TOPIC 4: Forces

Content

- Types of force
- Centre of gravity
- Turning effects of forces
- Equilibrium of forces
- Upthrust

Learning Outcomes

Candidates should be able to:

(a) recall and apply Hooke's law ($F=kx$, where k is the force constant) to new situations or to solve related problems

(b) describe the forces on mass, charge and current in gravitational, electric and magnetic fields, as appropriate

(c) show a qualitative understanding of normal contact forces, frictional forces and viscous forces including air resistance (no treatment of the coefficients of friction and viscosity is required)

(d) show an understanding that the weight of a body may be taken as acting at a single point known as its centre of gravity

(e) define and apply the moment of a force and the torque of a couple

(f) show an understanding that a couple is a pair of forces which tends to produce rotation only

(g) apply the principle of moments to new situations or to solve related problems

(h) show an understanding that, when there is no resultant force and no resultant torque, a system is in equilibrium

(i) use a vector triangle to represent forces in equilibrium

(j) Derive, from the definitions of pressure and density, the equation $p=\rho gh$

(k) solve problems using the equation $p=\rho gh$

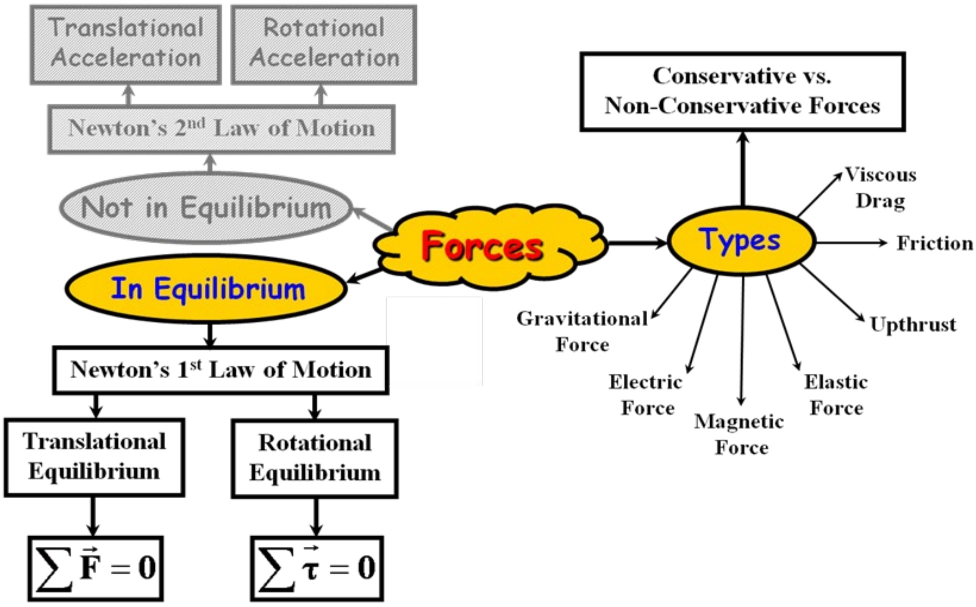
(l) show an understanding of the origin of the force of upthrust acting on a body in a fluid

(m) state that upthrust is equal in magnitude and opposite in direction to the weight of the fluid displaced by a submerged or floating object

(n) calculate the upthrust in terms of the weight of the displaced fluid

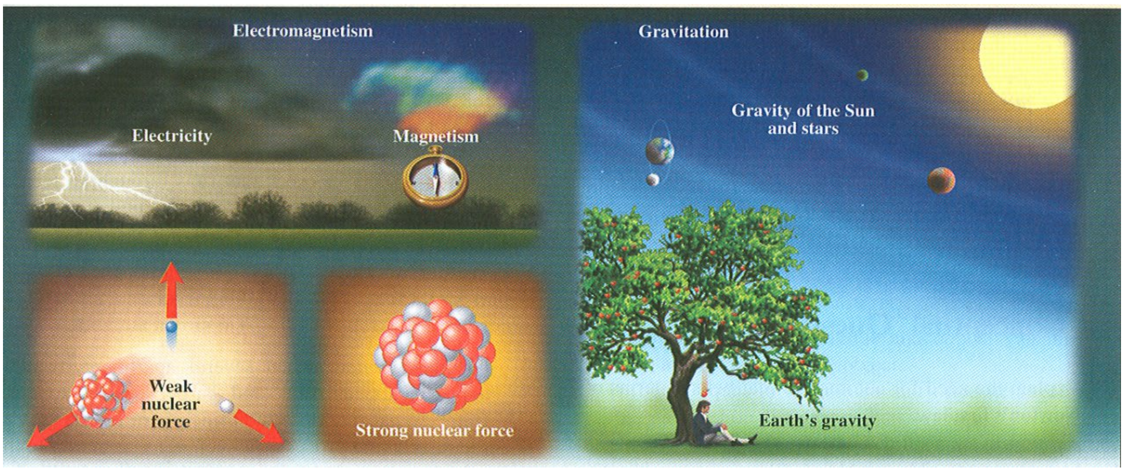
(o) recall and apply the principle that, for an object floating in equilibrium, the upthrust is equal in magnitude and opposite in direction to the weight of the object to new situations or to solve related problems

