Mathematics HL Mock Exam Paper 1

SECTION A

Answer all the questions in the spaces provided. Working may be continued below the lines, if necessary.

1. [*Maximum mark: 6*] **NOT IN SYLLABUS**

Let $A = \begin{pmatrix} 2 & 6 \\ k & -1 \end{pmatrix}$ and $B = \begin{pmatrix} h & 3 \\ -3 & 7 \end{pmatrix}$, where h and k are integers. Given that $\det A = \det B$ and that $\det AB = 256h$,

(a) show that h satisfies the equation $49h^2 - 130h + 81 = 0$; [2 marks]

(b) hence find the value of k. [4 marks]

2. [*Maximum mark:6*]

Find the area between the curves $y = 2 - 3x + x^2$ and $y = 2 + x - x^2$.

3. [*Maximum mark: 6*]

The random variable T has the probability density function

$$f(t) = \frac{\pi}{4}\cos\left(\frac{\pi t}{2}\right), \quad -a \le t \le a.$$

Find

(a) the value of a; [2 marks]

(b) an expression for the cumulative distribution function $F(x)$. [2 marks]

(c) the interquartile range. [2 marks]

4. [*Maximum mark: 6*]

The marks in an IB Mathematics HL exam are distributed normally with mean μ and standard deviation σ. If the cut off score for a 7 is a mark of 80%, and 10% of students get a 7, and the cut off score for a 6 is a mark of 65% and 30% of students get a 6 or 7, find the mean and standard deviation of the marks in this exam. Give your answers correct to **two** significant figures.

5. [*Maximum mark: 6*]

Find the **exact** value of

$$\frac{\arcsin(\sin 4)}{4} + \frac{\arccos(\cos 3)}{3} + \frac{\arctan(\tan 2)}{2} + \frac{\operatorname{arccot}(\cot 1)}{1}.$$

6. [*Maximum mark:6*]

Given that $z = (b + i)^2$, where b is real and positive, find the **exact** value of b when $\arg z = \frac{\pi}{3}$.

7. [*Maximum mark: 6*]

Let X be a random variable. By expanding the expression $E\left[\left(X - E(X)\right)^2\right]$ show that

$$E(X^2) \geq \left(E(X)\right)^2.$$

8. [*Maximum mark: 6*]

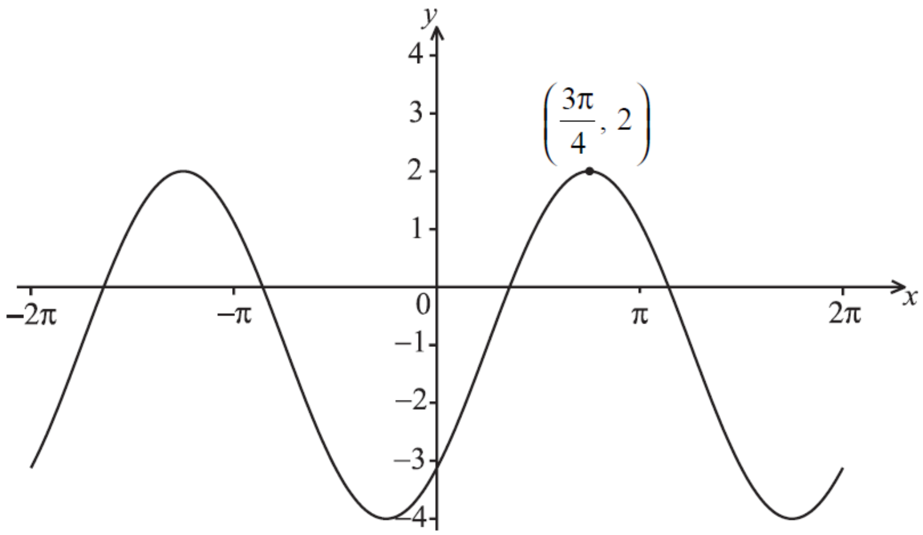
Find an equation of the plane containing the two lines

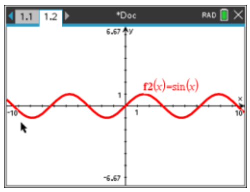
$$x - 1 = \frac{1-y}{2} = z - 2 \quad \text{and} \quad \frac{x+1}{3} = \frac{2-y}{3} = \frac{z+2}{5}.$$

9. [*Maximum mark: 6*]

(a) State the transformations from graph $y = a\sin(x+b) + c$ to graph $y = \sin x$. [3 marks]

(b) The graph below represents $y = a\sin(x+b) + c$, where a,b, and c are constant.





