2019 GCE A Level H2 Further Maths 9649 Paper 2 Solutions

Section A: Pure Mathematics [50 marks]

Question 1

For a given non-zero column vector $\mathbf{u} = \begin{pmatrix} x \\ y \end{pmatrix}$, let M be the set of all 2×2 matrices $\mathbf{X}$ for which there exists a real scalar constant λ such that $\mathbf{Xu} = \lambda\mathbf{u}$.

(i) Show that M contains the zero 2×2 matrix and also that it is closed under the usual operations of matrix addition and multiplication by a scalar. [4]

(ii) Write down a 2×2 matrix that is **not** in M for any non-zero $\mathbf{u}$. Justify your answer. [2]

[Solution]

(i) Given **M** is the set of 2×2 matrices $\mathbf{X}$ for which there exists a real scalar constant λ such that $\mathbf{Xu} = \lambda\mathbf{u}$, that is **u** is the eigenvector of **X.**

$\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = 0\begin{pmatrix} x \\ y \end{pmatrix}$, thus **M** contains the zero 2×2 matrix.

Let $\mathbf{X}_1$ and $\mathbf{X}_2 \in$ **M,** that is there exists real scalar constants λ_1 and λ_2 such that

$$\mathbf{X}_1\mathbf{u} = \lambda_1\mathbf{u} \text{ and } \mathbf{X}_2\mathbf{u} = \lambda_2\mathbf{u}$$

Then $(\mathbf{X}_1 + \mathbf{X}_2)\mathbf{u} = \mathbf{X}_1\mathbf{u} + \mathbf{X}_2\mathbf{u} = \lambda_1\mathbf{u} + \lambda_2\mathbf{u} = (\lambda_1 + \lambda_2)\mathbf{u}$, $\lambda_1 + \lambda_2$ is a real scalar constant.

$\mathbf{X}_1 + \mathbf{X}_2 \in \mathbf{M}$. Thus **M** is closed under matrix addition.

Consider $\mu \in \mathbb{R}$, $(\mu\mathbf{X}_1)\mathbf{u} = \mu(\mathbf{X}_1\mathbf{u}) = \mu\lambda_1\mathbf{u}$, $\mu\lambda_1$ is a real scalar. Thus $\mu\mathbf{X}_1 \in \mathbf{M}$.

Thus **M** is closed under matrix multiplication by a scalar.

(ii) Consider $\mathbf{X} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ then $\mathbf{Xu} = \lambda\mathbf{u}$

$$\Rightarrow \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \lambda\begin{pmatrix} x \\ y \end{pmatrix} \Rightarrow \begin{pmatrix} y \\ -x \end{pmatrix} = \begin{pmatrix} \lambda x \\ \lambda y \end{pmatrix}$$

No solution for λ for any non-zero $\mathbf{u} = \begin{pmatrix} x \\ y \end{pmatrix}$. Thus $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \notin \mathbf{M}$

Or any other 2x2 matrix such that $\mathbf{Xu} = \lambda\mathbf{u}$ has no real solution for λ.

Question 2

A conic section has polar equation $r=\dfrac{1}{2-\sin\theta}$, $0\le\theta<2\pi$.

(i) By finding the equation of this curve in a standard Cartesian form, show that it is an ellipse [5]

(ii) Determine the eccentricity of this curve and the coordinates of the two foci [4]

(i)	$r=\dfrac{1}{2-\sin\theta}$ $2r-r\sin\theta=1$ $2\sqrt{x^2+y^2}=1+y$ $4\left(x^2+y^2\right)=(1+y)^2$ $4\left(x^2+y^2\right)=1+2y+y^2$ $3y^2-2y+4x^2=1$ $3\left(y-\dfrac{1}{3}\right)^2-\dfrac{1}{3}+4x^2=1$ $3\left(y-\dfrac{1}{3}\right)^2+4x^2=\dfrac{4}{3}$ $\dfrac{9\left(y-\dfrac{1}{3}\right)^2}{4}+3x^2=1$ $\dfrac{x^2}{\left(\dfrac{1}{\sqrt{3}}\right)^2}+\dfrac{\left(y-\dfrac{1}{3}\right)^2}{\left(\dfrac{2}{3}\right)^2}=1$
(ii)	$e=\dfrac{1}{2};(0,0)$ and $\left(0,\dfrac{2}{3}\right)$