Concept Map



1 Introduction

One of the main goals of Physics has been to understand the immense variety of forces in the universe. Today, all forces are understood in terms of just four fundamental interactions as follows:



Force	Effects	Range/m
Gravitational Force	Weakest. Acts on all masses	Infinite
Electromagnetic Force	Acts on electric charges	Infinite
Strong Nuclear Force	Holds protons and neutrons together in a nucleus	10^{-15}
Weak Nuclear Force	Causes radioactive decay processes	10^{-17}

2 Types of Forces

Learning Outcomes *(b) describe the forces on mass, charge and current in gravitational, electric and magnetic fields, as appropriate* *(c) show a qualitative understanding of normal contact forces, frictional forces and viscous forces including air resistance (no treatment of the coefficients of friction and viscosity is required)*

In the past, we learn that a force is a **push or a pull** which one body exerts on another.

Now, we know that a force is defined as the <u>rate of change of momentum</u> of an object which is free to move. The <u>direction of the force</u> is in the <u>direction of the change in momentum</u>.

Conservative and Non Conservative Forces

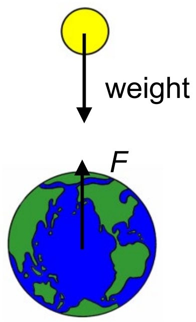
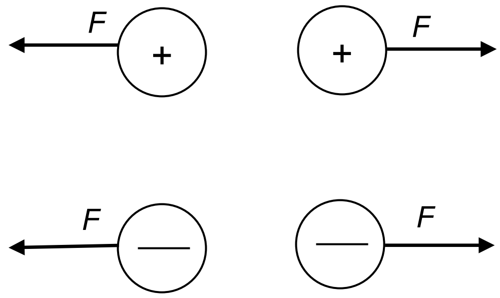
A force is <u>conservative</u> if the <u>work</u> it does on an object moving between two points is ______________________________ the object takes.

Gravitational force is an example of a conservative force.

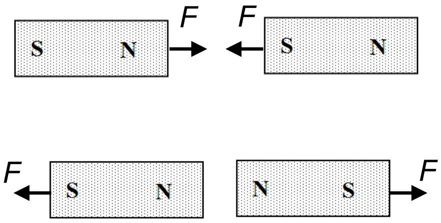
A force is <u>non-conservative</u> if the <u>work</u> it does on an object ________________ taken by the object between its initial and final points.

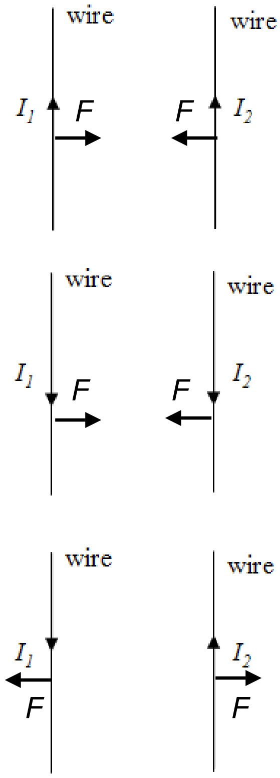
Friction is an example of a non-conservative force.

Example 1 Forces on a mass in a gravitational, electric and magnetic field

(1)	(2)
 Gravitational force: • is due to the interaction between ______ • on an object is called _____ if it is due to the gravitational attraction by the Earth	 Electric force: • is due to the interaction between ______

(3)





Magnetic force:

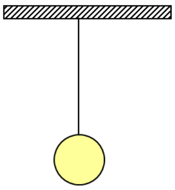
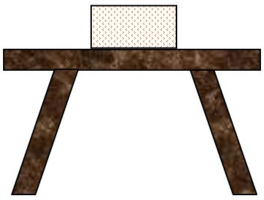
- is due to the interaction between

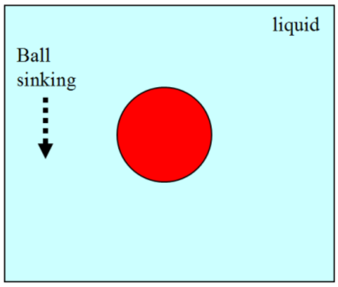
____

- acts on moving charges and

______________________________________ when they are placed in magnetic fields

Example 2 Contact forces

(1) Forces on the ball	(2) Forces on the block
	
T = tension **W** = weight of ball	**N** = normal contact force **W** = weight of block

(3) Forces on the block moving to the right	(4) Forces on the ball moving through a liquid
 **F** = forward force **f** = frictional force **N** = normal contact force **W** = weight of block	 **U** = upthrust **R** = drag force **W** = weight of ball

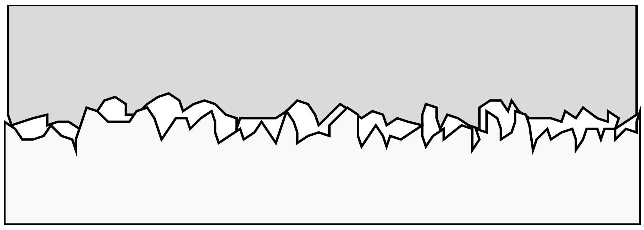
3 Frictional and Viscous Forces

Friction and viscous forces are known as *dissipative forces*.

