# *Topic 5 Work, Energy, Power*

***Guiding Questions:***

- *What is energy and how does it relate to work done?*
- *How does energy get transferred and transformed?*

***Content***

• Work
• Energy conversion and conservation
• Efficiency
• Potential energy and kinetic energy
• Power

***Learning Outcomes***

Candidates should be able to:

(a) define and use work done by a force as the product of the force and displacement in the direction of the force

(b) calculate the work done in a number of situations including the work done by a gas which is expanding against a constant external pressure: $W = p\Delta V$

(c) give examples of energy in different forms, its conversion and conservation, and apply the principle of energy conservation

(d) show an appreciation for the implications of energy losses in practical devices and use the concept of efficiency to solve problems

(e) derive, from the equations for uniformly accelerated motion in a straight line, the equation $E_k = \frac{1}{2}mv^2$

(f) recall and use the equation $E_k = \frac{1}{2}mv^2$

(g) distinguish between gravitational potential energy, electric potential energy and elastic potential energy

(h) deduce the elastic potential energy in a deformed material is related to the area under the force-extension graph

(i) show an understanding of and use the relationship between force and potential energy in a uniform field to solve problems

(j) derive, from the definition of work done by a force, the equation $E_p = mgh$ for potential energy changes near the Earth's surface

(k) recall and use the equation $E_p = mgh$ for gravitational potential energy changes near the Earth's surface

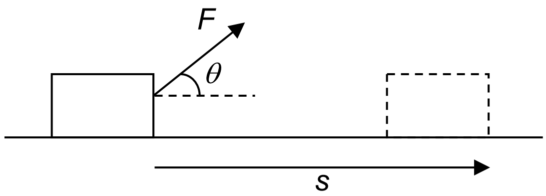
(l) define power as work done per unit time and derive power as the product of a force and velocity in the direction of the force.

(a) *define and use work done by a force as the product of the force and displacement in the direction of the force*

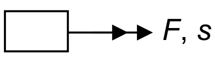
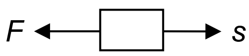
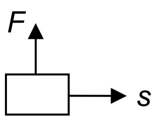
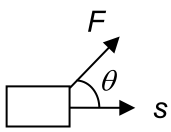
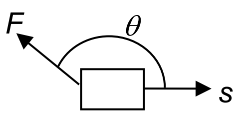
Work is a concept that provides a link between force and energy. Do not be misled by the many different meanings of the work in ordinary conversation – doing homework, going to work or having too much work to do. Not everything we call work in conversation is work in the physics sense of the word. We will now define work – in the physics sense.

| **Work done (by a constant force)** |
|---|
| Work done by a <u>constant force</u> on an object is defined as the product of the force and its displacement in the direction of the force. |

or $W = (F \cos \theta)s$
$W = F\,(s \cos \theta)$




where $F$ is the magnitude of the force, $s$ is the magnitude of the displacement and $\theta$ is the angle between the force and the displacement. Although force and displacement are vectors, work is a scalar quantity and the SI unit of work is the joule (J).

| | | |
|---|---|---|
| $\theta = 0^\circ$ |   | $W = F.s$ |
| $\theta = 180^\circ$ |   | $W =$ |
| $\theta = 90^\circ$ |   | $W =$ |
| $0^\circ < \theta < 90^\circ$ |   | $W =$ |
| $90^\circ < \theta < 180^\circ$ |   | $W =$ |

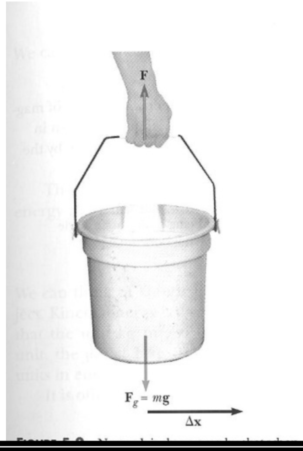
<u>Note:</u>

1. Work is an energy transfer. If the work done on a system is positive, energy is transferred <u>to</u> the system. If the work done is negative, energy is transferred <u>from</u> the system.

2. When discussing work, always use the phrase "Work done by ..… on ….."

**Example 1**

A man of mass 60 kg carrying a pail of 20 kg walks on a level road for 10 m. What is the work done by the man in carrying the load?



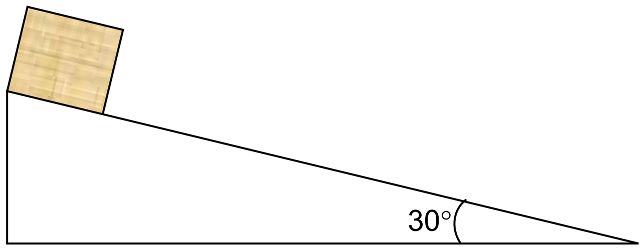

**Example 2**

A physics book is pushed 1.20 m along a horizontal tabletop by a horizontal force of 2.00 N. The opposing force of friction is 0.400 N.

(a) How much work is done on the book by the 2.00 N force?
(b) What is the work done on the book by the friction force?
(c) What is the total work done on the book?

**Example 3**

A 5.00 kg block slides from rest down a 2.50 m long rough incline of 30°. A friction force of 18.5 N acts on the block throughout the motion.




Determine

(a) the work done by the force of gravity.
(b) the work done by friction force between the block and the incline.
(c) the work done by the normal force.
(d) the total work done on the block.

**Note:**

1 When several forces act on an object, the total (or net) work done is the sum of the work done by each force separately, that is, $W_{total} = W_1 + W_2 + W_3 + \ldots = \Sigma W_i$

2 Another way to calculate the total work done is to find the work done by the net force as if there were a single force acting and then using the definition of work, that is,

$$W_{total} = (F_{net} \cos \theta)s$$

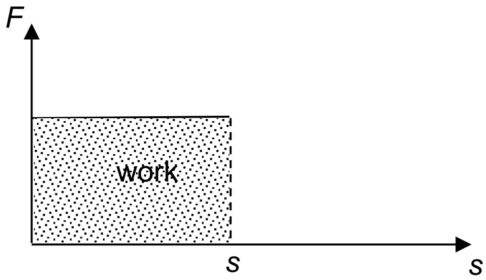
where $\theta$ is the angle between $F_{net}$ and the displacement $s$.