Question 3

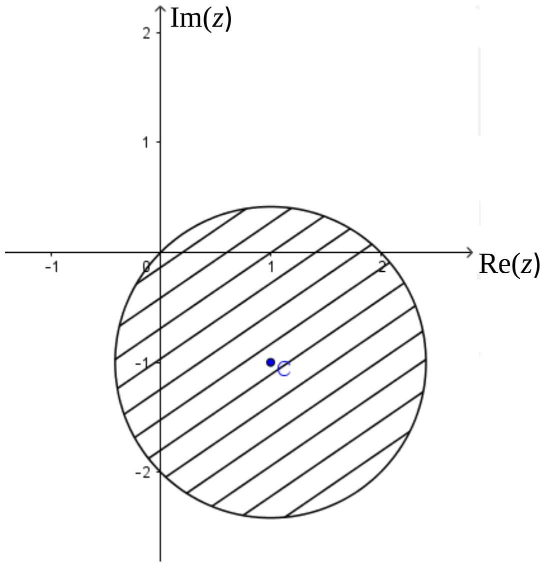
(i) On an Argand diagram, shade the region of the complex plane which represents the set of all complex numbers z for which $|z-1+\mathrm{i}| \le \sqrt{2}$. [3]

(ii) For this set of complex numbers, determine the maximum value of $\arg(z+2)$, where $-\pi < \arg(z+2) \le \pi$, giving your answer in the form $\tan^{-1} p$ for some rational number p. [7]

[Solution]

(i) $|z-1+\mathrm{i}| \le \sqrt{2} \Rightarrow |z-(1-\mathrm{i})| \le \sqrt{2}$

The shaded region is inside the circle with centre at 1 – i and radius = $\sqrt{2}$ units



(ii)

Let max arg(z + 2) = arg(z – (–2)) = θ

AB is tangent to the circle at B, thus $\angle ABC = 90^0$

AC = $|1-i-(-2)| = \sqrt{10}$ and BC = $\sqrt{2}$

Thus AB = $\sqrt{10-2} = 2\sqrt{2}$

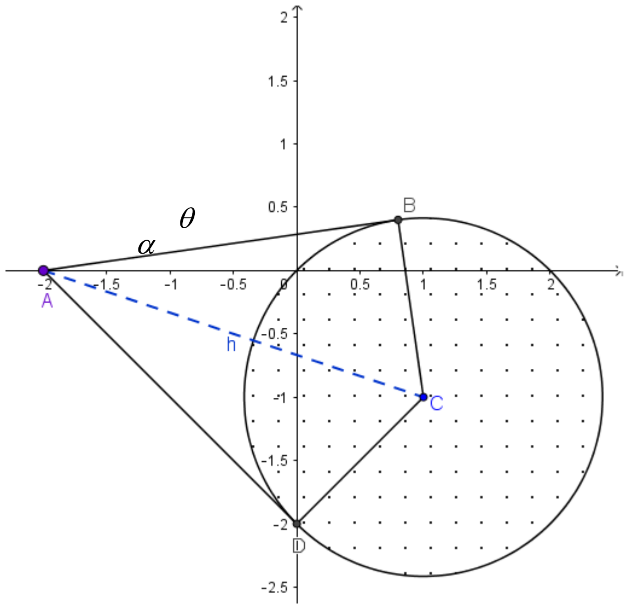
$\tan\angle CAB = \dfrac{\sqrt{2}}{2\sqrt{2}} = \dfrac{1}{2}$

Let α be the acute angle AC made with the Real axis, thus tan α $= \dfrac{1}{3}$.

Thus tan θ = tan(∠CAB – α)

$$= \frac{\tan\angle CAB - \tan\alpha}{1+\tan\angle CAB\tan\alpha} = \frac{\frac{1}{2}-\frac{1}{3}}{1+\frac{1}{2}\left(\frac{1}{3}\right)} = \frac{1}{7}$$

Thus max arg(z + 2) = $\tan^{-1}\dfrac{1}{7}$



Question 4

Let P_n be the proposition that

$$\frac{1+a^2+a^4+\ldots a^{2n}}{a+a^3+a^5+\ldots a^{2n-1}} > \frac{n+1}{n}$$

where $a>0, a\neq 1$, and n is a positive integer.

