Dunman High School (Senior High Physics Department)

H2 Topic 16

Electromagnetic Induction

Pickup of an electric guitar

A moving coil speaker cone

Learning Objectives

Content

- Magnetic Flux
- Laws of Electromagnetic Induction

Learning Outcomes

Candidates should be able to

a. define magnetic flux and the weber.

b. recall and solve problems using $\Phi = BA$.

c. define magnetic flux linkage.

d. infer from appropriate experiments on electromagnetic induction:
   i. that a changing magnetic flux can induce an e.m.f. in a circuit,
   ii. that the direction of the induced e.m.f. opposes the change producing it,
   iii. the factors affecting the magnitude of the induced e.m.f.

e. recall and solve problems using Faraday's law of electromagnetic induction and Lenz's law.

f. explain simple applications of electromagnetic induction.

16.0 Introduction

In Topic 15, you learnt two ways in which electricity and magnetism are related:
i. an electric current produces a magnetic field
ii. a magnetic field exerts a force on an electric current or moving electric charge.

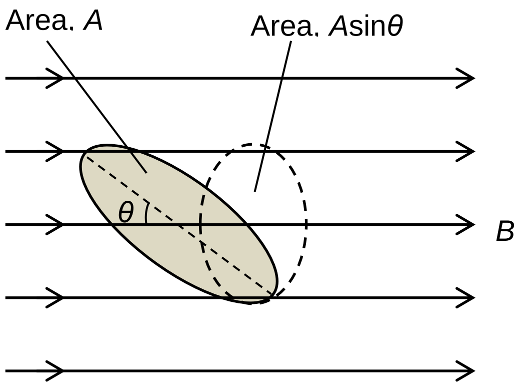
16.1 Magnetic Flux

Field lines tell us the strength and direction of the magnetic field. However, there are times where we will need to be able to calculate the amount of magnetic field that passes through a surface area. Hence, *magnetic flux* is the term used, and it is a measure of the number of field lines that pass through a given area *A*.

Magnetic Flux passing through any surface area is defined as
The product of the magnetic flux density and the area normal to the field through which the field is passing.

For a uniform magnetic flux density *B* at an angle *θ* to an area *A*, the magnetic flux *Φ* is given by

$$\boxed{\boldsymbol{\Phi = BA\sin\theta}}$$



Magnetic flux through an area

Alternatively, we can view magnetic flux as the product of the component of *B* perpendicular to the surface and the area of the surface.

$$\begin{aligned}\boldsymbol{\Phi} &= \boldsymbol{B_{\perp}A}\\ &= (\boldsymbol{B\sin\theta})\boldsymbol{A}\\ &= \boldsymbol{BA\sin\theta}\end{aligned}$$

Weber
The SI unit of magnetic flux is the *weber* (symbol: Wb) which is defined as the magnetic flux if a uniform magnetic flux density of 1 tesla passes perpendicularly through an area of 1 m^2.
(1 Wb = 1 T m^2)

16.1.1 Magnetic Flux Linkage

If the area *A* is bounded by a coil and the coil has *N* turns, then the total magnetic flux passing through the coil, otherwise known as *magnetic flux linkage* through the coil, is

$$\boxed{\boldsymbol{N\Phi = NBA\sin\theta}}$$

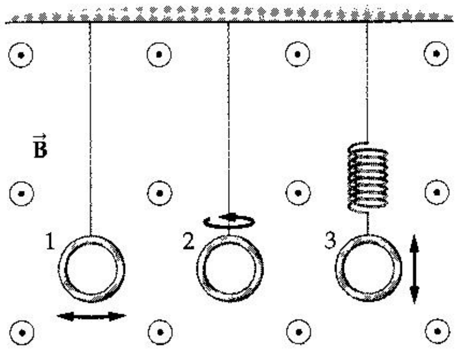
From the equation, magnetic flux linkage through a coil depends on
i. number of turns in the coil *N*
ii. magnitude of magnetic flux density *B*
iii. the surface area *A*
iv. the orientation of the coil *θ* with respect to the direction of *B*. (Note: *θ* is the angle between the magnetic flux density and the area of the coil)

A change in any of these variables results in a change of magnetic flux linkage.

Magnetic Flux Linkage
Magnetic Flux Linkage is the product of the magnetic flux passing through the coil and the number of turns on the coil.

Example 1

The three loops of wire shown are all in a region of uniform magnetic field. Loop 1 swings back and forth as the bob on a pendulum, loop 2 rotates about a vertical axis and loop 3 oscillates vertically on the end of a spring. Which loop(s) have a magnetic flux that changes with time?



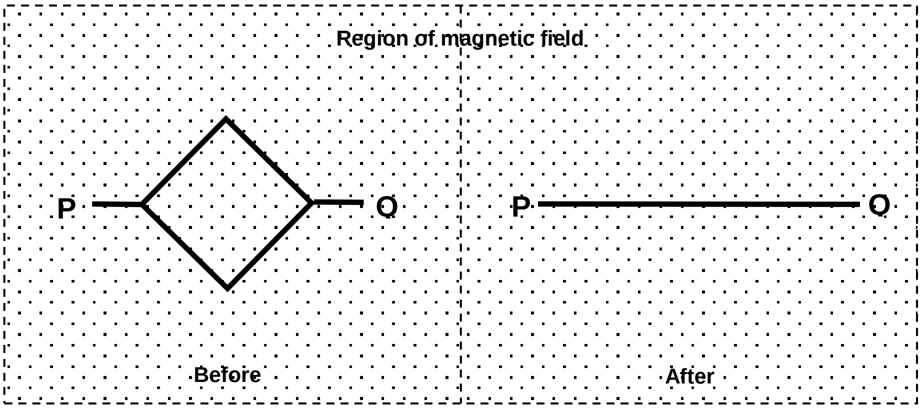
Solution

Only loop 2 has a changing magnetic flux as its orientation relative to the field changes as it rotates.

Since the magnetic field is uniform, moving loop 1 back and forth or moving loop 3 up and down does not change the magnetic flux through the loop, that is, the magnetic flux does not depend on the loop's position.

Example 2

A coil of 10 turns and area 0.35 m^2 is placed perpendicular to a magnetic field of density 2.0 T as shown in the diagram on the left. Points P and Q are pulled apart until the coil becomes a straight line as shown in the diagram on the right. What is the change in magnetic flux linkage through the coil?



