H2 Mathematics (9758)
Chapter 3 Functions
Discussion Questions (Suggested Solutions)

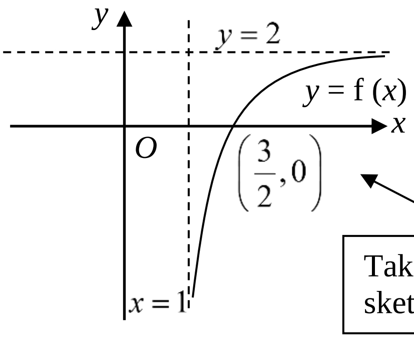
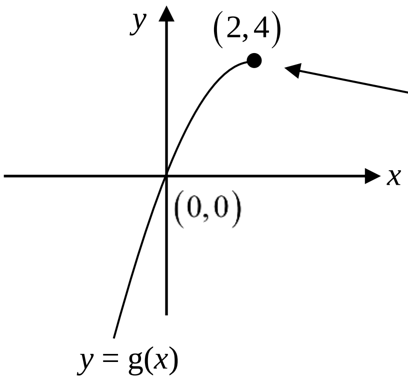
Level 1

1 Sketch the graphs of each of the following functions. State its domain and give its corresponding range.

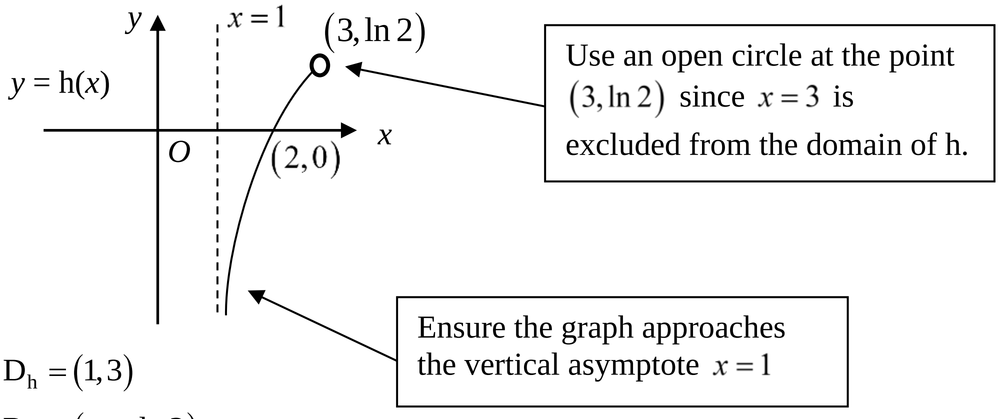
(a) $\mathrm{f}: x \mapsto \dfrac{2x-3}{x-1}, x \in \mathbb{R}, x > 1$

(b) $\mathrm{g}: x \mapsto -(x-2)^2 + 4, x \in \mathbb{R}, x \le 2$

(c) $\mathrm{h}: x \mapsto \ln(x-1), x \in \mathbb{R}, 1 < x < 3$

1	Solution
(a)	 $y = \dfrac{2x-3}{x-1}$ Vertical asymptote: $x = 1$ Horizontal asymptote: $y = 2$ Take the domain of f into consideration when sketching the graph of f $\mathrm{D_f} = (1, \infty)$ $\mathrm{R_f} = (-\infty, 2)$ Find $\mathrm{R_f}$ by referring to the graph of f to determine the range of possible y values. (i.e. minimum and maximum y values)
(b)	 Use a closed circle at the point $(2, 4)$ since $x = 2$ is included in the domain of g. $\mathrm{D_g} = (-\infty, 2]$ $\mathrm{R_g} = (-\infty, 4]$

(c)



$\mathrm{D_h} = (1, 3)$

$\mathrm{R_h} = (-\infty, \ln 2)$

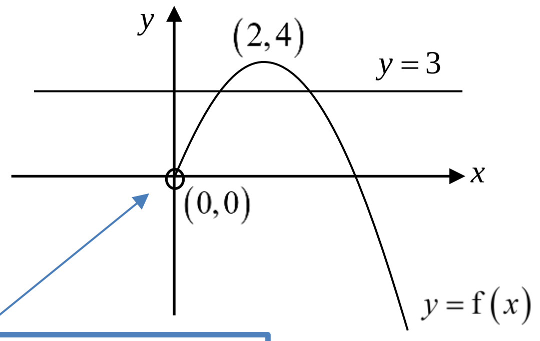
2 The function f is defined by

$$\mathrm{f} : x \mapsto -(x-2)^2 + 4,\ x \in \mathbb{R},\ x > 0.$$

(i) Sketch the graph of f.

(ii) State the domain and range of f.

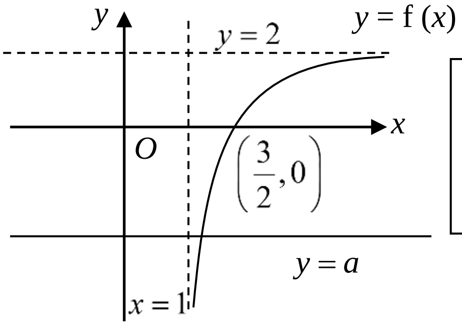
(iii) Explain why f^{-1} does not exist.

2	Solution
(i)	 Check both the rule and domain when sketching the graph. Open circle since $x = 0$ is not included.
(ii)	$\mathrm{D_f} = (0, \infty)$ $\mathrm{R_f} = (-\infty, 4]$
(iii)	To show that inverse does not exist, show that f is not a one-one function **Method 1: Horizontal Line Test** Since the horizontal line $y = 3$ cuts the graph of $y = \mathrm{f}(x)$ more than once, f is not a one-one function. Hence f^{-1} does not exist. Show that f is not a one-one function by giving a **specific example** of a horizontal line that cuts the graph the graph more than once. Remark: Any horizontal line $y = k$ with $0 < k < 4$ will be accepted. **Method 2: Algebraic Method** Since $\mathrm{f}(1) = \mathrm{f}(3) = 3$, f is not a one-one function. Hence f^{-1} does not exist. Show that f is not a one-one function by giving a **specific example** of two inputs that result in the same output.

