2023 JC1 H1 REVISION SET A-1
COMPLETE SOLUTIONS

Qn	EXPONENTIAL & LOGARITHMIC FUNCTIONS
1(a)	**VJC 2011 MYE Q2** $\lg(x-8)+\lg\left(\frac{9}{2}\right)=1+\lg\left(\frac{x}{4}\right)$ $\lg(x-8)-\lg\left(\frac{x}{4}\right)=\lg 10-\lg\left(\frac{9}{2}\right)$ $\lg\left(\frac{x-8}{x/4}\right)=\lg\left(\frac{10}{9/2}\right)$ $\frac{4x-32}{x}=\frac{20}{9}$ $36x-288=20x$ $\therefore x=18$
(b)	$\log_5(2x+1)-\log_5(3x-5)=1$ $\log_5\frac{(2x+1)}{(3x-5)}=1$ $\frac{(2x+1)}{(3x-5)}=5$ $(2x+1)=5(3x-5)$ $2x+1=15x-25$ $13x=26$ $x=2$
2	**SRJC 2016 PROMO Q3** **(a)** $\ln(pe^{25}) = \ln p + \ln(e^{25})$ $= k + 25\ln(e)$ $= k + 25$ **(b)** $3e^{x} - 4e^{-x} = 11$ $3e^{2x} - 4 = 11e^{x}$ Let $y = e^{x}$ $3y^2 - 4 = 11y$ $3y^2 - 11y - 4 = 0$ $(3y + 1)(y - 4) = 0$ $y = -\frac{1}{3}$ or $y = 4$ $e^{x} = -\frac{1}{3}$ (rejected $\because$ $e^{x} > 0$ for all x) or $e^{x} = 4$ $x = \ln 4$
3	**CJC 2012 PROMO Q3** (a) $2e^{3x}-5e^{x}=3e^{-x}$ $\Rightarrow 2e^{4x}-5e^{2x}=3$ $\Rightarrow 2e^{4x}-5e^{2x}-3=0$

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	Let $y = e^{2x}$. $\therefore 2y^2 - 5y - 3 = 0$ $(2y+1)(y-3) = 0$ $y = -\frac{1}{2}$ or $y = 3$ $\therefore e^{2x} = -\frac{1}{2}$ (rejected, since $e^{2x} > 0$) or $e^{2x} = 3$ So, $x = \frac{\ln 3}{2}$ (b) $\log_a(ax^2) = \log_a a + 2\log_a x = 1 + 4\log_a \sqrt{x} = 1 + 4(3) = 13$
4	**RI 2012 PROMO Q 2** $x = \ln(y+1) \Rightarrow y + 1 = e^x$ $\therefore y = (1 + y + 1)(1 - y - 1) \Rightarrow y^2 + 3y = 0$ $\Rightarrow y(y+3) = 0 \Rightarrow y = 0$ or $y = -3$ When $y = -3$, $x = \ln(-2)$ (N.A.) When $y = 0$, $x = \ln(1) = 0$
5	$\frac{4^{3x}}{64} = 4^y \Rightarrow \frac{2^{6x}}{2^6} = 2^{2y}$ i.e. $6x - 6 = 2y \Rightarrow y = 3x - 3$ $\lg(y-x) = 1 - \lg(x-1)$ $\lg(2x-3) = 1 - \lg(x-1)$ $\lg(2x-3) + \lg(x-1) = 1$ $\lg[(2x-3)(x-1)] = \lg 10$ $(2x-3)(x-1) = 10$ $2x^2 - 5x - 7 = 0$ $(2x-7)(x+1) = 0$ $x = -1$ or 3.5 Reject $x = -1$ since $\lg(x-1)$ is undefined. When $x = 3.5$, from $y = 3x - 3 = 10.5 - 3 = 7.5$ Therefore $x = 3.5$ and $y = 7.5$
6	**TJC 2016 PROMO Q3** For $\log_4 q^2 - \log_4 q - p = 0$ $\log_4 q^2 - \log_4 q - p = 0 \Rightarrow \log_4 \frac{q^2}{q} = p \Rightarrow \log_4 q = p$ or $2\log_4 q - \log_4 q = p \Rightarrow \log_4 q = p$ then $q = 4^p$ For $\sqrt{\log_p q} = 2 \Rightarrow \log_p q = 4 \Rightarrow p^4 = q$ $\therefore q = 4^p = p^4$ (shown)

