Example 1

A Blu-ray player is spinning a standard Blu-ray Disc (120 mm diameter) at 900 revolutions per minute (RPM). Determine
i) the frequency of the rotation (in revolutions per second)
ii) the period of the rotation
iii) the angular speed

i) $f = \dfrac{1}{T} = \dfrac{900}{60} = 15 \text{ rev s}^{-1}$

ii) $T = \dfrac{1}{f} = \dfrac{1}{15} = 0.0667 \text{ s}$

iii) $\omega = 2\pi f = 2\pi(15) = 94.2 \text{ rad s}^{-1}$

Example 2

Dishes are placed on a Lazy Susan at a reunion dinner during Chinese New Year. When the Lazy Susan is spun around to serve the guests, what can be said about the dishes in the inner circle compared to those in the outer circle in terms of their speed and angular velocity?

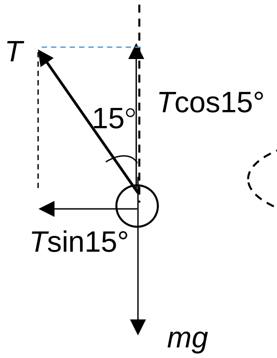
Angular velocity
The angular velocity ω is the same for all dishes. This should be clear if you realize that every dish undergoes the same angular displacement in the same amount of time. You may also notice that all dishes complete the same number of revolutions in the same duration of time. With the same T and f, they also have the same angular velocity $\omega = \dfrac{2\pi}{T} = 2\pi f$.

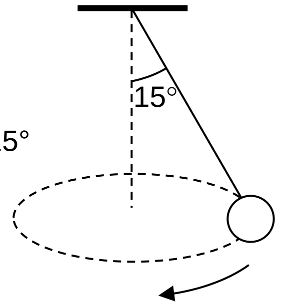
Speed
The dishes in the inner circle have a lower speed compared to the dishes in the outer circle. As $v = r\omega$, the smaller the radius, the lower the tangential velocity.

Example 3 - Conical pendulum

In a conical pendulum system, a small pendulum bob of mass 0.50 kg is rotating in a fixed horizontal plane. The string is 30 cm long and makes an angle of 15° to the vertical.

Calculate the (a) tension in the string;
(b) linear speed of the bob;
(c) period of rotation of the bob.





a) Vertically, object is stationary
Net force in the vertical direction = 0 N

$$T\cos\theta = mg$$
$$T\cos 15° = (0.50)(9.81)$$
$$T = 5.1 \text{ N}$$

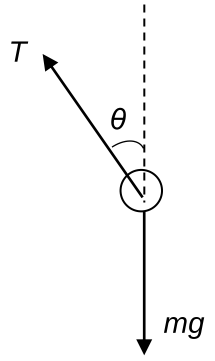
b) Net force in the horizontal direction provides the required centripetal force

$$T\sin\theta = \frac{mv^2}{r}$$
$$T\sin 15° = \frac{mv^2}{r} \Rightarrow v = \sqrt{\frac{rT\sin 15°}{m}} = \sqrt{\frac{(0.30\sin 15°)(5.078)(\sin 15°)}{0.50}} = 0.45 \text{ m s}^{-1}$$

c) Using $v = r\omega = r(\frac{2\pi}{T}) \Rightarrow T = \frac{2\pi r}{v} = \frac{2\pi(0.30\sin 15°)}{0.4498} = 1.1 \text{ s}$

Example 4

Explain, with the aid of a diagram, why the mass at the end of a light inelastic string cannot be whirled in uniform circular motion in such a way that the string is horizontal.



When string is horizontal, $\theta = 90°$
The tension in the string T will no longer have a vertical component.
There is no longer an upward vertical component to balance the weight of the bob.

