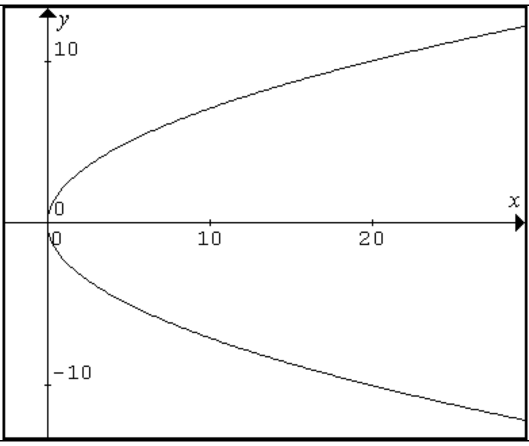
Hougang Secondary School
Mathematics Department
MARKSCHEME
Mathematics Syllabus Express/Normal Academic/Normal Technical

Subject : Additional Mathematics
Examination : Prelim 2
Level : 4E/~~5N/4AO~~
Paper : 1

Qn	Working	Marks	Remarks
1(i)	$4^{3x}\times 4^{y}=4^{0}$ $y=-3x$ ---------------Eq1	M1	Indices law
	$3^{y}\times\left(3^{4}\right)^{\frac{1}{x}}=3^{4}$ $y+\frac{4}{x}=4$ --------------Eq2	M1	Indices law
	Sub (1) into (2): $-3x+\frac{4}{x}=4$	M1	Perform substitution/elimination
	$-3x^{2}+4=4x$ $3x^{2}+4x-4=0$ $(x+2)(3x-2)=0$ $x=-2$ or $x=\frac{2}{3}$	A1	Answer for both *x*
	$y=6$ or $y=-2$	A1	
1(ii)		B1 B1	Shape, pass through origin
2(i)	$T_{r+1}=\binom{9}{r}\left(x^{9-r}\right)\left(-\frac{k}{x^{2}}\right)^{r}$ $=\binom{9}{r}(-k)^{r}(x)^{9-3r}$	M1	General term
	$9-3r=0$ $r=3$	M1	Term indep of *x*
	$\therefore\binom{9}{3}(-k)^{3}=-672$	M1	Form equation
	$k=2$	A1	

Qn	Working	Marks	Remarks
2(ii)	$(x+2)\left(\frac{1}{x}\text{ term + constant+...}\right)$ $9-3r=-1$ $r=\frac{10}{3}(rej)$ $(x+2)(\text{... + constant+...})$ $\therefore$ constant term = 2 (–672) = –1344	M1 M1 A1	Recognising the need to check for 1/*x* term
3	Base area = $\frac{1}{2}\left(4-\sqrt{5}\right)^2(2)$ $=\frac{1}{2}\left(16-8\sqrt{5}+5\right)(2)$ $=21-8\sqrt{5}cm^2$ $h=\frac{50\sqrt{5}-101}{21-8\sqrt{5}}$ $=\frac{50\sqrt{5}-101}{21-8\sqrt{5}}\times\frac{21+8\sqrt{5}}{21+8\sqrt{5}}$ $=\frac{242\sqrt{5}-121}{121}$ $-1+2\sqrt{5}cm$	M1 M1 M1√ A1	$\left(16-8\sqrt{5}+5\right)$ Forming equation of *h* using prism volume rationalising
4(i)	$R=500e^{-0.004(1000)}$ $R=9.1578$ $R=9.16g(3sf)$	M1 A1	Sub *t* = 1000
4(ii)	$\frac{10}{100}\times500g=50g$ $50=500e^{-0.004t}$ $\frac{1}{10}=e^{-0.004t}$ $\ln\frac{1}{10}=\ln e^{-0.004t}$ $t=\frac{\ln 0.1}{-0.004}$ $t=575.6years$ Year 2595.	M1 M1 A1	10percent of original Attempting to solve for t Do not round up

