HWA CHONG INSTITUTION
JC2 Preliminary Examination
Higher 3

CANDIDATE NAME	

CENTRE NUMBER						INDEX NUMBER				

PHYSICS 9814/01

Paper 1 **21 September 2023**

3 hours

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your Centre number, index number and name in the spaces at the top of this page.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams, graphs or rough working.
Do not use staples, paper clips, glue or correction fluid.

The use of an approved scientific calculator is expected, where appropriate.

Section A
Answer **all** questions.
You are advised to spend about 1 hour and 50 minutes on Section A.

Section B
Answer **two** questions only.
You are advised to spend about 35 minutes on each question in Section B.

The number of marks is given in brackets [ ] at the end of each question or part question.

For Examiner's Use		
Section A		
1		**7**
2		**9**
3		**11**
4		**8**
5		**6**
6		**19**
Section B		
7		**20**
8		**20**
9		**20**
Deductions		
Total		**100**

Data

speed of light in free space	c	=	3.00×10^{8} m s^{-1}
permeability of free space	μ_0	=	$4\pi \times 10^{-7}$ H m^{-1}
permittivity of free space	ε_0	=	8.85×10^{-12} F m^{-1}
			$(1/(36\pi)) \times 10^{-9}$ F m^{-1}
elementary charge	e	=	1.60×10^{-19} C
the Planck constant	h	=	6.63×10^{-34} J s
unified atomic mass constant	u	=	1.66×10^{-27} kg
rest mass of electron	m_e	=	9.11×10^{-31} kg
rest mass of proton	m_p	=	1.67×10^{-27} kg
molar gas constant	R	=	8.31 J K^{-1} mol^{-1}
the Avogadro constant	N_A	=	6.02×10^{23} mol^{-1}
the Boltzmann constant	k	=	1.38×10^{-23} J K^{-1}
gravitational constant	G	=	6.67×10^{-11} N m^{2} kg^{-2}
acceleration of free fall	g	=	9.81 m s^{-2}

Formulae

uniformly accelerated motion	s	=	$ut + \frac{1}{2}at^2$
	v^2	=	$u^2 + 2as$
moment of inertia of rod through one end	I	=	$\frac{1}{3}ML^2$
moment of inertia of hollow cylinder through axis	I	=	$\frac{1}{2}M\left(r_1^2 + r_2^2\right)$
moment of inertia of solid sphere through centre	I	=	$\frac{2}{5}MR^2$
moment of inertia of hollow sphere through centre	I	=	$\frac{2}{3}MR^2$
work done on/by a gas	W	=	$p\Delta V$
hydrostatic pressure	p	=	ρgh
gravitational potential	ϕ	=	$-Gm/r$
Kepler's third law of planetary motion	T^2	=	$\frac{4\pi^2 a^3}{GM}$
temperature	T / K	=	T / °C + 273.15

pressure of an ideal gas	p	=	$\frac{1}{3}\frac{Nm}{V}\langle c^2 \rangle$
mean translational kinetic energy of an ideal gas molecule	E	=	$\frac{3}{2}kT$
displacement of particle in s.h.m.	x	=	$x_0 \sin \omega t$
velocity of particle in s.h.m.	v	=	$v_0 \cos \omega t$
		=	$\pm\omega\sqrt{\left(x_0^2 - x^2\right)}$
electric current	I	=	$Anvq$
resistors in series	R	=	$R_1 + R_2 + \ldots$
resistors in parallel	$1/R$	=	$1/R_1 + 1/R_2 + \ldots$
capacitors in series	$1/C$	=	$1/C_1 + 1/C_2 + \ldots$
capacitors in parallel	C	=	$C_1 + C_2 + \ldots$
energy in a capacitor	U	=	$\frac{1}{2}CV^2$
electric potential	V	=	$\frac{Q}{4\pi\varepsilon_0 r}$
electric field strength due to a long straight wire	E	=	$\frac{\lambda}{2\pi\varepsilon_0 r}$
electric field strength due to a large sheet	E	=	$\frac{\sigma}{2\varepsilon_0}$
alternating current/voltage	x	=	$x_0 \sin \omega t$
magnetic flux density due to a long straight wire	B	=	$\frac{\mu_0 I}{2\pi d}$
magnetic flux density due to a flat circular coil	B	=	$\frac{\mu_0 N I}{2r}$
magnetic flux density due to a long solenoid	B	=	$\mu_0 n I$
energy in an inductor	U	=	$\frac{1}{2}LI^2$
RL series circuits	τ	=	$\frac{L}{R}$
RLC series circuits (underdamped)	ω	=	$\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$
radioactive decay	x	=	$x_0 \exp(-\lambda t)$
decay constant	λ	=	$\frac{\ln 2}{t_{1/2}}$

Section A

Answer **all** questions in this section.

You are advised to spend about 1 hour 50 minutes on this section.

1 **(a)** Explain what is meant by an inertial reference frame.

…….

…….

……… [2]

(b) Two identical carts A and B move toward each other on a low-friction track.

The speed of cart A is $2u$, while the speed of cart B is $3u$ in the Earth reference frame. The system of the two carts has kinetic energy K.

State and explain another inertial frame of reference in which the *kinetic energy* of the two-cart system has the same kinetic energy K.

………....

…….

…….

………………………………………………………………………………………………………..… [2]

(c) An object of mass m moving with a speed of u strikes a stationary object that has mass $0.5m$.

Determine the fraction of the initial kinetic energy of the system that is convertible to internal energy of the system.

[3]

[Total: 7]

2 A uniform solid reel of mass $2M$ is made of three discs of equal thickness d, the outer two being of radius $2R$ and the middle one of radius R, fixed together so that the discs have a common axis.

