5\cos\theta + 5 + 2\cos\theta = 5 + 9\cos\theta + 5\sin\theta \quad (shown)$	
5ii	$5 + 5\sin\theta + 9\cos\theta = 5 + \sqrt{106}\cos(\theta - \tan^{-1}\frac{5}{9}) = 5 + \sqrt{106}\cos(\theta - 0.507)$ max value $= \sqrt{106} + 5 = 15.3\ (3s.f)$ when $\theta = 0.507 \quad (3s.f.)$	
5iii	Area $= \frac{1}{2}(2\cos\theta + 2\cos\theta + 5\cos\theta)(5\sin\theta) = \frac{45}{4}\sin 2\theta$ maxArea occurs $at\ \theta = \frac{\pi}{4}, i.e. \sin 2(\frac{\pi}{4}) = 1$ but max P occurs when $\theta = 0.507$, hence max Perimeter does not imply max area	
6ia	$m = \pi + k$	
6ib	$l - k = \frac{\pi}{2}$	
6ii	The number of solutions of $5 - 3\sin 2x = 4 + \cos\frac{1}{2}x$ is determined by the number of intersections between the graphs of $y = 5 - 3\sin 2x$ and $y = 4 + \cos\frac{1}{2}x$.	
7i	$h^2 + 2h - 1680 = 0$ $(h-40)(h+42) = 0$ $h = 40 \quad , \quad h = -42 \quad$ (reject)	
7iia	$\frac{dh}{dt} = -\frac{1}{3}t$ $h = \int -\frac{1}{3}t\ dt = -\frac{1}{6}t^2 + c$ $t = 0, c = 40$ $\therefore h = -\frac{1}{6}t^2 + 40 \quad (shown)$	

7iib	$\frac{dV}{dh}=2h+2=82-\frac{t^2}{3}$ $\frac{dV}{dt}=\frac{dV}{dh}\times\frac{dh}{dt}=(82-\frac{t^2}{3})\left(-\frac{1}{3}t\right)=-\frac{82}{3}t+\frac{t^3}{9}$ $t=6,\ \frac{dV}{dt}=-140cm^3/s$	
7iic	$h=0$, $40=\frac{1}{6}t^2$ $t=15.49=15.5s\ (3s.f.),\quad -15.49$ (reject)	
8a	$\log_a 8\times\log_{16}a=\frac{\log_2 2^3}{\log_2 a}\times\frac{\log_2 a}{\log_2 2^4}=\frac{3}{4}$	
8bi	$\frac{\log_x x^2}{\log_x 4x^{-2}}=\frac{2}{\log_x 2^2-2}=\frac{2}{2p-2}=\frac{1}{p-1}\quad (shown)$	
8bii	$p+\frac{1}{p-1}=3$ $p^2-4p+4=0$ $(p-2)^2=0$ $p=2$ $2=\log_x 2$ $x^2=2$ $\therefore x=\sqrt{2}\quad \text{or}\quad -\sqrt{2}\quad$ (reject)	
9ai	$C=1000e^{0.15(12)}=6050$	
9aii	$m=12$ $\text{Value}=125800-1000e^{0.15\times 12}=119750.3=\$119750$	
9aiii	$28000=1000e^{0.15m}$ $m=\frac{\ln 28}{0.15}\quad =22.2$ Nearest month = 23	

9b	$2x\sqrt{3}+5x\sqrt{5}=3x\sqrt{5}+2\sqrt{3}$ $x=\frac{\sqrt{3}}{\sqrt{3}+\sqrt{5}} \quad =\frac{\sqrt{3}}{\sqrt{3}+\sqrt{5}}\times\frac{\sqrt{3}-\sqrt{5}}{\sqrt{3}-\sqrt{5}}=\frac{3-\sqrt{15}}{3-5} \quad =\frac{\sqrt{15}-3}{2}$ $\therefore a=-3$ and $b=15$	
10i	$F=kv^2$ $25=k(40^2)$ $k=\frac{1}{64}$ $\therefore F=\frac{1}{64}v^2$	
10ii	$C=\frac{\text{cost in \$/hr}}{\text{speed in km/h}}=\frac{\frac{v^2}{64}+100}{v}=\frac{v}{64}+\frac{100}{v} \quad (shown)$	
10iii	$\frac{dC}{dv}=\frac{1}{64}-\frac{100}{v^2}$ $\frac{dC}{dv}=0$ $\frac{1}{64}-\frac{100}{v^2}=0$ $v^2=6400$ $v=80 \quad or \quad -80\,(reject)$ $\frac{d^2C}{dv^2}=\frac{200}{v^3}$ $v=80\ ,\ \frac{d^2C}{dv^2}=\frac{200}{80^3}>0\Rightarrow$ minimum $\therefore v=80km/h$	
10iv	$C=\frac{80}{64}+\frac{100}{80}=\$2.50/km$	
10v	All costs remain constant, no changes, no delay or emergencies to delay time. Machine runs at constant speed.	