(i) Show that P_n can be written in the form $\mathrm{f}(n)>0$, where

$$\mathrm{f}(n)=\frac{n\left(a^{2n+2}-1\right)-a(n+1)\left(a^{2n}-1\right)}{a^2-1}. \qquad [3]$$

(ii) By considering $\mathrm{f}(n)-a\mathrm{f}(n-1)$, prove by induction that P_n is true for all positive integers *n*. [9]

[Solution]

(i) As $a>0$, the given proposition $\frac{1+a^2+a^4+\ldots a^{2n}}{a+a^3+a^5+\ldots a^{2n-1}} > \frac{n+1}{n}$

$\Rightarrow n(1+a^2+a^4+\ldots a^{2n}) > (a+a^3+a^5+\ldots a^{2n-1})(n+1)$

$\Rightarrow \frac{n((a^2)^{n+1}-1)}{a^2-1} > \frac{(n+1)a(a^{2n}-1)}{a^2-1}$

$\Rightarrow \mathrm{f}(n)=\frac{n(a^{2n+2}-1)-a(n+1)(a^{2n}-1)}{a^2-1}>0$

(ii) So P_n is the proposition $\mathrm{f}(n)=\frac{n(a^{2n+2}-1)-a(n+1)(a^{2n}-1)}{a^2-1}>0$, *n* is a positive integer.

$\mathrm{f}(1)=\frac{(a^4-1)-2a(a^2-1)}{a^2-1}=\frac{(a^2-1)(a^2+1-2a)}{a^2-1}=(a-1)^2>0$ as $a>0, a\neq 1$

Thus P_1 is true

Assume P_{k-1} is true that is $f(k-1)>0$ for some positive integer k

Consider $\mathrm{f}(k)-a\mathrm{f}(k-1)$

$$\mathrm{f}(k)-a\mathrm{f}(k-1)=\frac{k(a^{2k+2}-1)-a(k+1)(a^{2k}-1)-a(k-1)(a^{2k}-1)+a^2k(a^{2k-2}-1)}{a^2-1}$$

$$=\frac{k(a^{2k+2}-1+a^{2k}-a^2)-a(a^{2k}-1)(k+1+k-1)}{a^2-1}$$

$$=\frac{k(a^{2k+2}-1+a^{2k}-a^2)-2ak(a^{2k}-1)}{a^2-1}=\frac{k(a^{2k+2}-1+a^{2k}-a^2-2a^{2k+1}+2a)}{a^2-1}$$

$$=\frac{k(a^{2k}(a^2-2a+1)-(a^2-2a-1))}{a^2-1}=\frac{k(a-1)^2(a^{2k}-1)}{a^2-1}$$

$= k(a^2-1)(1+a^2+a^4+\ldots+(a^2)^{k-1})>0$ as $a>0, a\neq 1$

$f(k)=k(a^2-1)(1+a^2+a^4+\ldots+(a^2)^{k-1})+a\,f(k-1)>0$ as $f(k-1)>0$ by induction hypothesis.

Since P_1 is true and P_{k-1} is true $\Rightarrow P_k$ is true. Thus by Mathematical Induction, P_n is true for $n\in\mathbb{Z}^+$.

Question 5

Use the substitution $u = x\dfrac{\mathrm{d}y}{\mathrm{d}x} + 2y$ to solve the differential equation

$$x\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + (x+3)\frac{\mathrm{d}y}{\mathrm{d}x} + 2y = 3x+7$$

given that $y = 2$ and $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 1$ when $x = 1$. [13]

$u = x\dfrac{\mathrm{d}y}{\mathrm{d}x} + 2y$ ------ (1)

Differentiate w.r.t. x:

$$\frac{\mathrm{d}u}{\mathrm{d}x} = x\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + \frac{\mathrm{d}y}{\mathrm{d}x} + 2\frac{\mathrm{d}y}{\mathrm{d}x} = x\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + 3\frac{\mathrm{d}y}{\mathrm{d}x} \text{ ----- (2)}$$

$$x\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + (x+3)\frac{\mathrm{d}y}{\mathrm{d}x} + 2y = 3x+7$$

$$\left(x\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + 3\frac{\mathrm{d}y}{\mathrm{d}x}\right) + \left(x\frac{\mathrm{d}y}{\mathrm{d}x} + 2y\right) = 3x+7$$