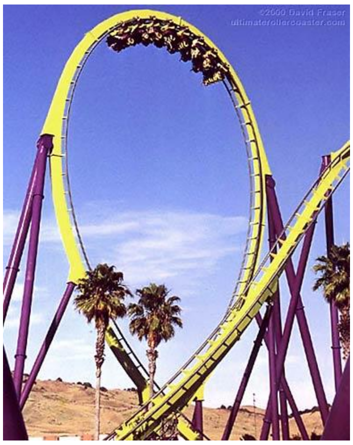
Altnernatively,

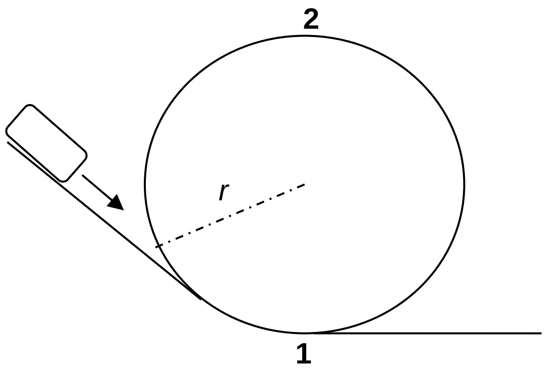
$T\cos\theta = mg$
As $\theta \to 90°$, $\cos\theta \to 0$

Thus $T \to \infty$
So infinitely large tension force is required to maintain vertical equilibrium if the string were horizontal.

<u>Example 5 - Roller coaster</u>

Some roller coasters have several loops along the track. The picture on the right illustrates such a coaster executing a loop-the-loop.



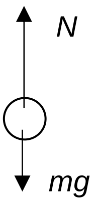


In the question below, assume that the total mass of one car plus its passengers is 170 kg and that friction can be ignored.

(a) A passenger car for a roller coaster enters a loop of radius 19 m at position 1, as indicated in the diagram, with a speed of 33 m s^{-1}. Determine the normal contact force the track exerts on the car at
  i) the bottom (position 1) and
  ii) the top (position 2) of the loop.

(b) Find the minimum speed at which the passenger car must travel while it is at the top of the loop, in order to clear the loop safely.

(a)(i) At position 1, required centripetal force is upward.



Applying N2L on car: $(F_{net} = ma)$

$$N - mg = \frac{mv^2}{r}$$

$$N = mg + \frac{mv^2}{r} = (170)(9.81 + \frac{33^2}{19}) = 1.1 \times 10^4 \text{ N}$$

(a)(ii) Firstly, we use PCOE to find the speed at position 2.

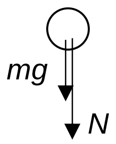
$$\text{loss in KE} = \text{gain in GPE}$$

$$\frac{1}{2}mu^2 - \frac{1}{2}mv_2{}^2 = mg(2r)$$

$$\frac{1}{2}(33^2 - v_2{}^2) = (9.81)(2 \times 19)$$

$$v_2 = 18.5 \text{ m s}^{-1}$$

At the top, the net force is downward (in the centripetal direction).

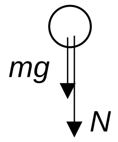


Applying N2L to the car: $(F_{net} = ma)$

$$N + mg = \frac{mv_2{}^2}{r}$$

$$N = \frac{mv_2{}^2}{r} - mg = (170)(\frac{18.53^2}{19} - 9.18) = 1.40 \times 10^3 \text{ N}$$

b) To just clear the loop safely, the car must not lose contact with track (i.e. $N > 0$).



Apply N2L to car: $(F_{net} = ma)$

$$N + mg = \frac{mv_{\min}^{2}}{r}$$

$$0 + mg = \frac{mv_{\min}^{2}}{r}$$

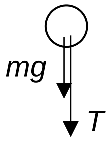
$$v_{\min} = \sqrt{rg} = \sqrt{(19)(9.81)} = 13.7 \text{ m s}^{-1}$$

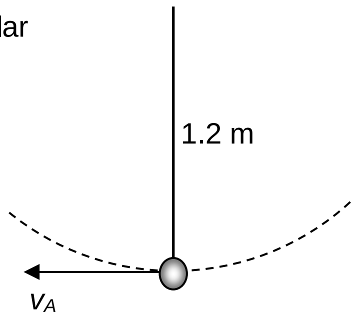
Example 6 – String and bob

A stone of mass 800 g is tied to one end of a string and is whirled in a vertical circle. The string is inextensible and of length 1.2 m. The stone has a certain speed v_A at the lowest point, as shown below.

(i) Determine the minimum speed the stone must have at the top of the circular motion if the string is to be taut at that instant.

(ii) Hence, show that the stone can complete a vertical circular motion if $v_A = 8.0 \text{ m s}^{-1}$.