(b) *calculate the work done in a number of situations including the work done by a gas which is expanding against a constant external pressure: $W = P\Delta V$*

(h) *deduce the elastic potential energy in a deformed material is related to the area under the force-extension graph*

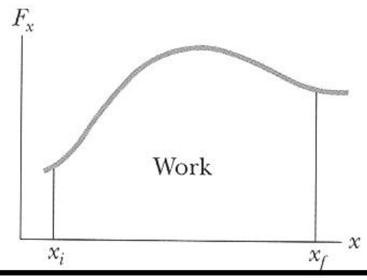
**Graphical interpretation of work done**

(a) If a constant force acts on an object over a displacement *s*, then the force-displacement graph would look like the one on the right.
Work done
$W = Fs$
= area under the force-displacement graph

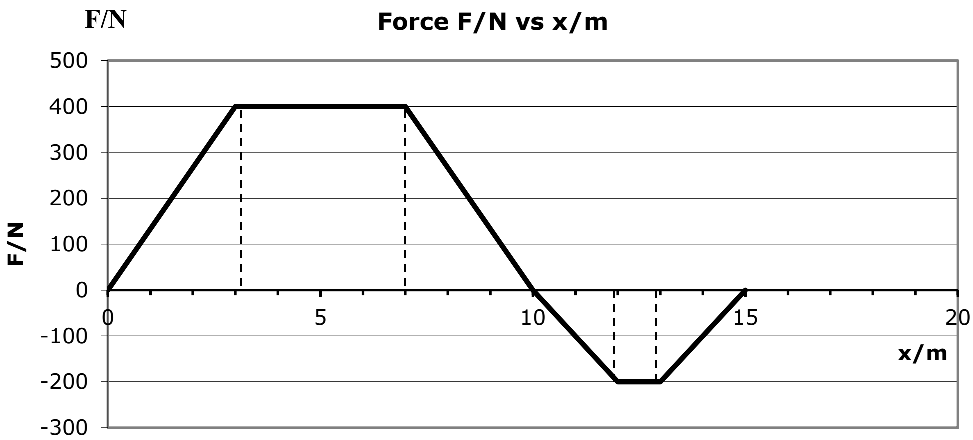



(b) For a variable force, the work done on an object that undergoes a displacement is still equal to the <u>area under the force–displacement graph</u>, that is,

$W = \int F dx$

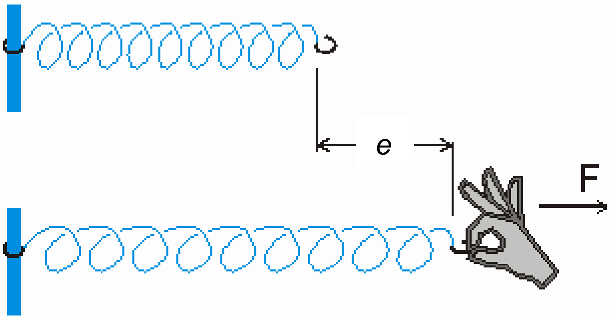


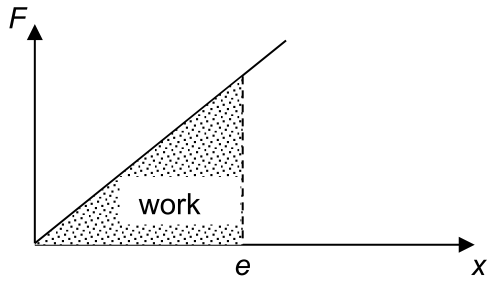

**Example 4**
The force on an object, acting along the *x* axis, varies as shown in the figure. Determine the work done by this force to move the object (a) from $x = 0$ to $x = 10$ m, and (b) from $x = 0$ to $x = 15$ m



## Work done to stretch a spring

If the spring obeys Hooke's law, $F = ke$, that is, the displacement $e$ of the spring is proportional to the force $F$ applied on it.



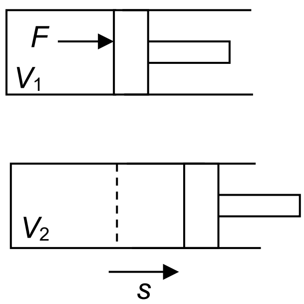




The work done $W$ by the force in stretching the spring is given by the area under the force–extension graph. This work done is stored as elastic potential energy $U$ in the spring.

$$\boxed{U = \frac{1}{2}Fe = \frac{1}{2}ke^2}$$

## Work done by a gas during expansion

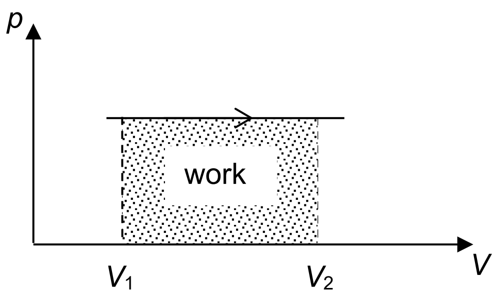
Consider a quantity of gas, at pressure $p$, in a syringe which has a frictionless piston as shown in the diagram. If the piston has cross-sectional area $A$, the force exerted by the gas on the piston is $F = pA$




If the gas expands slowly (so that we may consider the pressure of the gas to be constant) against a constant external pressure moving outwards a displacement $s$, then the force $F$ is constant.

The work done by the gas in expanding from $V_1$ to $V_2$

$$\boxed{\begin{aligned} W &= F.s = (pA)s = p(As) \\ &= p\Delta V \\ &= p(V_2 - V_1) \end{aligned}}$$




Note:

- ❖ When the gas expands ($V_2 > V_1$), work done by the gas $W$ is positive.
- ❖ When the gas is compressed ($V_2 < V_1$), work done by the gas $W$ is negative.
- ❖ Graphically, work done $W = \int pdV$, i.e., the area under the pressure–volume graph.

