NAME:		( )		CLASS:	

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YISHUN TOWN SECONDARY SCHOOL

PRELIMINARY EXAMINATION 2023
SECONDARY 4 NORMAL ACADEMIC
ADDITIONAL MATHEMATICS PAPER 2 (4051/02)

DATE	**:**	**15 August 2023**	**DAY**	**:**	**Tuesday**
DURATION	**:**	**1 hour 45 minutes**	**MARKS**	**:**	**70**

READ THESE INSTRUCTIONS FIRST

Do not turn over the cover page until you are told to do so.
Write your name, class and class index number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 70.

	MARKS	
	OBTAINED	FULL
1		4
2		4
3		5
4		5
5		6
6		8
7		9
8		10
9		10
10		9
TOTAL		70

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

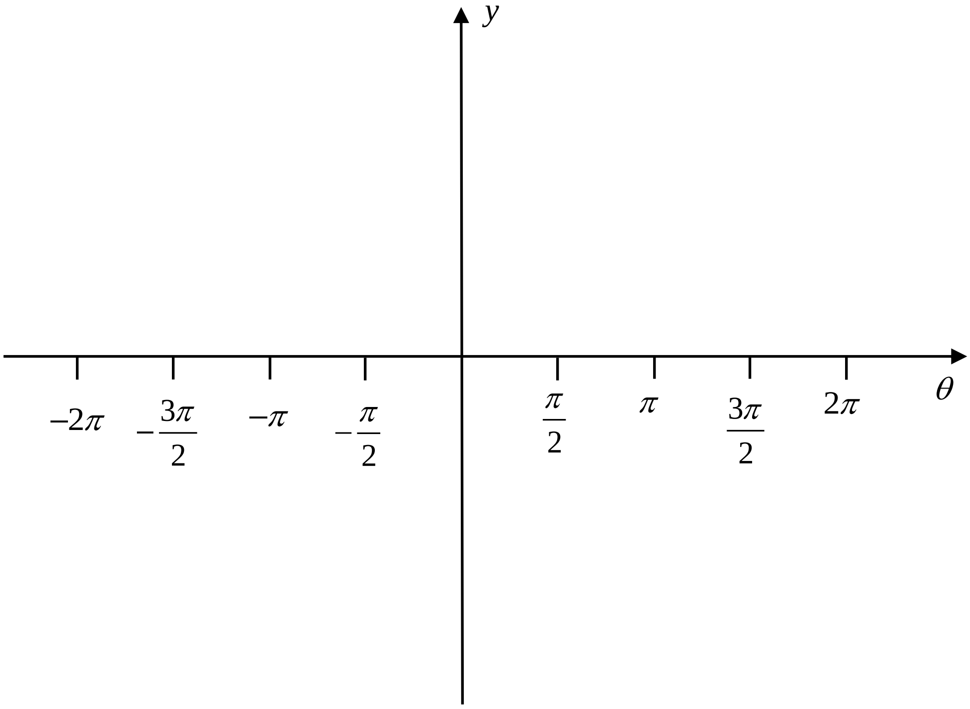
1 Integrate with respect to x

(a) $(x^2-1)^2$, [2]

(b) $\dfrac{x^3-x^2}{2x}$. [2]

2 **(a)** Write down the period of $y = 2\tan\frac{1}{2}\theta$. [1]

(b) On the axes below, sketch the graph of $y = 2\tan\frac{1}{2}\theta$ for $-2\pi \le \theta \le 2\pi$ radians. [3]



3 **(a)** Show that $\mathrm{f}(x)=3x^2-9x+7$ can be written as $\mathrm{f}(x)=a(x-b)^2+c$, where a, b and c are constants to be found. [4]

(b) Hence, explain why the function $\mathrm{f}(x)$ is always positive. [1]

4 A potato field is infested by insects. The percentage of the field that is infested, q %, t days after pesticide is sprayed is modelled by the equation

$$\frac{\mathrm{d}q}{\mathrm{d}t} = -\frac{5}{\sqrt{(2t+1)^3}}.$$

Just before the pesticide is sprayed, 5% of the field was already infested with insects. Once the percentage of the field that is infested decreases to 1% or less, the potatoes will be harvested. Find the minimum number of days for the potatoes to be harvested. [5]

5 The equation of a curve is $y=\dfrac{6x^2}{2-5x}$ for $x\neq\dfrac{2}{5}$.

(a) Express $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in the form $\dfrac{ax(4-bx)}{(2-5x)^2}$, where a and b are integers to be determined.

[3]

(b) State the range of values of x for which y is a decreasing function. [3]

6 A curve has the equation $y = x^3 - 2x + 5.$ The normal to the curve at $x = 1$ meets the y-axis at A and the x-axis at B.

