Motion In a Circle - Problem Set

Exercises

Kinematics of Uniform Circular Motion

E1. What is $\frac{\pi}{4} rad$ in degrees? [$45°$]

$\frac{\pi/4}{2\pi} \times 360°$

E2. What is $300°$ in radians? [5.23 rad]

$\frac{5}{3}\pi$ = 5.23 radians

E3. (2020 P1 Q11)

The minute hand of a large clock is 3.00 m long.

What is the magnitude of its angular velocity?

A 1.39×10^{-4} rad s^{-1}

B 1.75×10^{-3} rad s^{-1}

C 5.24×10^{-3} rad s^{-1}

D 1.05×10^{-1} rad s^{-1}

[B]

$\omega = \frac{2\pi}{T} = \frac{2\pi}{60\times60} = 1.75 \times 10^{-3}\ rad\ s^{-1}$

E4. (2016 P1 Q12)

What is the angular velocity of the Earth as it rotates on its axis?

A 1.75×10^{-3} rad s^{-1}

B 1.99×10^{-7} rad s^{-1}

C 4.36×10^{-3} rad s^{-1}

D 7.27×10^{-5} rad s^{-1}

[D]

$\omega = \frac{2\pi}{T} = \frac{2\pi}{24\times60\times60} = 7.27 \times 10^{-5}\ rad\ s^{-1}$

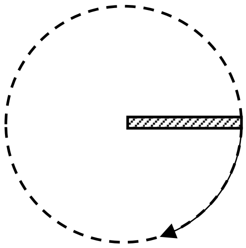
E5. Assuming that the earth moves in a circle at a constant rate around the sun, calculate the angular velocity of the earth around the sun. (Hint: how long does it take for the earth to go round the sun once?) [$1.99 \times 10^{-7} rad\ s^{-1}$]

$\frac{2\pi}{365\times24\times60\times60} = 1.99 \times 10^{-7} rad\ s^{-1}$ (The earth takes 365 days to go round the sun. The earth takes 24 hours to rotate about its own axis which results in the day and night cycle.)

E6. A car is moving around a circular track of radius $400\ m$ at constant angular velocity $0.050\ rad\ s^{-1}$. Calculate the total distance the car moves in 5 minutes. [6000 m]

Angular displacement in 5 min = $\omega t = 0.05 \times 5 \times 60 = 15\ rad$
Distance moved in 5 min = $r\theta = 15 \times 400 = 6000\ m$

E7. A rod is made to spin at a constant rate of 3 complete revolutions per second as shown below.

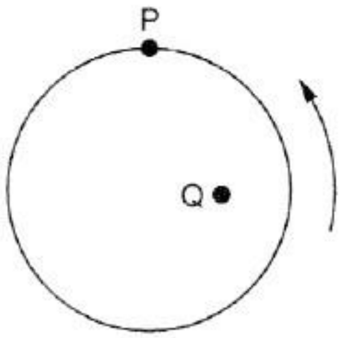


What is the difference in angle between the rod's current position and its position 0.50 seconds later? [$\pi\ rad$]

Angular displacement in 0.50 s = $\omega t = \frac{3\times2\pi}{1} \times 0.50 = 3\pi$
The rod would have moved 1.5 complete circles.
Hence the difference in angle = $3\pi - 2\pi = \pi\ rad = 3.14\ rad$

E8. (2017 P1 Q9)

P and Q are points on a disc that is rotating with uniform circular motion about its centre.

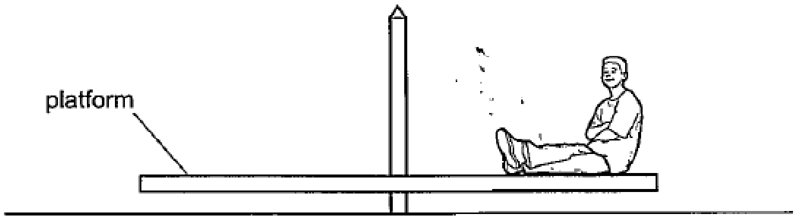


How do the angular velocities of P and Q compare and how do the angular displacements of P and Q compare after a quarter of a revolution?

