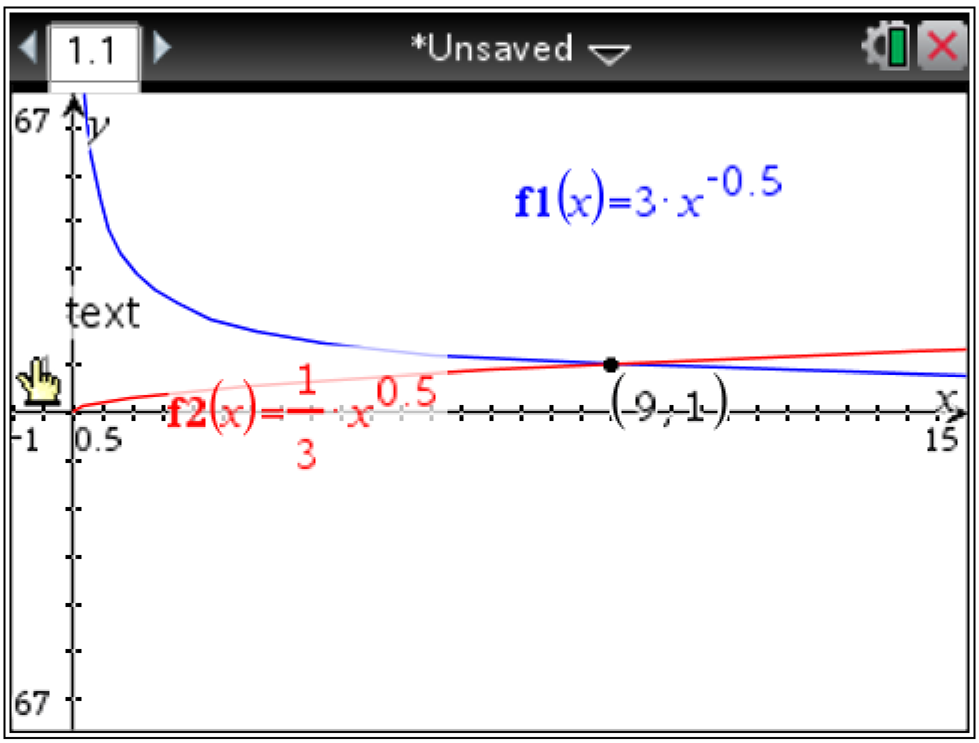
Solutions to AM Paper 1 EOY 2016

1. $3x^{-\frac{1}{2}} = \frac{1}{3}x^{\frac{1}{2}}$

$x = 9$

When x=9 $y = 3x^{-\frac{1}{2}}$ = $3(9)^{-0.5}$= 1

The point of intersection is (9 , 1)



2 (i) Area of triangle = $(\frac{11}{2}+3\sqrt{2})$

$$\frac{1}{2}\left(3+\sqrt{2}\right)\left(b+3\sqrt{2}\right)\sin 45^0=\frac{11}{2}+3\sqrt{2}$$

$$\left(3+\sqrt{2}\right)\left(b+3\sqrt{2}\right)\sin 45^0=2\sqrt{2}(\frac{11}{2}+3\sqrt{2})$$

Comparing both sides

$$3b+6=12$$

$b=2$ (shown)

(ii) Perpendicular distance from B to AC = $\dfrac{2(\frac{11}{2}+3\sqrt{2})}{(3+\sqrt{2})}$

$$= \frac{(11+6\sqrt{2})\left(3-\sqrt{2}\right)}{(9-2)}$$

$$= \frac{(33-12-11\sqrt{2}+18\sqrt{2})}{7}$$

$$= \frac{21+7\sqrt{2})}{7}=3+\sqrt{2}$$

3. (a) $P = P_o e^{kt}$

From question given when t = 0 , P = 50

Therefore $P = 50e^{kt}$

Also P = 960 when t = 8

$$960 = 50e^{8k}$$

$$e^{8k} = \frac{96}{5}$$

$$8k = \ln\left(\frac{96}{5}\right)$$

$$k = \frac{1}{8}\ln\left(\frac{96}{5}\right) = 0.369$$

(b) $P = 50e^{0.369(24)} = 351000$

(c) $15(50) = 50e^{kt}$

$$15 = e^{0.369t}$$

$t = \frac{1}{0.369}\ln(15)$ = 7.33 hours

4 Let $\frac{2(x^3-3)}{(x+3)(x-1)^2} = \frac{A}{x+3} + \frac{B}{x-1} + \frac{C}{(x-1)^2}$

$$2(x^2-3) = (A)(x-1)^2 + B(x+3)(x-1) + C(x+3)$$

Let x=1 : -4 = 4C

C= -1

Compare coefft of x^2 : 2 = A + B _

Let x = -3 : 12 = $A(-4)^2$

A = 3/4

Sub in 1 B= 5/4

$$\frac{2(x^2-3)}{(x\ \ +3)(x-1)^2} = \frac{3}{4(x\ \ +3)} + \frac{5}{4(x-1)} - \frac{1}{(x-1)^2}$$

5 (a) let $f(x) = x^3 + 3x^2 + hx + k$

By Factor Theorem : $f(2) = 0$

$$8 + 12 + 2h + k = 0$$
$$2h + k = -20$$

By Remainder Theorem : $f(-1) = 30$

$$-1 + 3 - h + k = 30$$
$$-h + k = 28$$

Solving the equations simultaneously $2h + k = -20$ --------------1

$-h + k = 28$ ---------------2

1 (-) 2

$$3h = -48$$
$$h = -16$$

When $h = -16$, $k = 28 - 16$ =12

(b) (i) Remainder = $f(3) = (3-2)(3+6)(3-1) = 18$

(ii) Let $f(x) = (x-2)(x^2 + bx - 6)$ (By inspection)