*(c) give examples of energy in different forms, its conversion and conservation, and apply the principle of energy conservation*

*(g) distinguish between gravitational potential energy, electric potential energy and elastic potential energy*

| |
|---|
| ***Principle of the conservation of energy***: |
| Energy can neither be destroyed nor created in any process. It can be transformed from one form to another, and transferred from one body to another but the total amount remains constant. |

Energy exists in different forms. Some of the common forms of energy are given below:

| Form/Type of Energy | Common Examples/Descriptions |
|---|---|
| Chemical Energy | Energy stored in the bonds of chemical compounds and the energy may be released during a chemical reaction. E.g. chemical potential energy stored in fuels such as oil, wood, coal, electric cells, food. |
| Nuclear Energy | Energy stored in the nucleus of an atom and released through fission, fusion or radioactivity. E.g. atomic bombs, nuclear reactors. |
| Radiant Energy | Energy that is transmitted in the form of electromagnetic radiation. E.g. X-rays, visible light, radio waves. |
| Internal Energy | Energy associated with motion and interaction of atoms and molecules in solids, liquids and gases, in the form of kinetic and potential energy, related to our sensation of temperature. |
| Rest / Mass Energy | Mass is a form of energy as given by Einstein's famous equation: $\Delta E = \Delta mc^2$ |
| Mechanical Energy<br>(a) Kinetic Energy<br>(b) Potential energy | (a) Energy possessed by all objects in motion.<br>(b) (i) Gravitational Potential Energy – waterfall, raised objects<br>(ii) Elastic Potential Energy – compressed or stretched springs, bent condition of a diving board, stretched elastic band<br>(iii) Electric Potential Energy – exists between charges |

**Examples of Conversion of Energy**

- *Swinging pendulum*: gravitational potential energy of body converted to kinetic energy and vice versa.
- *Diver on springboard:* Chemical energy stored in body of diver allows him to compress diving board causing the bent springboard to store elastic potential energy which is then reconverted into kinetic energy for the diver by giving him an upward push.
- *Burning of fuels:* Stored chemical energy in the fuels is converted into heat and radiant energy.
- *Connecting a battery to a filament lamp:* Stored chemical energy in the cells is converted into electrical energy, which in turn is converted in the filament into heat and radiant energy.

*(e)* *derive, from the equations for uniformly accelerated motion in a straight line, the equation* $E_k = \frac{1}{2}mv^2$

*(f)* *recall and use the equation* $E_k = \frac{1}{2}mv^2$

The energy that a body possesses solely due **to its motion** is called kinetic energy.

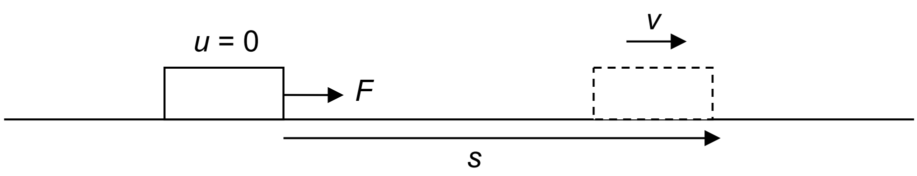
If a body of mass *m* is moving with speed *v*, then its

$$\boxed{\text{kinetic energy } E_k = \frac{1}{2}mv^2}$$

Kinetic energy is a scalar quantity and its SI unit is the joule (J).

**<u>To derive $E_k = ½mv^2$ from the equations of motion</u>**

Consider a stationary body of mass *m* which moves a horizontal displacement *s* under the action of a constant net force *F*.




Since the force is constant, the body moves with a constant acceleration *a* given by Newton's second law of motion:

$$F = ma$$

When the body undergoes a displacement *s*, its final velocity *v* can be found from the following equation of motion:

$$v^2 = u^2 + 2as$$

or $$s = \frac{v^2}{2a} \qquad \text{since } u = 0$$

The work done on the body $W = F.s$

$$= ma\left(\frac{v^2}{2a}\right)$$

$$= \frac{1}{2}mv^2$$

The work done by force *F* increases the kinetic energy of the body. Hence the kinetic energy of the body at speed *v* is $E_k = \frac{1}{2}mv^2$.

In general, when work is done by a force $F$ acting over a distance $s$ on a body, and the speed of a body, with mass $m$, increases from $u$ to $v$ then

$$Fs = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

$$W = (E_k)_{final} - (E_k)_{initial}$$

$$W = \Delta E_k$$

➔ <u>the work done by a resultant external force on a body = change in kinetic energy of the body</u>. When the work done is positive, the speed of the body increases, increasing the kinetic energy. When the work done is negative, the speed of the body decreases, decreasing the kinetic energy.

**Example 5**

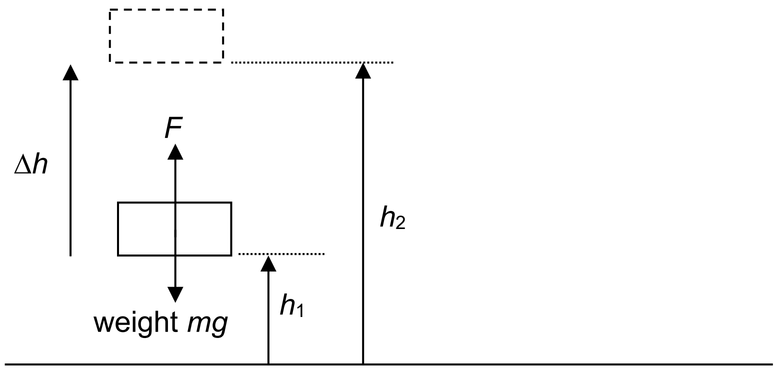
A car of mass 800 kg and moving at 30 m $s^{-1}$ along a horizontal road is brought to rest by a constant retarding force of 5000 N. Calculate the distance the car moves while coming to rest.

*(j)* *derive, from the definition of work done by a force, the equation $E_p = mgh$ for potential energy changes near the Earth's surface*

*(k)* *recall and use the equation $E_p = mgh$ for gravitational potential energy changes near the Earth's surface*

The energy that a body possesses due to its position relative to other objects, stresses within itself, its electric charge or state is called potential energy. Potential energy (p.e.) is a scalar quantity and its SI unit is the joule (J). Examples of potential energies are gravitational p.e., elastic p.e., and electric p.e.