Solution

Initial magnetic flux linkage $(N\Phi)_i$ = $NBA\ sin\theta$ = 10(2.0)(0.35) sin 90°= 7.0 Wb

Final magnetic flux linkage $(N\Phi)_f$ = 0

Change in magnetic flux linkage =$(N\Phi)_f - (N\Phi)_i$ = – 7.0 Wb

16.2 Electromagnetic Induction

Ten years after the discovery that an electric current could produce a magnetic field, experiments conducted independently by Joseph Henry in United States and Michael Faraday in England showed that a changing magnetic field could induce an electric current in a circuit. The results of these experiments led to a basic and important law known as Faraday's law.

16.2.1 Faraday's Law

Faraday's Law of electromagnetic induction states that the induced e.m.f. is **proportional** to the **rate of change** of the magnetic flux linkage.

If the magnetic flux linkage is measured in weber-turns and the e.m.f. in volts, then the constant of proportionality is unity.

Mathematically:

$$\boxed{\boldsymbol{\varepsilon = -\frac{d(N\Phi)}{dt}}}$$

The negative sign is involved with the direction of the e.m.f. and is explained as part of the Len'z law.

16.2.2 Lenz's Law

Lenz's Law states that the induced current or e.m.f. is in a direction so as to produce effects which oppose the change that is producing it.

With the minus sign indicating that the induced e.m.f. opposes the change in magnetic flux linkage.

Replacing magnetic flux $\boldsymbol{\Phi = BAsin\theta}$

$$\boxed{\boldsymbol{\varepsilon = -\frac{d(NBA\sin\theta)}{dt}}}$$

Hence the e.m.f. induced depends on quantities such as *N* number of coils, magnetic flux density *B*, area of coil *A* and angle θ between *B* and *A*.

Example 3

A square coil of side 5.0 cm lies perpendicular to a magnetic field of flux density 4.0 T. The coil consists of 200 turns of wire.

i. What is the magnetic flux linkage through the coil?

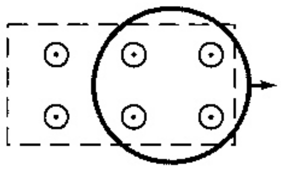
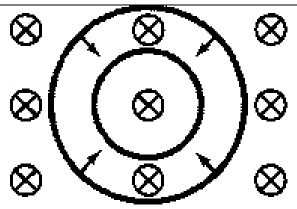
$$\begin{aligned} N\Phi &= BA\sin\theta \\ &= (200)(4.0)(0.050)^2(\sin 90^\circ) \\ &= 2.0 \text{ Wb} \end{aligned}$$

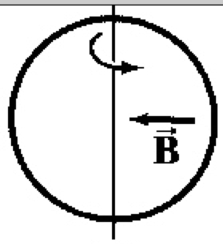
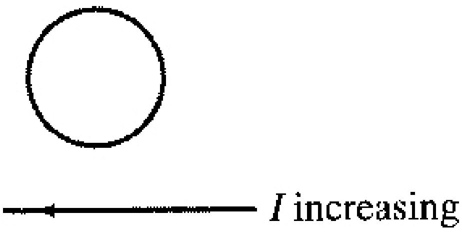
ii. The coil is rotated through an angle of 90° in 0.20 seconds. Calculate the magnitude of the average e.m.f. induced in the coil while it is being rotated.

$$\begin{aligned} \varepsilon &= -\frac{d(N\Phi)}{dt} \\ &= -\frac{d(NBA\sin\theta)}{dt} \\ &\approx -NBA\frac{\Delta \sin\theta}{\Delta t} \\ &= -(200)(4.0)(0.050^2)\left(\frac{\sin 0^o - \sin 90^o}{0.20}\right) \\ &= 10\ V \end{aligned}$$

Example 4

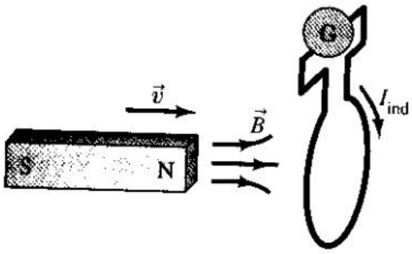
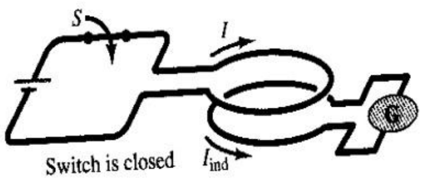
Predict the direction of the induced current in the circular coil for each of the following cases:

Action	Direction of induced current
 Pulling the coil to the right out of a magnetic field pointing out of page	**If the coil is pulled out of the field, magnetic flux linkage through the coil decreases. The induced current will be in a direction to oppose this decrease and hence will flow <u>counter-clockwise</u> to produce a magnetic field pointing out of the page.**
 Shrinking a coil in a magnetic field pointing into page	**Since the coil area gets smaller, the magnetic flux linkage through the coil decreases. Hence the induced current will be <u>clockwise</u> to produce its own magnetic field into the page to make up for this decrease in magnetic flux linkage.**

Action	Direction of induced current
 Rotating the coil by pulling the left side towards us and pushing the right side in with the magnetic field pointing right to left.	**Initially, the flux through the coil is zero. When the loop is rotated, the flux linkage (pointing to left) through the coil begins to increase. To oppose this increase in flux linkage, the current induced in the coil will flow <u>counter-clockwise</u> (to produce its own magnetic field to the right).**
 Increasing the current in a wire that is placed below a coil in the plane of the page	**The magnetic field produced within the coil by the wire points into the page (plane of the loop). As the current increases, the magnetic flux linkage through the coil increases. To oppose this increase in flux, the induced current in the coil will flow <u>counter-clockwise</u> (to produce a magnetic field out of the page).**