Find values for a, b and c. [3 marks]

10. [*Maximum mark: 6*]

Given that $(1-2x)^5(1+3x)^4 = a + bx + cx^2 + \cdots$, find the values for a, b and c.

Section B

Answer all the questions on the answer sheets provided. Please start each question on a new page.

11. [*Maximum mark: 13*]

(a) The function f is defined by $f(x) = (x+2)^2 - 3$.

The function g is defined by $g(x) = ax + b$, where a and b are constants.

Find the value of a, $a > 0$ and the corresponding value of b, such that

$$f\big(g(x)\big) = 4x^2 + 6x - \frac{3}{4}.$$

[8 marks]

(b) The functions h and k are defined by $h(x) = 5x + 2$ and $k(x) = cx^2 - x + 2$ respectively. Find the value of c such that $h\big(k(x)\big) = 0$ has equal roots. [5 marks]

12. [*Maximum mark: 14*]

(a) Sketch the graphs of

$$y = \frac{x+1}{x-1} \text{ and } y = |3x - 5|.$$

State intercepts, asymptotes, maximum and minimum points of each graph clearly if there is any. [8 marks]

(b) Hence, find the range(s) of x for which

$$\frac{x+1}{x-1} < |3x - 5|$$

where $x \neq 1$. [6 marks]

13. [*Maximum mark: 16*]

(a) Using Mathematical Induction, prove that $\frac{d^n}{dx^n}(\cos x) = \cos\left(x + \frac{n\pi}{2}\right)$, for all positive integer values n. [7 marks]

(b) Solve the equation $\sin 4x = \cos x$ for $-\frac{2\pi}{3} < x < \frac{\pi}{4}$. [9 marks]

14. [*Maximum mark: 17*]

Let A be the point $(2, -1, 0)$, B the point $(3, 0, 1)$ and C the point $(1, m, 2)$, where $m \in \mathbb{Z}$, $m < 0$.

(a) Given that $A\hat{B}C = \arccos\frac{\sqrt{2}}{3}$, show that $m = -1$. [6 marks]

(b) Determine the Cartesian equation of the plane ABC. [4 marks]

(c) The line L is perpendicular to plane ABC and passes through A. Find a vector equation of L.

[3 marks]

(d) The point D $(6, -7, 2)$ lies on L. Find the volume of the pyramid $ABCD$. [4 marks]

Answers

1(b) k = -3

2. 8/3

3(a) a = 1 or 1 + 4k

(b) $F(x) = \frac{1}{2}\left(\sin\left(\frac{x\pi}{2}\right) + 1\right)$

(c) 2/3

4. μ=55 σ=-20

5. $2 - \frac{\pi}{4}$

6. $\sqrt{3}$.

8. $-7x - 2y + 3z = -3$ or $\boldsymbol{r} = \begin{pmatrix}1\\1\\2\end{pmatrix} + \mu\begin{pmatrix}1\\-2\\1\end{pmatrix} + \lambda\begin{pmatrix}3\\-3\\5\end{pmatrix}$

9(a)

- Translate the graph along negative y-axis by c;
- Translate the graph along positive x-axis by b;
- Stretch vertically by factor $\frac{1}{a}$.

(b)

$$a = 3, b = -\frac{\pi}{4} + 2k\pi, c = -1$$

10. $a = 1, b = 2, c = -26$

11(a)

$$a = 3, b = \frac{1}{2}$$

(b) $c = \frac{5}{48}$

12. $x < 1 \quad or \quad x > \frac{9+\sqrt{33}}{6}$

13(b) $x = -\frac{\pi}{2}, -\frac{3\pi}{10}, \frac{\pi}{10}, \frac{\pi}{6}$

14(a)(i) $\overrightarrow{BA} \cdot \overrightarrow{BC} = 1 - m$ and show m = -1

(b) $-2x + 3y - z = -7$

(c) $\frac{\sqrt{14}}{2}$

(d)(i) $r = \begin{pmatrix}2\\-1\\0\end{pmatrix} + \lambda\begin{pmatrix}-2\\3\\-1\end{pmatrix}$

(ii) 14/3