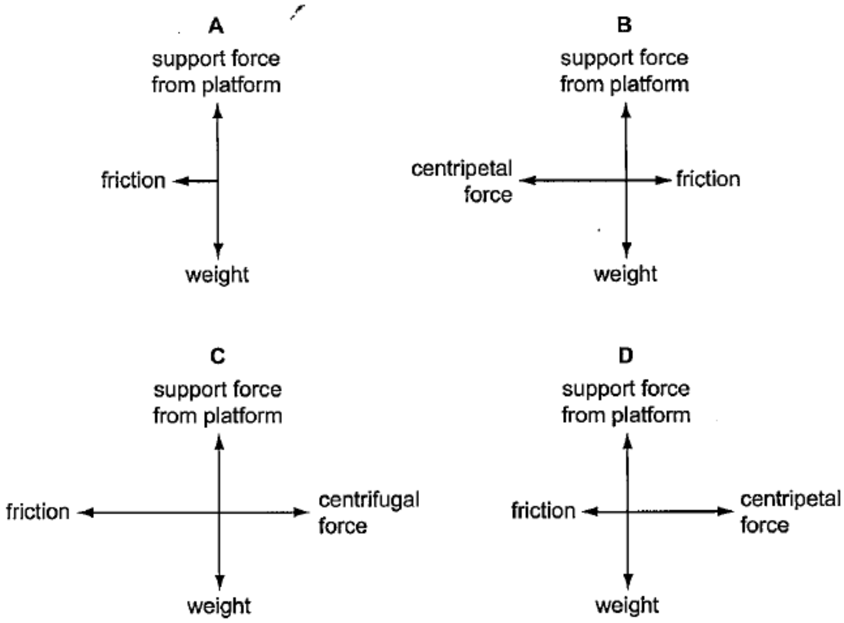
	angular velocity after a quarter of a revolution	angular displacement after a quarter of a revolution
A	different	different
B	different	the same
C	the same	different
D	the same	the same

D (both have the same angular velocity, hence angular displacement will be the same, although actual distance travelled will be larger for P)

E9. The diagram shows a child sitting on a playground turntable, which is turning with constant angular velocity.



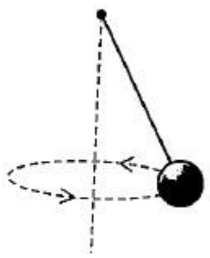
Which diagram shows the forces acting on the child when in the position shown?



A (centripetal force is a resultant force)

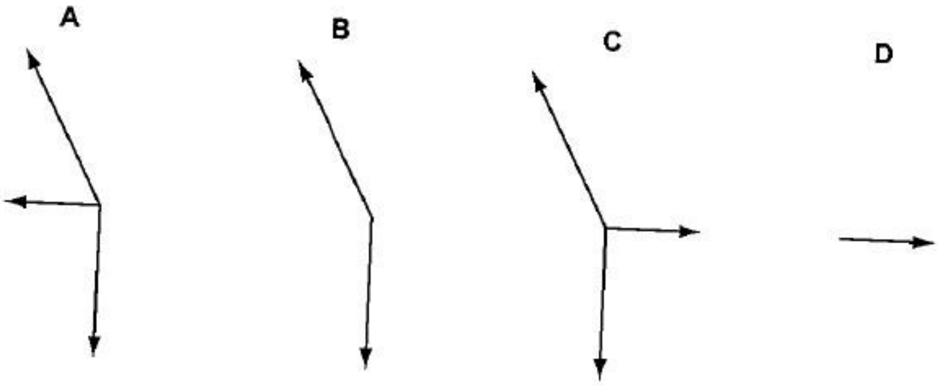
E10. (2013 P1 Q11)

11 A small ball suspended from a light thread moves in a horizontal circle at a constant speed.



A student draws the forces acting on the ball but fails to label them.