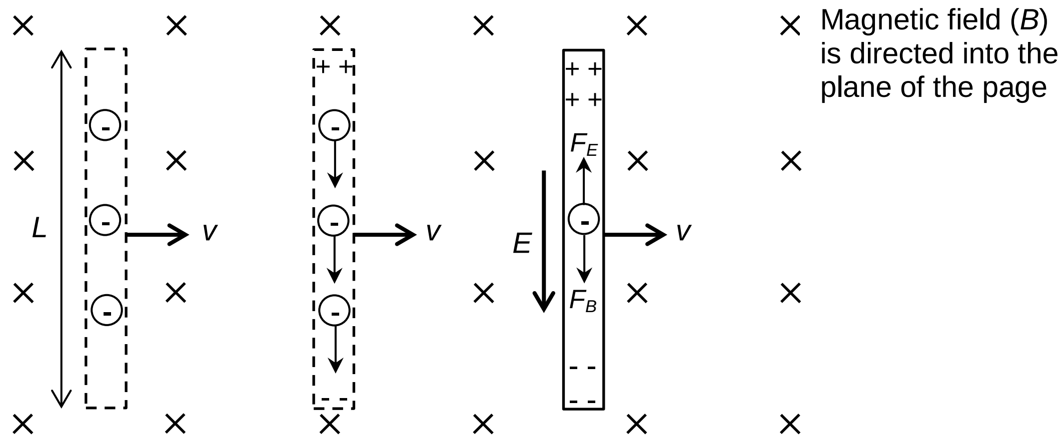
Lenz's Law is a consequence of the *principle of conservation of energy*. Consider moving a magnet towards the face of a solenoid. According to Lenz's Law, if the magnet's pole facing the solenoid was North, the induced current in the solenoid would flow such that the end facing the magnet would be a North pole as well. Suppose the induced currents' directions were opposite to those prescribed by Lenz's Law, the north pole of an approaching magnet would induce a south pole in the near face of the solenoid. The attractive force between these poles would accelerate the magnet's approach and make the magnetic field increase more quickly. This in turn would increase the current induced in the solenoid, strengthening the magnetic field, increasing the attraction and acceleration, and so on. Both the kinetic energy of the magnet and the rate of energy dissipation in the solenoid (due to heat dissipation) would increase. A small energy input would produce a large energy output, violating the law of conservation of energy

16.2.3 Experimental Observations Explained using the Laws of Electromagnetic Induction

Experimental Observation	Explanation
 If the bar magnet is moved towards the wire loop, an induced current is observed in the wire as shown as deflection in the galvanometer. A faster movement results in a larger current than a slower movement. If the magnet's motion is reversed (i.e. moved away from the loop), then the current reverses.	When the magnet moves towards or away from the loop, the magnetic flux linkage through the loop changes. According to Faraday's law, an e.m.f. is induced in the loop. A faster movement gives rise to a more rapidly changing magnetic flux linkage through the loop leading to a larger induced e.m.f. and hence a larger current. When the magnet move towards the loop, the magnetic flux through the wire increases, and by Lenz's law, the current flows in a direction to decrease the flux linkage through the loop. However, when the motion is reversed, the magnetic flux linkage through the loop decreases and current flows in the opposite direction so as to increase the flux linkage through the loop.
 When switch is closed in the upper loop, an induced current momentarily flows in the lower loop shown by deflection in the galvanometer. When the switch is subsequently opened, the deflection of the galvanometer is in the opposite direction.	When the switch is closed, current flow in the upper loop produces a magnetic field. As the current in the upper loop builds up from zero to its steady value, it produces a change in the magnetic field, resulting in a changing magnetic flux linkage through the lower loop. According to Faraday's law, the changing magnetic flux linkage through the lower loop induces an e.m.f. in the lower loop and hence an induced current flows within it. From Lenz's law, since the magnetic flux through the lower loop increases, the current in it flows in a direction such that it produces magnetic fields that oppose this increase in magnetic flux linkage. When the switch is opened and the flux linkage through the lower loop decreases, current flows in the opposite direction so as to increase the flux linkage through the lower loop.

16.2.4 Motional EMF (Cutting of magnetic field)

In earlier sections, we considered cases in which an e.m.f. is induced when there is a change in the magnetic flux linkage. In this section, we will study what is called motional e.m.f., which is the e.m.f. induced in a conductor moving through a constant magnetic field.



When a conductor (wire) moves in a uniform magnetic field, the conductor cuts across the magnetic field and an induced e.m.f. is generated. In the above figure, the conductor is being pushed to move to the right cutting across a uniform magnetic field (B) which is directed into the page. Now, think of the free electrons in the conductor. They are moving to the right, so they are in effect an electric current. Because electrons are negatively charged, the conventional current is flowing to the left. Based on *Fleming's left-hand rule*, each electron experiences a downward magnetic force, $F_B = Bqv$. Under the influence of this force, the electrons move to the lower end of the conductor and accumulate there, leaving a net positive charge at the upper end. As a result of this charge separation, an electric field E (directed downward) is produced inside the conductor. The charges accumulate at both ends until the downward magnetic force on charges remaining in the conductor is balanced by the upward electric force, $F_E = qE$.

Electric force on remaining charge = Magnetic force on the charge

$$F_E = F_B$$
$$qE = qvB$$
$$E = vB$$

Since the electric field in the conductor is uniform, the potential difference across the ends of the conductor of length L is given by

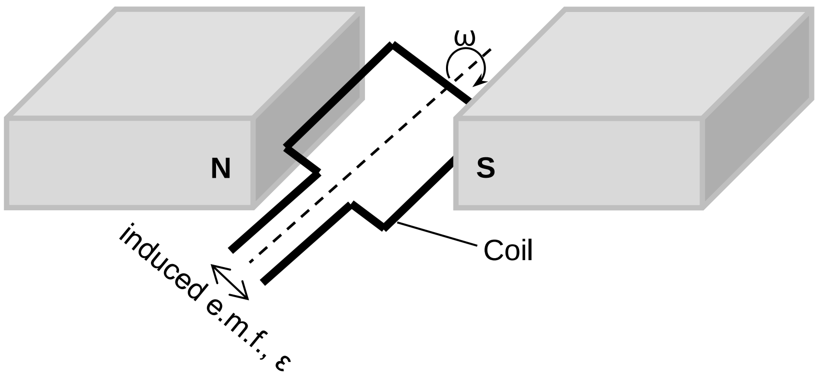
$$\varepsilon = EL$$
$$= BLv$$

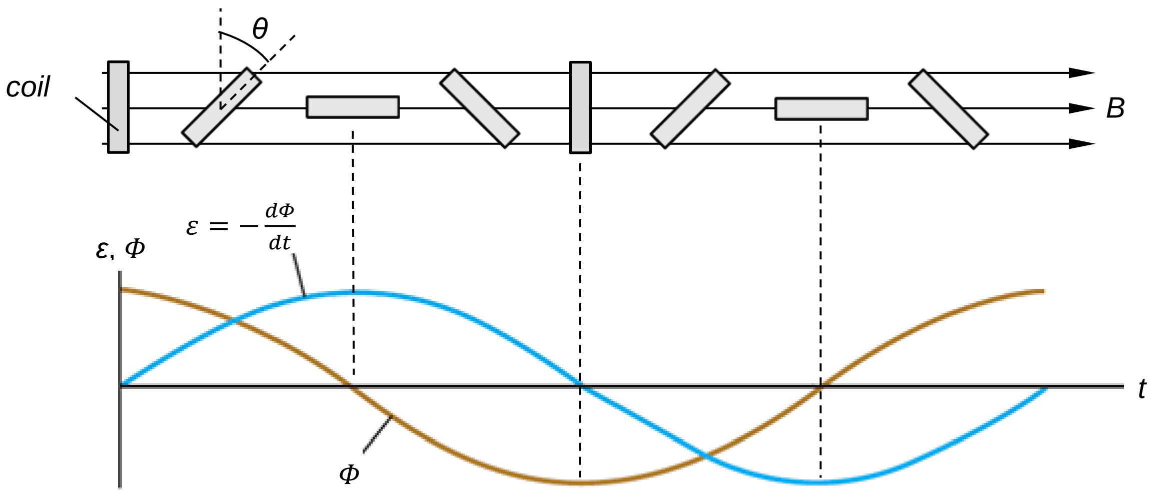
Therefore an e.m.f. (ε) is induced as long as the conductor continues to move through the uniform magnetic field. If the direction of the motion is reversed, the polarity of the induced e.m.f. is also reversed.