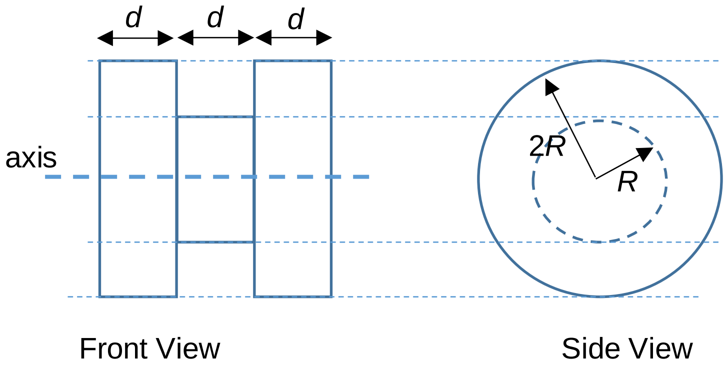


Fig. 2.1

(a) **(i)** Express the density of the reel in terms of M, R and d.

[1]

(ii) Hence, show that the moment of inertia I of the reel about its axis is given by $I = \frac{11}{3}MR^2$.

[3]

(b) A light inextensible string, wound several times around the rough central disc of the reel, passes over a fixed smooth peg at B and carries a box of mass *M* at its free end. The reel stands on a rough plane inclined at 30° to the horizontal. The straight portion of the string from B to the reel is parallel to the slope of the plane and the portion BC is vertical.

The box falls vertically with the string taut, and the reel, whose axis is horizontal, rolls up the inclined plane without slipping.

Find an expression for the acceleration of the box in terms of the acceleration of free fall, *g*.

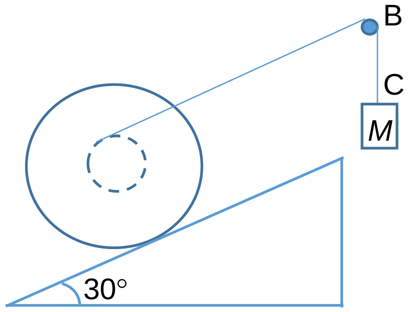


Fig. 2.2

[5]

[Total: 9]

3 **(a)** State Kepler's first law of planetary motion.

………

……… [1]

(b) A rocket is to be fired from Earth, to reach Venus by means of a Hohmann transfer orbit. While in the transfer orbit, the only force on the rocket will be the gravitational force due to the Sun.

(i) On Fig. 3.1, draw and label the transfer orbit from Earth to Venus.

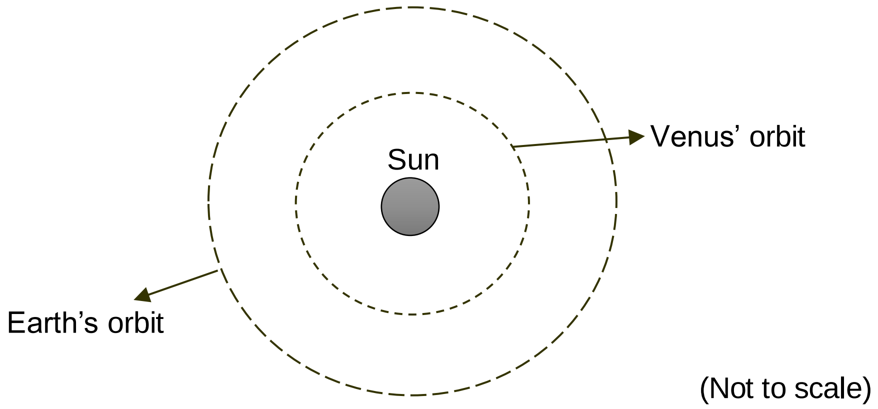


Fig. 3.1 [2]

(ii) It can be shown that for a small mass *m* in an elliptical orbit of semi-major axis *a* about a large mass *M*, the total mechanical energy *E* is $-\dfrac{GMm}{2a}$.

Show that the rocket should enter the transfer orbit with a speed of 27.2 km s^{-1}, if it is to reach the orbit of Venus.
Assume that Earth and Venus move in coplanar circular orbits centred at the Sun.

The radius of Earth's orbit about the Sun is 1.50×10^{11} m
The radius of Venus' orbit about the Sun is 1.08×10^{11} m
The mass of the Sun is 1.99×10^{30} kg

[4]

(iii) Calculate the speed of the rocket when it reaches the orbit of Venus.

speed = …………………….. m s^{-1} [2]

(iv) Calculate the time of flight for the transfer from Earth to the orbit of Venus.

time = ………………… s [2]

[Total: 11]

4 Two identical electromagnetic point sources P and Q emit in-phase vertically-polarised microwaves of wavelength 1.0 cm. A microwave detector moves through the four points G, H, J and K as shown in Fig. 4.1. The detector detects destructive interference, constructive interference, constructive interference, destructive interference and constructive interference at points G, H, J, K, respectively.