Some energy of the object moving through the fluid is *dissipated as heat* to the surroundings.

3.1 Frictional Force

> Frictional force is the force exerted by one body on another body when two bodies slide over one another.
>
> It is caused by irregularities in the surfaces in mutual contact and depends on the surfaces in contact as well as how much they are pressed against each other.
> **It is the _____________________________ along surface of contact**.



Close examination of the flattest and most highly polished surface reveals hollows and humps more than one hundred atoms high.

When one solid is placed over on another, contact occurs only at a few places of small areas. The pressure at the points of contact is extremely high and causes the humps to flatten out until the increased area of contact enables the upper solid to be supported.

At the points of contact, small, cold-welded 'joints' are formed by the strong adhesive forces between molecules which are very close together. These joints have to be broken before one surface can move over the other.

3.2 *Viscous / Drag Force*

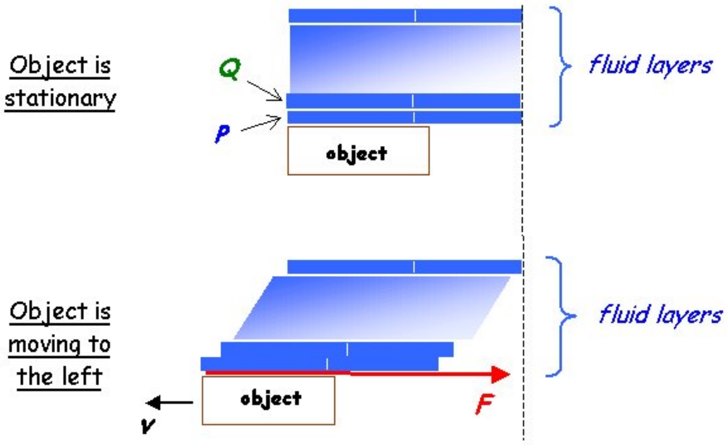
The frictional force experienced either by an object as it moves through a fluid or by a fluid as it moves over a surface.

i.e it is resistance when an object move in a fluid and the force is dependent on the speed of the object.

In a viscous flow, fluid can be regarded as a stack of very thin layers, each moving with different speeds due to internal friction between the layers.

When an object moves relative to a fluid with velocity *v*, the layer of fluid *P* adjacent to the object is pulled along. But the next layer *Q* slows layer *P* down. There is a gradual decrease in velocity of the layers with distance away from the object.

The overall effect is a net retarding force on the object. This retarding force, *F* which opposes the motion acts to the right.



The magnitude of the viscous force __________ with increase in speed of the object.

The exact relationship depends also on the shape, size, texture of surface, viscosity of fluid, etc.

Example 3

The mass of a diver is 61 kg. In a free-fall dive from an aircraft, before the parachute opened, the diver reached a terminal speed of 90 m s^{-1}.

Ignoring upthrust, calculate

(i) the weight of the diver

(ii) the force of air resistance on the diver when at the terminal speed

(iii) the magnitude and unit of the constant k in the expression *air resistance* = kv^2

(iv) using the expression given in (iii), estimate the air resistance on the diver when he is moving with a speed of 25 m s^{-1}

(d) *show an understanding that the weight of a body may be taken as acting at a single point known as its centre of gravity*

4 Centre of Gravity (C.G.)

Every body of mass is attracted toward the centre of the Earth by the force of gravity.

The entire *weight of a body* may be taken as acting at a *single point known as its centre of gravity (C.G.).*

The C.G. of a *homogeneous, symmetric body* must lie on its *axis of symmetry*.

5 Free-Body Diagrams

A mechanical system may consist of more than one object or body. A free-body diagram takes into account *all the forces acting on a given body*, and these are marked out on the diagram.