16.3 Applications of Electromagnetic Induction

16.3.1 Generator

Electric generators take in energy by work and transfer it out by electrical transmission.





When a coil of *N* turns rotates with a constant angular velocity ω in a uniform magnetic field *B*, an e.m.f. ε is induced. The flux and the resulting induced e.m.f. between the terminals of the coil during one complete rotation are shown in the above figure. According to Faraday's law, the induced e.m.f. is equal to negative the gradient of the flux linkage against time graph.

- When the flux linking the coil is maximum, the rate of change of flux is zero, and hence the induced e.m.f. is zero.
- When the flux linking the coil is zero, the rate of change of flux is maximum (the graph is steepest) and hence the induced e.m.f. is also maximum.

Assume that at time t = 0, the coil is perpendicular to the *B* field and maximum flux linkage through the coil. At any other point in time, the flux linkage through the coil is

$N\Phi = NBAcos\theta$ where *θ* is the angle between the coil and vertical axis.

For uniform circular motion, $\theta = wt$

$$N\Phi = NBAcos(\omega t)$$

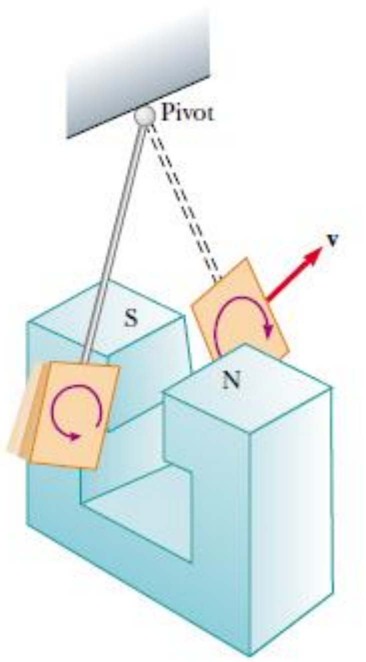
From laws of induction, induced e.m.f. is given by

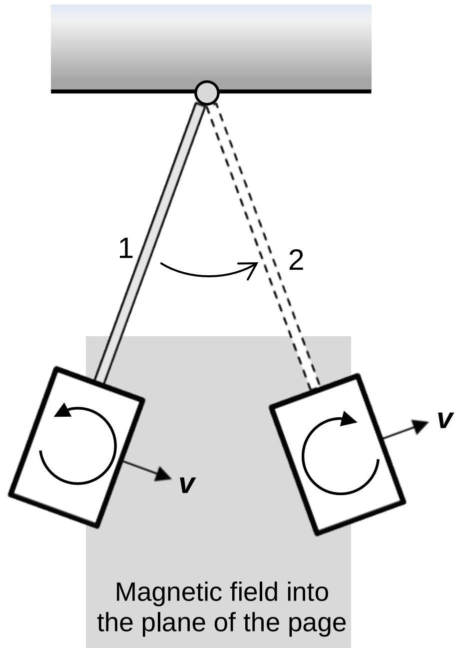
$$\varepsilon = -\frac{d[NBAcos(\omega t)]}{dt} = NBA\omega \sin(wt)$$

Factors affecting the magnitude of the induced e.m.f. are (i) magnetic flux density *B*, (ii) area of the coil *A*, number of turns in the coil *N* and angular speed *ω* or frequency of rotation *f*.

16.3.2 Eddy Currents

When a conductor is subjected to a changing magnetic flux, the induced e.m.f. causes currents to flow. If the conductor is in the shape of a loop, the induced current will flow around the loop. However, if the conductor is a solid plate, induced currents, known as Eddy currents, flow simultaneously along many different paths in swirls. This can easily be demonstrated by allowing a flat copper or aluminium plate attached to the end of a rigid bar to swing back and forth through a magnetic field. As the plate enters the field, the plate cuts the magnetic field/flux. The changing magnetic flux induces an e.m.f. in the plate, which in turn causes the free electrons in the plate to move, producing the swirling eddy currents. According to Len'z law, the direction of the eddy current is such that they create magnetic fields that oppose the change that causes the currents.





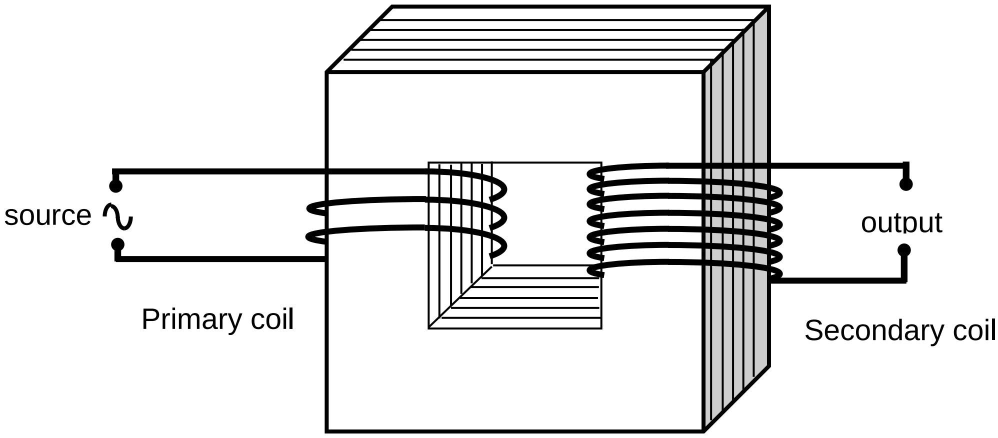
As the swinging plate enters the field at position 1, the flux due to the external magnetic field into the page through the plate is increasing, hence by Len'z law the induced eddy current must provide its own magnetic field out of the page. Direction of the induced current is thus anticlockwise. The opposite is true as the plate leaves the field at position 2, where the current is clockwise.