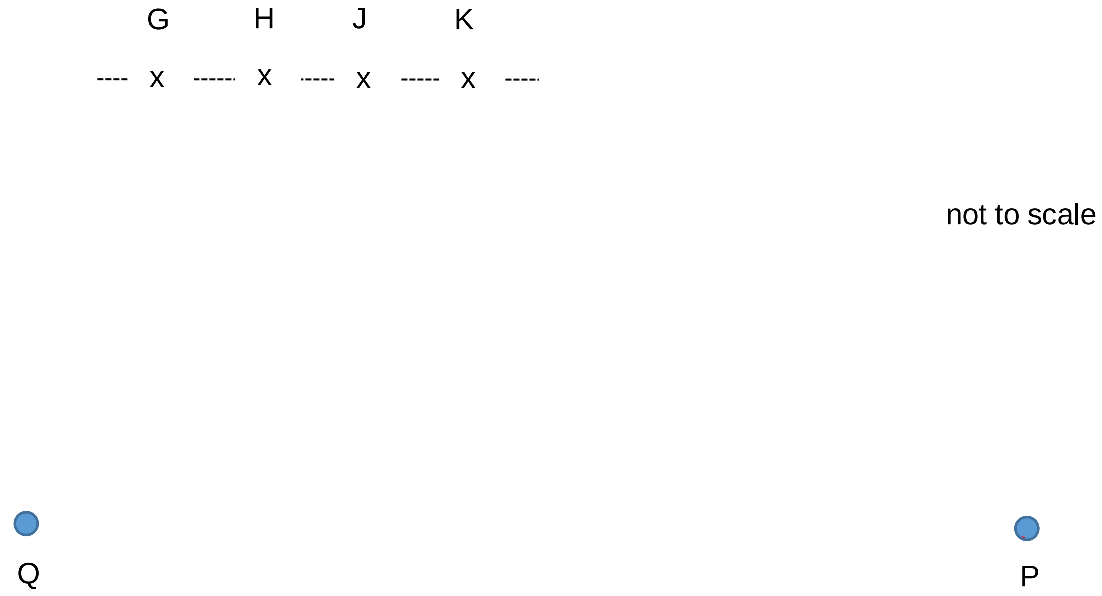


Fig. 4.1

The amplitude and intensity of the microwaves from point source P alone at point H are A_o and I_o, respectively. The intensity due to a point source at a distance *d* from the source is inversely proportional to the square of the distance *d*.

(a) The distances between point H and point source Q and between point H and point source P are 7.0 cm and 10.0 cm, respectively.

Show that the resultant intensity at point H due to the interference of the waves emitted from the point sources is 5.90 I_o.

[2]

(b) The distance between the point J and the point source Q is 7.3 cm.

Determine the resultant intensity at point J, in terms of I_o, due to the interference of the two point sources.

resultant intensity = ………………… I_o [4]

(c) Sketch on Fig 4.2 the variation with distance of the resultant intensity, starting from point G, through points H and J, and ending at point K. Distinguish peaks and troughs if any.

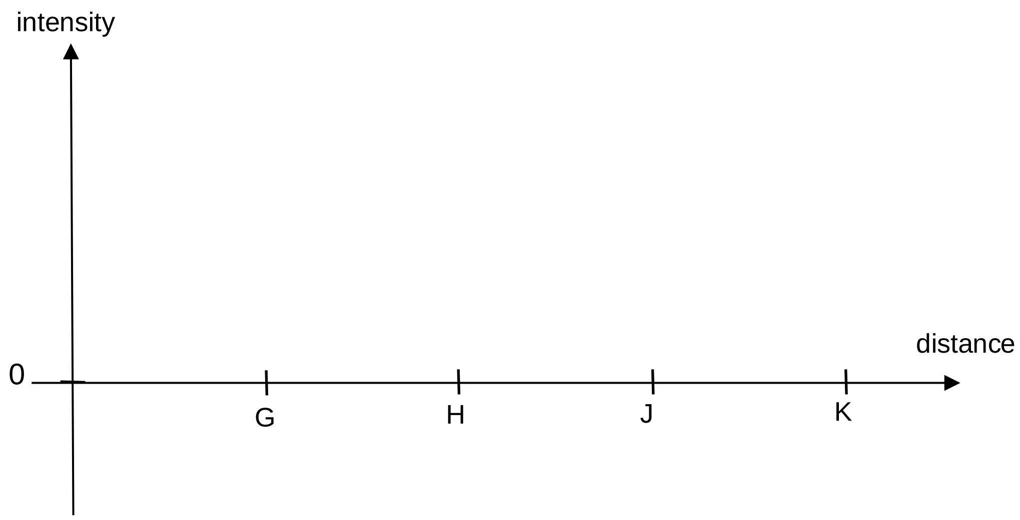


Fig. 4.2

[2]

[Total: 8]

5 The visible spectrum of a gas contains four lines. The photons corresponding to the mentioned lines are emitted when the outermost electron in the atom undergoes a transition from a higher excited energy level to a lower energy level. The lower energy level is the same for the four lines.

Assume there are only seven energy levels, and three out of the four visible lines have wavelengths of 410 nm, 434 nm and 656 nm.

A theory suggests that the wavelengths λ in the spectrum of this gas are given by the equation

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

where R is a constant, n_1 and n_2 are positive integers and $n_2 > n_1$.

Based on the above equation, the graph of $\left(\frac{1}{\lambda}\right)$ against $\left(\frac{1}{n_2^2}\right)$ is a straight line for a fixed value of n_1. Using the graph, the values of R and n_1 can be determined.

Tables 5.1 and 5.2 show the values of λ and n_2 for some of the transitions.

λ / nm	$\left(\frac{1}{\lambda}\right)$ / m^{-1}
410	2.44×10^6
434	2.30×10^6
656	1.52×10^6

Table 5.1

n_2	$\left(\frac{1}{n_2^2}\right)$
3	0.111
4	0.0625
5	0.0400
6	0.0277
7	0.0204

Table 5.2

(a) On the following graph,

(i) draw 3 horizontal lines which correspond to the values of $\left(\frac{1}{\lambda}\right)$ in Table 5.1, [1]

(ii) draw 5 vertical lines which correspond to the values of $\left(\frac{1}{n_2^2}\right)$ in Table 5.2. [1]

(iii) The values of $\left(\frac{1}{\lambda}\right)$ with corresponding $\left(\frac{1}{n_2^2}\right)$ which satisfy the equation are the coordinates of **some** of the points of intersections of the lines drawn in **(i)** and **(ii)**. Draw a line that passes through these points. [1]

(b) Using the graph, determine

(i) the value of R and n_1.

