SINGAPORE CHINESE GIRLS' SCHOOL
PRELIMINARY EXAMINATION 2017
SECONDARY FOUR
O-LEVEL PROGRAMME

ADDITIONAL MATHEMATICS **4047/02**
Paper 2

Friday **4 August 2017** **2 hours 30 minutes**

Additional Materials: Answer Paper
Cover Page

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved electronic scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 100.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2\sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$\Delta = \frac{1}{2}bc\sin A$$

1. **(i)** Express $\dfrac{7x+9}{2x^2-9x-5}$ in partial fractions. [3]

(ii) Hence find $\displaystyle\int_6^7 \frac{7x+9}{2x^2-9x-5}\,\mathrm{d}x$ [3]

2. Given that $\mathrm{f}(x) = 2x^3 - 3x^2 - 3x + 22$ is exactly divisible by $2x^2 + bx + c$,

(i) find the value of b and of c, [4]

(ii) show that $2x^3 - 3x^2 - 3x + 22 = 0$ has only one real root, [3]

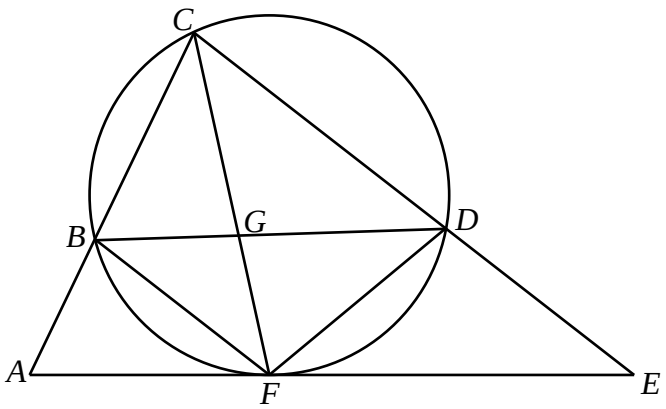
(iii) find the remainder when $\mathrm{f}(x)$ is divided by $(2x+3)$. [2]

3. The roots of the quadratic equation $2x^2 - 3x - 4 = 0$ are α and β.

(i) Without using a calculator, show that $\dfrac{\alpha}{\beta^2} + \dfrac{\beta}{\alpha^2} = \dfrac{99}{32}$. [5]

(ii) Hence find the quadratic equation whose roots are $\dfrac{\alpha}{\beta^2}$ and $\dfrac{\beta}{\alpha^2}$. [2]

4. The diagram below shows a quadrilateral *BCDF* whose vertices lie on the circumference of a circle. The tangent to the circle at the point *F* meets *CB* extended at *A* and *CD* extended at *E*. The lines *CF* and *BD* intersect at *G* and *BG = GF*.



(i) State an angle which is equal to angle *BFA*. [1]

(ii) Prove that angle *FDE* = angle *FAB* + angle *FCB*. [2]

(iii) Prove that quadrilateral *BCDF* is a trapezium. [3]

5. Solve the following equations.

(i) $(2^{3x+2})(3^{x-1}) = 32$ [2]

(ii) $7(3^{1-x}) = 3^{x+1} + 2$ [4]

(iii) $\log_3(x+5) - \log_{\sqrt{3}}(x-1) = \log_3 2$ [4]

6. A curve has the equation $y = \frac{x}{e^{2x}}$. The point (p, q) is a stationary point on the curve.

Determine

(i) the exact value of p and of q, [4]

(ii) the nature of the stationary point (p, q). [3]

Hence

(iii) write down the values of x for which y is increasing. [1]

7. The population, P, of a certain species of fish after t years is given by

$$P = 1750(1 + 2e^{-kt}).$$

When $t = 2$, the population is 4600.

(i) Find the value of k. [3]

Any species is considered as an "Endangered Species" if its population falls below 2500.

(ii) Determine, with working, whether this species of fish will become an "Endangered Species" after 10 years. [2]

(iii) Hence sketch the population-time graph. [2]

8. The depth of water, d metres, at a pier, t hours after low tide, can be modelled by the formula

$$d = c - a\cos(bt),$$

where a, b and c are positive constants.

(i) If low tides occur every 12 hours, find the value of b. [1]

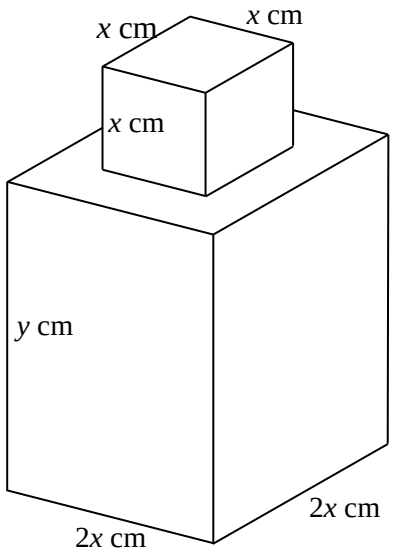
Given that the depth of water at the pier was 2 metres during low tide and 8 metres during high tide,

(ii) find the value of a and of c. [2]

The pier will be open when the depth of the water is more than 4 m.