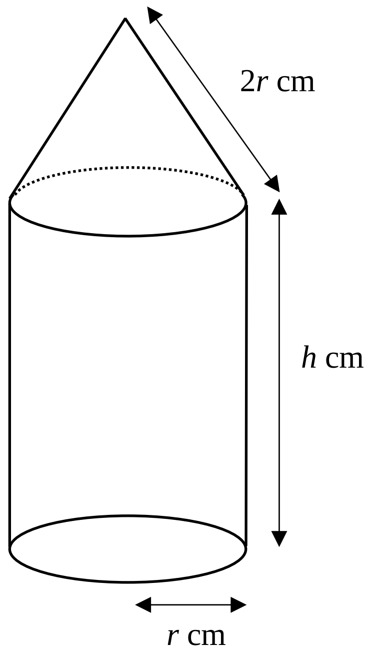
(a) Find the area of triangle AOB, where O is the origin. [6]

(b) The tangent to the curve at $x = 1$ is parallel to the tangent at point Q. Find the coordinates of Q. [2]

7 **(a)** By substituting $3\theta = 2\theta + \theta$, show that $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$. [3]

(b) Hence solve the equation $4\cos^3\theta - 3\cos\theta = \frac{1}{2}$ for $-180^o < \theta < 180^o$. [6]

8 The diagram shows a container used to contain a chocolate product. It is made up of a cone fixed to the top of a right circular cylinder of radius r cm and height h cm. The slant height of the cone is $2r$ cm.



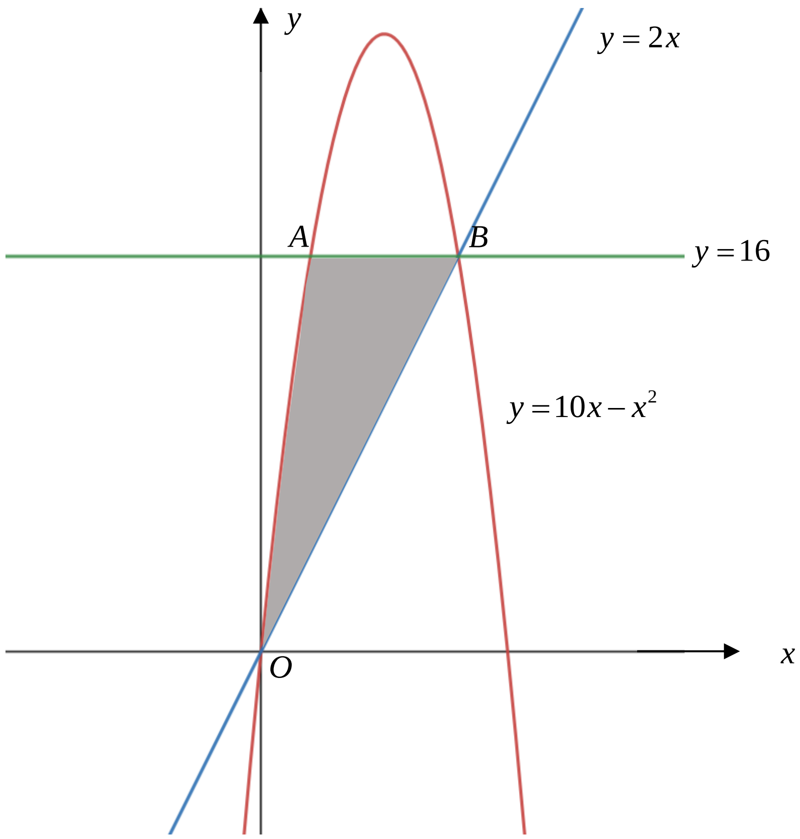
(a) Given that the volume of the **cylindrical part** is $108\pi\ \text{cm}^3$, express h in terms of r. [1]

(b) Show that the total surface area, A cm^2, of the container is given by $A = 3\pi\left(r^2 + \dfrac{72}{r}\right)$. [2]

(c) Given that r can vary, find the value of r for which A has a stationary value and explain why this value of r gives the minimum value of surface area. [5]

(d) For aesthetic purposes, a shop owner would only display the container in his shop only if the height of the container is less than 35 cm and has minimum surface area. Determine whether the container can be displayed in the shop. [2]

9 The diagram shows the curve $y=10x-x^2$, the line $y=2x$ and the line $y=16$. The line $y=16$ intersects the curve at the points A and B. The line $y=2x$ also intersects the curve at point B.