(i) At the top of the circle, According to Newton's 2nd Law,

$T + mg = \dfrac{mv^2}{r}$ (The slower the speed at the top, the lower the tension T, when speed is a minimum, $T = 0$.)

$$0 + mg = \frac{mv_{\min}^{2}}{r} \Rightarrow v_{\min} = \sqrt{rg} = \sqrt{(1.2)(9.81)} = 3.43 \text{ m s}^{-1}$$

(ii) Firstly, we use PCOE to calculate the speed at which the stone arrives at the top.

$$\text{loss in KE} = \text{gain in GPE}$$

$$\tfrac{1}{2}\cancel{m}v_A^{2} - \tfrac{1}{2}\cancel{m}v_B^{2} = \cancel{m}g(2r)$$

$$\frac{1}{2}(8.0^2 - v_B^{2}) = (9.81)(2\times 1.2)$$

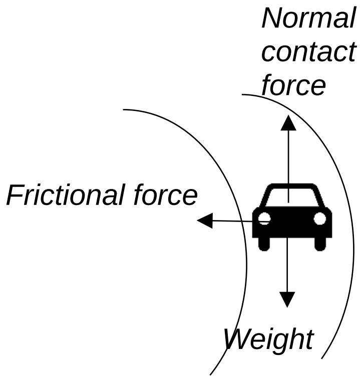
$$v_B = 4.11 \text{ m s}^{-1}$$

Since the stone arrives at the top at a speed higher than 3.43 m s^{-1}, it is able to complete the vertical circular motion.

Example 7 - Car going around a bend

(a) A bend in the road has a 50 m radius of curvature. A car of mass 600 kg takes the bend at 45 km h^{-1}.
   i) What is the centripetal acceleration of the car?
   ii) What is the centripetal force experienced by the car and what provides it?
   iii) What will happen to the car if driver decides to take the bend at 60 km h^{-1} instead? (Given that the maximum friction between the tyres and road surface is 3000 N.)

(i) $$a_c = \frac{v^2}{r} = \frac{(\frac{45\times10^3}{60\times60})^2}{50} = 3.1 \text{ m s}^{-2}$$



(ii) $$F_c = \frac{mv^2}{r} = ma_c = (600)(3.125) = 1875 \text{ N} = 1900 \text{ N (2 s.f)}$$

The centripetal force is provided by the friction exerted by the road surface on the car tyres.

(iii) If car travels at 60 km h^{-1}

Centripetal force required , $F_c = \frac{mv^2}{r} = \frac{(600)(\frac{60\times10^3}{60\times60})^2}{50} = 3300 \text{ N (2 s.f)}$

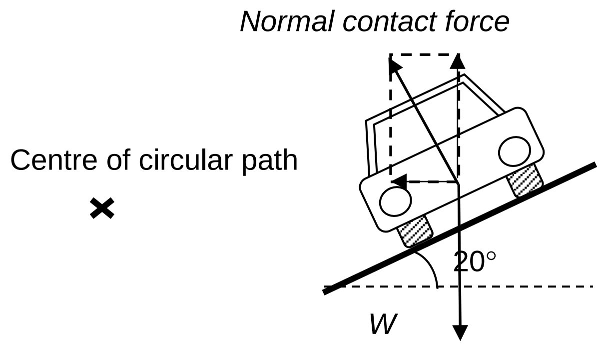
As the maximum frictional force between road surface and tyres is 3000 N, the frictional force is insufficient to provide the required centripetal force of 3300 N, the car will skid off the road from the circular curvature path.

(b) When the car negotiates a corner on horizontal ground, the frictional force between the tyres and the ground is the only force providing the centripetal force. As there is a limit to this frictional force for a particular road surface, there is a maximum speed which the car can make the turn safely, above which skidding will occur.

Hence, some corners (especially at race-tracks) have raised embankments to increase the maximum speed at which a vehicle can take the corner than if on a level road. It does so by making the **normal contact force** contribute a component to the centripetal force.

For an embankment inclined at 20° to the horizontal, find the speed at which the normal contact force is able to **completely provide the centripetal force** (i.e. no frictional force required).