Eddy currents are dissipated as heating in the conductor (i.e. the conductor gets heated up). Eddy currents can be reduced by eliminating paths for the current flow, for example, by cutting slits in the plate. These slits will prevent large eddy currents from occurring.

Sometimes eddy currents can be a good thing, for example, it can be used to set up a braking system which can rapidly change kinetic energy to other forms of energy. This can be taken a step further if a circuit can be built to channel the electrical energy from the kinetic energy back into the battery. This is what most hybrid cars do.

16.3.3 Transformer

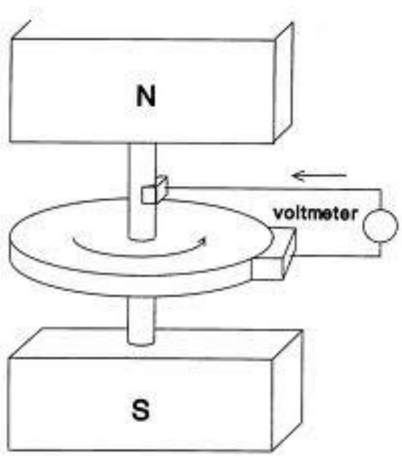
Our entire system for distributing electricity is based on transformers, devices that use electromagnetic induction to change a.c. voltages. Transformers raise voltages for transmission over long distances across power lines. Transformers then reduce voltages for safe use in businesses and homes. The figure on the right shows a simple transformer which consists of two coils wound on a common soft iron core so that nearly all the magnetic field lines produced by the primary coil pass through each turn of the secondary coil. We will learn more about transformer in the next topic, Alternating Currents.



Annex

A Rotating Disc Perpendicular to a Uniform Magnetic Field

When a disc of area A perpendicular to a uniform magnetic field B is rotated with constant frequency f, an e.m.f. is induced between the center and the rim of the disc.



Based on Fleming's LHR, the electrons in the disc experience a magnetic force in a radial direction. Due to this force, the electrons move towards the rim of the disc, leaving a net positive charge at the center of the disc. An induced e.m.f. is thus generated and if the circuit is completed, an induced current is produced.

To calculate the induced e.m.f, we can regard the disc as a many-spoked wheel. The radius of the disc is sweeping through the magnetic field.

Area swept out by a radius per second = $\pi r^2 f$
Change in flux per second = $BA = B\pi r^2 f$

By Faraday's law, $\boxed{|\varepsilon| = B\pi r^2 f}$

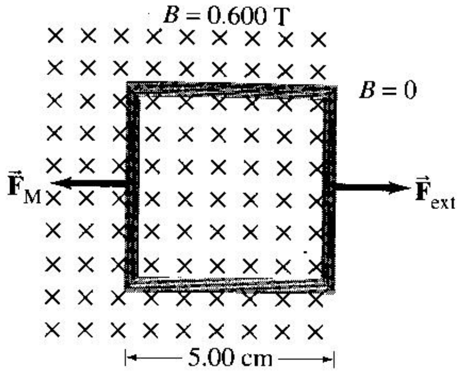
EMI Tutorial

Self-Attempt Questions

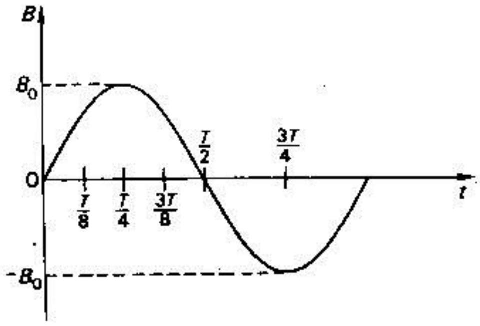
S1 A square coil of wire with side 5.00 cm contains 100 loops and is positioned perpendicular to a uniform magnetic field of 0.600 T. It is quickly pulled from the field at constant speed (moving perpendicular to the magnetic field) to a region of zero magnetic flux density. At t = 0, the right edge of the coil is at the edge of the magnetic field. It takes 0.100 s for the whole coil to reach the field-free region. The coil's total resistance is 100 Ω. Find

(a) the change in magnetic flux through the coil
(b) the e.m.f. induced
(c) the current induced
(d) the energy dissipated in the coil
(e) F_{ext} required to pull the coil

[-0.15 Wb, 1.5 V, 15 mA, 0.00225 J, 0.045N]



S2 A magnetic field is applied perpendicular to the plane of a flat coil of copper wire. The time variation of the magnetic flux density is as shown graphically below.



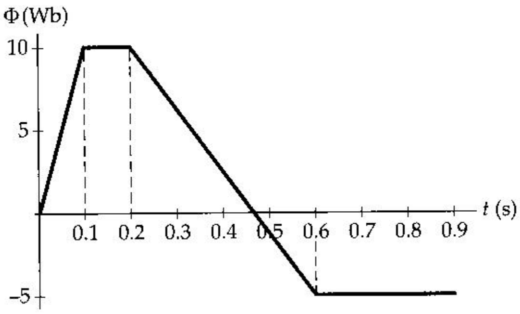
At which of the following values of t is the magnitude of the emf induced in the coil a maximum?

A $\frac{T}{8}$ B $\frac{T}{4}$ C $\frac{3T}{8}$ D $\frac{T}{2}$

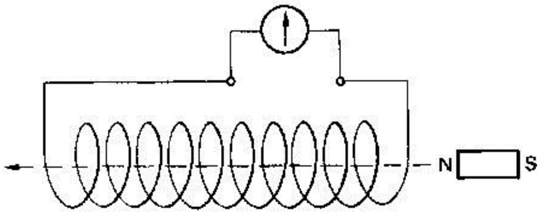
(N87/I/20)

S3 The graph shows the magnetic flux through a coil as a function of time. What is the induced e.m.f. in the coil at (a) t = 0.05 s, (b) t = 0.15 s, (c) t = 0.5 s?