**To derive $E_p = mgh$ from the defining equation $W = Fs$**




Consider an object at a certain height $h_1$ from the ground raised by a constant force $F$ equal and opposite to the weight of the object. The object moves at constant speed to a height $h_2$ (to ensure that no work is used to increase its kinetic energy)

*Weight* $mg = F$

Work done by the force $F$ is

$W$ = force x displacement in the direction of the force

$= F\,\Delta h$

$= F\,(h_2 - h_1)$

$= mg\,(h_2 - h_1)$

$= mgh_2 - mgh_1$

= final GPE − initial GPE

Work done by a force in raising an object through a vertical height $h$ at a constant speed results in a **change** in gravitational potential energy of the object.

| Hence gravitational potential energy is defined as $E_p = mgh$ |
|---|

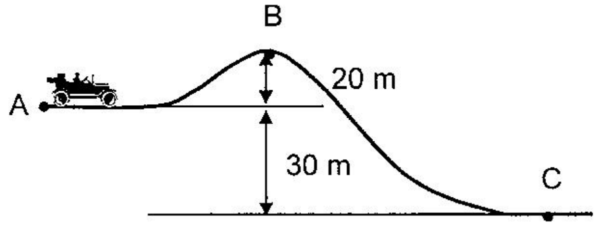
In solving problems involving gravitational potential energy, it is important to choose a reference level where the $E_p = 0$ J. The choice of zero level is arbitrary because the important quantity is the *difference* in potential energy and this difference is independent of the choice of zero level.

Note:

1) $E_p = mgh$ is the potential energy of a body mass $m$ at a height $h$ above the Earth's surface. This value of $E_p$ for the body is not absolute but depends on the reference level chosen. In this case, $E_p$ is taken to be zero on the Earth's surface (ground).
2) $E_p = mgh$ is valid for small distances $h$ near the Earth's surface where $g$ can be taken to be constant.

**Example 6**

A vehicle of mass 1000 kg moves from point A to point B then to point C. What is the gain in gravitational potential energy (a) at B? (b) at C?

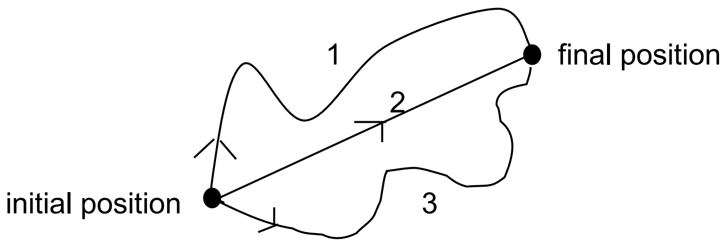



*(i) show an understanding of and use the relationship between force and potential energy in a uniform field to solve problems*

Two categories of forces exist in nature – conservative and non-conservative forces.

A force is conservative if the work the force does on an object moving between two points is independent of the path the object takes between the points.

In other words, the work done on an object by a conservative force depends only on the initial and final positions of the object. Examples of conservative forces: gravitational force, electric force between charges, elastic force in springs.




A force is non-conservative if the work the force does on an object depends on the path taken by the object between its starting and final points. Examples of non-conservative forces: friction, drag force. These are also called dissipative forces.

In the derivation of the formula for potential energy,

Work done by $F = F\Delta h$ = *final GPE – Initial GPE*

$F\Delta h = \Delta E_p$

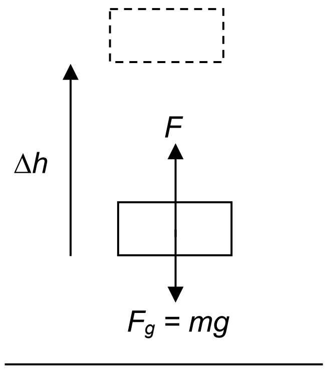
$F = \Delta E_p / \Delta h$

*Since* $F = -F_g$ (weight of the object),

$$F_g = -\frac{\Delta E_p}{\Delta h}$$

Taking the limit that $\Delta h \rightarrow 0$,

$$F_g = -\frac{dE_p}{dh}$$




This result can be generalized to all conservative forces such that the conservative force *F* is related to the potential energy *U* such that

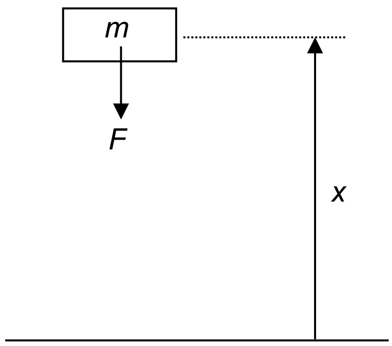
$$\boxed{F = -\frac{dU}{dx}}$$

The gradient of the potential energy – displacement graph gives the magnitude of the conservative force. In addition, the force points in the direction of decreasing potential energy.

**Example 7**

The gravitational potential energy of a mass *m* with respect to a reference level is $U = mgx$.

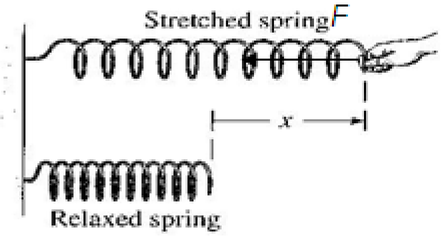
Sketch the graphs of the (a) gravitational potential energy *U* against *x*. (b) gravitational force *F* against *x*.




This is an example of a uniform field in which the force on the mass is independent of the position of the mass in the field.

**Example 8**

The elastic potential energy in a spring $U = \frac{1}{2}kx^2$ Sketch the graph of the elastic force $F$ against $x$.




The elastic force $F = -\frac{dU}{dx} = -\frac{d}{dx}\left(\frac{1}{2}kx^2\right) = -kx$

**Note**:

The elastic force is negative because the force acts in opposite direction to displacement *x*.

*(c) give examples of energy in different forms, its conversion and conservation, and apply the principle of energy conservation*

The sum of kinetic energy and potential energy is called mechanical energy.

The principle of conservation of mechanical energy states that in the absence of any net external force, the total mechanical energy of the system is conserved.