3 The function f is defined as $\mathrm{f}: x \mapsto \dfrac{2x-3}{x-1},\ x \in \mathbb{R}, x > 1$.

(i) Explain why f^{-1} exists.

(ii) Find f^{-1} in a similar form.

<table>
<tr><th>3</th><th>Solution</th></tr>
<tr><td>(i)</td><td>

To explain that f^{-1} exists $\Leftrightarrow$
show f is one-one (horizontal line test)
Note: you must draw the graph if it is not given by the question.

Since any line $y = a$, where $a \in \mathbb{R}$, cuts the graph of f at most once, f is one-one.
Hence f^{-1} exists.</td></tr>
<tr><td>(ii)</td><td>Let $y = \dfrac{2x-3}{x-1}$
$y(x-1) = 2x-3$
$x(y-2) = y-3$
$x = \dfrac{y-3}{y-2}$
$x = \mathrm{f}^{-1}(y) = \dfrac{y-3}{y-2}$
$\mathrm{f}^{-1}(x) = \dfrac{x-3}{x-2}$
$D_{\mathrm{f}^{-1}} = R_{\mathrm{f}} = (-\infty, 2)$
$\mathrm{f}^{-1}: x \mapsto \dfrac{x-3}{x-2},\ x \in \mathbb{R}, x < 2$

To write the expression of f^{-1} in similar form, state the rule and domain of f^{-1}</td></tr>
</table>

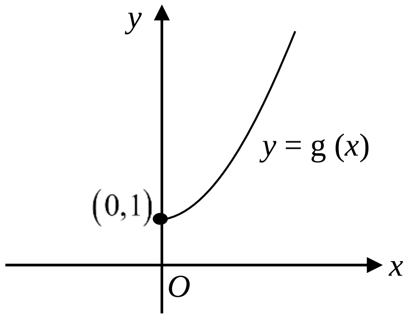
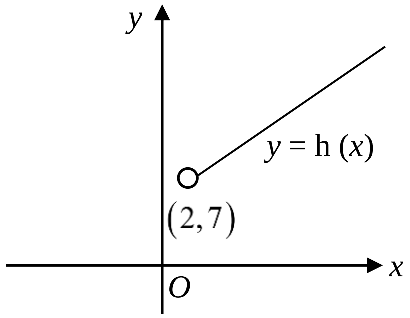
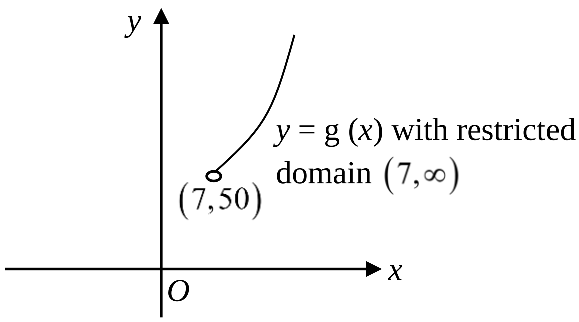
4 Functions g and h are defined by

$$\mathrm{g} : x \mapsto x^2+1, \; x\in\mathbb{R}, x\geq 0,$$
$$\mathrm{h} : x \mapsto 2x+3, \; x\in\mathbb{R}, x> 2.$$

(i) Show that gh exists.

(ii) Find gh in a similar form.

(iii) Find the range of gh.

4	**Solution**
(i)	 $\mathrm{R_h} = (7, \infty)$ and $\mathrm{D_g} = [0, \infty)$ Since $\mathrm{R_h} \subseteq \mathrm{D_g}$, gh exists.
(ii)	$\mathrm{g}(\mathrm{h}(x)) = (2x+3)^2+1$ $\mathrm{gh} : x \mapsto (2x+3)^2+1, x\in\mathbb{R}, x>2$
(iii)	**Method 1:** Using rule and domain of gh $\mathrm{R_{gh}} = (50, \infty)$ **Method 2:** Using mapping method  $\mathrm{D_h} = (2,\infty) \xrightarrow{\mathrm{h}} \mathrm{R_h} = (7,\infty) \xrightarrow{\mathrm{g}} \mathrm{R_{gh}} = (50, \infty)$ $\mathrm{R_{gh}} = (50, \infty)$

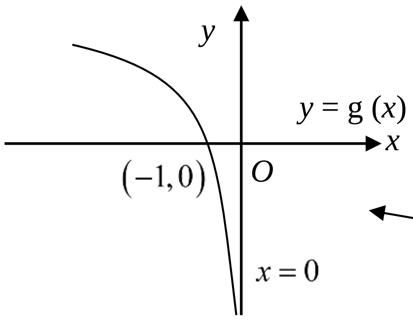
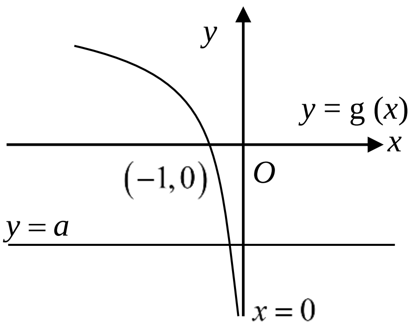
<u>Level 2</u>

5 The function g is defined as $\mathrm{g}: x \mapsto \ln\left(x^2\right), x \in \mathbb{R}, x < 0$.

(i) State the domain and range of g.