Sub eqn (1) and (2) into DE:

$$\frac{\mathrm{d}u}{\mathrm{d}x} + u = 3x+7 \text{ ----- (3)}$$

Integrating factor = $\mathrm{e}^{\int \mathrm{d}x} = \mathrm{e}^x$

Multiply equation (3) throughout by integrating factor:

$$\mathrm{e}^x\frac{\mathrm{d}u}{\mathrm{d}x} + u\mathrm{e}^x = \mathrm{e}^x(3x+7)$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(u\mathrm{e}^x\right) = \mathrm{e}^x(3x+7)$$

$$u\mathrm{e}^x = \int \mathrm{e}^x(3x+7)\,\mathrm{d}x$$

$$u\mathrm{e}^x = \mathrm{e}^x(3x+7) - \int \mathrm{e}^x(3)\,\mathrm{d}x$$

$$u\mathrm{e}^x = \mathrm{e}^x(3x+7) - 3\mathrm{e}^x + C_1 \text{ ----- (4)}$$

When $y = 2$ and $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 1$ when $x = 1$

From eqn (1): $u = 5$

Sub $x = 1$, $u = 5$, $5\mathrm{e} = 10\mathrm{e} - 3\mathrm{e} + C_1 \Rightarrow C = -2\mathrm{e}$

$$\therefore u\mathrm{e}^x = \mathrm{e}^x(3x+4) - 2\mathrm{e}$$

$$\therefore \mathrm{e}^x\left(x\frac{\mathrm{d}y}{\mathrm{d}x} + 2y\right) = \mathrm{e}^x(3x+4) - 2\mathrm{e}$$

$$x\frac{\mathrm{d}y}{\mathrm{d}x} + 2y = (3x+4) - \frac{2\mathrm{e}}{\mathrm{e}^x}$$

	$\frac{\mathrm{d}y}{\mathrm{d}x}+\frac{2y}{x}=\left(3+\frac{4}{x}\right)-\frac{2\mathrm{e}^{1-x}}{x}$ Integrating factor: $\mathrm{e}^{\int\frac{2}{x}}=\mathrm{e}^{2\ln x}=x^2$ Multiply throughout by integrating factor: $x^2\frac{\mathrm{d}y}{\mathrm{d}x}+2xy=\left(3x^2+4x\right)-2x\mathrm{e}^{1-x}$ $\frac{\mathrm{d}}{\mathrm{d}x}\left(x^2y\right)=\left(3x^2+4x\right)-2x\mathrm{e}^{1-x}$ $x^2y=\int\left(3x^2+4x\right)-2x\mathrm{e}^{1-x}\ \mathrm{d}x$ $x^2y=\left(x^3+2x^2\right)+\left(2x\mathrm{e}^{1-x}-\int 2\mathrm{e}^{1-x}\mathrm{d}x\right)$ $x^2y=\left(x^3+2x^2\right)+\left(2x\mathrm{e}^{1-x}+2\mathrm{e}^{1-x}\right)+C_2$ Sub $x=1, y=2,$ $2=3+(2+2)+C_2\Rightarrow C_2=-5$ $y=x+2+2\mathrm{e}^{1-x}\left(\frac{1}{x}+\frac{1}{x^2}\right)-\frac{5}{x^2}$

Section B: Probability and Statistics [50 marks]

Question 6

Hand grip strength is measured using a dynamometer. Measurements are given in kilograms. For males aged between 30 and 39 the mean grip strength for the dominant hand is known to be 46 kg.