R = ………………. m^{-1} [1]
n_1 = …………..…. [1]

(ii) the wavelength of fourth visible line.

wavelength = ……………… nm [1]

[Total: 6]

6 The reality of the forces acting on the moving charges in a conductor in a magnetic field is strikingly demonstrated by the Hall effect, an effect analogous to the transverse deflection of an electron beam in a magnetic field in a vacuum.

To describe this effect, let's consider a conductor in the form of a flat strip, as shown in Fig. 6.1a. The current is in the direction of the +x-axis and there is uniform magnetic field B_y perpendicular to the plane of the strip, in the +y-direction. The drift velocity of the moving charges has magnitude v_d . Fig. 6.1a shows the case of negative charges, such as electrons in a metal, and Fig. 6.1b shows positive charges.

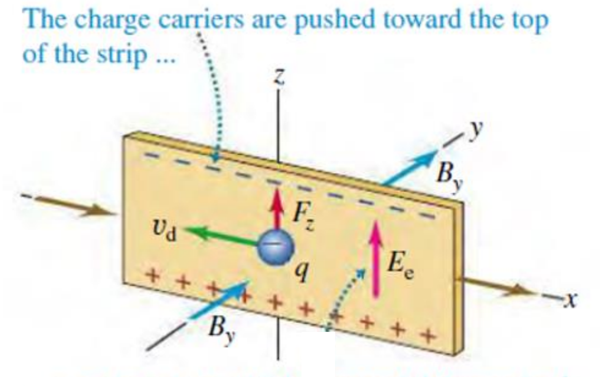


Fig. 6.1a (negative charge carriers)

In both cases the magnetic force is upward, just as the magnetic force on a conductor is the same whether the moving charges are positive or negative. In either case a moving charge is driven toward the upper edge of the strip by the magnetic force $F_z = |q| v_d B_y$ where q is the elementary charge.

If the charge carriers are electrons, as in Fig. 6.1a, an excess negative charge accumulates at the upper edge of the strip, leaving an excess positive charge at its lower edge. This accumulation continues until the resulting transverse electrostatic field E becomes large enough to cause a force that is equal and opposite to the magnetic force. After that, there is no longer any net transverse force to deflect the moving charges. This electric field causes a transverse potential difference between opposite edges of the strip, called the Hall voltage. The polarity depends on whether the moving charges are positive or negative. Experiments show that for metals the upper edge of the strip in Fig. 6.1a does become negatively charged, showing that the charge carriers in a metal are indeed negative electrons.

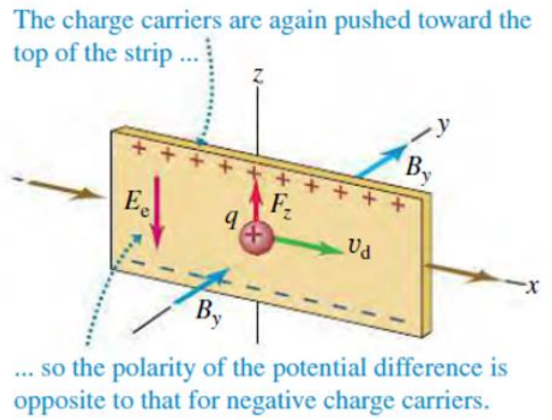


Fig. 6.1b (positive charge carriers)

However, if the charge carriers are positive, as in Fig. 6.1b, then positive charge accumulates at the upper edge, and the potential difference is opposite to the situation with negative charges.

Measuring the Hall voltage for a material can provide information about the sign of the mobile charges and about the number of them per unit volume. For example, Hall effect measurements have confirmed that in some metals and semiconductors, it is positive holes in the electron sea rather than the electrons themselves that are the dominant charge carriers.

(a) **(i)** Explain why the directions of magnetic force in Fig. 6.1a and Fig. 6.1b are both upwards.

....

....

....

.... [2]

(ii) Explain the role of applying the magnetic field to the conductor and creating a Hall voltage.

……

……

……

………………………………………………………………………………………………… [2]

(b) A bar made from a new conducting material is 9.0 cm long with a rectangular cross section 4.0 cm wide and 2.0 cm deep. The bar is inserted into a circuit with a 1.5 V battery and carries a constant current of 0.70 A. The resistance of the copper connecting wires and the ammeter, and the internal resistance of the battery, are all negligible compared to the resistance of the bar.

Using large coils not shown in the diagram, a uniform magnetic field of 1.7 T is applied perpendicular to the bar (to the left as shown in Fig. 6.2). A voltmeter is connected across the front and back of the bar. The voltmeter reads -0.29 mV. There is only one kind of mobile charge carrier in the bar material.