(ii) Give a reason why g^{-1} exists.

(iii) Find the rule, domain and range of g^{-1}.

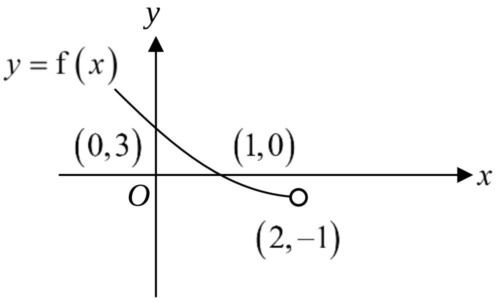
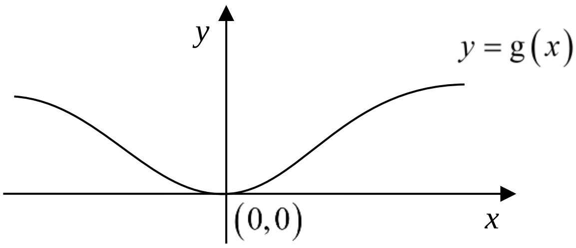
5	**Solution**
(i)	 Take the domain of g into consideration when sketching the graph of g Ensure the graph approaches the vertical asymptote $x = 0$ $\mathrm{D_g} = (-\infty, 0)$ $\mathrm{R_g} = (-\infty, \infty)$ Find $\mathrm{R_g}$ by referring to graph of g to determine the range of possible *y* values. (i.e. minimum and maximum *y* values)
(ii)	 To explain that g^{-1} exists $\Leftrightarrow$ show g is one-one (horizontal line test) Since any line $y = a$, where $a \in \mathbb{R}$, cuts the graph of g at most once, g is one-one. Hence g^{-1} exists.
(iii)	Let $y = \ln\left(x^2\right)$ $e^y = x^2$ $x = \pm\sqrt{e^y}$ Use the domain of g to decide which expression to pick. Since $x < 0$, $x = -\sqrt{e^y}$ $\mathrm{g}^{-1}(x) = -\sqrt{e^x}$ $\mathrm{D}_{\mathrm{g}^{-1}} = \mathrm{R_g} = (-\infty, \infty) = \mathbb{R}$ $\mathrm{R}_{\mathrm{g}^{-1}} = \mathrm{D_g} = (-\infty, 0)$

6 2013/CJC Prelim/II/3 (modified)

Functions f and g are defined by

$$\mathrm{f} : x \mapsto (x-2)^2 - 1, \quad x \in \mathbb{R},\ x < 2$$
$$\mathrm{g} : x \mapsto \ln(x^2 + 1), \quad x \in \mathbb{R}$$

Only one of the composite functions fg and gf exists. Give the rule and domain of the composite function that exists, and explain why the other composite does not exist.

6	Solution
	 
	$\mathrm{R_g} = [0, \infty)$ $\mathrm{D_f} = (-\infty, 2)$ Since $\mathrm{R_g} \not\subseteq \mathrm{D_f}$, fg does not exist
	$\mathrm{R_f} = (-1, \infty)$ $\mathrm{D_g} = \mathbb{R}$ Since $\mathrm{R_f} \subseteq \mathrm{D_g}$, gf exists.
	$\mathrm{gf}(x) = \mathrm{g}\left((x-2)^2 - 1\right)$ $= \ln\left[\left((x-2)^2 - 1\right)^2 + 1\right]$ $\mathrm{D_{gf}} = \mathrm{D_f} = (-\infty, 2)$ $\mathrm{gf} : x \mapsto \ln\left[\left((x-2)^2 - 1\right)^2 + 1\right],\ x \in \mathbb{R},\ x < 2$

7 2018/ACJC Promo/Q9(part)

Functions f and g are defined by

$$\mathrm{f} : x \mapsto \frac{x+3}{4-x}, \quad x \in \mathbb{R},\ x \neq 4,$$

$$\mathrm{g} : x \mapsto \frac{1}{x}, \qquad x \in \mathbb{R},\ x < 0.$$

(i) Show that the composite function fg exists. [1]

(ii) Find the range of fg. [2]

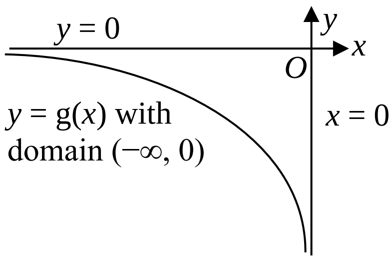
(iii) Find an expression for $\mathrm{fg}(x)$ and hence, or otherwise, find $(\mathrm{fg})^{-1}\left(\frac{1}{2}\right)$. [3]

<table>
<tr><th>7</th><th>Solution</th></tr>
<tr><td>(i)</td><td>

$\mathrm{R_g} = (-\infty, 0) \quad \mathrm{D_f} = \mathbb{R} \setminus \{4\}$

$\mathrm{R_g} \subseteq \mathrm{D_f}$

Therefore, fg exists.



