

VICTORIA JUNIOR COLLEGE

# JC 2 PRELIMINARY EXAMINATION 2022

CANDIDATE	
NAME	

CLASS

INDEX NUMBER

## H1 MATHEMATICS

Paper 1

8865/01 3 hours

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF26)

## READ THESE INSTRUCTIONS FIRST

Write your class and name on all the work you hand in.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.

Write your answers in the spaces provided in the question paper.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use an approved graphing calculator.

Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [] at the end of each question or part question. The total number of marks for this paper is 100.

This document consists of **23** printed pages and **1** blank page.

#### Section A: Pure Mathematics [40 marks]

- 1 (a) Find the set of values of k for which the graphs of  $y = 2x^2 kx + k$  and y = kx intersect at most once. [4]
- (b) MARS Café serves three types of coffee: Flat, Cappuccino and Macchiato. On 23 December 2021, a customer paid \$17.30 for two cups of Flat, one cup of Cappuccino and three cups of Macchiato after using a \$10 cash voucher. On the same day, the Café owner recorded a total sale of \$1151.30 from selling 84 cups of Flat, 123 cups of Cappuccino and 65 cups of Macchiato. The total price of three cups of Macchiato is equivalent to the total price of five cups of Flat.

Express this information as 3 linear equations and hence find the price of one cup of Macchiato. [4]

- 2 The curve  $C_1$  has equation  $y = 4 2^x$ . The curve  $C_2$  has equation  $y = \frac{x^2}{4} 3$ .
  - (i) Sketch C<sub>1</sub> and C<sub>2</sub> on the same diagram, stating the exact coordinates of any points of intersection with the axes and the equations of any asymptotes. [4]
  - (ii) Find the x-coordinates of the points of intersection of  $C_1$  and  $C_2$ , giving your answers correct to 4 decimal places. [1]
  - (iii) Write down as an integral an expression for the area of the region bounded by the curves  $C_1$  and  $C_2$ , the line x = -5 and the *y*-axis. Evaluate this integral, giving your answer correct to 3 decimal places. [2]

3 (i) Differentiate 
$$\ln(5xe^{x^2-3})$$
. [3]

(ii) Integrate 
$$\left(\frac{1}{2\sqrt{x}} + x\right)^2$$
. [3]

- 4 A curve has equation  $y = 5 + x + 3x^2 x^3$ .
  - (i) The point *P* is on the curve such that the *x*-coordinate of *P* is positive. The tangent to the curve at *P* is parallel to the line y = -8x+3. Find the equation of the tangent at *P*, giving your answer in the form ax + by + c = 0 where *a*, *b* and *c* are integers. [4]
  - (ii) The tangent at *P* meets the *x*-axis at point *T*. Find the area of triangle *PTO*, where *O* is the origin. [2]

5 A fish farm which supplies sea bass to restaurants in Singapore started operations in January 2001. Based on an expert model, the population of sea bass in the fish farm, *P* thousands, at time *t* years is given by

 $P = N - 15e^{-0.2t}$ , where N is a positive real number.

The fish farm had a population of 15000 sea bass when it started operations.

(i) Show that 
$$N = 30$$
. [1]

(ii) Find 
$$\frac{\mathrm{d}P}{\mathrm{d}t}$$
. [2]

- (iii) Using the expression for  $\frac{dP}{dt}$ , explain why the sea bass population in the fish farm will keep increasing. [1]
- (iv) Sketch the graph of P against t. [2]
- (v) The fish farm is designed to have a maximum capacity of 25000 fishes. Determine the year in which the fish farm is estimated to reach its maximum capacity. [1]

The owner of the fish farm is also interested in modelling his revenue, R thousand dollars per year, at any point in time. The model he uses is

$$R = \frac{t^3}{3} - \frac{9t^2}{2} + 14t + 15, \quad \text{for } 0 \le t \le 5.$$

(vi) Use differentiation to find the stationary value of *R* and justify whether this value is a minimum or maximum. [4]

(vii) Use your calculator to find the value of  $\int_{0}^{5} \left(\frac{t^{3}}{3} - \frac{9t^{2}}{2} + 14t + 15\right) dt$ . In the context of the question, what does this value represent? [2]

#### Section B: Probability and Statistics [60 marks]

6 A and B are events such that P(A|B) = 0.3, P(B|A) = 0.6 and  $P(A \cup B) = 0.72$ .

(i) Find 
$$P(A \cap B)$$
. [3]

- (ii) Determine whether the events *A* and *B* are independent. [2]
- (iii) Describe in words what is meant by the event represented by the shaded region in the Venn diagram and find its probability. [2]



7 Jar A contains three red marbles and five green marbles. Two marbles are drawn from the jar at random, without replacement.