In equation form, this means that the final value of the mechanical energy is the same as its initial value:

$$(E_K + E_P)_i = (E_K + E_P)_f$$

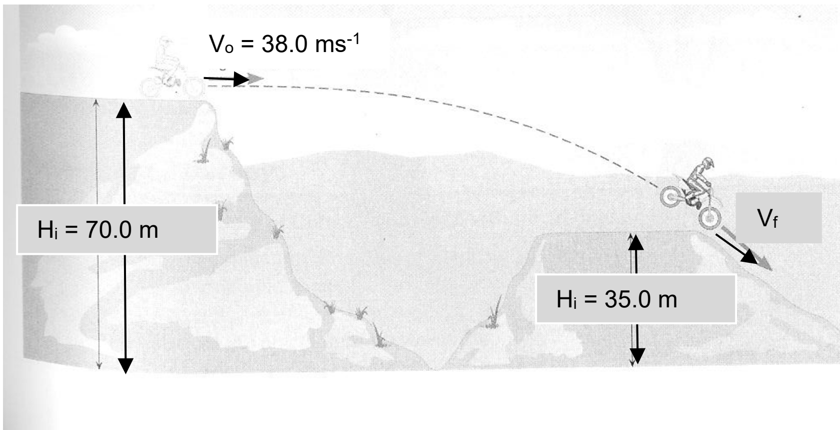
This principle offers keen insight into the way in which the physical universe operates. While the sum of the kinetic and potential energies at any point is conserved, the two forms may be inter-converted or transformed into one another.

**Example 9**

A motorcyclist is trying to leap across the canyon by diving horizontally off the cliff at a speed of 38 m $s^{-1}$. Ignoring air resistance, determine the speed with which the cyclist strikes the ground on the other side.

Hint:

Once the cyclist leave the ground, only the weight acts on the cycle since air resistance is ignored. So work done by non-conservative forces is zero. Applying conservation of mechanical energy, total mechanical energy is the same at the initial and final position of cyclist.




*Additional: Try solving this question using kinematics equations you learnt in Topic 2.

Most moving objects experience dissipative forces e.g. friction, drag and the work done by these forces is not zero. In these situations, the total mechanical energy is <u>not</u> conserved. The work done by dissipative forces equal the loss in the mechanical energy, that is,

Initial mechanical energy + work done <u>by</u> dissipative forces = final mechanical energy

$$(E_K + E_P)_i + W_d = (E_K + E_P)_f$$

Note:

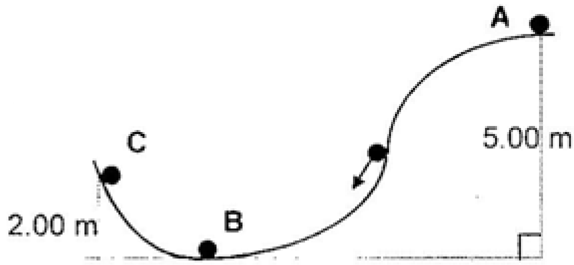
The work done by the dissipative forces $W_d$ is negative (as the forces always act in opposite direction to the displacement).

**Example 10**

Consider a ball of mass 200 g falling from rest through air. If its speed is 15 m $s^{-1}$ after falling through a distance of 20 m. (a) How much energy is lost as work done against drag? (b) What is the average drag experienced by the ball?

**Example 11**

A 0.400 kg bead slides on a curved wire, starting from rest at point A. (a) If the wire is frictionless, determine the speed of the bead at B and C. (b) If there is a constant friction force of 2.0 N, determine the speed of the bead at B and C. The length of wire AB is 6.0 m and BC is 3.0 m.



*(l)* *define power as work done per unit time and derive power as the product of a force and velocity in the direction of the force.*

In many situations, the time it takes to do work is just as important as the amount of work that is done. Power incorporates both concept of work and time.

**Power is defined as work done per unit time.**

Power is a scalar quantity and the SI unit is the watt (W). From the definition of power,

The instantaneous power is $P = \frac{dW}{dt} = \frac{d(F.s)}{dt} = F\left(\frac{ds}{dt}\right) = Fv$

The above expression can be used to calculate power if a constant force $F$ is acting on an object moving at velocity $v$.

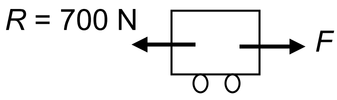
The average power is $\langle P \rangle = \frac{\text{work done}}{\text{time taken}} = \frac{W}{t} = \frac{F.s}{t} = F\left(\frac{s}{t}\right) = F \langle v \rangle$

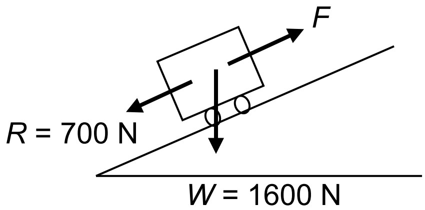
**Example 12**

A car of mass 1500 kg starts from rest and accelerates uniformly to 18 m s$^{-1}$ in 12 s. Assume air resistance is constant at 400 N during this time. Determine (a) the average power developed by the engine, (b) the instantaneous power output of the engine at 12 s.

**Example 13**

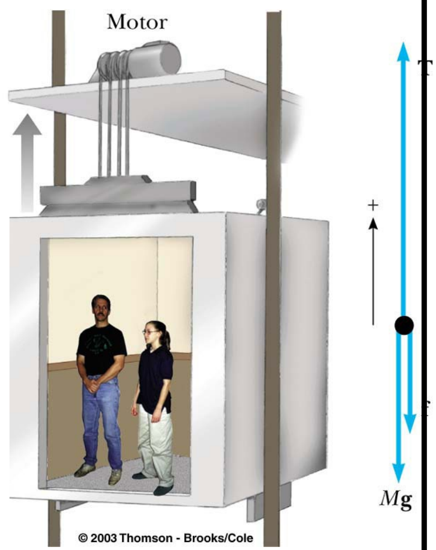
Find the power developed by the engine of 1600 kg car moving at 90 km h$^{-1}$ on a level road if the total resistive force is 700 N. What is the power required if the car is to move with the same speed, up a slope that is inclined at 10° to the horizontal?