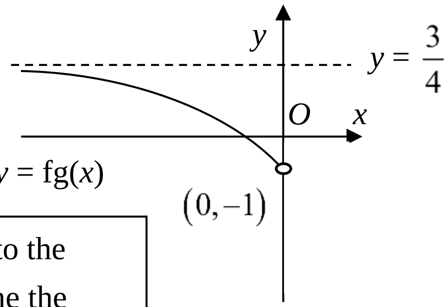
</td></tr>
<tr><td>(ii)</td><td>

Method 1: Using rule and domain of fg

$$\mathrm{fg}(x) = \frac{\frac{1}{x}+3}{4-\frac{1}{x}} = \frac{1+3x}{4x-1}$$

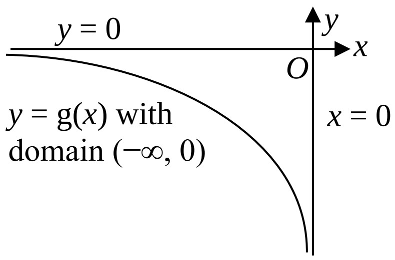
$\mathrm{D_{fg}} = \mathrm{D_g} = (-\infty, 0)$

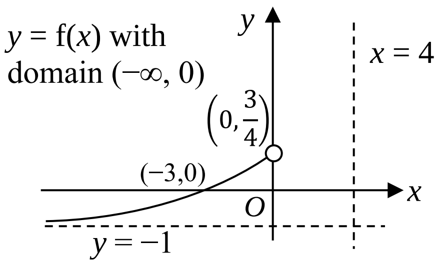
$\mathrm{R_{fg}} = \left(-1, \frac{3}{4}\right)$



Find $\mathrm{R_{fg}}$ by referring to the graph of fg to determine the range of possible *y* values. (i.e. minimum and maximum *y* values)

Method 2: Using mapping method





$$\mathrm{D_g} = (-\infty, 0) \xrightarrow{\mathrm{g}} \mathrm{R_g} = (-\infty, 0) \xrightarrow{\mathrm{f}} \mathrm{R_{fg}} = \left(-1, \frac{3}{4}\right)$$

</td></tr>
</table>

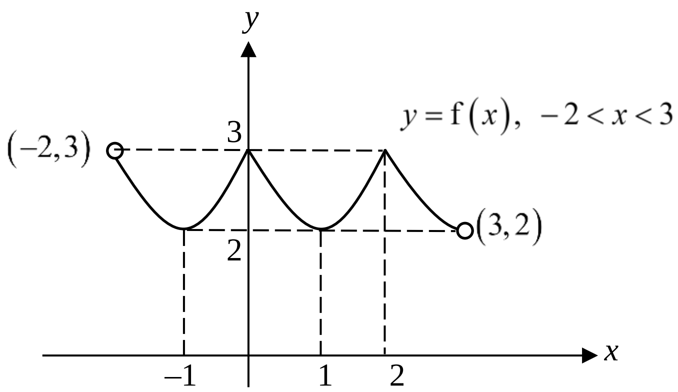
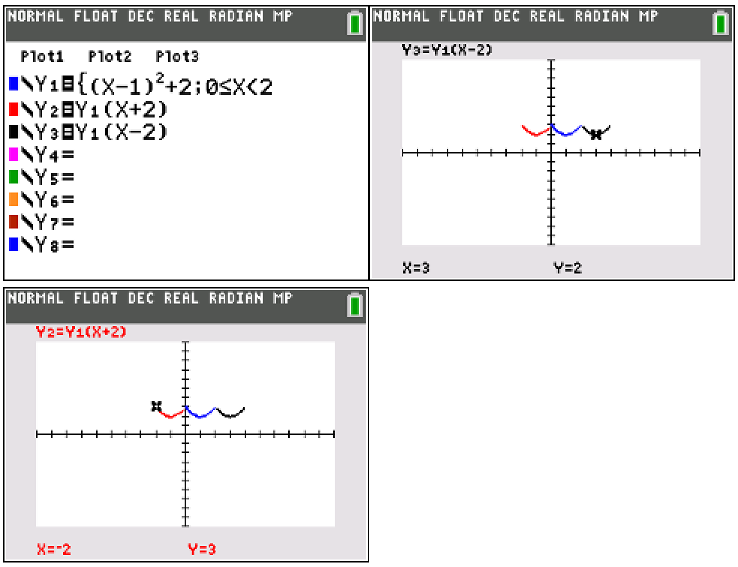
(iii)	$\mathrm{fg}(x)=\dfrac{\frac{1}{x}+3}{4-\frac{1}{x}}=\dfrac{1+3x}{4x-1}$ Let $(\mathrm{fg})^{-1}\left(\dfrac{1}{2}\right)=a \quad \Rightarrow \quad \mathrm{fg}(a)=\dfrac{1}{2}$ $\dfrac{1+3a}{4a-1}=\dfrac{1}{2}$ $2(1+3a)=4a-1$ $2a=-3$ $a=-\dfrac{3}{2}$ $\therefore (\mathrm{fg})^{-1}\left(\dfrac{1}{2}\right)=-\dfrac{3}{2}$ $(\mathrm{fg})^{-1}\left(\dfrac{1}{2}\right)=a$ $\Rightarrow \mathrm{fg}(\mathrm{fg})^{-1}\left(\dfrac{1}{2}\right)=\mathrm{fg}(a)$ $\Rightarrow \dfrac{1}{2}=\mathrm{fg}(a)$ [since $\mathrm{fg}(\mathrm{fg})^{-1}(x)=x$]

8 It is given that $\mathrm{f}(x)=(x-1)^2+2,\ x\in\mathbb{R},\ 0\le x<2$ and that $\mathrm{f}(x)=\mathrm{f}(x+2)$ for all real values of x.

(i) State the period of f.

(ii) Evaluate $\mathrm{f}(1)$ and $\mathrm{f}(-2)$.

(iii) Sketch the graph of $y=\mathrm{f}(x)$ for $-2<x<3$.