Which diagram shows the correct forces?



B (The tension and weight result in a centripetal force to the left pointing to the center of the horizontal circle)

Kinematics of Uniform Circular Motion

P1. A body rotates with uniform speed in a circle of radius r and takes time T to complete one revolution.

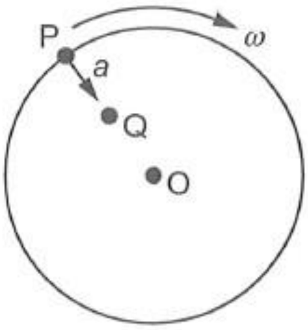
What are the magnitudes of the angular velocity ω, the linear velocity v, and the acceleration a?

	angular velocity ω	linear velocity v	acceleration a
A	$\frac{1}{T}$	$\frac{4\pi r}{T}$	$\frac{2\pi r}{T^2}$
B	$\frac{2\pi}{T}$	$\frac{2\pi r}{T}$	$\frac{2\pi r}{T^2}$
C	$\frac{2\pi}{T}$	$\frac{2\pi r}{T}$	$\frac{4\pi^2 r}{T^2}$
D	$\frac{2\pi}{T}$	$\frac{4\pi r}{T}$	$\frac{4\pi^2 r}{T^2}$

C ($v = r\omega$, $a = \omega^2 r$)

P2. (2017 P1 Q9)

A disc of radius 4.0 cm rotates about its centre O. P is a point on the circumference and Q is halfway between O and P.



At P, the angular velocity ω is 2.0 rad s^{-1} and the centripetal acceleration a is 16 cm s^{-2}.

Which row gives the values of these quantities at Q?

	ω / rad s^{-1}	a / cm s^{-2}
A	1.0	0.5
B	1.0	2.0
C	2.0	2.0
D	2.0	8.0

D (ω is the same for both points
$a = \omega^2 r$ Since radius is halved, the centripetal acceleration is also halved to 8 cm s^{-2})

P3. (2020 P1 Q10)

The orbit of the Moon around the Earth is circular with a radius of 3.85×10^8 m. The time for one orbit is 27.3 days.

What is the centripetal acceleration of the Moon?

A 1.84×10^{-20} m s^{-2}

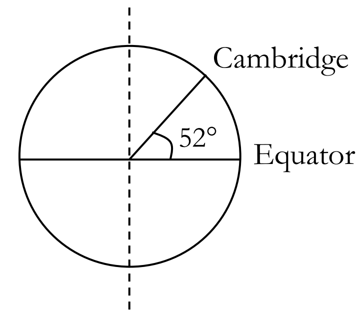
B 2.73×10^{-3} m s^{-2}

C 1.03×10^{3} m s^{-2}

D 1.57×10^{3} m s^{-2}

B $(a = \omega^2 r = \frac{4\pi^2}{(27.3\times24\times60\times60)^2} \times 3.85 \times 10^8 = 2.73 \times 10^{-3}\ ms^{-2})$

P4. Singapore is on the equator. Cambridge is at a latitude of 52° N, as shown in the figure.