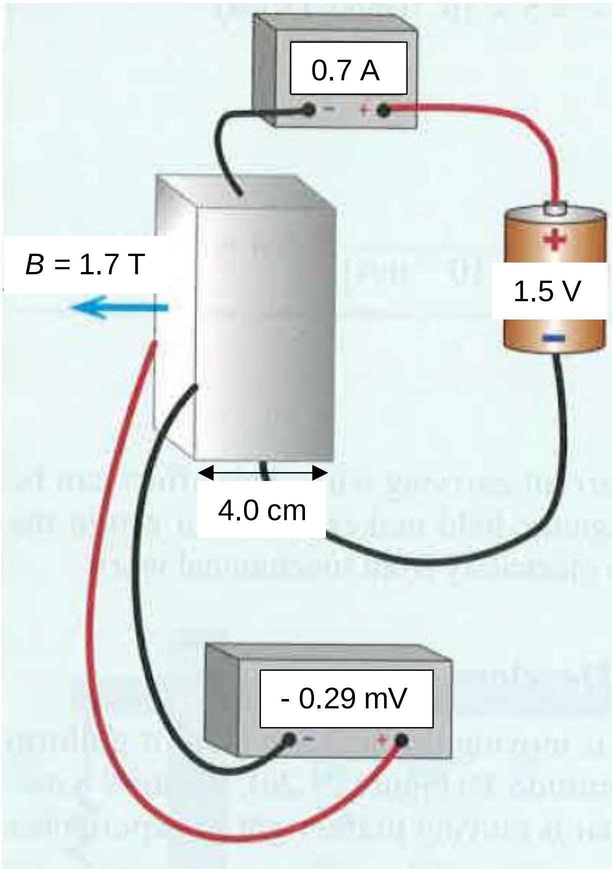


Fig. 6.2

(i) Explain why the voltmeter shows negative reading -0.29 mV.

……

………………………………………………………………………………………………… [1]

(ii) Hence, or otherwise, state and explain the polarity of mobile charge carriers in the bar.

……

……

……

………………………………………………………………………………………………… [2]

(iii) Calculate the magnitude of the transverse E-field $E_{\perp}$ from the front to the back of the bar.

$E_{\perp}$ = …………………….. V m^{-1} [2]

(iv) Determine the drift velocity, v_{drift} of the charge carriers.

v_{drift} = …………………….. m s^{-1} [2]

(v) Given that the mobile charge carriers are single charged, calculate the number density of mobile charge carriers, n.

n = …………………….. m^{-3} [2]

(vi) Hall resistance R_{Hall} is the ratio of Hall voltage to the current passing through the bar. Show that R_{Hall} is given by

$$R_{Hall} = B\frac{d}{nAq}$$

where A is the cross-sectional of the bar and d is the depth of the bar.

[2]

(c) The Quantum Hall Effect (QHE) is a quantized version of the Hall effect, which is observed in two-dimensional electron systems subjected to low temperatures and strong magnetic fields, in which the Hall resistance R_{Hall} exhibits steps that take on the quantized values as shown in Fig. 6.3.

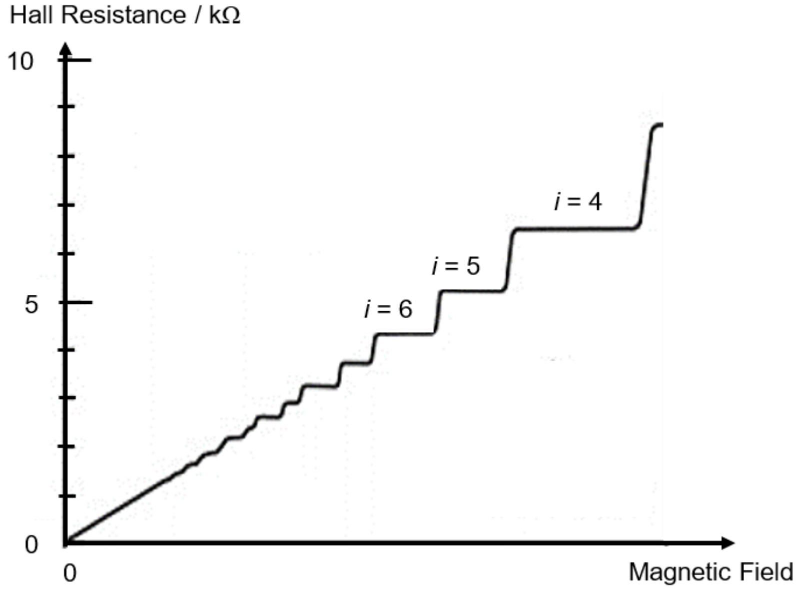


Fig. 6.3

The quantized version of R_{Hall} is given by

$$R_{Hall} = \frac{h}{ie^2}$$

where e is the elementary charge, h is Planck's constant and i is an integer and the value of R_{Hall} is approximately equal to 25,813 Ω.

(i) Show that the SI base unit of $\frac{h}{ie^2}$ is the same as ohms (Ω).

[2]

(ii) How does the Fig. 6.3 tell you that the Quantum Hall Effect is observed at large magnetic fields?

....

....

....

.... [2]

[Total: 19]

Section B

Answer **two** questions from this section.

You are advised to spend about 35 minutes on each question.

7 (a) Fig. 7.1 shows the cross-section of two long coaxial cylinders in electrostatic equilibrium.

The inner cylinder is an insulator, with radius *a* and a uniform volume charge density ρ_0. The outer cylinder shell is a conductor with some unknown non-zero charge, and has an inner radius *b* and outer radius *c.* The volume between the two cylinders ($b < r < c$) is a vacuum.

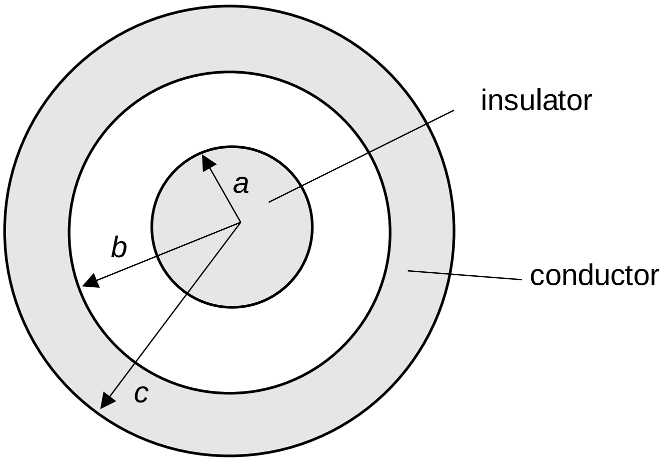


Fig. 7.1

Let *r* be the radial distance of an arbitrary point from the common axis of the cylinders.