(a) Find the coordinates of A. [2]

(b) Find the area of the shaded region *OAB*. [8]

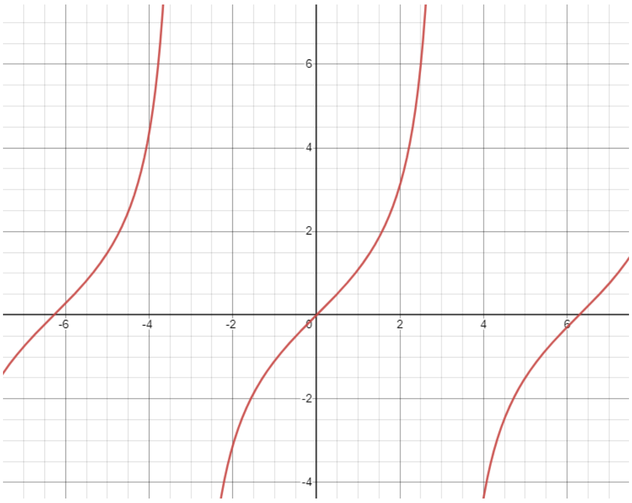
10 The curve $y = x\left(\frac{1}{2}x - k\right)^3$ where $k < 4$, has a gradient of 0 at $x = 2$.

(a) Show that $k = 1$. [3]

(b) Explain why $x = 2$ is a point of inflexion on the curve and determine the nature of the other turning point. [6]

END OF PAPER

Answer Key

1a	$\frac{x^5}{5}-\frac{2}{3}x^3+x+c$	**7a**	$\begin{aligned}\cos 3\theta &= \cos(2\theta+\theta)\\ &= \cos 2\theta\cos\theta-\sin 2\theta\sin\theta\\ &= (2\cos^2\theta-1)\cos\theta-(2\sin\theta\cos\theta)\sin\theta\\ &= 2\cos^3\theta-\cos\theta-2\sin^2\theta\cos\theta\\ &= 2\cos^3\theta-\cos\theta-2(1-\cos^2\theta)\cos\theta\\ &= 2\cos^3\theta-\cos\theta-2\cos\theta+2\cos^3\theta\\ &= 4\cos^3\theta-3\cos\theta\end{aligned}$
1b	$\frac{x^3}{6}-\frac{x^2}{4}+c$	**7b**	$\theta=20^o,100^o,140^o,-20^o,-100^o,-140^o$
2a	2π or 360^o	**8a**	$\pi r^2h=108\pi$ $h=\frac{108}{r^2}$
2b	 	**8b**	$3\pi\left(r^2+\frac{72}{r}\right)$
3a	$3\left(x-\frac{3}{2}\right)^2+\frac{1}{4}$	**8c**	$r=3.30$ $\frac{\mathrm{d}^2A}{\mathrm{d}r^2}=3\pi\left(2+\frac{144}{r^3}\right)$ >0 $\therefore$ A is a minimum at r = 3.30
3b	Since $3\left(x-\frac{3}{2}\right)^2\geq 0,\ 3\left(x-\frac{3}{2}\right)^2+\frac{1}{4}\geq\frac{1}{4}(>0)$.	**8d**	Height of container = $9.9058+\sqrt{(2(3.3019)^2-(3.3019)^2}$ $=15.6$ Since 15.6 < 35 cm, the container can be displayed in the shop.
4	$t=12$	**9a**	$A(2,16)$

<table>
<tr><td>5a</td><td>$a = 6$ and $b = 5$</td><td>9b</td><td>$49\frac{1}{3}\text{ units}^2$</td></tr>
<tr><td>5b</td><td>$x < 0 \ \ or \ \ x > \frac{4}{5}$</td><td>10b</td><td>
$\frac{dy}{dx} = \left(\frac{1}{2}x - 1\right)^2 (2x - 1)$
When $\frac{dy}{dx} = 0,$
$x = 2, \ \frac{1}{2}$
<table>
<tr><td></td><td>2^-</td><td>2</td><td>2^+</td></tr>
<tr><td>$\frac{dy}{dx}$</td><td>>0</td><td>0</td><td>>0</td></tr>
</table>
By 1st derivative test, $x = 2$ is a point of inflexion.
<table>
<tr><td></td><td>$\frac{1}{2}^-$</td><td>$\frac{1}{2}$</td><td>$\frac{1}{2}^+$</td></tr>
<tr><td>$\frac{dy}{dx}$</td><td><0</td><td>0</td><td>>0</td></tr>
</table>
By 1st derivative test, the curve has a minimum point at $x = \frac{1}{2}$.
</td></tr>
<tr><td>6a</td><td>12.5 units^2</td><td></td><td></td></tr>
<tr><td>6b</td><td>$3x^2 - 2 = 1$
$3x^2 = 3$
$x = \pm 1$
When $x = -1,$
$y = 6$
$Q(-1, 6)$</td><td></td><td></td></tr>
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