**Solutions**

1 $$\frac{V}{\pi} = (3+2\sqrt{3})^2 \times (5\sqrt{3} + \frac{1}{\sqrt{3}})$$

$$= (9+12\sqrt{3}+12) \times (5\sqrt{3} + \frac{\sqrt{3}}{3})$$

$$= (21+12\sqrt{3}) \times (5\sqrt{3} + \frac{\sqrt{3}}{3})$$

$$= 105\sqrt{3} + 7\sqrt{3} + 180 + 12$$

$$= 112\sqrt{3} + 192$$

2(i) $$\frac{5}{x^2} = 3x^{\frac{1}{2}}$$

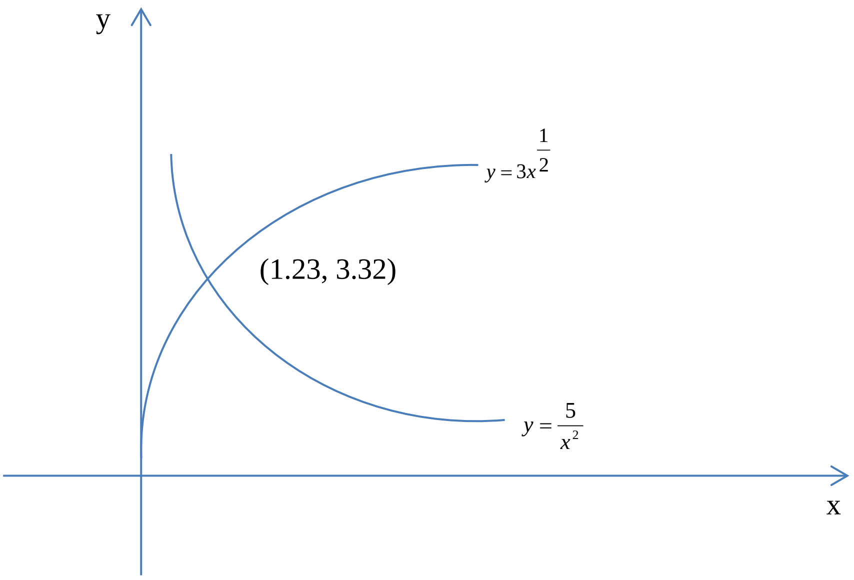
$$x^{\frac{5}{2}} = \frac{5}{3}$$

$$x^5 = \frac{25}{3}$$

$$x = 1.23$$

$$y = 3.32$$

(ii)



3 $2x-15xy+6y=0$

$$2x-15x(\frac{1}{3x})+6(\frac{1}{3x})=0$$

$$2x-5+\frac{2}{x}=0$$

$$2x^2-5x+2=0$$

$$(x-2)(2x-1)=0$$

$x=2$ or $x=\frac{1}{2}$

$y=\frac{1}{6}$ $y=\frac{2}{3}$

Grad of AB $=-\frac{1}{3}$

Grad of perpendicular $=3$

Midpoint of AB $=(\frac{5}{4},\frac{5}{12})$

Equation: $y=3x-\frac{10}{3}$

4 $2x^2-3x-6=0$

(i) $\alpha+\beta=\frac{3}{2}$

$\alpha\beta=-3$

(ii) $\alpha^3\beta+\alpha\beta^3$

$=\alpha\beta(\alpha^2+\beta^2)$

$=-3[(\frac{3}{2})^2-2(-3)]$

$=-\frac{99}{4}$

$(\alpha^3\beta)(\alpha\beta^3)=\alpha^4\beta^4$

$=(-3)^4$

$=81$

$$x^2+\frac{99}{4}x+81=0$$

$$4x^2+99x+324=0$$

5(i) amplitude = 4

Period = $180°$

(ii) $4\sin 2x+2=0$

$$\sin 2x=-\frac{1}{2}$$

$2x = 210, 330$

$x = 105, 165$

6(i) $x(x+k) > k$

$x^2 + kx - k > 0$

$D < 0$

$k^2 - 4(-k) < 0$

$k(k+4) < 0$

$-4 < k < 0$

(ii) $x^2 + (2x+p)^2 - 5 = 0$

$x^2 + 4x^2 + 4px + p^2 - 5 = 0$

$5x^2 + 4px + p^2 - 5 = 0$

$D > 0$

$(4p)^2 - 4(5)(p^2 - 5 > 0$

$16p^2 - 20p^2 + 100 > 0$

$p^2 - 25 < 0$

$(p-5)(p+5) < 0$

$-5 < p < 5$

7(i) $x^2 + y^2 - 10x - 6y - 15 = 0$

$(x-5)^2 + (y-3)^2 - 15 - 5^2 - 3^3 = 0$ M1

$(x-5)^2 + (y-3)^2 = 49$

Centre = (5, 3)

Rad = 7

(ii) Distance = $\sqrt{6^2 + 1^2}$

$= \sqrt{37} < 7$

It lies in the circle.

(iii) New centre = (-7, 3)

$(x+7)^2 + (y-3)^2 = 49$

$x^2 + y^2 + 14x - 6y + 9 = 0$

8(i) $\log_3(18-7x) - \log_3 2 = \log_{\sqrt{3}}(x-2)$

$$\log_3(18-7x)-\log_3 2=\frac{\log_3(x-2)}{\log_3\sqrt{3}}$$

$$\log_3\frac{18-7x}{2}=\log_3(x-2)^2$$

$18-7x=2(x-2)^2$

$2x^2-x-10=0$

$(2x-5)(x+2)=0$

$x=\frac{5}{2}$ or $x=-2$ (NA)

(ii) $e^{x-1}+4=21\left(\frac{1}{e}\right)^{x-1}$

$e^{2(x-1)}+4e^{x-1}-21=0$

Let $y=e^{x-1}$

$y^2+4y-21=0$

$(y-3)(y+7)=0$

$e^{x-1}=3$

$x=2.10$

$e^{x-1}=-7$ (NA)

9(a)(i) Let $f(x)=3x^3-2x^2-7x-2$

$=(x+1)(3x^2+ax-2)$

$=3x^3+(a+3)x^2+(a-2)x-2$

$a+3=-2$

$a=-5$

$f(x)=(x+1)(3x^2-5x-2)$

$=(x+1)(3x+1)(x-2)=0$

$x=-1,-\frac{1}{3},2$

(ii) $\sin^2\theta(3\sin\theta-2)=7\sin\theta+2$

$3\sin^3\theta-2\sin^2\theta-7\sin\theta-2=0$

$\sin\theta=-1$

$\theta=270°$

$\sin\theta=-\frac{1}{3}$

$\theta=199.5°,340.5°$

$\sin\theta=2(NA)$

(b) $f(x)=x^3+14x+6$

$f(a)=a^3+14a+6$

$$g(x) = x^3 - 2x^2 - 8x - 30$$

$$g(a) = a^3 - 2a^2 - 8a - 30$$

$$a^3 - 2a^2 - 8a - 30 = a^3 + 14a + 6$$

$$2a^2 + 22a + 36 = 0$$

$$(a+2)(a+9) = 0$$

$$a = -2 \;\; or \;\; -9$$