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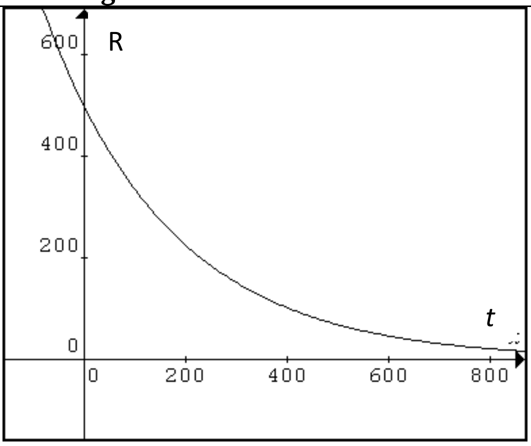
4(iii)		B1	Shape, include R-intercept 500 No need to penalise the negative t portion of the graph
5(i)	$\sec A = \frac{1}{\cos A}$ $\sec A = -\frac{5}{4}$	M1 A1	Knowing reciprocal identity
5(ii)	$\sin(A+B) = \sin A \cos B + \cos A \sin B$ $= \frac{3}{5}\left(\frac{15}{17}\right) + \left(-\frac{4}{5}\right)\left(-\frac{8}{17}\right)$ $=$ $= \frac{77}{85}$	M1√ A1	Correct use of formulae
6	$2x^2 + 4kx + 5k + 10 > 6k$ $2x^2 + 4kx - k + 10 > 0$ for all values of x $(4k)^2 - 4(2)(10-k) < 0$ $16k^2 - 80 + 8k < 0$ $(k-2)(2k+5) < 0$ $-2.5 < k < 2$	M1 M1 M1 M1 A1	Forming quad ineq (Allow slips) Correct a,b,c Using D < 0 Factorisation
7(i)	$m_{BC} = m_{AD}$ $= \frac{6-10}{6-0}$ $= -\frac{2}{3}$ $m_{DC} = \frac{3}{2}$ Eqn of *DC* is $y - 6 = \frac{3}{2}(x-6)$ $y = 1.5x - 3$ At *C*, $0 = 1.5x - 3$ $x = 2$ $\therefore C(2,0)$	M1 M1√ M1√ M1√	Gradient Perpendicular gradient Eqn DC Alternatively, accept using gradient of DC. Find point C

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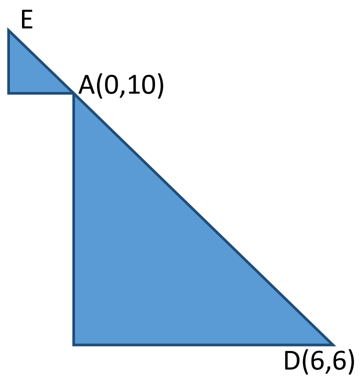
	$M_{BC} = \left(\frac{-10+2}{2}, \frac{8+0}{2}\right)$ $= (-4, 4)$ Eqn of perpendicular bisector is $y - 4 = \frac{3}{2}(x+4)$ $y = \frac{3}{2}x + 10$	M1√ A1	Midpoint of BC Eqn
7(ii)	Area *ABD* $= \frac{1}{2}\begin{vmatrix} 0 & -10 & 6 & 0 \\ 10 & 8 & 6 & 10 \end{vmatrix}$ $= \frac{1}{2}[0 - 60 + 60 + 100 - 48 - 0]$ $= 26 units^2$	M1 A1	Correct use of shoelace method
7(iii)	$E\left(0 - \frac{6-0}{4}, \frac{10-6}{4} + 10\right)$ $= E\left(-\frac{3}{2}, 11\right)$  	B1	
8(i)	$Vol = \frac{1}{3}\pi r^2 h$ $V = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h$ $V = \frac{1}{12}\pi h^3 (shown)$	M1 M1	r = h/2 correct volume formulae
8(ii)	$\frac{dV}{dh} = \frac{1}{4}\pi h^2$ $\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$ $2.5 = \frac{1}{4}\pi(3)^2 \times \frac{dh}{dt}$ $\frac{dh}{dt} = \frac{10}{9\pi} m/\min$	M1 M1√ A1	 Able to form chain rule and use h = 3 Accept 0.354m/s