8	**Solution**
(i)	2
(ii)	$\mathrm{f}(1)=(1-1)^2+2=2$ $\mathrm{f}(-2)=\mathrm{f}(-2+2)=\mathrm{f}(0)=(0-1)^2+2=3$
(iii)	$\mathrm{f}(3)=\mathrm{f}(1)=2$  

9 2010/MJC JC1 MYE/I/5 (Modified)

The function f is defined by

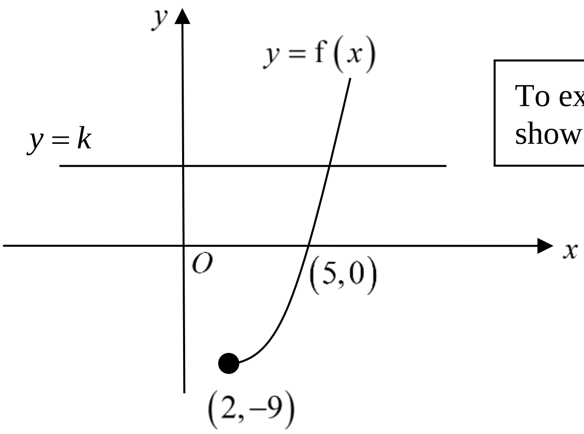
$$\mathrm{f} : x \mapsto x^2 - 4x - 5, \quad x \geq 2 .$$

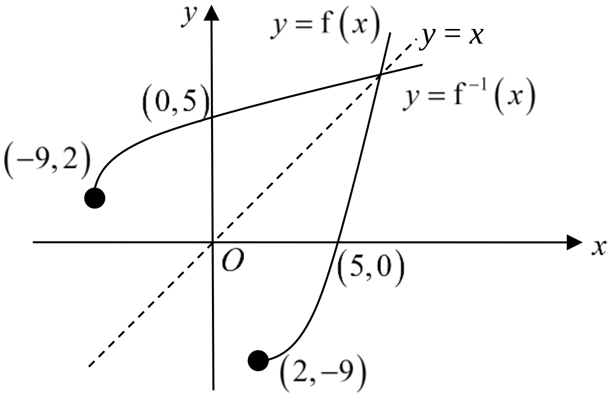
(i) Show that f^{-1} exists. [1]

(ii) Find f^{-1} in a similar form. [3]

(iii) Write down the equation of the line in which the graph of $y = \mathrm{f}(x)$ must be reflected to obtain the graph of $y = \mathrm{f}^{-1}(x)$. [1]

(iv) Sketch the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$ on the same diagram. Hence find the exact solution of the equation $\mathrm{f}(x) = \mathrm{f}^{-1}(x)$. [4]

9	**Solution**
(i)	 To explain that f^{-1} exists $\Leftrightarrow$ show f is one-one (horizontal line test) Since any horizontal line $y = k, k \in \mathbb{R}$ cuts the graph of $y = \mathrm{f}(x)$ at most once, f is one-one and f^{-1} exists.
(ii)	Let $y = x^2 - 4x - 5$ $y = (x-2)^2 - 9$ $(x-2)^2 = y + 9$ $x = 2 \pm \sqrt{y+9}$ Since $x \geq 2$, $x = 2 + \sqrt{y+9}$ $\therefore \ \mathrm{f}^{-1}(x) = 2 + \sqrt{x+9}$ $\mathrm{D}_{\mathrm{f}^{-1}} = \mathrm{R}_{\mathrm{f}} = [-9, \infty)$ Find R_{f} by referring to the graph of f to determine the range of possible *y* values. (i.e. minimum and maximum *y* values) $\therefore \ \mathrm{f}^{-1} : x \mapsto 2 + \sqrt{x+9}, \quad x \geq -9$ To write the expression of f^{-1} in similar form, state the rule and domain of f^{-1}

(iii)	The graph of $y=\mathrm{f}(x)$ is reflected in the line $y=x$ to obtain the graph of $y=\mathrm{f}^{-1}(x)$.
(iv)	 **Note:** $y=\mathrm{f}(x)$ and $y=\mathrm{f}^{-1}(x)$ should be symmetrical about the line $y=x$. The graphs must also satisfy both the vertical and horizontal line test as f and f^{-1} are both one-one functions Solving $\mathrm{f}(x)=\mathrm{f}^{-1}(x)$ can be seen as solving $\mathrm{f}(x)=x$. $(x-2)^2-9=x$ $x^2-5x-5=0$ $x=\dfrac{5\pm3\sqrt{5}}{2}$ Since $x\geq 2$, $x=\dfrac{5+3\sqrt{5}}{2}$. Note that the x-coordinate of the intersection of the graphs of f and f^{-1} is an element of the set $\mathrm{D}_{\mathrm{f}}\cap\mathrm{D}_{\mathrm{f}^{-1}}=[2,\infty)$ Therefore, choose the correct value of x such that $x\in[2,\infty)$

<u>Level 3</u>

10 2018/ACJC Promo/Q9(modified)

The function h is defined by

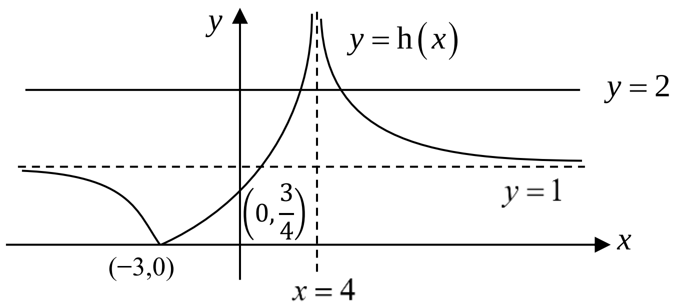
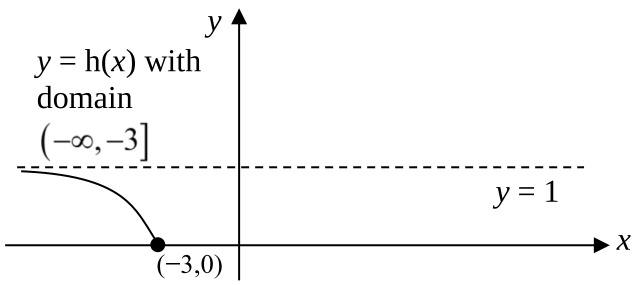
$$\mathrm{h} : x \mapsto \left|\frac{x+3}{4-x}\right|, \quad x \in \mathbb{R},\ x \neq 4.$$