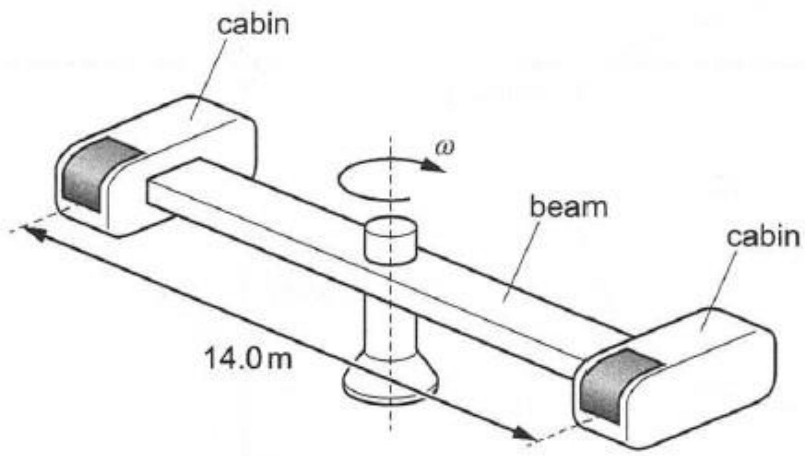
A student in Singapore has a centripetal acceleration a_s because of the earth's rotation about its axis. The centripetal acceleration of another student at Cambridge is a_c.
What are the magnitudes of the centripetal accelerations a_s and a_c?

(Radius of earth = 6.4×10^6 m;
angular velocity of earth about its axis = 7.3×10^{-5} rad s^{-1})

Radius of the circular path followed by student in Cambridge = $6.4 \times 10^6 \cos 52° = 3.94 \times 10^6\ m$
$a_c = \omega^2 r = (7.3 \times 10^{-5})^2 \times 3.94 \times 10^6 = 2.10 \times 10^{-2}\ m\ s^{-2}$
$a_s = \omega^2 r = (7.3 \times 10^{-5})^2 \times 6.4 \times 10^6 = 3.41 \times 10^{-2}\ m\ s^{-2}$

P5. (2019 P1 Q11)

As part of their training, astronauts are subjected to large forces by being placed in cabins at either end of a horizontal beam and rotated about a vertical axis.



The centres of the cabins are 14.0 m apart. The astronauts experience a centripetal acceleration of 20*g*.

What is the angular velocity ω of the astronauts?

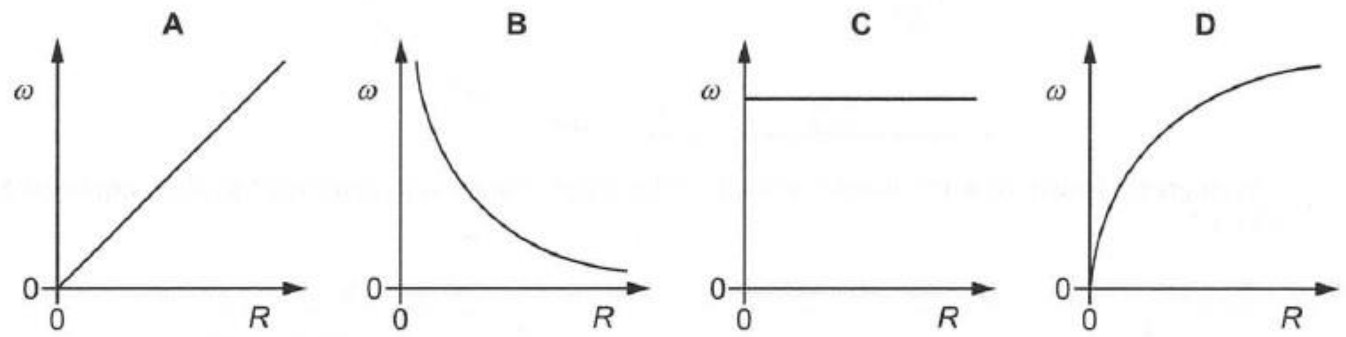
A 1.2 rad s^{-1} B 1.7 rad s^{-1} C 3.7 rad s^{-1} D 5.3 rad s^{-1}

D $(a = \omega^2 r \rightarrow \omega = \sqrt{\frac{a}{r}} = \sqrt{\frac{20g}{7.0}} = 5.3\ rad\ s^{-1})$

P6. (2019 P1 Q10)

10 A small ball on the end of a long string moves in a horizontal circle of radius R. As the ball moves, R increases but the speed remains constant.

Which graph best represents the relationship between the angular velocity ω of the ball and the radius of the circle?