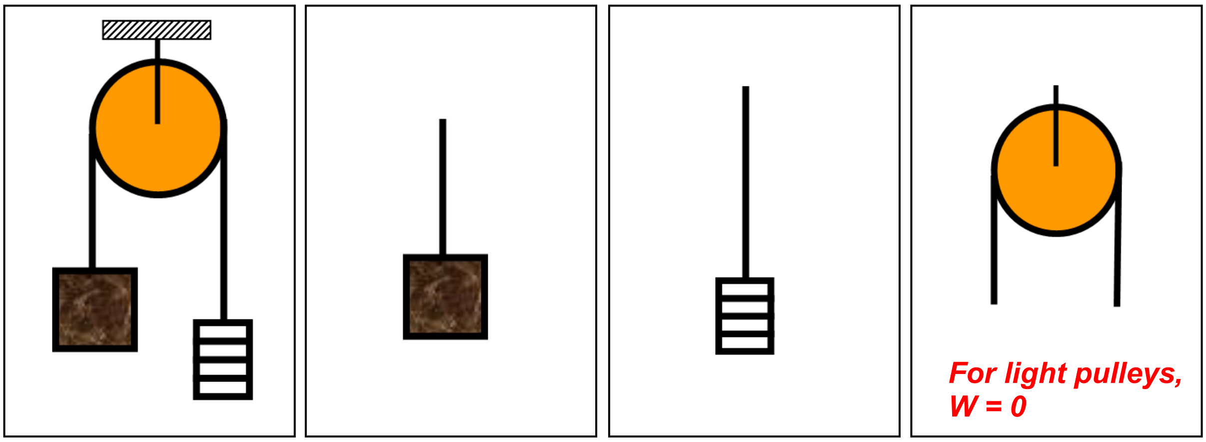
Example 4 A single-body system

L = lift force
W = weight
T = thrust
D = drag force

When a system consists of more than one object, you can clarify the situation by isolating each object and drawing separate free-body diagrams.

Example 5 A multiple-body system

T = tension in string
W = weight of object

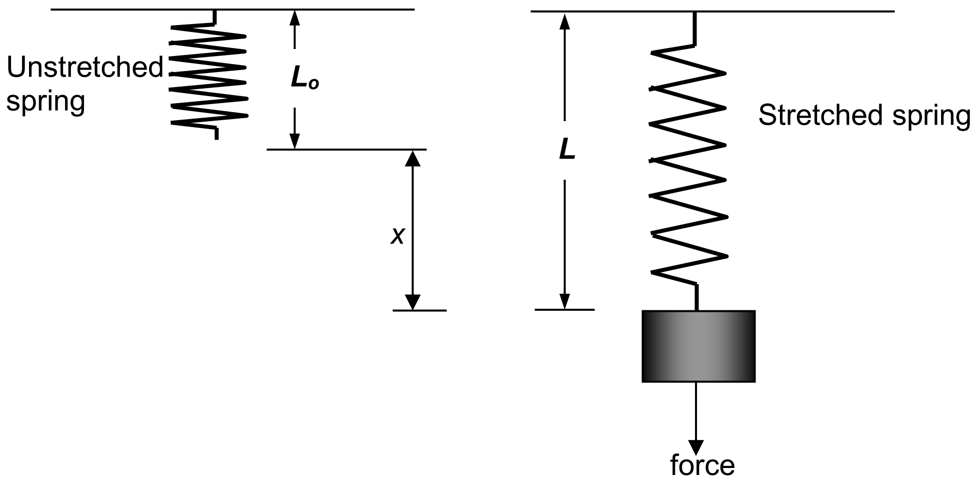


Learning Outcomes

(a) recall and apply Hooke's law (F=kx, where k is the force constant) to new situations or to solve related problems

5 Hooke's Law

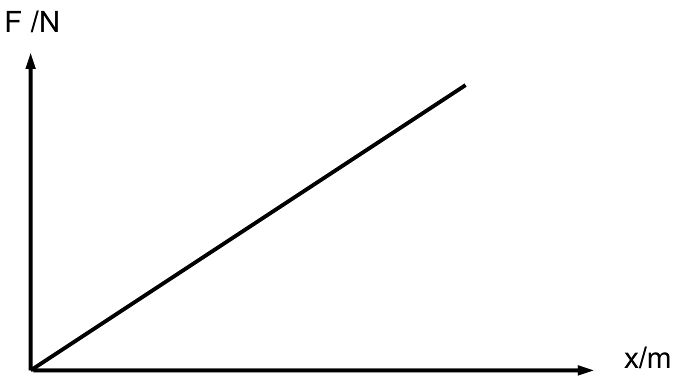
Hooke's law states that force is directly proportional extension, provided that the elastic limit has not been exceeded.



$$\boxed{F = kx}$$

where F is the force applied to the material
L_o is the unstretched length of material
L is final length of material
x is the extension of the material = $L - L_o$
k is the proportionality (force) constant [unit: N m^{-1}]

Graph to show how F, the force applied, depends on the extension, x assuming it obeys Hooke's law.



Note:

(i) The law applies to *springs* as well as *metals in the form of wires*.