7 (b) Vertically, car is in translational equilibrium



Vertical component of normal contact force = Weight

$$N\cos\theta = mg$$

$$N = \frac{mg}{\cos\theta} = \frac{(600)(9.81)}{\cos(20)} = 6264 \text{ N}$$

Horizontally, according to Newton's 2nd Law

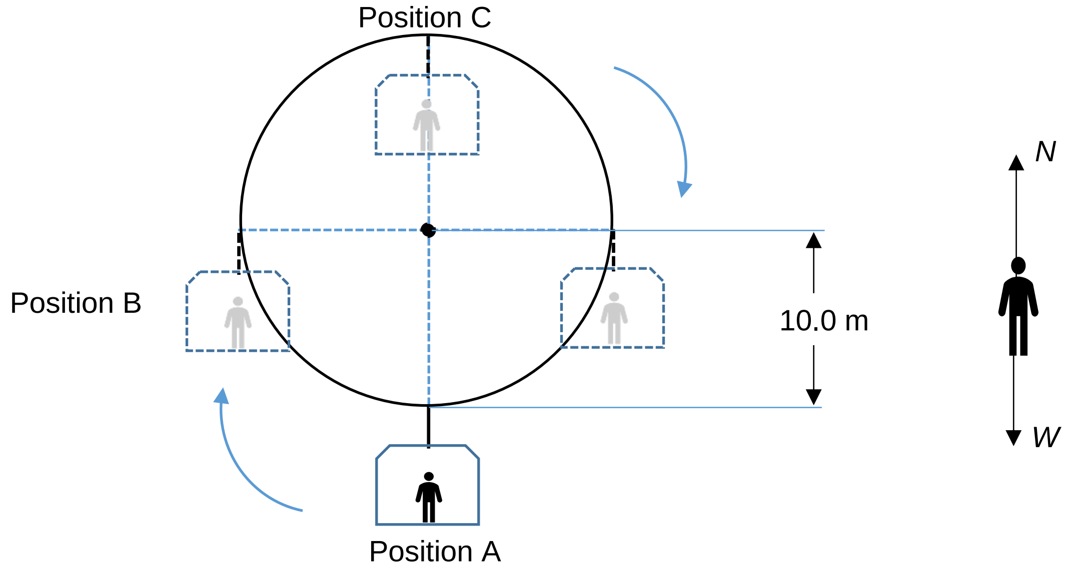
Horizontal component of normal contact force $= \frac{mv^2}{r}$

$$N\sin\theta = \frac{mv^2}{r} \Rightarrow v = \sqrt{\frac{rN\sin\theta}{m}} = \sqrt{\frac{50(6264)\sin(20)}{600}} = 13 \text{ m s}^{-1}$$

The speed at which normal contact force is able to completely provide the centripetal force is 13 m s^{-1}.

Example 8 - Person on a Ferris wheel

The figure below is a simplified diagram of a Ferris wheel. A single carriage with a passenger is shown.



The Ferris wheel takes 0.500 mins to make one complete revolution. Each carriage goes through a circular path of radius 10.0 m. The passenger's mass is 65.0 kg.

Determine the normal contact force exerted by the carriage on the passenger at
(a) position A,
(b) position B, and
(c) position C.
(d) If the passenger were to be standing on a weighing scale, what can you infer about the readings at positions A, B and C respectively?

(a) Position A, by Newton's 2nd Law,

$$N - mg = mr\omega^2$$

$$N = mr\omega^2 + mg = mr\left(\frac{2\pi}{T}\right)^2 + mg = (65)(10.0)\left(\frac{2\pi}{0.500 \times 60}\right)^2 + 65(9.81) = 666 \text{ N}$$

(b) Position B, as man is moving in a uniform circular motion, net force in vertical direction = 0

$$N = mg = 65(9.81) = 638 \text{ N}$$

(c) Position C, by Newton's 2nd Law,

$$mg - N = m\omega^2 r$$

$$N = mg - mr\omega^2 = mg - mr\left(\frac{2\pi}{T}\right)^2 = 65(9.81) - (65)(10.0)\left(\frac{2\pi}{0.500 \times 60}\right)^2 = 609 \text{ N}$$

(d) The normal contact force calculated in (a) to (c) is the force exerted by carriage on man. By Newton's third law, the force which the man exerts on the carriage is equal in magnitude and opposite in direction. Answers (a) to (c) imply that the force exerted by the man on the carriage is the lowest at the top and highest at the bottom of the Ferris wheel. If instead of standing on the carriage directly, the man steps on a weighing scale, the reading will show a corresponding change.