(iii) For how long will the pier be open in a 12-hour period after low tide. [3]

9. The diagram shows a solid which consists of a cube fixed on top of a cuboid. The cube has sides x cm. The cuboid has a square base of side $2x$ cm and a height of y cm.



Given that the volume of the solid is 270 cm^3,

(i) show that the total surface area, A cm^2, of the solid is given by

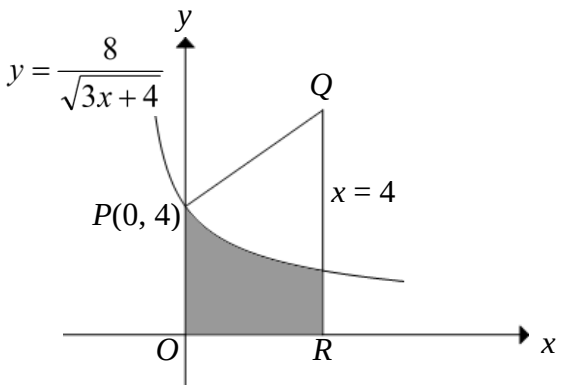
$$A = 10x^2 + \frac{540}{x}.$$ [5]

Given that x can vary,

(ii) find the value of x for which A has a stationary value and determine whether this value of A is a maximum or a minimum. [5]

10. The diagram shows part of the curve $y = \frac{8}{\sqrt{3x+4}}$. The curve intersects the y-axis at $P(0, 4)$.

The normal to the curve at P intersects the line $x = 4$ at the point Q and the line segment QR is parallel to the y-axis.



(i) Find the coordinates of Q. [5]

(ii) Find the ratio of the area of the shaded region to the area of the trapezium $OPQR$ in the form 1: n. [5]

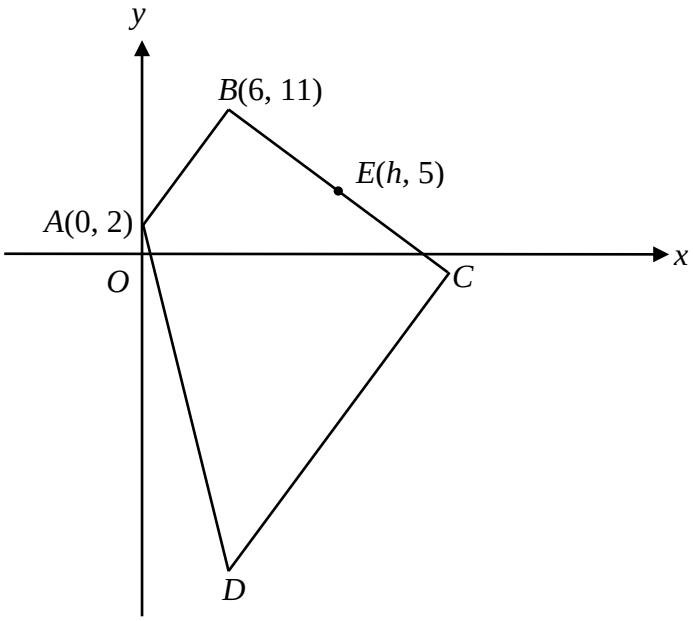
11. (a) (i) Prove that $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$. [3]

(ii) Find all the angles between $-\frac{\pi}{2}$ and π which satisfy the equation

$$\frac{4\cos^3\theta - 3\cos\theta}{\cos\frac{3\theta}{2}} = 4\sin\frac{3\theta}{2}.$$ [4]

(b) Solve the equation $6\cos x - 3\sec x = 7$ for $-180^\circ \le x \le 180^\circ$. [4]

12. Solutions to this question by accurate drawing will not be accepted.



The diagram, which is not drawn to scale, shows a trapezium *ABCD* in which *AB* is parallel to *DC* and *BD* is parallel to the *y*–axis. The coordinates of *A* and of *B* are (0, 2) and (6, 11) respectively.
E(*h*, 5) is the midpoint of *BC* such that *AB* = *BE* and *h* > 0.

(i) Show that the value of *h* is 15. [2]

Find

(ii) the coordinates of *C*, [2]

(iii) the equation of *DC*, [3]

(iv) the coordinates of *D*. [1]

Given that the area of triangle $ABE = 58\frac{1}{2}$ units^2,

(v) find the perpendicular distance from *B* to the line segment *AE*. [2]

End of Paper 2