(ii) If two springs of spring constant k_1 and k_2 are connected *in parallel*, they can be replaced by a single spring of spring constant

$$k_{eff} = k_1 + k_2.$$

If the two springs are connected *in series*, they can be replaced by a single spring of spring constant

$$\frac{1}{k_{eff}} = \frac{1}{k_1} + \frac{1}{k_2}$$

Example 6

A spring obeying Hooke's law has an un-stretched length of 50 mm and a spring constant of 400 N m^{-1}. What is the tension in the spring when the overall length is 70 mm? (8.0 N)

6 Elastic Potential Energy

An external force F_{ext} acting on a wire causing it to extend from x_1 to x_2 performs work given by

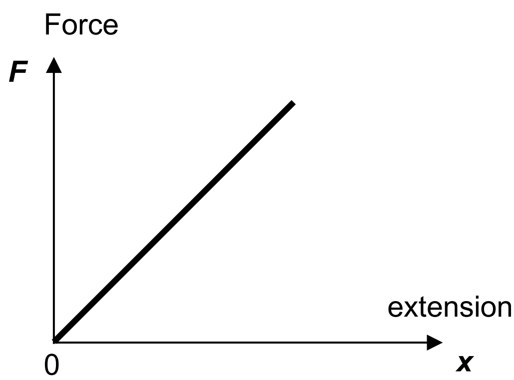
$$W = \int_{x_1}^{x_2} F_{ext} \, dx$$

This work is stored as elastic potential energy in the wire. The elastic potential energy is equal to the area under the force-extension curve, between the limits x_1 and x_2.

The elastic potential energy stored in a deformed material that obeys Hooke's law, such as a spring or wire, can be found from the **area under the force-extension graph**.

Elastic potential energy (PE):

$$U = \frac{1}{2}Fx = \frac{1}{2}kx^2 = \frac{F^2}{2k}$$



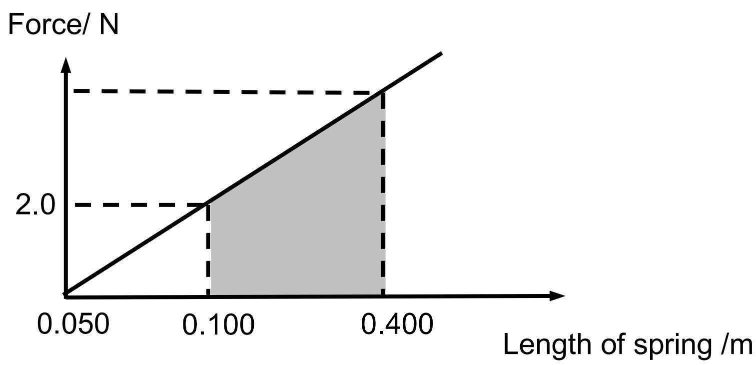
Example 7

A force of 10 N acting on a certain spring gives an extension of 40 mm. Two such springs are connected end to end and this double-length spring is extended by 40 mm. Assuming that the springs conform to Hooke's Law, what is the strain energy? (0.10 J)

Example 8

The extension of a rubber band when a variable force is applied is shown by the graph below.

The original length of the spring is 0.050 m. What is the increase in potential energy stored in the rubber band when it extends from 0.100 m to 0.400 m?



Learning Outcomes

(j) derive, from the definitions of pressure and density, the equation p= ρgh
(k) solve problems using the equation p= ρgh
(l) show an understanding of the origin of the upthrust acting on a body in a fluid
(m) state that upthrust is equal to the weight of the fluid displaced by a submerged or floating object
(n) calculate the upthrust in terms of the weight of the displaced fluid
(o) recall and apply the principle that, for an object floating in equilibrium, the upthrust is equal to the weight of the object to new situations or to solve related problems

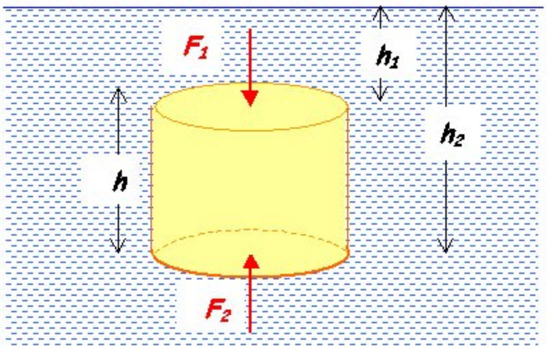
7 Upthrust

Upthrust is the vertical upward force exerted by the surrounding fluid when a body is submerged fully or partially in a fluid.

It is equal in magnitude and opposite in direction to the weight of fluid displaced by the body.

In other words, it is the resultant force due to the *difference in pressure* exerted by the fluid at the top and bottom surfaces of the body.

Consider a solid cylinder of height *h* and cross-sectional area *A*, submerged in a fluid of density ρ as shown.



By definition, pressure $$p = \frac{\text{Normal Force } F}{\text{Cross-sectional Area } A}$$

Hence, pressure on the top surface,

$$p_1 = \frac{\text{Weight of Column of Fluid Above}}{\text{Cross} - \text{sectional Area } A} + p_{atm}$$
$$= \frac{m_1 g}{A} + p_{atm}$$
$$= \frac{\rho_f A h_1 g}{A} + p_{atm}$$
$$= \rho_f h_1 g + p_{atm}$$

Similarly, the pressure on the bottom surface,

$$p_2 = \rho_f h_2 g + p_{atm}$$

Thus, the downward force on the cylinder, $F_1 = (\rho_f h_1 g + p_{atm})A$

The upward force on the cylinder, $F_2 = (\rho_f h_2 g + p_{atm})A$

Hence, the resultant upward force (upthrust) on the cylinder due to pressure difference between the top and bottom surfaces,

$$
\begin{aligned}
U &= F_2 - F_1 \\
&= \rho_f g(h_2 - h_1)A \\
&= \rho_f ghA \\
&= \rho_f gV_{displaced} \\
&= m_f g \qquad \text{(weight of the fluid displaced by the object)}
\end{aligned}
$$

The above result is known as the ***Archimedes' Principle*** which states that for any object immersed partially or fully in a fluid, the upthrust is equal in magnitude and opposite in direction to the weight of fluid displaced by the object.