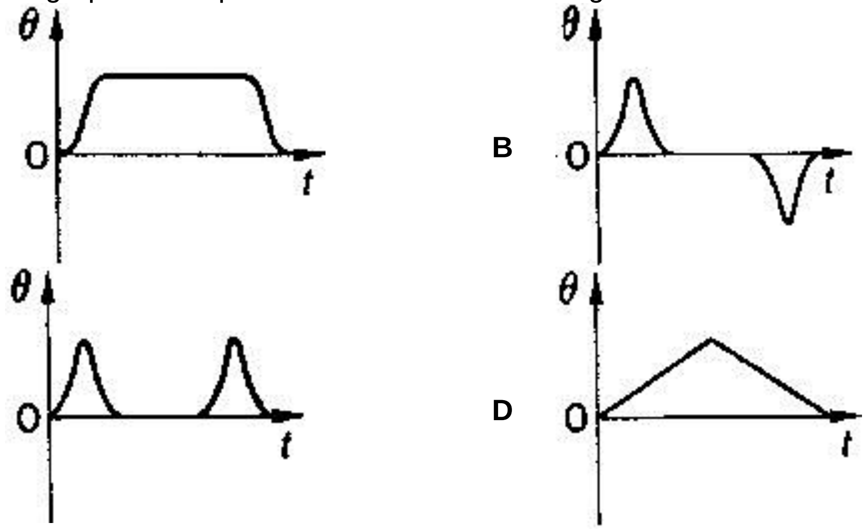
[-0.1 kV, 0, 0.04 kV]



S4 A short bar magnet passes at a steady speed right through a long solenoid. A galvanometer is connected across the solenoid.



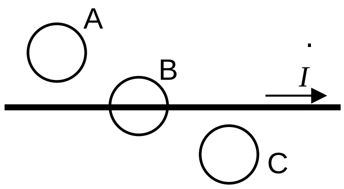
Which graph best represents the variation of the galvanometer deflection θ with time t?



(J91/I/18)

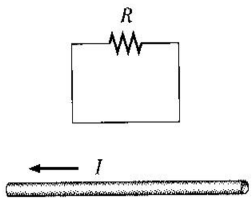
S5 The area of a coil of 100 turns oriented with its plane perpendicular to a magnetic field of density 0.20 T is 0.050 m^2. Find the e.m.f. induced in this coil if the magnetic field reverses its direction in 0.30 s. [6.7 V]

S6 If the current is the wire is increased, indicate the direction of the induced current in each loop A, B and C.



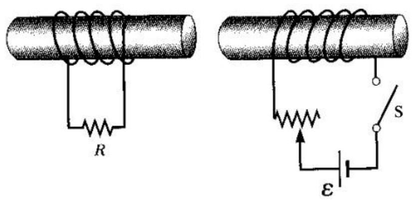
S7 Determine the direction of the induced current in R in each of the following cases:

(a)



current I decreases rapidly to zero

(b)



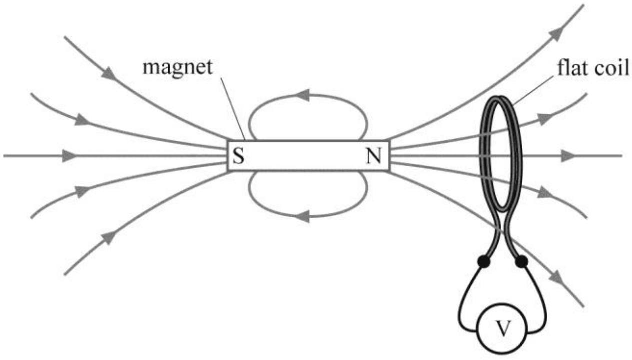
i. first, the switch is closed
ii. then the variable resistor in series with the cell is decreased
iii. then the circuit containing resistor R is moved to the left
iv. then the switch is opened.

Tutorial Discussion

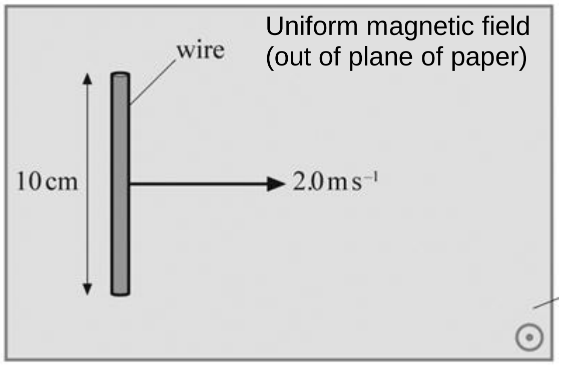
T1 The diagram shows a magnet placed close to a flat circular coil.

(a) Explain why there is no induced e.m.f. even though there is magnetic flux linking the coil. [1]

(b) Explain why there is an induced e.m.f. when the magnet is pushed towards the coil. [2]



T2 The diagram shows a straight wire of length 10 cm moved at a constant speed of 2.0 m s^{-1} in a uniform magnetic field of flux density 0.050 T.



For a period of 1 second, calculate:

(a) the distance travelled by the wire [1]

(b) the area swept by the wire [1]

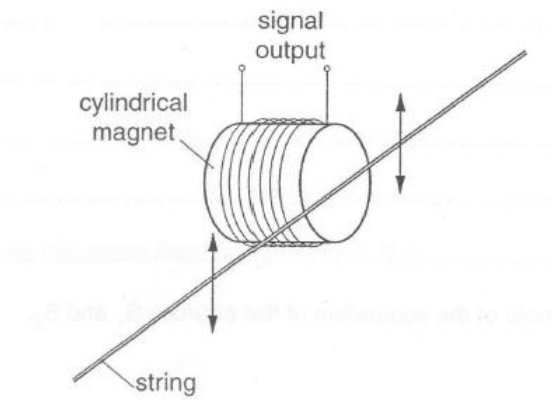
(c) the change in the magnetic flux for the wire (or the magnetic flux 'cut' by the wire) [2]

(d) the e.m.f. induced across the ends of the wire using your answer to (c) [2]

(e) the e.m.f. induced across the ends of the wire using $E = BLv$. [1]

T3 The pickup on an electric guitar produces an electrical signal from the vibrations of the guitar strings. The pickup consist of a small coil of insulated wire wound round a small cylindrical bar magnet as illustrated in the figure.

The strings of the guitar are made of steel. When a string vibrates, an electrical signal is generated between the terminals of the coil.



(a) i. State Faraday's law of electromagnetic induction

ii. Use Faraday's law to explain why an electrical signal is generated [4]

(b) Suggest why

i. the electrical signal must be amplified before connection to a loud speaker

ii. the design of the pickup would be inappropriate for use on a guitar with nylon strings

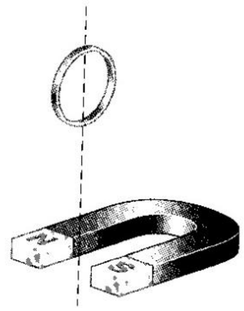
[3]

(N01/III/6)

T4 (a) A loop of wire is allowed to fall between the poles of a magnet along the dotted line as shown. Determine the direction of the induced current in the loop when it is

i above the magnet,

ii. below the magnet.

