P1 Q1	The transformation T is a linear mapping from $\mathbb{R}^4$ to $\mathbb{R}^3$ defined by the matrix **A**. **(a)** Explain why the null space of **A** is not the zero vector space. [2] It is given that the matrix **A** is defined as follows. $$\mathbf{A}=\begin{pmatrix}2&3&4&-8\\1&1&1&-3\\3&6&9&-15\end{pmatrix}$$ **(b)** The *kernel* of T, $\ker(\mathrm{T})$, is defined as the set of vectors $\begin{pmatrix}x\\y\\z\\w\end{pmatrix}$ in $\mathbb{R}^4$ that maps to the zero vector in $\mathbb{R}^3$. Showing all necessary working, use row operations on **A** to find $\ker(\mathrm{T})$. [4]

P1 Q2	Let $I=\int_1^3 \mathrm{e}^{2x}\sin x\,\mathrm{d}x$. **(a)** Use a calculator to find the value of *I*, giving your answer to 2 decimal places. [1] **(b)** Use Simpson's Rule with 3 ordinates to approximate the value of *I*, correct to 2 decimal places. Hence find the approximate percentage error when using this method. [3] **(c)** Explain with the aid of a suitable diagram, why Simpson's Rule does not give a good approximation of the value of *I* in **(b)** and suggest how the method can be improved to give a better approximation. [3]

P1 Q3	The sequence $\{M_n\}$ is given by the recurrence relation $$M_{n+2}=\left(\frac{M_{n+1}}{2M_n}\right)^2+\frac{2M_n}{M_{n+1}} \text{ for } n\geq 1,$$ and the set of positive initial values M_1 and M_2. **(a)** For the case when $M_1=1$ and $M_2=2$, by considering the *subsequences* $\{M_{2k}\}$ and $\{M_{2k-1}\}$ where $k\in\mathbb{Z}^+$, of the sequence $\{M_n\}$, determine the behaviour of the sequence $\{M_n\}$ as $n\to\infty$. (A *subsequence* of $\{M_n\}$ is a sequence obtained by removing some terms without changing the order of the remaining terms.) [3] **(b)** For the case when $M_1=2$ and $M_2=2$, explain how the behaviour of the sequence will differ from part **(a)**. [2] **(c)** For the case when $M_1=M_2=\alpha$, the sequence $\{M_n\}$ becomes a constant sequence. Find the exact value of α. [2]

P1 Q4	The function f is given by $\mathrm{f}(x)=8\sin\left(\dfrac{x}{2}\right)-2x+5$. It is known that the equation $\mathrm{f}(x)=0$ has a single root $x=\alpha$.
	(a) Show that $4.5<\alpha<5$. [2]
	(b) Show that the iterative process defined by $x_0=4.5$, $x_{n+1}=4\sin\left(\dfrac{x_n}{2}\right)+\dfrac{5}{2}$ is not suitable to find a good approximation to α. [2]
	(c) Using the Newton-Raphson method, write down an iterative formula of the form $x_{n+1}=\mathrm{F}\left(x_n\right)$ that can be used to approximate the value of α. Hence calculate the value of α, correct to 3 decimal places. [3]
	(d) By considering the stationary points of the curve $y=\mathrm{f}(x)$, explain whether $x_0=2$ is a suitable starting value for using the Newton-Raphson method to find an approximation to α. [2]

P1 Q5	A curve C is given by the equation $x^4+y^4=2x^2y(1-y)$.
	(a) Show that C has the polar equation $r=\sin 2\theta\cos\theta$ for $0\le\theta\le\pi$. [3]
	(b) Sketch C, stating the equation of the line of symmetry (if any). [2]
	(c) Find the length of curve C. [4]

P1 Q6	A *Bernoulli Equation* is a differential equation of the form $$\frac{\mathrm{d}y}{\mathrm{d}x}+\mathrm{p}(x)y=\mathrm{q}(x)y^n,$$ where $\mathrm{p}(x)$ and $\mathrm{q}(x)$ are functions of x and n is a real number.
	(a) Show that the substitution $u=y^{1-n}$ reduces the equation into the form $$\frac{\mathrm{d}u}{\mathrm{d}x}+(1-n)\mathrm{p}(x)u=(1-n)\mathrm{q}(x).$$ [2]
	(b) Use the result in part **(a)** to obtain the solution of the differential equation $$\frac{\mathrm{d}y}{\mathrm{d}x}-xy+\frac{1}{2}xy^2=0,$$ given that $y=0.1$ when $x=0$. Obtain, correct to 4 decimal places, the values of y when $x=0.1$ and $x=0.2$. [4]
	Now consider the differential equation $$\frac{\mathrm{d}y}{\mathrm{d}x}-x\ln\left(1+y\right)=0,$$ where $y=0.1$ when $x=0$.
	(c) Use the improved Euler method with step length 0.1 to estimate the values of y when $x=0.1$ and $x=0.2$, correct to 4 decimal places. [3]
	(d) Comment on your numerical answers from parts **(b)** and **(c)**. [1]

x

2024 H2 FM 9649 Prelim Paper 1 (ACJC_EJC_NJC_RVHS)

P1 Q7	A city is facing a virus outbreak. The number of infected individuals in the city, x thousands, at time t days can be modelled by the equation $$\frac{\mathrm{d}x}{\mathrm{d}t}=\frac{1}{20}x^2\left(1-\frac{x^2}{100}\right),$$ where $0<x<10$ and $t\geq 0$. **(a)** Given that there were 2000 infected individuals initially, find t in terms of x. [5] **(b)** Sketch the solution curve for x against t, labelling any key features including the coordinates of the point of inflexion. What is the significance of this point of inflexion in the context? [4] An attempt to improve the model was made with the updated equation $$\frac{\mathrm{d}x}{\mathrm{d}t}=\frac{1}{20}x^2\left(1-\frac{x^2}{100}\right)-\mathrm{h}(x),$$ where $\mathrm{h}(x)>0$ for $0<x<10$. **(c)** Give, in context, two possible interpretations for what $\mathrm{h}(x)$ can represent. [2]