Compare coefft of *x* on both sides

$$-16 = -6 - 2b$$
$$b = 5$$

Therefore $f(x) = (x-2)(x^2 + 5x - 6)$

$$f(x) = (x-2)(x+6)(x-1)$$

6 (a) $4\cos ec^2 x = 3\cot x + 5$

$4(1+\cot^2 x) = 3\cot x + 5$

$4\cot^2 x - 3\cot x - 1 = 0$

$(4\cot x + 1)(\cot x - 1) = 0$

$(4\cot x + 1) = 0 \quad or \quad (\cot x - 1) = 0$

$\cot x = -\frac{1}{4} \quad or \quad \cot x = 1$

$\tan x = -4 \quad or \quad \tan x = 1$

$x = 104.0^o,\ 284.0^o \quad or \quad x = 45^o,\ 225^o$

(b) $\tan(2z-1) = 0.6$

$-1 < 2z - 1 < 9$

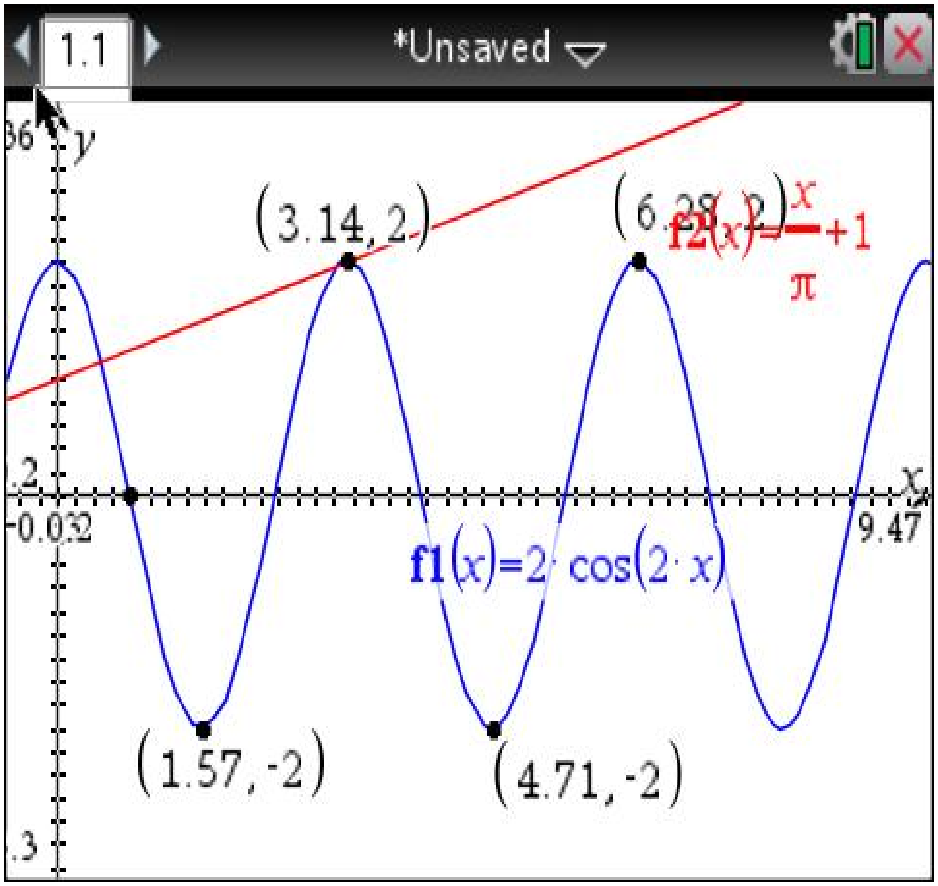
(2z-1) lies in the I and III quadrant.

Basic Reference angle = $\tan^{-1}(0.6)$

Max (2z-1) = 2π+ $\tan^{-1}(0.6)$)

= 3.91 (3 sf)

7 (a)



(ii) $2\pi\cos 2x - \pi = x$

$$\frac{2\pi\cos 2x}{\pi} - 1 = \frac{x}{\pi}$$

$$2\cos 2x = \frac{x}{\pi} + 1$$

The line to be inserted is y = (x/π)+1

(iii)

(iv) from sketch there are 2 solutions to the given equation.

(b) A lies in the II quadrant-------------------------M1

(i) $\sec A = -\frac{4}{\sqrt{7}}$

(ii) $\cos(-A) = -\frac{\sqrt{7}}{4}$

(iII) $\sin(\frac{\pi}{2} - A)$

$= \cos A$

$= -\frac{\sqrt{7}}{4}$

8. (a) $\log_2(x+1) - \log_4(x-3) = 2$

$\log_2(x+1) - \frac{1}{2}\log_2(x-3) = 2$

$\log_2(x+1) - \log_2(x-3)^{\frac{1}{2}} = 2$

$\log_2 \frac{(x+1)}{(x-3)^{\frac{1}{2}}} = 2$

$\frac{(x+1)}{(x-3)^{\frac{1}{2}}} = 4$

$(x+1)^2 = 16(x-3)$

$x^2 - 14x + 49 = 0$

$(x-7)^2 = 0$

$x = 7$

(b) $e^x(e^x + 3) = 18$

$e^{2x} + 3e^x - 18 = 0$

$(e^x + 6)(e^x - 3) = 0$

$e^x = -6 \quad or \quad e^x = 3$

X = *ln* (3)= 1.10 (3 sf)

(c) $4(\lg x)^2 + (\lg y)^2 = 4(\lg x)(\lg y)$

$(2\lg x - \lg y)^2 = 0$

$2\lg x = \lg y$

$\lg y = \lg x^2$

$y = x^2$