A researcher wished to investigate whether or not hand grip strength is affected by obesity. She took a random sample of obese males aged between 30 and 39, and measured their dominant hand grip strength, x kg. The data are summarised as follows.

$$n = 22 \qquad \sum x = 1031 \qquad \sum x^2 = 49410$$

(i) Construct a 95% confidence interval for the mean grip strength for the dominant hand based on the data, giving the end-points of the interval correct to 2 decimal places. State an assumption required for your method to be valid. [5]

(ii) State, with a reason, what conclusion the researcher should reach. [2]

(i)	$s^2 = \frac{1}{21}\left[49410 - \frac{1031^2}{22}\right] = \frac{3437}{66}$ 95% confidence limits for the mean grip strength for the dominant hand = $\bar{x} \pm t_{(\frac{\alpha}{2},\, n-1)} \frac{s}{\sqrt{n}}$ $= \frac{1031}{22} \pm 2.07961\left(\sqrt{\frac{3437}{66 \times 22}}\right)$ 95% confidence interval for the mean grip strength for the dominant hand = (43.66, 50.06) Assumption: Assume that the grip strength for the dominant hand follows a normal distribution.
(ii)	Since $\mu = 46$ lies within the 95% confidence interval, there is insufficient evidence at 5% level of significance to conclude that hand grip strength is affected by obesity.

Question 7

A lifestyle magazine in the UK asked a random sample of 200 of its readers to state their favourite colour for a smartphone. A similar magazine in Singapore asked a random sample of 400 of its readers the same question. The responses are summarised in the table.

	White	**Silver**	**Black**	**Gold**	**Red**	**Other**
UK	51	22	49	29	11	38
Singapore	87	76	77	51	34	75

(i) Carry out a chi-squared test, using a 10% significance level, to investigate whether the data give any reason to support that preferences for colours of smartphones in the UK differ from those in Singapore. [5]

(ii) Give the contributions to the test statistic, and hence state the colour for which the preferences in the UK and Singapore differ the most. [3]

(i)

H_0: The preferences for colours of smartphone are independent of the 2 countries, UK and Singapore

H_1: The preferences for colours of smartphone are not independent of the 2 countries, UK and Singapore

Level of significance: 10%

If H_0 is true, the expected frequencies should be as follows:

	White	**Silver**	**Black**	**Gold**	**Red**	**Other**
UK	46	32.667	42	26.667	15	37.667
Singapore	92	65.333	84	53.333	30	75.333

Degree of freedom = (6 – 1)(2 – 1) = 5,

Test statistic: $\sum \frac{\left(O_{ij} - E_{ij}\right)^2}{E_{ij}} \sim \chi_5^2$

Level of significance: 10%

From GC, we have $\chi_{cal}^2 = 9.70$ and p - value = 0.0842 < 0.1

Since the p-value < Level of significance, we reject H_0

Hence there is sufficient evidence at the 10% level of significance to conclude that the preferences for colours of smartphone in the two countries are different.

(ii)

Contributions of the test statistics, $\frac{\left(O_{ij} - E_{ij}\right)^2}{E_{ij}}$:

	White	**Silver**	**Black**	**Gold**	**Red**	**Other**
UK	0.543	3.483	1.167	0.204	1.067	0.0030
Singapore	0.272	1.742	0.583	0.102	0.533	0.0015

	The colour Silver has the largest contribution in the two countries. Hence the colour preferences which differed the most was Silver. (Do not just state the colour but provide simple reasons/justifications.)

Question 8

A company has a fleet of 24 similar vehicles used by its sales staff. The fleet manager wishes to investigate whether or not switching from conventional engine oil to synthetic engine oil will give improved fuel economy. He randomly chooses 10 of the vehicles to be switched to synthetic oil. The fuel economy, measured in litres per 100 kilometre, of all 24 vehicles is monitored for a month. The data are summarised as follows.

Conventional oil fuel economy, x	$n = 14$	$\bar{x} = 8.04$	$\Sigma\left(x-\bar{x}\right)^2 = 4.53$
Synthetic oil fuel economy, y	$n = 10$	$\bar{y} = 7.40$	$\Sigma\left(x-\bar{y}\right)^2 = 6.41$

(i) Carry out an appropriate t-test on the data, using a 1% significance level. State the assumptions required for the test to be valid. [8]

A statistician advises that it would have been better to measure the fuel economy of all 24 vehicles, once with conventional oil and once with synthetic oil.