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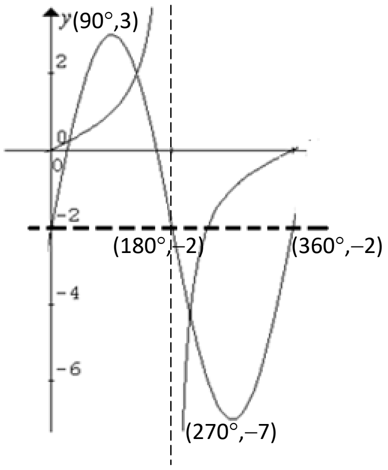
8(iii)	$A = \pi r^2$ $A = \pi\left(\frac{h}{2}\right)^2 = \frac{1}{4}\pi h^2$ $\frac{dA}{dh} = \frac{1}{2}\pi h$ $\frac{dA}{dt} = \frac{dA}{dh} \times \frac{dh}{dt}$ $= \frac{1}{2}\pi(3) \times \frac{10}{9\pi}$ $= 1\frac{2}{3} m^2 / \text{min}$	M1 M1 M1√ A1	Forming eq of area Differentiate A Forming chain rule (rates)
9(i)	Least value of g(x) = –7 Greatest value of g(x) = 3	B1 B1	
9(ii)	Period of f(x)= 360°	B1	Accept 2π
9(iii)	Period of g(x)= 360°	B1	Accept 2π
9(iv)		Sine graph B1 B1√ tan graph B1 B1√	Shape, (90°,3),(270°,–2) (0°,–2), (180°, –2), (360°,–2) shape asymptote at x = 180°
9(v)	3 solutions	B1	No ecf
10(i)	$y = \ln\left(3 + x^2\right)$ $\frac{dy}{dx} = \frac{2x}{3 + x^2}$ $\frac{d^2y}{dx^2} = \frac{\left(3 + x^2\right)(2) - (2x)(2x)}{\left(3 + x^2\right)^2}$ $= \frac{2x^2 + 6 - 4x^2}{\left(3 + x^2\right)^2}$ $= \frac{-2x^2 + 6}{\left(3 + x^2\right)^2}$	B1 M1√ A1	differentiation quotient law

Qn	Working	Marks	Remarks
10(ii)	$\frac{dy}{dx}=\frac{2x}{3+x^2}=0$ $x=0$ $y=\ln\left(3+(0)^2\right)$ $y=\ln 3$ $(0,\ln 3)$ When $x = 0$, $\frac{d^2y}{dx^2}=\frac{0+6}{(0+3)^2}=\frac{2}{3}$ Since $\frac{d^2y}{dx^2}>0$, $(0,\ln 3)$ is a <u>minimum point.</u>	M1√ A1 M1√ A1	$\frac{dy}{dx}=0$ Stationary point Using second derivative test Min pt
11i(a)	$s=5t^2-40t$ $v=10t-40$ When t =1,v = –30 m/s $a=10\,\text{m/s}^2$	M1 A1 A1	$v=\frac{ds}{dt}$
11i(b)	When v = 0, $10t-40=0$ $t=4$ P changed its direction when t = 4.	M1√ A1	v = 0
11i(c)	When t = 4, $s=5(4)^2-40(4)$ $s=-80m$ When t = 10, $s=5(10)^2-40(10)$ $s=100m$ Total distance travelled = 80+180m Average speed = 26m/s	M1√ M1√ M1√ A1	ECF up to 3 method marks
11ii(a)	$9=3(e^{0.5(0)}+k)$ $3=1+k$ $k=2$	M1 A1	
11ii(b)	$d=3\left(e^{0.5t}+2\right)$ $d'=3(0.5)\left(e^{0.5t}\right)$ $d'=1.5e^{0.5t}$ Since $e^{0.5t}>0$, for all values of t, $1.5e^{0.5t}>0$, $\therefore d$ is increasing.	M1√ M1√	Differentiate d Reasonable explanation