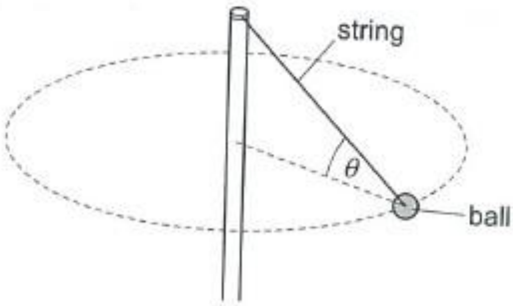


B ($v = \omega r$, since v is constant, $\omega \propto \frac{1}{R}$)

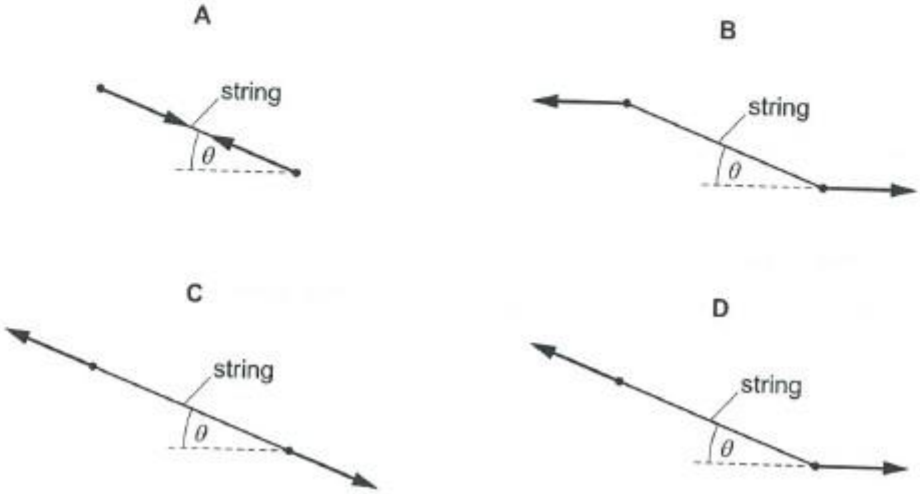
<u>Centripetal Force</u>

P7. (2015 P1 Q10)

10 A ball is attached to a string of negligible mass and moves in a horizontal circle. The string makes an angle θ with the horizontal, as shown.

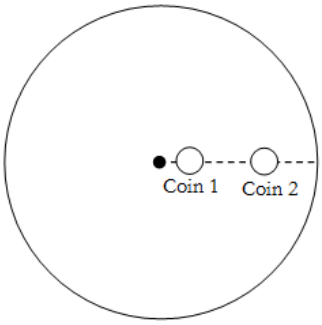


Which force diagram correctly shows the forces on the string?



C (String is in tension so it must be being pulled, string assumed to have negligible weight. Note this is for forces on string not the ball. See E10 for forces on the ball)

P8. Two identical coins are placed on a flat horizontal turn-table as shown in the figure below.



Explain which coin would slip first as the angular velocity of the turn-table is increased.

Centripetal force required to maintain circular motion = $m\omega^2 r$

Since both coins are rotating along a common radial line from the centre of the circle, their angular velocities ω are constant.

But Coin 2 requires a larger centripetal force to maintain its circular motion because the radius *r* of its circular path is larger than that of Coin 1.

Since the frictional force by the turn-table on each coin is fixed, we expect that Coin 2 would slip first as the turn-table is rotated.

P9. (2016 P1 Q13)

A bus on a horizontal road travels round a corner. The corner is an arc of a circle. When the road is dry, the maximum safe speed is $16\,\text{m}\,\text{s}^{-1}$.

When the road is wet, the maximum frictional force between the surface of the road and the wheels of the bus is halved.

What is the maximum safe speed of the bus for it to travel round the corner when the road is wet?