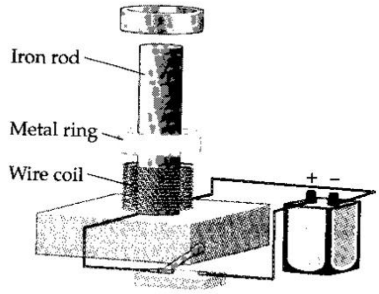


(b) Rather than let the loop fall downwards, suppose we attach a string to it and raise it upwards with constant speed along the path indicated by the dotted line. How does the tension in the string compare with the weight of the loop when the loop is

i. below the magnet,

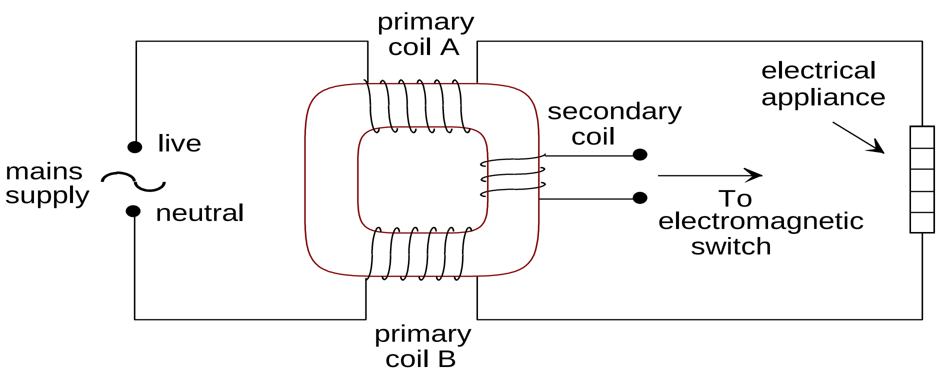
ii. above the magnet.

T5 The diagram shows a vertical iron rod with a wire coil of many turns wrapped around its base. A metal ring slides over the rod and rests on the wire coil. Initially, the switch connecting the coil to a battery is open but when it is closed, the ring flies into the air. Explain.



T6 (a) Explain the potential danger associated with switching off the current in a large electromagnet.

(b) A protective device in a mains circuit consists of a transformer with two primary coils A and B and a secondary coil as shown. The primary coils each have the same number of turns and are wound in opposite directions on the core. The mains supply is connected in series with the two primary coils and the electrical appliance. The secondary coil is connected to an electromagnetic switch.

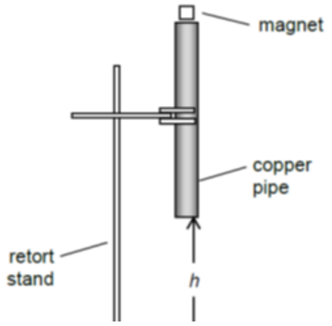


i At one particular moment, the live lead is positive with respect to the neutral lead. Explain why there is no e.m.f. induced in the secondary coil.

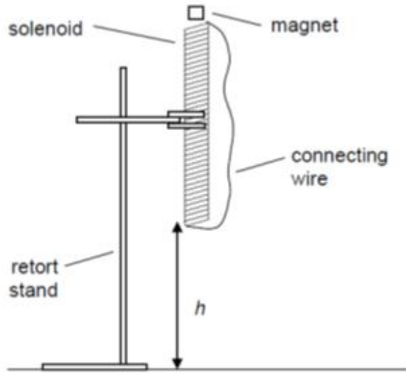
ii A fault develops in the electrical appliance such that there is a current to earth. As a result, the current in primary coil A is no longer equal to that in coil B. Explain why there is now an e.m.f. induced in the secondary coil.

T7 (a) A magnet is released from rest from the top of a copper pipe as shown below.

Explain why the time taken for the magnet to fall is considerably longer than when the magnet is released from the same height without the copper pipe.



(b) The copper pipe is then replaced by a solenoid as shown in the Figure. The ends of the solenoids are connected by a wire. Explain why the time taken for the magnet to fall is shorter as compared to the case in (a) when the copper pipe was used.



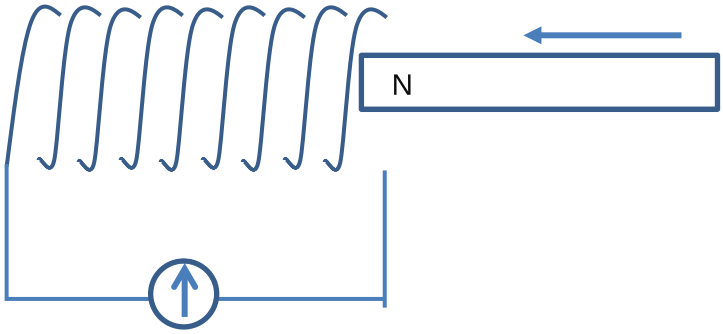
Assignment Questions

A1 (a) Distinguish between *magnetic flux density* and *magnetic flux* [2]

(b) (i) State *Faraday's law of electromagnetic induction*. [2]

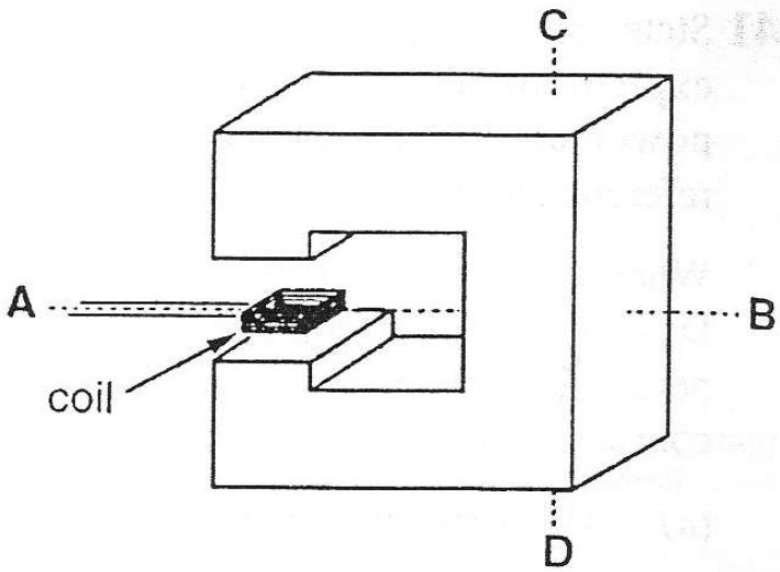
(ii) State *Lenz's law* and explain why it is an example of the law of conservation of energy [4]

(c) The north pole of a bar magnet is pushed into a solenoid to which is connected a galvanometer as shown in Figure below.