(i) State the magnitude of the electric field strength *E* when $b < r < c$. Give a reason for your answer.

………

……… [2]

(ii) Using Gauss' law, find an expression for the magnitude of the electric field strength E where $a \le r \le b$ in terms of ρ_0, ε_0, a and r .

[3]

(iii) Using Gauss' law, show that the surface charge density σ on the inner surface of the outer cylinder (i.e. at $r = b$)

$$\sigma = -\frac{\rho_0 a^2}{2b}$$

[3]

(b) A solenoid carrying a current I, has N turns of wire, where N is a large number. It is bent into a circle to form a toroid, with inner radius a, and outer radius b as shown in Fig. 7.2. The radial distance of a point from the centre of the toroid is r.

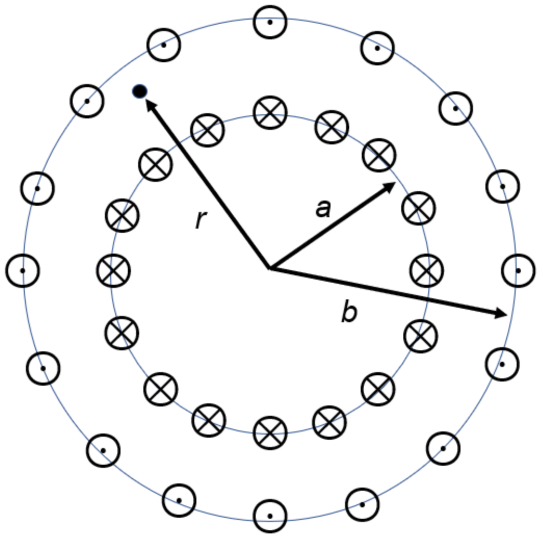


Fig. 7.2

Just like for a long solenoid, the windings may be assumed to form circular loops (i.e. ignore the slight pitch of the helical windings), and thus the magnetic flux density B is negligible outside the toroid (i.e. when $r < a$ and $r > b$).

(i) Using Ampere's Law, find an expression for the magnetic flux density B for $a \le r \le b$. Show your working clearly.

[3]

(ii) In a real toroid, the turns are not precisely circular loops, but are segments of a helix. Estimate the magnitude of the magnetic flux density outside the toroid (i.e. when $r < a$ and $r > b$), and hence explain why we can neglect this.

………………………………………………………………………………………………………

………………………………………………………………………………………………………

………………………………………………………………………………………………………

……………………………………………………………………………………………………[2]

(c) A water molecule consists of two hydrogen atoms and an oxygen atom, shown in Fig. 7.3 below. Due to the uneven distribution of charges, the two hydrogen atoms each have a small positive charge $+q$ and the oxygen atom has a charge of $-2q$.

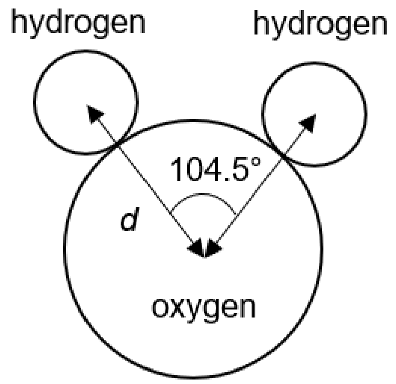


Fig. 7.3

The angle between the two hydrogen atoms is 104.5°, and the distance between the centre of each hydrogen atom from the centre of the oxygen atom is d.

The water molecule is placed in a uniform electric field with electric field strength E at an angle of θ as shown in Fig. 7.4.

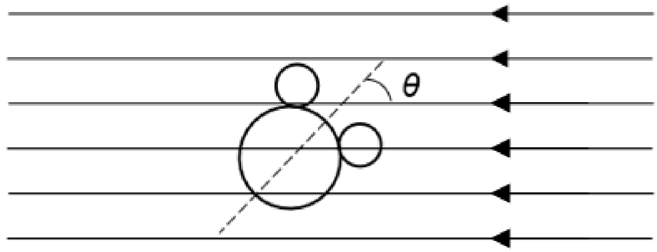


Fig. 7.4

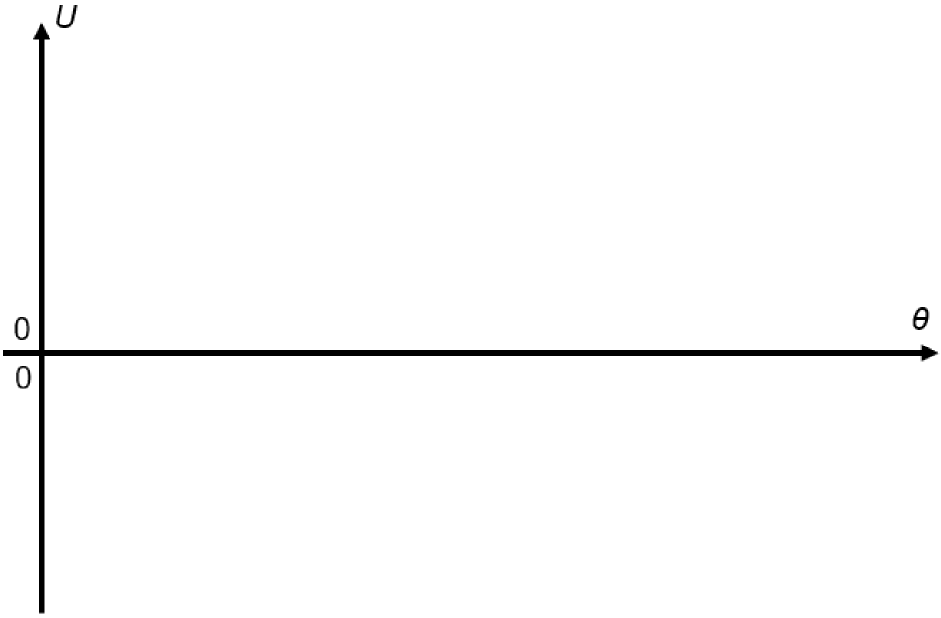
(i) Write down an expression for the electric dipole moment p in terms of q and the given quantities.

p = ... [2]

(ii) Write down an expression for the torque τ on the dipole.