(i) Sketch the graph of h and state its range. [2]

(ii) Explain why the inverse function h^{-1} does not exist. [1]

(iii) The function h^{-1} exists if the domain of h is restricted to $x \le k$. State the greatest value of *k*. [1]

(iv) Using the domain in **(iii)**, find $\mathrm{h}^{-1}(x)$ and state the domain of h^{-1}. [4]

10	Solution
(i)	 $\mathrm{D}_{\mathrm{h}^{-1}} = \mathrm{R}_\mathrm{h} = [0,\infty)$
(ii)	Since the horizontal line $y=2$ cuts the graph of $y=\mathrm{h}(x)$ more than once, h is not a one-one function. Hence h^{-1} does not exist.
(iii)	For h^{-1} to exist, *h* must be 1-1, $\therefore x \le k$. Greatest $k=-3$.
(iv)	$y=\mathrm{h}(x)=\left\lvert\frac{x+3}{4-x}\right\rvert$ Use the domain of h to decide which expression to pick. For $x \le -3$, $y=-\frac{x+3}{4-x}$ $(x-4)y=x+3$ $x(y-1)=4y+3$ $x=\frac{4y+3}{y-1}$ $\therefore\ \mathrm{h}^{-1}(x)=\frac{4x+3}{x-1}$  $\mathrm{D}_{\mathrm{h}^{-1}} = \mathrm{R}_\mathrm{h}$ with $\mathrm{D}_\mathrm{h} = (-\infty,-3]$ $= [0,1)$

11 2013/VJC Prelim/II/3 (Modified)

The function f is defined by

$$\mathrm{f}: x \mapsto x^2 - 2x + 2, \quad 1 < x \le 3.$$

(i) Sketch the graphs of $y = \mathrm{f}(x)$, $y = \mathrm{f}^{-1}(x)$ and $y = \mathrm{ff}^{-1}(x)$ on a single diagram, indicating clearly the domains of the respective functions. [3]

(ii) Without using the graphing calculator, find the exact solution of the equation $\mathrm{f}(x) = \mathrm{f}^{-1}(x)$. [2]

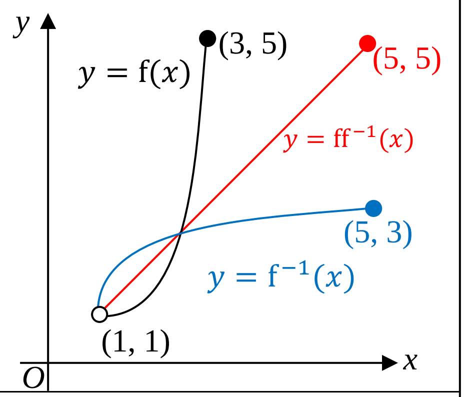
(iii) State the range of values of x satisfying the equation $\mathrm{f}^{-1}\mathrm{f}(x) = \mathrm{ff}^{-1}(x)$. [1]

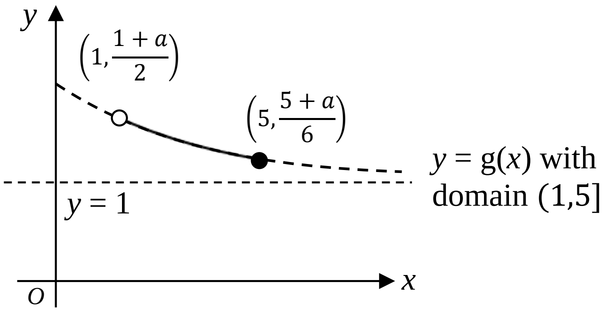
The function g is defined by

$$\mathrm{g}: x \mapsto \frac{x+a}{x+1}, \quad x \ge 0,$$

where a is a constant and $a > 1$.

(iv) Show that the composite function gf exists and find, in exact form, the range of gf. [4]

11	**Solution**
(i)	 Graphs of f and f^{-1} should be symmetrical about the line $y = x$ (Use the same scale for x- and y-axis) Graphs must pass vertical and horizontal line test. The graph of ff^{-1} is the line $y = x$ where $D_{\mathrm{ff}^{-1}} = D_{\mathrm{f}^{-1}} = (1, 5]$ Be careful with the curvature, ensure that the graph of $\mathrm{f}^{-1}(x)$ does not curve downwards
(ii)	$\mathrm{f}(x) = \mathrm{f}^{-1}(x)$ $\mathrm{f}(x) = x$ $x^2 - 2x + 2 = x$ $x^2 - 3x + 2 = 0$ $(x-2)(x-1) = 0$ $x = 2$ or $x = 1$ (rejected $\because x > 1$) Solving $\mathrm{f}(x) = \mathrm{f}^{-1}(x)$ can be seen as solving $\mathrm{f}(x) = x$
(iii)	$\mathrm{f}^{-1}\mathrm{f}(x) = \mathrm{ff}^{-1}(x)$. $\mathrm{D}_{\mathrm{f}^{-1}\mathrm{f}} = \mathrm{D}_{\mathrm{f}} = (1, 3]$ $\mathrm{D}_{\mathrm{ff}^{-1}} = \mathrm{D}_{\mathrm{f}^{-1}} = \mathrm{R}_{\mathrm{f}} = (1, 5]$ Find R_{f} by referring to graph of f to determine the range of possible y values. (i.e. minimum and maximum y values) So, $\mathrm{f}^{-1}\mathrm{f}(x) = \mathrm{ff}^{-1}(x)$ when $1 < x \le 3$