Use Lenz's law to predict the direction of the current induced in the solenoid. Explain your reasoning. [4]

(d) A small square coil has its plane set at right angles to the uniform magnetic field between the pole pieces of a horseshoe magnet as shown below.

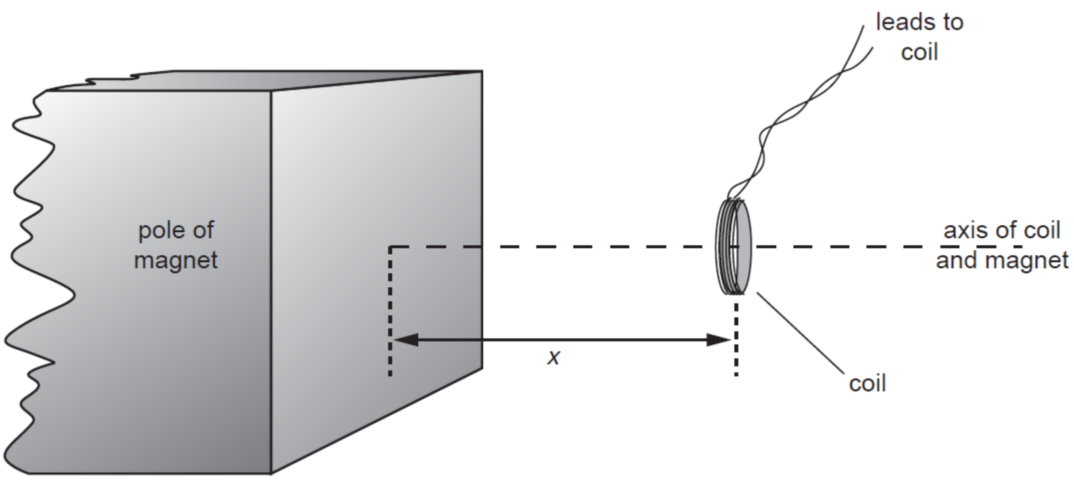


The magnet is now rotated at constant angular velocity about the axis **AB**. Draw sketch graph, on the same time axis, to show the variation of,

(i) the magnetic flux through the coil
(ii) the e.m.f. induced in the coil. [3]
Explain clearly how your graphs are obtained.

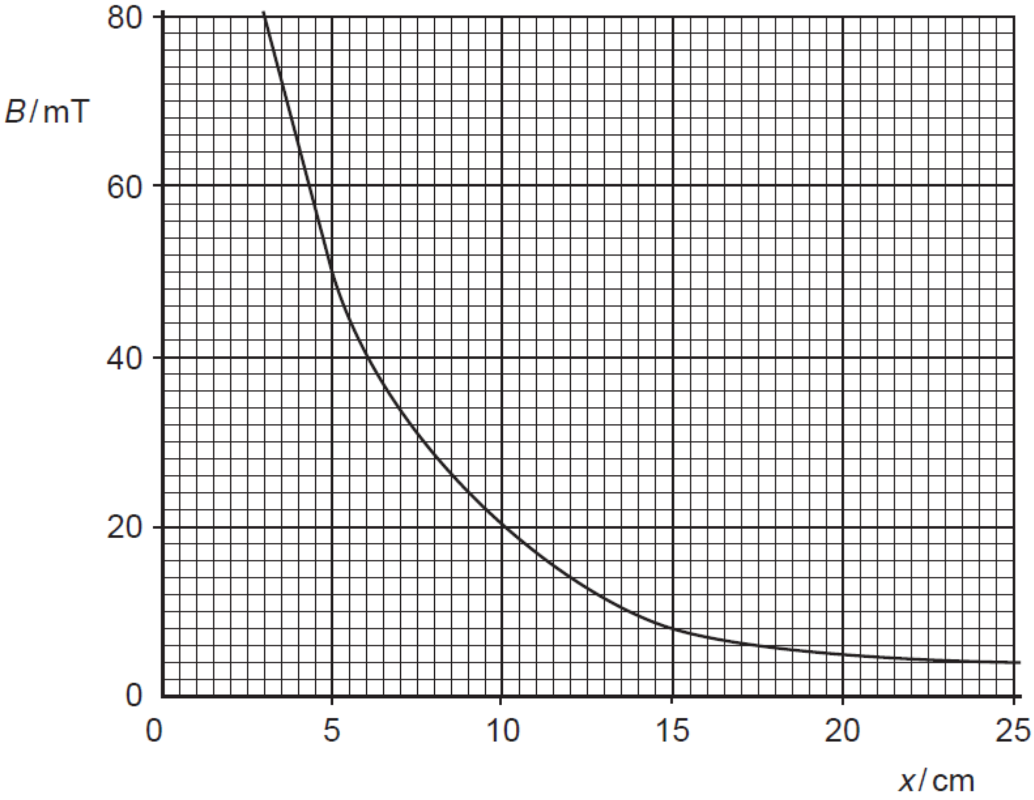
(e) Draw a second set of graphs for the case where the magnet rotates at constant angular velocity about axis **CD**, to show variation of (You may assume that the magnetic flux density region outside the pole pieces is zero.),
(i) the magnetic flux through the coil,
(ii) the e.m.f. induced in the coil.
Explain clearly how your graphs are obtained. [7]
(N89/III/12)

A2 A small coil is positioned so that its axis lies along the axis of a large bar magnet, as shown in the figure.



The coil has a cross-sectional area of 0.40 cm^2 and contains 150 turns of wire.

The average magnetic flux density *B* through the coil varies with distance *x* between the face of the magnet and the plane of the coil as shown in the figure.



(a) (i) The coil is 5.0 cm from the face of the magnet. Use the figure to determine the magnetic flux density in the coil.

(ii) Hence show that the magnetic flux linkage of the coil is 3.0×10^{-4} Wb. [2]

(b) The coil is moved along the axis of the magnet so that the distance *x* changes from *x* = 5.0 cm to *x* = 15.0 cm in a time of 0.30 s. Calculate

(i) the change in flux linkage of the coil, [2]

(ii) the average e.m.f. induced in the coil. [2]

(c) State and explain the variation, if any, of the speed of the coil so that the induced e.m.f. remains constant during the movement in (b). [3]