**Example 14**

A 1000 kg lift carries a maximum load of 800 kg. A constant frictional force of 4000 N retards its motion upward. What minimum power must the motor deliver to lift the fully loaded lift at a constant speed of 3.00 m s$^{-1}$?



(d) *show an appreciation for the implications of energy losses in practical devices and use the concept of efficiency to solve problems*

In machines, due to inevitable work done by frictional forces, the energy conversion can never be 100%; the (useful) energy output is always less than the energy input.

**Energy Input = Useful Energy Output + Energy Loss**

The efficiency of a machine in its conversion of energy from one form to another is defined as:

efficiency $\eta = \dfrac{\textit{useful energy output}}{\textit{total energy input}} x100\%$

Alternatively, $\eta = \dfrac{\textit{useful power output}}{\textit{total power input}} x100\%$

**Example 15**

Water flows over a section of Niagara Falls at a rate of $5.0 \times 10^6$ kg each second and falls 50 m.

(a) How much power is generated by the falling water?

(b) Assuming a efficiency of 95% in converting mechanical energy to electrical energy, how many 100 W bulbs could this light?

**Example 16**

A car has a mass of 800 kg and its efficiency is rated at 18% (i.e. 18% of the available fuel energy is delivered to the wheels). Determine the amount of petrol used to accelerate the car from rest to 30 m s$^{-1}$. Use the fact that 1 kg of petrol supplies $5.0 \times 10^7$ J of energy.

**Definition List**

| | |
|---|---|
| Principle of the conservation of energy | Energy can neither be destroyed nor created in any process. It can be transformed from one form to another, and transferred from one body to another but the total amount remains constant. |
| Work-Kinetic Energy Theorem | States that the net work done by forces acting on a particle on a level surface is equal to the change in kinetic energy of the particle. |
| Mechanical Energy | The mechanical energy of a system is the sum of the kinetic energy and all the potential energies present at an instant. |
| Principle of conservation of mechanical energy | states that in the absence of any net external force or any internal non-conservative force, the total mechanical energy of the system is conserved.<br>(not applicable if non-conservative forces are present $\Rightarrow$ will have to use law of conservation of energy ) |
| Work Done<br>(by constant Force) | The work done by a constant force on an object is defined as the force *x* displacement in the direction of the force. |
| Work Done<br>(by variable Force) | Area under the Force-Displacement curve. |
| Chemical Energy | Energy associated with motion and interaction of electrons in atoms and molecules e.g. in fuels such as oil, wood, coal, electric cells, food |
| Nuclear Energy | Energy associated with motion and interaction of protons and neutrons in nuclei e.g. atomic bombs, nuclear reactors |
| Electromagnetic Energy | Energy associated with interaction of electric charges and currents e.g. energy of electromagnetic waves such as visible light, radio waves etc |
| Mass Energy | The energy equivalence of mass as given by Einstein's famous equation $E = mc^2$. |
| Kinetic energy | The kinetic energy of an object is the energy it possesses by virtue of its motion. |
| Potential energy | Ability to do work<br>as a result of the position, shape or state of an object.<br>(includes Gravitational potential energy, electrical potential energy, chemical and Elastic potential energy). |
| Gravitational potential energy | (In WEP) Ability to do work due to<br>height or position of mass<br>OR<br>moving mass from one point to another. |
| Elastic potential energy | Ability to do work as a result of a change of shape of an object. |
| Power | Work done per unit time. OR rate of doing work. |
| Instantaneous Power | Calculated by using $P = F\,v$ where $F$ = force applied and $v$ = velocity. |
| Average Power | Total work done divided by total time taken. |

**Topic 5: Tutorial Questions**

**Part 1: Work**

1 Can a body have energy without having momentum? Can a body have momentum without having energy?

2 If there is a net non-zero force on a moving object, is it possible for the total work done on the object to be zero? Explain, with an example that illustrates your answer.

3 In a race between two toy cars A and B, with masses 0.50 kg and 0.60 kg respectively, both cars begin from rest at the starting line.
(a) If the same force is supplied to the cars over the course of the 3.0 m race.
(i) Which car finishes first? Why?
(ii) Which car has a larger kinetic energy when it crosses the finishing line? Why?
(iii) Which car has a larger momentum when it crosses the finishing line? Why?
(b) If the same force is supplied to the cars over the course of a 30 seconds race.
(i) Which car travelled further after 30 seconds? Why?
(ii) Which car has a larger momentum after 30 seconds? Why?
(iii) Which car has a larger kinetic energy after 30 seconds? Why?

4 As a simple pendulum swings back and forth, the forces acting on the suspended object are the weight, the tension in the string and air resistance. (a) Which of these forces does no work on the pendulum? (b) Which of these forces does negative work at all times during its motion? (c) Describe the work done by weight while the pendulum is swinging.

5 A box of 10 kg rests on the floor. How much work is required to move it at constant speed (a) 5.0 m along the frictionless floor, (b) 5.0 m along the floor against a frictional force of 230 N, (c) 4 m vertically upwards? (0 J, 1150 J, 392 J)

6 A car travelling at a speed $v$ skids a distance $d$ after its brakes locked. How far will it skid if it is travelling at a speed $2v$ when its brakes lock? ($4d$)

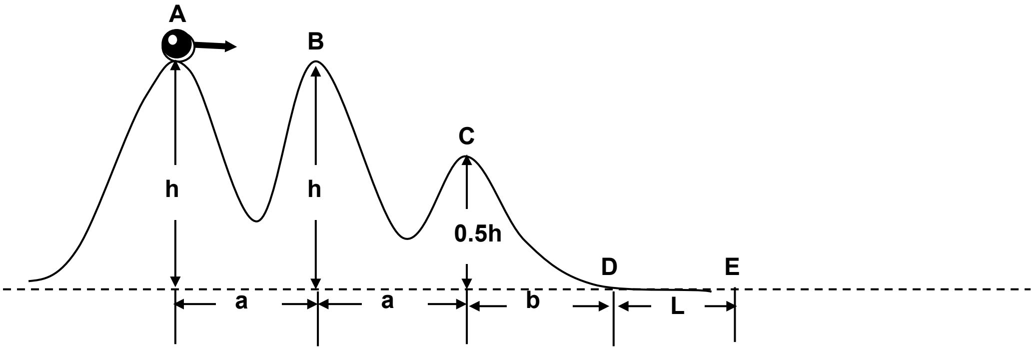
7 A bullet of mass 0.030 kg has a velocity of 500 m $s^{-1}$. If it penetrates 0.40 m into a target, determine the average resistive force offered by the target. ($9.4 \times 10^3$ N)

**Part 2: Conservation of Mechanical energy**

8 Three identical balls are thrown from the top of a building, all with the same initial speed. The first ball is thrown horizontally, the second at some angle above the horizontally and the third at some angle below the horizontal. Neglecting air resistance, rank the speeds of the balls as they reach the ground.