(iv)	$R_f = (1,5]$ $D_g = [0,\infty)$ Since $R_f \subseteq D_g$, the composite function gf exists. Using $R_f = (1,5]$ as the restricted domain for g,  $g(1) = \frac{1+a}{2}$ and $g(5) = \frac{5+a}{6}$ Use the mapping method to find R_{gf} since the expression for g consists of unknowns. $D_{gf} = D_f = (1,3] \xrightarrow{f} R_f = (1,5] \xrightarrow{g} R_{gf} = \left[\frac{5+a}{6}, \frac{1+a}{2}\right)$ So $R_{gf} = \left[\frac{5+a}{6}, \frac{1+a}{2}\right)$

12 At a local meteorological station, the daily average temperature recorded by the instrument is in degrees Fahrenheit, $^{\circ}\mathrm{F}$. The meteorological station master wants to record the temperature in degrees Celsius, $^{\circ}\mathrm{C}$ and he uses the following function c to do the conversion:

$$\mathrm{c}: x \mapsto \frac{5}{9}(x-32), \quad x > -459.67.$$

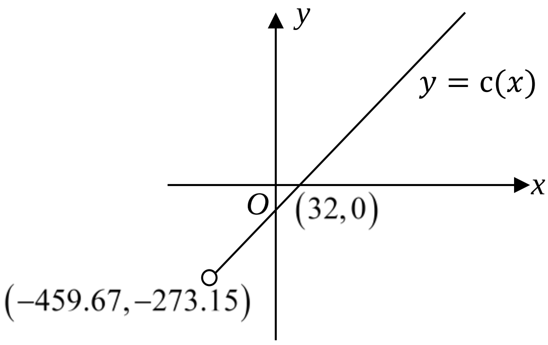
(i) Given that the average temperature of a particular day is given as $50\,^{\circ}\mathrm{F}$, express the temperature in terms of $^{\circ}\mathrm{C}$. [1]

(ii) Define c^{-1} and explain the significance of this function in the context of the question. [3]

A physicist wants to record the temperature in Kelvin, K and he has the following function k which converts temperature in degrees Celsius, $^{\circ}\mathrm{C}$ to Kelvin, K:

$$\mathrm{k}: x \mapsto x + 273.15, \quad x > -273.15.$$

(iii) The physicist wants to convert the temperature from degrees Fahrenheit, $^{\circ}\mathrm{F}$ to Kelvin, K directly. Define a composite function to meet his requirement [2]

12	Solution
(i)	$\mathrm{c}(50) = \frac{5}{9}(50-32) = 10$ The average temperature is $10\,^{\circ}\mathrm{C}$.
(ii)	Let $y = \frac{5}{9}(x-32)$ $x = \frac{9}{5}y + 32$ $\mathrm{c}^{-1}(x) = \frac{9}{5}x + 32$ $\mathrm{D}_{\mathrm{c}^{-1}} = \mathrm{R}_c = (-273.15, \infty)$ $\therefore\ \mathrm{c}^{-1}: x \mapsto \frac{9}{5}x + 32, \quad x > -273.15$  The function c^{-1} converts temperature in $^{\circ}\mathrm{C}$ to $^{\circ}\mathrm{F}$.
(iii)	$\mathrm{kc}(x) = \frac{5}{9}(x-32) + 273.15$ $\mathrm{D}_{\mathrm{kc}} = \mathrm{D}_{\mathrm{c}} = (-459.67, \infty)$ $\mathrm{kc}: x \mapsto \frac{5}{9}(x-32) + 273.15, \quad x > -459.67$

13 2014/DHS Promo (Modified)

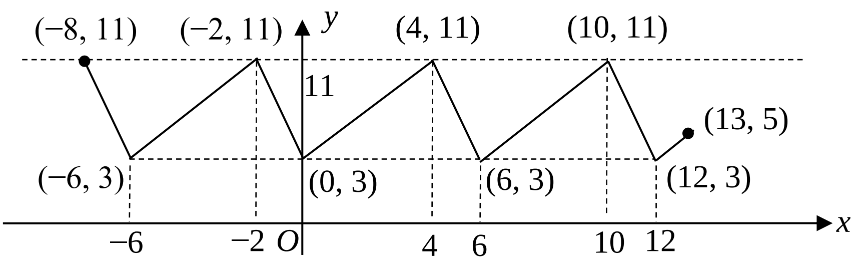
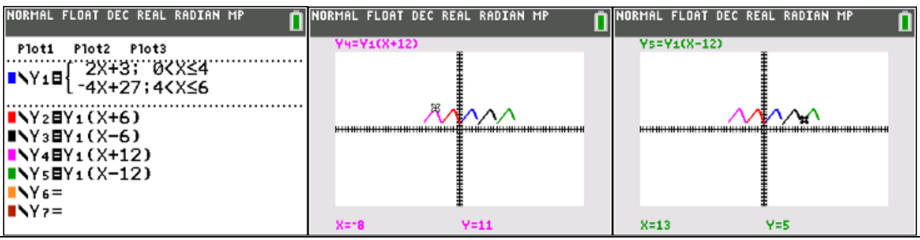
The function f is defined by

$$f(x)=\begin{cases}2x+3 & \text{for } 0<x\le 4,\\ -4x+27 & \text{for } 4<x\le 6,\end{cases}$$

and that $f(x)=f(x+6)$ for all real values of x.