τ = ... [1]

(iii) On the axes below, sketch the variation with θ of the potential energy U of the water molecule. Indicate any coordinates of interest.



[2]

(iv) State the orientation that the water molecule is likely to settle in. Explain your answer.

……

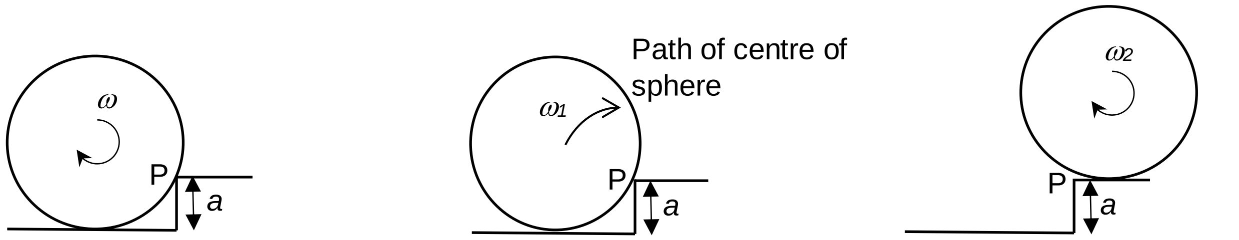
……

……

………………………………………………………………………………………………...... [2]

[Total: 20]

8 A solid sphere of mass m and radius $2a$ is rolling on level ground, with angular speed ω, towards a step of height a. It mounts the step without slipping. The process occurs in three stages.
In the first stage, as shown in Fig. 8a, the sphere goes through an inelastic collision with the step, causing a change in the velocity of its centre of mass.
In the second stage, the sphere rotates about P without slipping, as shown in Fig. 8b.
In the third stage, the sphere continues to roll on level ground, as shown in Fig. 8c.



Before colliding with step — **Fig. 8a**

Mounting the step — **Fig. 8b**

Moving on after the step — **Fig. 8c**

(a) Find an expression for the angular momentum about P before the sphere collides with the step, in terms of m, a, and ω,

(i) due to the rotational motion of the sphere,

[2]

(ii) due to the translational motion of the sphere.

[2]

(b) Hence, show that the total angular momentum about P before the sphere collides with the step is given by $\frac{18}{5}ma^2\omega$.

[1]

(c) **(i)** Immediately after the sphere collides with the step, the motion of the sphere becomes a rotation about the point P, as shown in Fig. 8b.
Using parallel-axis theorem, find an expression for the moment of inertia of the sphere about point P in terms of *m* and *a*.

[2]

(ii) Hence, use the principle of the conservation of angular momentum to show that $\omega_1 = \dfrac{9}{14}\omega$ immediately after the collision.

[2]

(iii) Calculate the percentage loss in energy due to the collision of the sphere with the step.

[4]

(iv) Explain why there is a decrease in the angular speed of the sphere as it mounts the step.

....

....

....

....

.... [2]

(d) Use the principle of conservation of energy to show that $\omega_2^2 = \omega_1^2 - \dfrac{5g}{14a}$, where ω_2 is the angular speed of the sphere, after it has mounted the step as shown in Fig. 8c.

[3]

(e) Hence show that in order for the sphere to mount the step, $\omega > \dfrac{1}{9}\sqrt{\dfrac{70g}{a}}$

[2]

[Total: 20]

9 **(a)** A voltage source is connected to a diode *D*, a capacitor of capacitance *C* and a resistor of resistance *R*, as shown in Fig. 9.1. The source supplies an alternating square-wave voltage, with a maximum voltage V_o and a minimum voltage of zero, as shown in Fig. 9.2. The voltage across the resistor fluctuates periodically, with a period T = 0.10 ms.

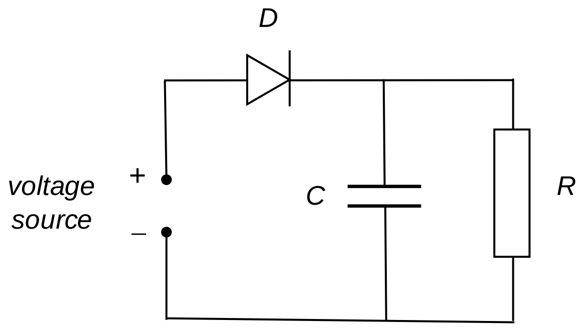


Fig. 9.1

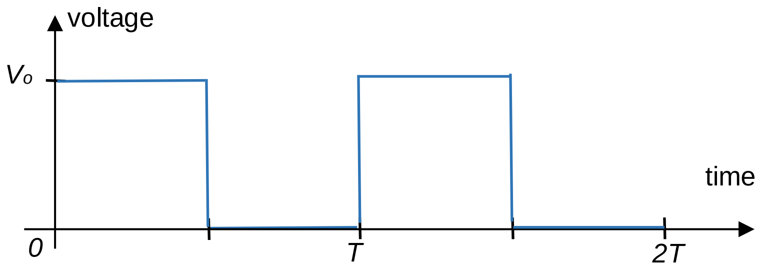


Fig. 9.2

(i) Suggest the purpose of diode.

....