9 A block of mass 0.250 kg is placed on a light vertical spring (spring constant $5.00 \times 10^3$ N $m^{-1}$) and pushed downwards, compressing the spring 0.100 m. After the block is released, it leaves the spring and continues to travel upward. If air resistance is ignored, what height above the point of release will the block reach? (10.2 m)

10 A frictionless roller coaster of mass $m$ starts at point A with speed $v_o$, as shown in the figure. Assume that the roller coaster can be considered as a particle and that it always remains on the track.



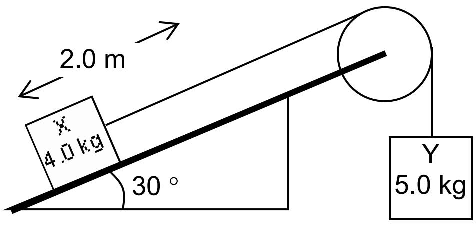

(a) What will be the speed of the roller coaster at point B and C?

(b) What constant acceleration is required to stop it at E if the brakes are applied at point D?

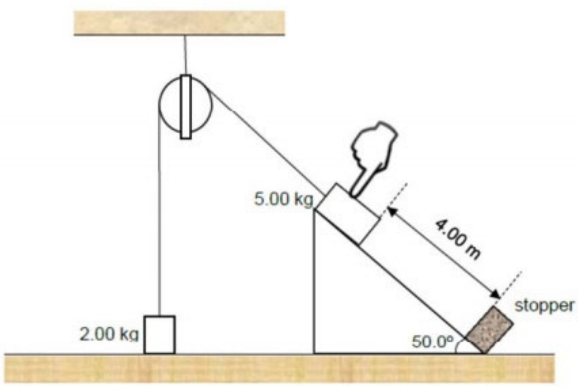
( (a) $v_B = v_o$; $v_C = \sqrt{v_o^2 + gh}$ (b) $-(v_o^2 + 2gh)/(2L)$ )

11 In the dangerous sport of bungee jumping, a daring student jumps from a balloon with an elastic cord attached to him. The unstretched length of the cord is 25.0 m and the student weighs 700 N. The balloon is 36.0 m above the surface of the river below. Calculate the required force constant of the cord if the student is to stop safely 4.00 m above the surface of the river. Ignore air resistance. (914 N $m^{-1}$)

12 The diagram shows two bodies X and Y connected by a light cord passing over a light, free-running pulley, X starts from rest and moves on a smooth plane inclined at 30° to the horizontal. Calculate the total kinetic energy of the system when X has travelled 2.0 m along the plane. (59 J)




13 Two objects, of masses 2.00 kg and 5.00 kg respectively, are connected by a light inextensible string passing over a light frictionless pulley. The 2.00 kg object was initially on the ground and the 5.00 kg object was held at rest on a rough slope as shown. A rigid stopper is fixed at the bottom of the slope. The 5.00 kg object is released and it slides down the slope, experiencing a constant frictional force of 10 N in the process. Assume air resistance to be negligible.




(a) Describe the energy conversions for the system of two masses as the 5.00 kg slides along the slope. [2]

(b) Calculate the speed of the 2.00 kg object just before the 5.00 kg object hits the stopper. [2]

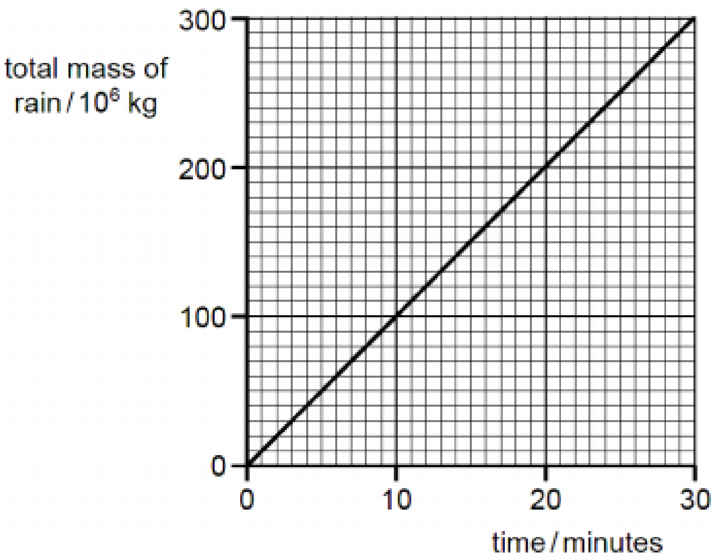
(c) Determine the maximum height above the ground to which the 2.00 kg object eventually rises. [2]

14 A missile of mass 60 kg is projected vertically upwards with a speed of 180 m $s^{-1}$ from a submarine which is 150 m below the surface. Assuming the water offers a constant resistance of 200 N to the motion of the missile, calculate the velocity of the missile just before it leaves the water. Ignore upthrust on the missile. (169 m $s^{-1}$)

**Part 3: Power**

15 A student of mass 50.0 kg climbs a 5.00 m long rope and stops at the top. (a) What must his average speed be in order to match the power output of a 200 W light bulb? (b) How much work does he do? (0.408 m $s^{-1}$, 2.45 kJ)

16 Rain from a thunderstorm reaches the ground at a speed of 12 m $s^{-1}$. The graph shows how the total mass of deposited rain increases with time.