Example 9

(a) What is the buoyancy force (upthrust) on a human body of volume 7.4 x 10^{-2} m^3 when totally immersed in

(i) air of density 1.3 kg m^{-3}

(ii) sea water of density 1030 kg m^{-3}?

(b) Hence explain why, the upthrust acting on a human body when in air is normally ignored.

7.1 *Principle of Floatation*

Principle of Floatation states that when an object is floating in equilibrium in a fluid, the weight of the object is equal to the weight of fluid displaced by the object.

Hence, an object floats because the upthrust acting on it is equal in magnitude and opposite in direction to the weight of the object. The object sinks when the upthrust acting on it is less than its weight.

A ship made of steel can float because its internal hollow volume displaces a large amount of water and produces sufficient upthrust to keep the ship floating.

Example 10

A student rolled a lump of plasticine into the shape of an air-tight sphere. He dropped the plasticine into a bucket of water and observed that the plasticine sphere sank to the bottom of the water.

Using Archimedes' Principle and the Principle of Floatation, show that the density of the plasticine must be greater than the density of water.

How would it possible for the student to make the lump of plasticine float in the bucket of water, even though the density of the plasticine is greater than that of water?

Example 11

A boat floating in fresh water displaces 35.6 kN of water.

(a) What weight of water would this boat displace if it were floating in salt water of density 1024 kg m^{-3}?

(b) What is the volume of salt water displaced?

8 Equilibrium of Forces

Learning Outcomes

(h) show an understanding that, when there is no resultant force and no resultant torque, a system is in equilibrium

8.1 Conditions for Equilibrium

When a *body* is in equilibrium,

(1) resultant force on it must be zero in any direction

(2) resultant torque on it must be zero about any axis of rotation

Learning Outcomes

(i) use a vector triangle to represent forces in equilibrium

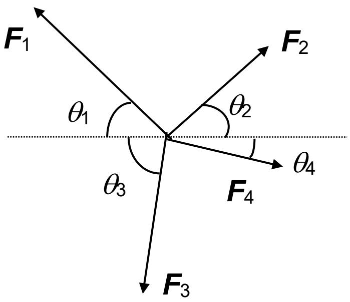
8.1.1 Translational Equilibrium

When the resultant force on a body is zero in any direction, there is no acceleration of its centre of mass and the body is said to be in *translational equilibrium*.

Method 1: Summation of forces in the x, y (and z) direction is zero

If the forces can be resolved into components in two chosen perpendicular directions, say x and y-axis, then both $\Sigma F_x = 0$ and $\Sigma F_y = 0$.

If a body is stationary under the influence of the forces as shown below, then at equilibrium,

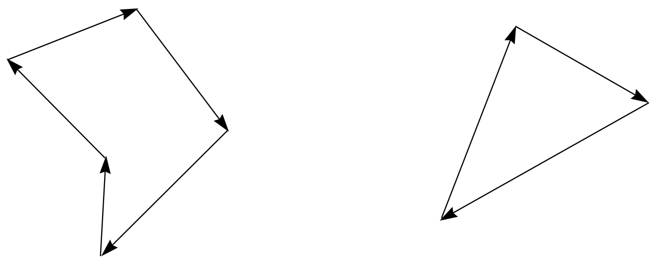


$\Sigma F_x = 0$: $\quad -F_1 \cos\theta_1 + F_2 \cos\theta_2 - F_3\cos\theta_3 + F_4\cos\theta_4 = 0$

$\Sigma F_y = 0$: $\quad F_1 \sin\theta_1 + F_2 \sin\theta_2 - F_3\sin\theta_3 - F_4\sin\theta_4 = 0$

Method 2: Forces must form a closed polygon

Since the resultant force acting on a body is zero, the *vector* sum of forces acting on the body is zero in any direction. The vector diagram showing the addition of all forces acting on the body will be a closed polygon.



Learning Outcomes

(e) define and apply the moment of a force and the torque of a couple.
(f) show an understanding that a couple is a pair of forces which tends to produce rotation only.
(g) apply the principle of moments to new situations or to solve related problems.