.... [1]

(ii) Explain, with reference to the capacitor, how the voltage across the resistor varies with time when the voltage of the source is equal to zero, from $t = T/2$ to $t = T$.

....

....

....

....

....

.... [2]

(iii) Derive an expression for the instantaneous voltage V across the capacitor in terms of t, V_o, R and C, as described in **(a)(ii)**.

[3]

(iv) Sketch on Fig. 9.3 the variation with time of the voltage across the resistor from $t = 0$ to $t = 2T$.

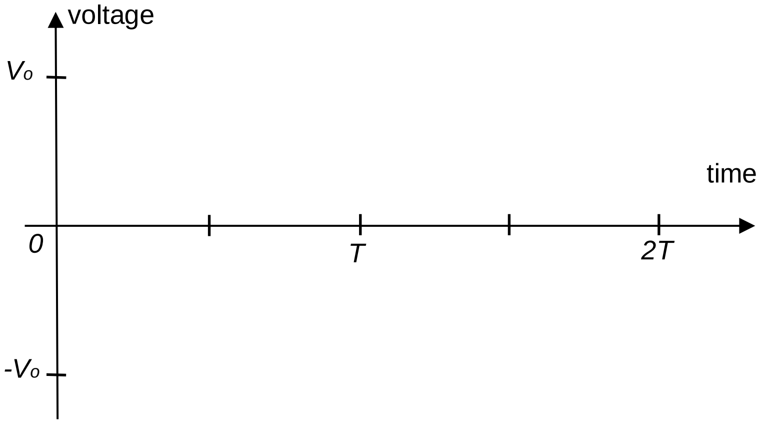


Fig. 9.3

[2]

(b) The same voltage supply is connected to two identical diodes, an inductor *L* and the same resistor *R*, as shown in Fig. 9.4. The voltage across the resistor fluctuates periodically with a period *T* of 0.10 ms and fluctuates between a maximum current i_{max} and a minimum current i_{min} which is greater than 0.

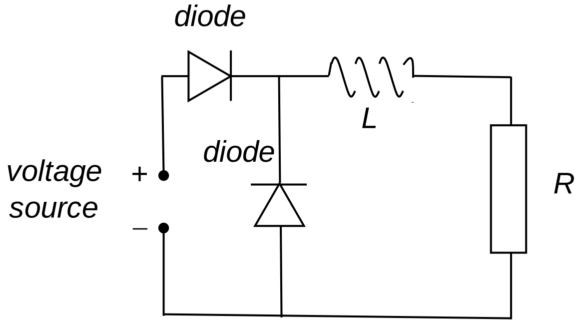


Fig. 9.4

(i) Explain, with reference to the diodes and the inductor, how the voltage across the resistor varies with time when the voltage of the source is equal to zero, from $t = T/2$ to $t = T$.

....

....

....

....

....

.... [3]

(ii) Derive an expression for the instantaneous current i in the inductor in terms of t, i_o, *R* and *L*, for $t = T/2$ to $t = T$.

[3]

(iii) Sketch,

1. the variation with time of the voltage across the inductor from $t = 0$ to $t = 2T$ on Fig. 9.5.

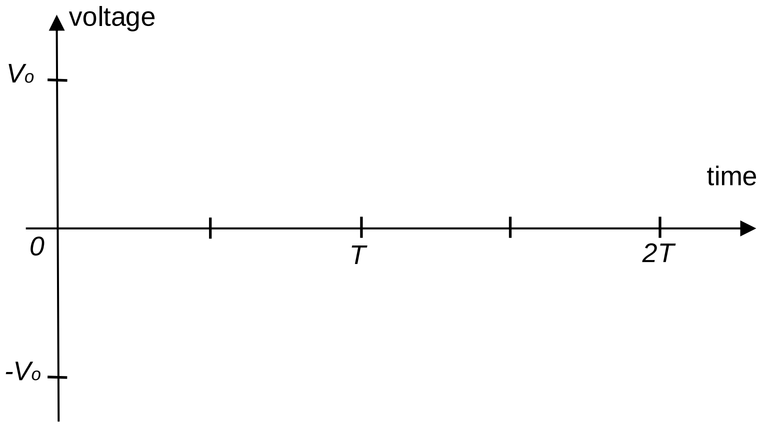


Fig. 9.5

2. the variation with time of the voltage across the resistor from $t = 0$ to $t = 2T$ on Fig. 9.6.

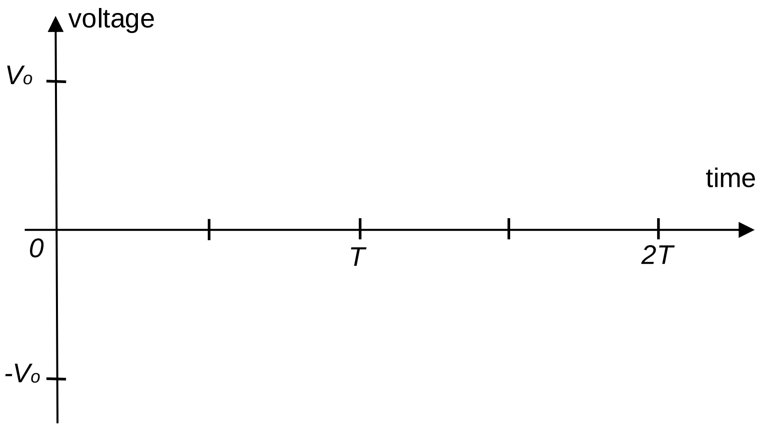


Fig. 9.6

[4]

(iv) Derive an expression for the period T of the voltage source in terms of R and L when $i_{\text{min}} = 0.95 i_{\text{max}}$.

[2]

[Total: 20]

End of Paper