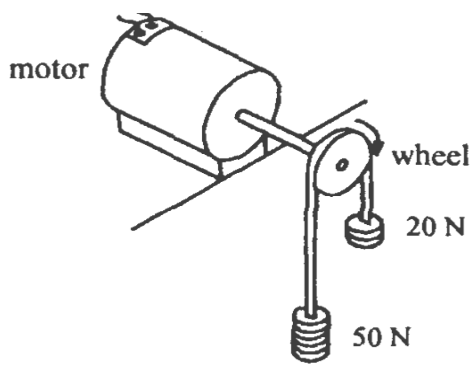



What is the average power delivered by the rain as it hits the ground? (1.2 x $10^7$ W)

17 A car of weight 2500 N operating at a rate of 130 kW develops a maximum speed of 31 m $s^{-1}$ on a level road. Assuming that the resistive force (due to friction and air resistance) remains constant, what is the car's maximum speed on an incline of 10°? (28 m $s^{-1}$)

18 The force resisting the motion of a car is proportional to the square of the speed of the car. The magnitude of the force at a speed of 20 m $s^{-1}$ is 800 N. What effective power is required for the car's engine to maintain a steady speed of 40 m $s^{-1}$? (128 kW)

19 The diagram shows an arrangement used to find the output power of an electric motor. The wheel attached to the motor's axle has a circumference of 0.5 m and the belt which passes over it is stationary when the weights have the values shown. If the wheel is making 20 revolutions per second, calculate the output power. (300 W)



20 **Data based Question**

In the USA, cars account for about half the oil consumed, half the urban pollution and a quarter of emission of greenhouse gases. Vehicle usage is set to continue its growth globally. With improved technology over the past decade, electric vehicle is touted to be a greener alternative to conventional cars powered by fossil fuels.

The following tables gives the specifications of variants of the same model of car, Ford Focus, one oil-powered (named 'Classic') and one electric-powered (named 'Electric').

Specifications for two variants of Ford Focus:

Electric:

| | |
|---|---|
| Capacity of lithium-ion battery | 23 kWh |
| Average distance per full charge | 122 km |
| Carbon dioxide emission per MWh of electricity generated from fossil fuel | 601 kg |

Table 1

Classic

| | |
|---|---|
| Distance per litre of petrol | 29.1 km |
| Carbon dioxide emission per km travelled | 105 g |
| Energy released in combustion of 1 litre of petrol | 9.7 kWh |

Table 2

(a) One of the key advantages of electric vehicles over conventional oil-powered vehicles is energy efficiency. This can be measured by comparing the average energy expenditure per kilometre for each variant.

Using the data quoted in both Tables 1 and 2,

(i) show that for the Electric variant, the energy stored in the lithium-ion battery when fully charged is $8.28 \times 10^7$ J.

(ii) hence, calculate the average energy expenditure per kilometre travelled for the Electric variant. (680 kJ $km^{-1}$)

(iii) calculate the average energy expenditure per kilometre travelled for the Classic variant. (1200 kJ $km^{-1}$)

(b) However, in many countries, electricity is generated through the burning of fossil fuel. Electricity generated through such a method has a yield of about 30%.

(i) What do you understand by a '*yield of about 30%*'?

(ii) Assuming that the electricity used for charging the lithium-ion battery in the Electric variant is generated through the burning of fossil fuel, using (a)(ii), determine the actual energy expenditure travelled per kilometre for the Electric variant. (2300 kJ $km^{-1}$)

(c) There have been calls by members of public for the Singapore government to provide more tax rebates for consumers purchasing electric cars as part of government's effort to encourage environment conservation efforts.

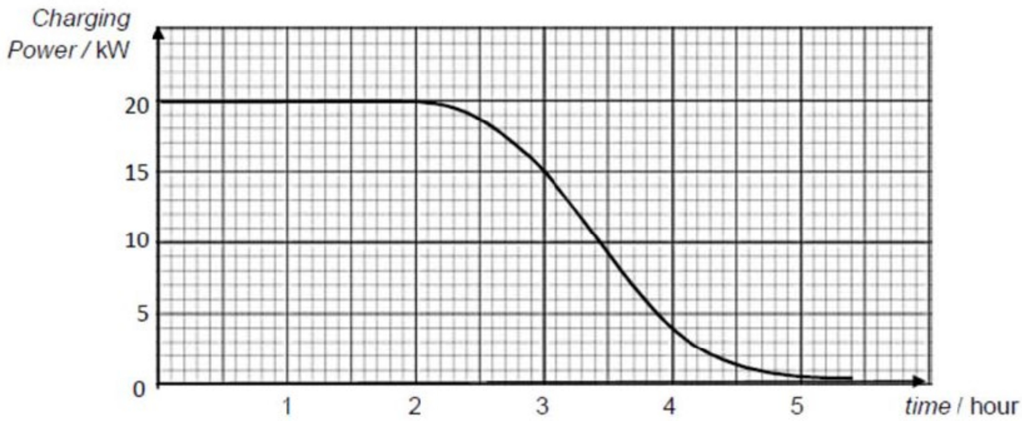
One way to measure how environmentally friendly a car is to look at its carbon footprint (carbon dioxide emitted per kilometre).

(i) Assuming that the electricity in Singapore's power plants is generated from fossil fuels, using Table 1, calculate the amount of carbon dioxide emitted per kilometre for the Electric variant. (113 g)

(ii) Comment on whether the Electric variant of Ford Focus is more environmentally friendly than the Classic variant in the context of Singapore.

(d) One of the key challenges facing electric car manufacturers is how to reduce the long charging time for the vehicle battery pack.

A particular electric car has a charging power rating of 20 kW. However, the charging power will decay as the battery gets charged up. It takes about 5 hours for the battery pack to charge fully from empty state. The figure below shows how the charging power varies with time.




(i) Estimate the amount of energy that the battery can store at full capacity.

(69 kWh)

(ii) Another manufacturer has invented a better charging technology that allows the decay in charging power to be delayed.
Assuming that the maximum charging power of this charger is still 20 kW and that the capacity of the battery remains the same, sketch, in the graph above, its corresponding charging curve for the same battery. Label this graph A.