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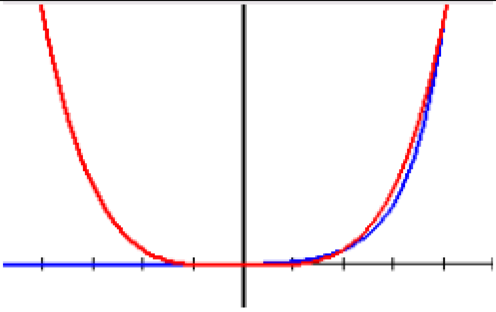
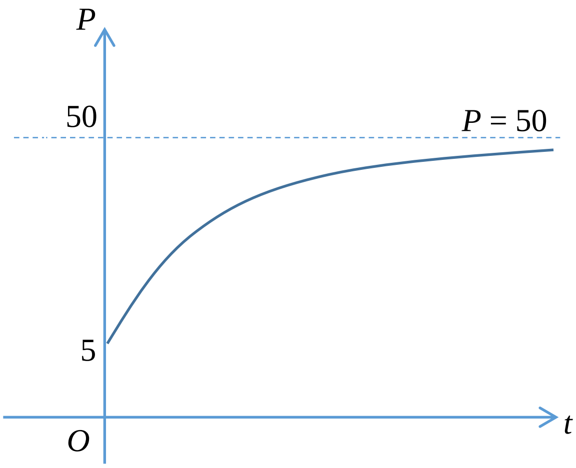
	 [G1] Shape and intersections From graph, $p=2$ or 4 (since $p > 0$) $\therefore q=16$ or 256
7	**JJC 2013 Promo Q4** (i) $P=-45e^{At}+B$ Given $t = 0$, $P = 5$, $5=-45e^{A(0)}+B$ $B=5+45(1)$ $=50$ (Shown) Hence, $P=-45e^{At}+50$ Given $t = 10$, $P = 35$, $35=-45e^{A(10)}+50$ $-15=-45e^{10A}$ $e^{10A}=\frac{-15}{-45}$ $e^{10A}=\frac{1}{3}$ $10A=\ln\left(\frac{1}{3}\right)$ $10A=-\ln 3$ $A=-\frac{1}{10}\ln 3$ where $k=-\frac{1}{10}$ (ii) $P=-45e^{-\frac{1}{10}(\ln 3)t}+50$ When $t = 20$, $P=-45e^{-\frac{1}{10}(\ln 3)(20)}+50=45$ Population = 45 000 (iii) $P=-45e^{-\frac{1}{10}(\ln 3)t}+50$ After a very long time, $e^{-\frac{1}{10}(\ln 3)t}\to 0$, $P\to 50$. Population = 50 000
	(iv) Sketch $P=-45e^{-\frac{1}{10}(\ln 3)t}+50$ 

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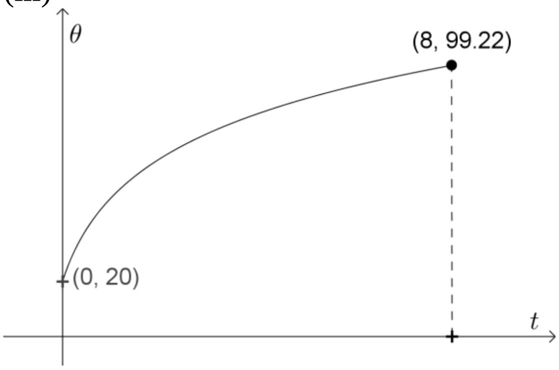
8	**RVHS 2016 PROMO Q11** (i) When $t=0,\ \theta=20$ $20=b+k\ln(0+1)$ $b=20$ When $t=2,\ \theta=65$ $65=20+k\ln(2(2)+1)$ $45=k\ln(5)$ $k=\dfrac{45}{\ln 5}$ (ii) When $\theta=50$, $50=20+\dfrac{45}{\ln 5}\ln(2t+1)$ $30=\dfrac{45}{\ln 5}\ln(2t+1)$ $\dfrac{2}{3}\ln 5=\ln(2t+1)$ $2t+1=e^{\frac{2}{3}\ln 5}$ $t=\dfrac{e^{\frac{2}{3}\ln 5}-1}{2}=0.962$ minutes (3sf)	(iii)  (iv) No. Suggested reason: Boiling point of water is $100^{\circ}\mathrm{C}$ but the graph is an increasing curve, so the model will be inaccurate once the temperature rises above $100^{\circ}\mathrm{C}$ as $t\to\infty$.