A $\frac{16}{4}$ **B** $\frac{16}{(2\sqrt{2})}$ **C** $\frac{16}{2}$ **D** $\frac{16}{\sqrt{2}}$

D (The frictional force provides the centripetal force. Hence $F = \frac{mv^2}{r} = \frac{m(16)^2}{r}$

The radius r is constant. When the maximum frictional force is halved, v^2 where v is the maximum safe speed is halved too.

Wet road, $F_{wet} = \frac{mV_{new}^2}{r} = ½\,F$

$\frac{mv_{new}^2}{r} = ½\,\frac{m(16)^2}{r}$

Hence new $v_{new} = \sqrt{\frac{16^2}{2}} = \frac{16}{\sqrt{2}}$

P10. On a normal day, the maximum friction between the wheels of a 1000 kg car and the road is 6500 N.

(a) Calculate the maximum speed in km h^{-1} the car can go round a sharp circular bend of radius 6.0 m without skidding.

The friction on the road provides the centripetal force for the car to turn.

Hence maximum centripetal force $= 6500 = m\frac{v^2}{r}$

Maximum speed = $\sqrt{\frac{6500\times r}{m}} = \sqrt{\frac{6500\times 6.0}{1000}} = 6.25ms^{-1} = 22.5km\,h^{-1}$

(b) A circular bend of radius 6.0 m is indeed a very sharp bend. Usually in Singapore, most of the bends have a speed limit of $50\ km\,h^{-1}$. Calculate the minimum radius of the bend for a speed limit of $50\ km\,h^{-1}$ to be safe.

$F_C = m\frac{v^2}{r}$

$F_C \leq 6500$

$m\frac{v^2}{r} \leq 6500$

$r \geq m\frac{v^2}{6500}$

$r \geq 1000\frac{13.89^2}{6500}$ → minimum radius 29.7 m

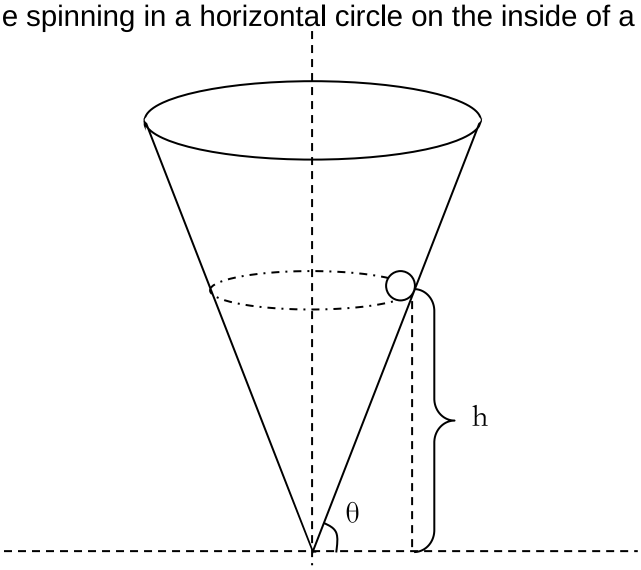
(c) Explain what will happen to your calculated value of the minimum radius if you have to take into account rainy weather.

Wet weather will result in less maximum friction between the tyres and the road. As such the maximum centripetal acceleration provided will be decreased and hence the minimum radius must be larger.

(d) On a racing track, cars or bicycles need to go on a speed as fast as possible. Suggest how a sharp bend can be made safer for faster speeds without increasing the friction between the tyres and the road.

The bend can be a banked road and the normal contact force from the banked road will provide additional centripetal acceleration.

P11. Consider a marble spinning in a horizontal circle on the inside of a cone as shown below



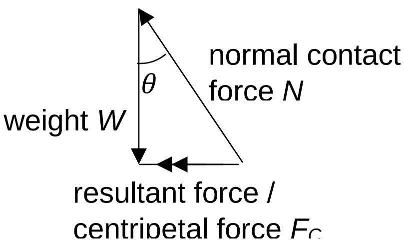
(a) Show that the speed of the marble for circular motion at height h is $\sqrt{gh}$.