(ii) Explain briefly what the test procedure would have been if the statistician's advice had been followed. Explain also why that procedure would have been better. [2]

(i)	Let μ_X and μ_Y be the population mean conventional oil fuel economy and synthetic oil fuel economy respectively. Both sample sizes $n_X = 14$, $n_Y = 10$ are small, two-sample t-test is used. We need to make the following **assumptions**: 1. The conventional oil fuel economy and synthetic oil economy follows a normal distribution 2. The variances of the two populations are the same. $H_0: \mu_X = \mu_Y$ $H_1: \mu_X > \mu_Y$ Level of significance: 1% $\bar{x} = 8.04$, $\bar{y} = 7.40$, $s_p^2 = \dfrac{\Sigma\left(x-\bar{x}\right)^2 + \Sigma\left(y-\bar{y}\right)^2}{n_x + n_y - 2} = \dfrac{4.53+6.41}{14+10-2} = \dfrac{547}{1100}$ Under H_0, Test statistic: $\dfrac{(\bar{X}-\bar{Y})-0}{\sqrt{\dfrac{547}{1100}\left(\dfrac{1}{14}+\dfrac{1}{10}\right)}} \sim t_{n_X+n_Y-2} = t_{22}$ Using GC, $t_{cal} = \dfrac{(8.04-7.40)-0}{\sqrt{\dfrac{547}{1100}\left(\dfrac{1}{14}+\dfrac{1}{10}\right)}} = 2.192$ Using GC, p-value $= 0.0196 > 0.01$ Since p-value > level of significance, **we do not reject H_0.**

	There is **insufficient evidence** at 1% level of significance to conclude that switching from conventional engine oil to synthetic engine oil will give improved fuel economy.
(ii)	The test procedure would be changed to a pair-sample *t*-test if the statistician's advice had been followed. The pairing will eliminate any differences or variabilities between different vehicles and thus better compare the fuel economy between conventional oil and synthetic oil.

Question 9

A recent news report stated that the average number of babies born per hour in the UK is 90.

(i) State the conditions under which the number of babies born per hour can be well modelled by a Poisson distribution. [1]

You should now assume these conditions hold.

(ii) Calculate the probability that, in a randomly chosen hour, more than 100 babies will be born in the UK. [1]

(iii) Calculate the probability that, in a randomly chosen minute, no babies will be born in the UK. [2]

(iv) Let the number of babies born per day in the UK be denoted by the random variable Y. Find integers a and b, each a multiple of 10, such that $\mathrm{P}(Y<a)\approx 0.025$ and $\mathrm{P}(Y>b)\approx 0.025$. [3]

You should now assume that the number of twin births in a given interval of time follows a Poisson distribution.

(v) Records for a particular maternity hospital show that, on average, there are no twin births in 41% of weeks.

(a) Calculate the average number of twin births per year at this hospital. (Take a year to be 52 weeks.) [3]

(b) Find the average time interval, in days, between twin births at this hospital. [2]

<table>
<tr><td>(i)</td><td>Babies are born randomly and independently in the UK
The mean rate of birth is a constant.</td></tr>
<tr><td>(ii)</td><td>Let X be the number of babies born in an hour. X~$\mathrm{P_0}(90)$
$\mathrm{P}(X>100)=1-\mathrm{P}(X\le 100)$ = 0.13490 ≈ 0.135 (to 3 s.f.)</td></tr>
<tr><td>(iii)</td><td>Let W be the number of babies born in a minute. W~$\mathrm{P_0}(\frac{90}{60})$, i.e. W~$\mathrm{P_0}(\frac{3}{2})$
$\mathrm{P}(W=0)=0.223$ (to 3 s.f.)</td></tr>
<tr><td>(iv)</td><td>$Y\sim \mathrm{P_0}(90\times 24)$ i.e. $Y\sim \mathrm{P_0}(2160)$
$\mathrm{P}(Y<a)\approx 0.025$
$\mathrm{P}(Y\le a-1)\approx 0.025$
Using GC,
<table>
<tr><td>a</td><td>$\mathrm{P}(Y<a)$</td></tr>
<tr><td>2060</td><td>0.0148</td></tr>
<tr><td>2070</td><td>$0.0251\approx 0.025$</td></tr>
<tr><td>2080</td><td>0.041</td></tr>
</table>
Hence a = 2070 (multiple of 10)