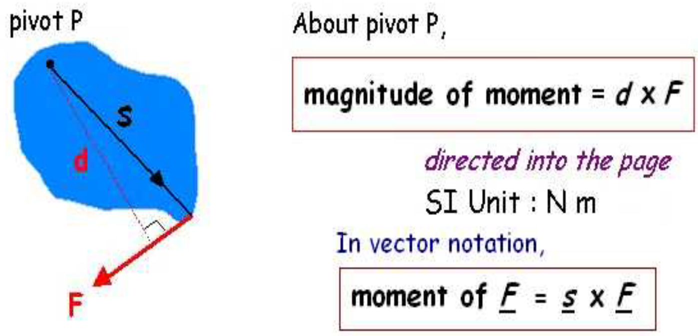
8.1.2 Rotational Equilibrium

When the resultant torque of a body is zero any axis of rotation, there is no angular acceleration of the object and the body is said to be in *rotational equilibrium*.

Moment of a Force

The **moment of a force** about a point is the product of the magnitude of the force and the perpendicular distance of the force from that point (pivot).

The direction of the moment by the force follows the Right-Hand Corkscrew Rule.



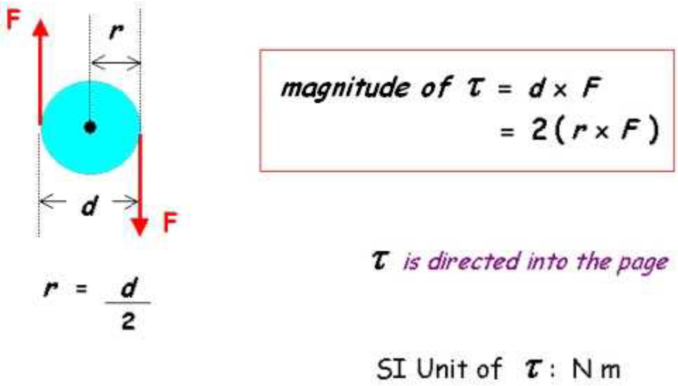
Torque of a Couple

The **torque of a couple** is the product of one of the forces and the perpendicular distance between the forces.

(Must include definition of couple if asked to define torque of a couple)

A **couple** consists of a pair of forces of equal magnitude but acting in opposite directions whose lines of action are parallel but separate.

The magnitude of the *torque of a couple* τ is defined as the product of one of the forces *F* and the perpendicular distance *d* between the lines of action of the forces.



Principle of Moments

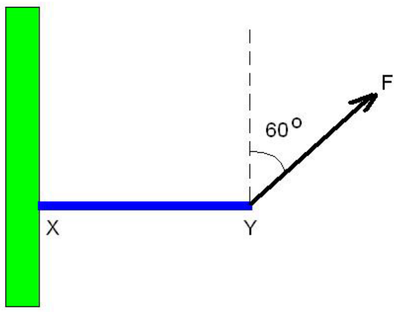
When the resultant torque of a body is zero about any axis of rotation, it further implies that the sum of moments of all forces acting on the body would be zero about any axis.

Hence, the principle of moments may be used to deal with bodies in rotational equilibrium.

The **Principle of Moments** states that when a system is in equilibrium, the sum of clockwise moments about any axis must be equal to the sum of anticlockwise moments about the same axis.

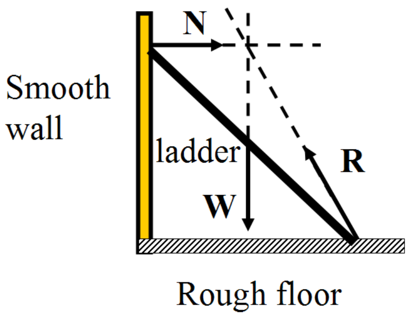
Example 12

A uniform rod XY of weight 10.0 N is freely hinged to a wall at X. It is held horizontal by a force F acting from Y at an angle of 60° to the vertical as shown in the diagram. What is the value of F?

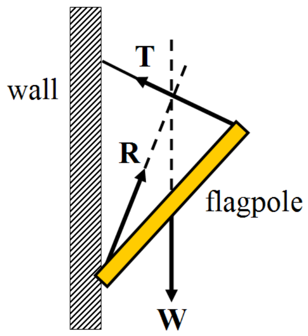


8.2 System in Equilibrium

For systems in equilibrium, notice that the lines of action of the **non-parallel** forces pass through a common point (in the physical diagram).



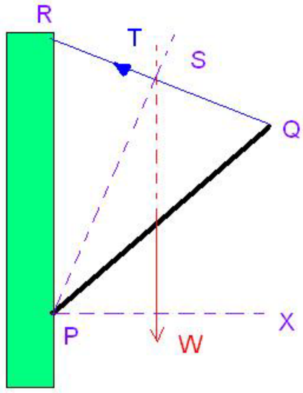
Physical Diagram



Physical Diagram

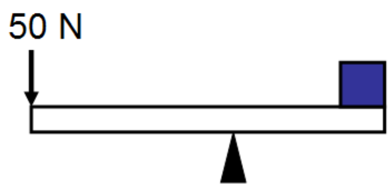
Example 13

The diagram below shows a heavy flagpole PQ hinged at a vertical wall at end P and held by a wire connected to the end Q and a point R on the wall. The weight of the flagpole is *W* and the tension in the wire is *T*. What is the direction of the force exerted by the wall on the flagpole?