Recalling our vector diagram

$W = mg = N\cos\theta$

$F_C = m\frac{v^2}{r} = N\sin\theta$

$\tan\theta = \frac{m\frac{v^2}{r}}{mg} = \frac{v^2}{gr}$



$tan\theta = \frac{h}{r} = \frac{v^2}{gr}$

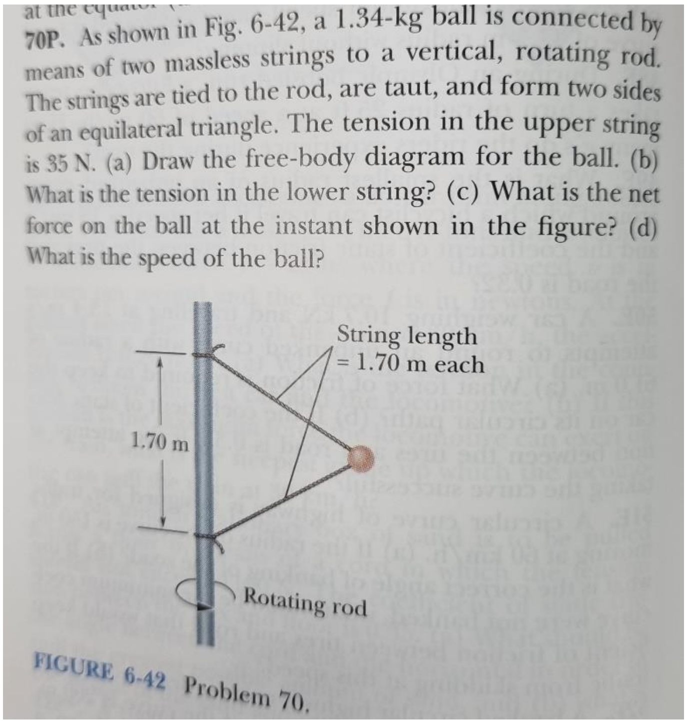
$v = \sqrt{gh}$

(b) Hence, state how the speed of the marble must be adjusted such that the marble is moving in a horizontal circle at a greater height.

Based on the equation, a larger h will require a larger speed v.

<u>Challenging Question</u>

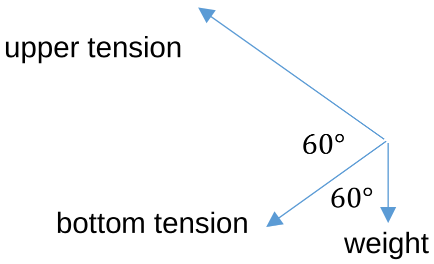
C1.



70P. As shown in Fig. 6-42, a 1.34-kg ball is connected by means of two massless strings to a vertical, rotating rod. The strings are tied to the rod, are taut, and form two sides of an equilateral triangle. The tension in the upper string is 35 N. (a) Draw the free-body diagram for the ball. (b) What is the tension in the lower string? (c) What is the net force on the ball at the instant shown in the figure? (d) What is the speed of the ball?

FIGURE 6-42 Problem 70.

(a)



(b) Vertical components of the forces must add up to zero since this is horizontal circle

$T_{upper}\ sin30° = T_{bottom} \sin 30° + mg$

$35\ sin30° = T_{bottom} \sin 30° + 1.34g$

$T_{bottom} = 8.71\ N$

(c) Net force is the sum of the horizontal components of the tensions

$F_c = T_{upper}\ cos30° + T_{bottom} \cos 30° = 35\ cos30° + 8.71 \cos 30° = 37.9\ N$

(d) $F_c = \frac{mv^2}{r} = 37.9$

$\frac{1.34 \times v^2}{1.70 cos30°} = 37.9$

$v = 6.45\ m\ s^{-1}$