P1 Q8	A curve C_1 is given parametrically by $x=2\cos t$, $y=2\sin 2t$, where $-\frac{1}{2}\pi\leq t\leq\frac{1}{2}\pi$. When C_1 is rotated 2π radians about the y-axis, a surface area of revolution is formed with area S. **(a)** Determine the value of S. [5] Another curve C_2 has equation $\left(x-\frac{3}{2}\right)^2+y^2=1$. The region bounded by C_1 and C_2 containing the point $(1,0)$ is denoted by R. **(b)** Find the coordinates of the points of intersection between C_1 and C_2. [3] **(c)** Hence find, to 3 significant figures, the volume of solid of revolution formed when R is rotated 2π radians about the y-axis. [5]

P1 Q9

Cockroaches are common household pests that reproduce at an alarming rate and will plague any residential area if left uncontrolled. The natural growth, ΔP of the pest population can be modelled as a proportion of the current population and written as the recurrence relation

$$\Delta P = P_{n+1} - P_n = kP_n \text{ for } n \geq 0,$$

where k is a positive constant, and P_n denotes the population (in thousands) in the nth month after the initial check on the pest situation. It is given that P_0 denotes the initial population.

(a) Explain why the sequence $\{P_n\}$ is geometric. Hence identify a limitation to this model in the long run. [2]

It is known that a more realistic model for the natural growth of the pest population is the *logistic growth model*, represented by the recurrence relation

$$\Delta P = P_{n+1} - P_n = a\left(1 - \frac{P_n}{b}\right)P_n \text{ for } n \geq 0,$$

where a and b are positive constants.

(b) Determine the behaviour of the sequence $\{P_n\}$ for each of the cases when

(i) $P_n > b$, **(ii)** and $0 < P_n < b$. [3]

(c) State the two possible values for the limit of the sequence $\{P_n\}$. Hence explain the significance of the constant b in this context. [2]

The management committees of large residential estates usually deploy intervention measures to control the population of the pests. For a certain residential estate, the management committee proposes that after the intervention measures they have taken, the population of pests in their estate can be modelled by the recurrence relation

$$Q_{n+2} = cQ_{n+1} + dQ_n \text{ for } n \geq 1,$$

where c and d are constants, and Q_n denotes the population (in thousands) in the nth month after the start of the intervention process. It is given that Q_0 denotes the initial population.

(d) For the case where $c = \sqrt{2}$ and $d = -1$, solve the recurrence relation to obtain a general solution, leaving your answer in a form that involves only real numbers.

Comment on whether these values will provide a feasible model. [3]

It is now given that $c = 1.9$ and $d = -0.9$.

(e) Solve the recurrence relation to obtain a general solution. [2]

(f) Given that $Q_0 = 50$, find the range of values of Q_1 such that Q_n is positive for all possible values of n. [2]

P1 Q10

In Farmtown, an economist sought to model the production from the grain and cattle sectors of the economy using an input-output model. In this model, the amount of grain and cattle available for external consumption is modelled as the difference between the amount of grain and cattle produced, and the amount consumed by the respective sectors during production using the equation

$$\begin{pmatrix} e_g \\ e_c \end{pmatrix} = \begin{pmatrix} g \\ c \end{pmatrix} - \mathbf{P}\begin{pmatrix} g \\ c \end{pmatrix},$$

where

- g is the number of units of grain produced,
- c is the number of units of cattle produced,
- $\mathbf{P}$ is the 2×2 matrix known as the internal consumption matrix,
- e_g is the number of units of grain available for external consumption, and
- e_c is the number of units of cattle available for external consumption.

It is given that $\mathbf{P} = \begin{pmatrix} \alpha & \beta \\ 0.01 & 0.06 \end{pmatrix}$, where α and β are positive real constants.

(a) Give an interpretation of α and β. [2]

The economist uses the model to predict the amount of production required to satisfy a demand for external consumption of 300 units of grain and 100 units of cattle.

(b) By finding the matrix $(\mathbf{I}-\mathbf{P})^{-1}$, find the corresponding values of g and c in the cases where

(i) $\alpha = 1.02$, $\beta = 0.04$, and where

(ii) $\alpha = 0.04$, $\beta = 1.02$. [5]

(c) Hence comment on whether the economy will be able to cope with the demand for external consumption for each of the cases in **(b)**. [2]

(d) With reference to your interpretation in **(a)**, explain why the outcomes for both cases in **(b)** are so different. [2]

To model an isolated agricultural society, the economist also considered the case where the economy has no resources available for external consumption, and where $\alpha = \beta$.

(e) Find the value of α such that there is a solution where g and c are not both zero. [3]