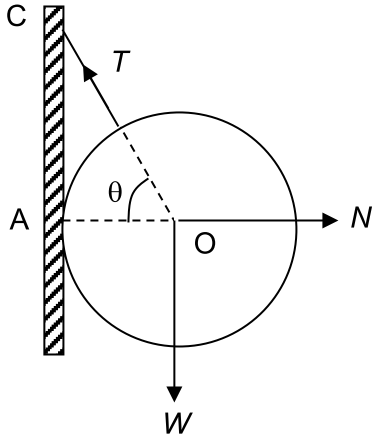
Example 14

A block of mass 20 kg is to be lifted by a light horizontal level of length 1.0 m. Where should the lever be pivoted so that a force of 50 N can balance the block as shown by the figure on the right?



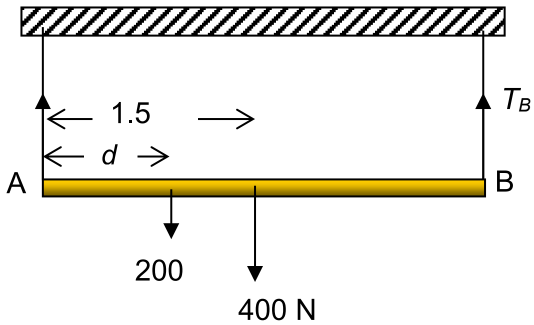
Example 15

A smooth sphere of weight W is supported by a string and is in contact with a smooth vertical wall. One end of the string is fastened to a point on the surface of the sphere and the other end is attached to a point on the wall. If the length of the string is equal to the radius of the sphere, find the tension T in the string and the reaction N of the wall on the sphere in terms of its weight, W.



Example 16

A heavy uniform beam AB is 3.0 m long and has a weight of 400 N. It is suspended in a horizontal position by two vertical strings attached to its ends. Each of the strings can withstand a maximum tension of 350 N. How far from A can an object of weight 200 N be placed on the beam so that one of the strings is on the verge of breaking?



Problem-solving tip:

Take moments about a point through which an unknown force acts.

Definition List

Force	Force is directly proportional to the rate of change of momentum. The direction of the force is in the direction of the change of momentum.
Field of force	A region of space where a force is felt.
Hooke's Law	Force is directly proportional to extension, provided that the elastic limit has not been exceeded.
Elastic limit	Point beyond which (the spring) does not return to its original length when the load is removed.
Pressure	Pressure is force per unit area.
Hydrostatic pressure	The pressure at a certain depth or at a point in the liquid.
Density	Density = mass / volume
Upthrust, *U*	Upthrust is the vertical upward force exerted by the surrounding fluid when a body is submerged fully or partially in a fluid. It is equal in magnitude and opposite in direction to the weight of fluid displaced by the body.
Origin of upthrust	Due to the difference in pressure exerted by the fluid at the top and bottom surfaces of the body.
Archimedes' Principle	States that for any object immersed partially or fully in a fluid, the upthrust is equal in magnitude and opposite in direction to the weight of fluid displaced by the object.
Principle of floatation	States that when an object is floating in equilibrium in a fluid, the weight of the object is equal to the weight of fluid displaced by the object.

Frictional Force	Force exerted by one body on another body when two bodies slide over one another. It is caused by irregularities in the surfaces in mutual contact and depends on the surfaces in contact as well as how much they are pressed against each other. It is the component of contact force along surface of contact.
Drag Force	The frictional force experienced either by an object as it moves through a fluid or by a fluid as it moves over a surface.
Tensile Forces	Two forces acting in opposite directions on an object so as to tend to increase its length along the direction of the forces.
Compressive Force	Two forces acting in opposite directions on an object so as to tend to reduce its length along the direction of the forces.

Conservative Force	A force is conservative if the work it does on an object moving between two point is independent of the path the object takes e.g. gravitational, electrostatic, elastic forces.
Non-Conservative Force	A force is non-conservative if the work it does on an object depends on the path taken by the object between its initial and final points e.g. friction, drag force, propulsive force.
Equilibrium	No resultant force acting on the body in any direction and no resultant moment acting on the body about any point.
Centre of Gravity	The single point at which the entire weight of the body can be considered to act.
Centre of Mass	The centre of mass is the mean location of all the mass in a system. It is the point through which a single force must act for the object to accelerate linearly without any rotation.
Moment of a Force	The moment of a force about a point is the product of the magnitude of the force and the perpendicular distance of the force from the point (pivot).
Couple	A couple consists of a pair of forces equal in magnitude but acting in opposite directions whose lines of action are parallel but separate.
Torque of a Couple	The torque of a couple is the product of one of the forces and the perpendicular distance between the forces. Note: must include definition of couple too.
Principle of Moments	When a system is in equilibrium, the sum of the clockwise moments about any point is equal to the sum of the anti-clockwise moments about the same point.
Terminal velocity	The constant velocity achieved by an object when it is falling under gravity through a viscous fluid.