(i) Find the value of $f(-17)+f(17)$. [2]

(ii) Sketch the graph of $y=f(x)$ for $-8\le x\le 13$. [3]

13	Solution
(i)	$f(-17)=f(-11)=f(-5)=f(1)=2(1)+3=5$ $f(17)=f(11)=f(5)=-4(5)+27=7$ $\therefore f(-17)+f(17)=5+7=12$ $f(x)=f(x+6)$ $\Rightarrow$ The period of the graph is 6 units. The graph repeats itself every 6 units.
(ii)	Check for continuous/disjoint at $x=4$: When $x=4$, $f(x)=2(4)+3=11$ When $x=4$, $f(x)=-4(4)+27=11$ Thus, the graph is continuous at $x=4$. Calculating the coordinates of end-points for $-8\le x\le 13.$: $f(13)=f(7)=f(1)=2(1)+3=5\Rightarrow$ closed circle $f(-8)=...=f(4)=2(4)+3=11\Rightarrow$ closed circle  

<u>Things to note:</u>

- Proper scale for x-axis (e.g. 1cm denote 1 unit)
- **Open circles for excluded endpoints** and **closed circles for included endpoints**
- Draw dotted reference lines for critical y-values to ensure you always end at the same height

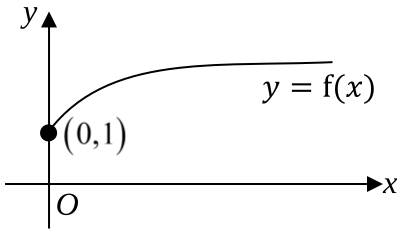
14 2016(9740)/I/10(b)

The function g, with domain the set of non-negative integers, is given by

$$g(n)=\begin{cases}1 & \text{for } n=0,\\ 2+g\left(\frac{1}{2}n\right) & \text{for } n \text{ even},\\ 1+g(n-1) & \text{for } n \text{ odd}.\end{cases}$$

(i) Find $g(4)$, $g(7)$ and $g(12)$. [3]

(ii) Does g have an inverse? Justify your answer. [2]

14	Solution
((i)	$g(0)=1$ $g(1)=1+g(0)=1+1=2$ $g(2)=2+g(1)=2+2=4$ $g(3)=1+g(2)=1+4=5$ $g(4)=2+g(2)=2+4=6$ $g(5)=1+g(4)=1+6=7$ $g(6)=2+g(3)=2+5=7$ $g(7)=1+g(6)=1+7=8$ $g(12)=2+g(6)=2+7=9$ $\therefore g(4)=6$ $g(7)=8$ $g(12)=9$  **Comment:** For unseen question, it is a good practice for students to try listing out the first few "terms" of the function to gain some understanding of the function.
(ii)	$\because g(5)=g(6)$ from (b)(i) Therefore g is not 1-1, implying that g does not have an inverse. To prove that a function is not 1-1, we need to give counter-example. i.e. Find 2 different values of *x* that give the same *y*-value.

Answer Key
1(a) $\mathrm{D_f}=(1,\infty)$, $\mathrm{R_f}=(-\infty,2)$ **(b)** $\mathrm{D_g}=(-\infty,2]$, $\mathrm{R_g}=(-\infty,4]$ **(c)** $\mathrm{D_h}=(1,3)$, $\mathrm{R_h}=(-\infty,\ln 2)$
2 (ii) $\mathrm{D_f}=(0,\infty)$, $\mathrm{R_f}=(-\infty,4]$
3 (ii) $\mathrm{f}^{-1}:x\mapsto\dfrac{x-3}{x-2},\ x\in\mathbb{R},\ x<2$
4 (ii) $\mathrm{gh}:x\mapsto(2x+3)^2+1,\ x\in\mathbb{R},\ x>2$ **(iii)** $(50,\infty)$
5 (i) $\mathrm{D_g}=(-\infty,0)$, $\mathrm{R_g}=(-\infty,\infty)$ **(iii)** $\mathrm{g}^{-1}:x\mapsto-\sqrt{\mathrm{e}^x},\ x\in\mathbb{R}$, $\mathrm{R_{g^{-1}}}=(-\infty,0)$
6 $\mathrm{gf}:x\mapsto\ln\left[\left((x-2)^2-1\right)^2+1\right],\ x\in\mathbb{R},\ x<2$
7 (ii) $\mathrm{R_{fg}}=\left(-1,\dfrac{3}{4}\right)$ **(iii)** $(\mathrm{fg})^{-1}\left(\dfrac{1}{2}\right)=-\dfrac{3}{2}$
8 (i) 2 **(ii)** $\mathrm{f}(1)=2$, $\mathrm{f}(-2)=3$
9 (ii) $\mathrm{f}^{-1}:x\mapsto 2+\sqrt{x+9},\quad x\geq-9$ **(iii)** $y=x$ **(iv)** $x=\dfrac{5+3\sqrt{5}}{2}$
10 (iii) Greatest $k=-3$ **(iv)** $\mathrm{h}^{-1}(x)=\dfrac{4x+3}{x-1}$, $\mathrm{D_{h^{-1}}}=[0,1)$
11 (ii) $x=2$ **(iii)** $1<x\leq 3$ **(iv)** $\mathrm{R_{gf}}=\left[\dfrac{5+a}{6},\ \dfrac{1+a}{2}\right)$
12 (i) $10\ ^\circ\mathrm{C}$ **(ii)** $\mathrm{c}^{-1}:x\mapsto\dfrac{9}{5}x+32,\quad x>-273.15$ **(iii)** $\mathrm{kc}:x\mapsto\dfrac{5}{9}(x-32)+273.15,\quad x>-459.67$
13 (i) 12
14 (b)(i) $\mathrm{g}(4)=6$, $\mathrm{g}(7)=8$, $\mathrm{g}(12)=9$