$\mathrm{P}(Y>b)\approx 0.025$
$\mathrm{P}(Y\le b)\approx 0.975$
Using GC,
<table>
<tr><td>b</td><td>$\mathrm{P}(Y\le b)$</td></tr>
<tr><td>2240</td><td>0.9348</td></tr>
<tr><td>2250</td><td>$0.9737\approx 0.975$</td></tr>
<tr><td>2260</td><td>0.9842</td></tr>
</table>
Hence b = 2250 (multiple of 10)</td></tr>
<tr><td>(v) (a)</td><td>Let λ be the average number of twin births per year.
Let T be the number of twin births in a week at this hospital. $T\sim \mathrm{P_0}\left(\frac{\lambda}{52}\right)$</td></tr>
</table>

	$P(T=0)=0.41$ $e^{-\frac{\lambda}{52}}=0.41$ $\lambda=-52\ln 0.41=46.3631\approx 46.4$
(b)	Let A be the number of days between twin births at this hospital. $A\sim \text{Exp}\left(\frac{\lambda}{52}\div 7\right)$ i.e. $A\sim \text{Exp}\left(\frac{\lambda}{364}\right)$ E(A) = $\frac{364}{\lambda}=\frac{364}{-52\ln 0.41}\approx 7.85$ (to 3.s.f.) Hence, the average time interval, in days, between twin births at this hospital is 7.85.

Question 10

At a weather station, the cloud cover is measured at noon each day. The cloud cover is recorded as the proportion of the sky that is obscured by cloud. This proportion is modelled by a random variable X with probability density function f(x) defined as follows.

$$\mathrm{f}(x)=\begin{cases} k\left(x^2-0.8x+0.3\right) & \text{for } 0\le x\le 1, \\ 0 & \text{otherwise.} \end{cases}$$

(i) **(a)** Find the value of the constant k and sketch f(x). [3]

(b) Explain what the shape of the density function indicates about the cloud cover at noon at the weather station. [2]

(c) Find the most likely and least likely values of cloud cover at noon at the weather station. [2]

The average thickness of the cloud is related to the cloud cover, x. Thin clouds are associated with low cloud cover, and thick clouds are associated with high cloud cover.

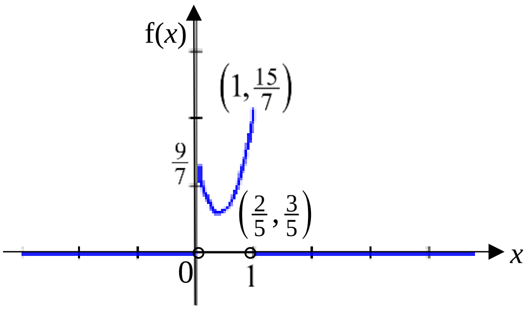
(ii) A model is proposed in which the average cloud thickness, Y m, is given by

$$Y=500\left(\mathrm{e}^X-1\right).$$

Use this model to find

(a) E(Y), [2]

(b) the probability that the average cloud thickness at the weather station at noon exceeds 250 metres. [4]

(i)(a)	$k\int_0^1 x^2-0.8x+0.3\,\mathrm{d}x=1$ $k\left(\frac{7}{30}\right)=1$ (using GC) $k=\frac{30}{7}$ 
(b)	From the shape of the density function, we observed that extremes of cloud cover (cloud cover with proportion of 0 and 1) are more common than intermediate values, with a high proportion of cloud cover being more common than a low proportion of cloud cover. Also cloud cover with a proportion of $\frac{2}{5}$ is the least observed.
(c)	Most likely cloud cover at noon is 1. Least likey cloud cover at noon is $\frac{2}{5}$ (or 0.4).
(ii)(a)	$\mathrm{E}(Y)=\int_0^1 500\left(\mathrm{e}^x-1\right)\mathrm{f}(x)$ $=\int_0^1 500\left(\mathrm{e}^x-1\right)\left(\frac{30}{7}\right)\left(x^2-0.8x+0.3\right)\mathrm{d}x$ ≈ 429 m (to 3.f.) (using GC)

(b)	$\mathrm{P}(Y > 250) = \mathrm{P}\left(\mathrm{e}^{X} - 1 > \frac{1}{2}\right)$ $= \mathrm{P}\left(\mathrm{e}^{X} > \frac{3}{2}\right)$ $= \mathrm{P}\left(X > \ln\frac{3}{2}\right)$ $= \frac{30}{7}\int_{\ln\frac{3}{2}}^{1} x^2 - 0.8x + 0.3\,\mathrm{d}x$ ≈ 0.665 (to s.f.) (using GC)