Find the probability that

- (i) none of the marbles are green, [1]
- (ii) at least one of the marbles is green. [2]

All the marbles are put back into jar A. Jar B contains six red marbles and two green marbles. A fair six-sided die is tossed. If the score is 1 or 2, a marble is drawn from jar A. Otherwise, a marble is drawn from jar B.

- (iii) Find the probability that a green marble is drawn. [2]
- (iv) Given that a red marble is drawn, find the probability that it was drawn from jar A. [2]
- 8 (a) Different arrangements of all the letters of the word COPYRIGHTABLE are formed.
  - (i) Find the number of different arrangements that can be formed consisting of the words COPY and TABLE. [1]
  - (ii) Find the number of different arrangements that can be formed such that no vowel is next to another vowel. [2]
  - (b) Passwords are formed using 5 letters from the word COPYRIGHTABLE. Find the number of different passwords that can be formed with at least 3 vowels. [3]

- **9** A technician repairs two types of electronic devices, laptops and tablets. The times taken by the technician to repair laptops are normally distributed with mean 2 hours and standard deviation 0.55 hours. The times taken by the technician to repair tablets are normally distributed with mean 1.5 hours and standard deviation 0.3 hours. The times taken to repair the electronic devices are independent of each other.
  - (i) Find the probability that the time taken by the technician to repair a laptop is more than 2.5 hours. [1]
  - (ii) Find the probability that the time taken by the technician to repair a laptop exceeds the total time taken to repair two tablets. [3]

On any workday, the technician will repair two laptops and three tablets. The technician earns \$50 per hour for repairing a laptop and \$35 per hour for repairing a tablet.

(iii) Find the probability that the total earnings of the technician on a workday is between \$300 and \$400.

The technician repairs another type of electronic device, mobile phones. The times taken by the technician to repair mobile phones are recorded. The records show that 25% of the mobile phones take more than 1.5 hours to be repaired and 10% of the mobile phones take less than 0.5 hours to be repaired.

- (iv) Assuming that the times taken by the technician to repair mobile phones are normally distributed, find the mean and variance of the distribution. [4]
- 10 A confectionary produces gummy candies in different colours. The probability that any randomly chosen gummy candy is red is 0.6. The gummy candies are sold in packets of 24. The number of red gummy candies in a packet is the random variable *X*.
  - (i) Find the probability that, in a randomly selected packet of gummy candies, at most half of the gummy candies are red. [1]
  - (ii) Find the probability that the number of red gummy candies in a packet is more than the expected value of *X*. [3]

A packet of gummy candies is accepted if at most half of the gummy candies are red.

Two packets of gummy candies are selected one after another.

- (iii) Find the probability that the first packet of gummy candies selected is accepted and the second packet of gummy candies selected is not accepted. [2]A customer buys 12 packets of gummy candies.
- (iv) Find the probability that, for exactly 4 of these packets, at most half of the gummy candies are red.

11 The following table shows the age, *x* years, and the maximum heart rate, *y* bpm (beats per minute), for a random sample of 8 adult males.

Adult male	Α	В	С	D	Ε	F	G	Н
x	63	34	21	49	26	53	74	42
у	157	187	196	168	204	171	149	177

- (i) Give a sketch of the scatter diagram for the data, as shown on your calculator. [2]
- (ii) Find the product moment correlation coefficient and comment on its value. Interpret this value in the context of the data. [3]
- (iii) Find the equation of the regression line of y on x, in the form y = mx + c, giving the values of m and c correct to 3 significant figures. Sketch this line on your scatter diagram. [2]
- (iv) Calculate an estimate of the maximum heart rate of a randomly chosen 61-year-old adult male. [1]
- (v) Explain why you would expect this estimate to be reliable. [1]
- 12 A manufacturer states that a jar of jam that he produces has a mean mass of 200 grams. A food standards inspector wishes to investigate whether the mean mass of these jars of jam is actually less than 200 grams. A random sample of 70 jars of jam is taken. The masses of the jars of jam, *x*, in grams, are summarised by

$$\sum x = 13950, \quad \sum x^2 = 2781000.$$

(i) Determine the conclusion of the investigation if the food standards inspector carries out a test at the 5% significance level.

The manufacturer now produces a new recipe of jam and finds that the population standard deviation is 15.3 grams.

A new random sample of 70 jars of jam is taken and the mean mass of the jars of jam in this sample is m grams. A test at the 10% significance level shows that there is sufficient evidence to suggest that the population mean mass of the jars of jam differs from 200 grams.

[4]

(ii) Find the range of possible values of *m*.