CATHOLIC JUNIOR COLLEGE
General Certificate of Education Advanced Level
Higher 2
JC2 Preliminary Examination

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| CANDIDATE NAME | |
| CLASS | |
| INDEX NUMBER | |

# MATHEMATICS 9758/01

Paper 1

**1 September 2020**

**3 hours**

Candidates answer on the Question Paper.
Additional Materials: List of Formulae (MF26)

**READ THESE INSTRUCTIONS FIRST**

Write your name and class on all the work you hand in.

Write in dark blue or black pen on both sides of the paper.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.

Write your answers in the spaces provided in the Question Paper.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use an approved graphing calculator.

Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

| Question | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | Total |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Marks | | | | | | | | | | | | |
| Total | 5 | 6 | 7 | 8 | 8 | 8 | 9 | 10 | 12 | 13 | 14 | 100 |

This document consists of **27** printed pages and **1** blank page.

**1.** A function f is defined by $\mathrm{f}(x)=\dfrac{2x+3}{3x+5}$.

**(i)** Show that $\mathrm{f}(x)$ can be written in the form $a+\dfrac{b}{3x+5}$, where $a$ and $b$ are constants to be determined. [1]

**(ii)** Hence describe a sequence of transformations which transforms the graph of $y=\dfrac{1}{x}$ on to the graph of $y=\mathrm{f}(x)$. [4]

**2.** The complex number $z$ has modulus 2 and argument $\dfrac{\pi}{8}$. It is also given that $w=1+\mathrm{i}$.

**(i)** Given that $n$ is an integer, find $\dfrac{z^n}{w^*}$ in terms of $n$, giving your answer in the form $r\mathrm{e}^{\mathrm{i}\theta}$. [3]

**(ii)** Hence, find the smallest two positive integers $n$ such that $\dfrac{z^n}{w^*}$ is real and negative. [3]

**3.** **Do not use a calculator in answering this question.**

The equation $2z^3+5z^2+pz+q=0$ has a root $z=-1+\mathrm{i}$, where $p$ and $q$ are real constants. Find $p$ and $q$ and all the other roots of the equation. [7]

**4.** The function f is given by

$$\mathrm{f}(x)=\begin{cases}\dfrac{5}{2}-x, & x\in\mathbb{R},\ x<2\\[2mm] \dfrac{1}{x}, & x\in\mathbb{R},\ x\geq 2\end{cases}$$

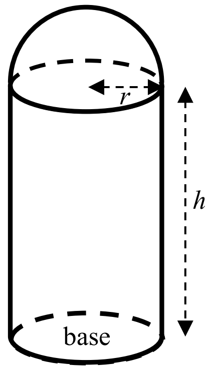
**(i)** Sketch the graph of $y=\mathrm{f}(x)$, stating the equation of any asymptotes and the coordinates of any points where the curve crosses the $x$- and $y$-axes. [2]

**(ii)** Explain why f has an inverse. Hence find $\mathrm{f}^{-1}(x)$, giving your answer in a similar form. [4]

Let $\mathrm{g}(x)=x^3$, $x\in\mathbb{R},\ x\geq 2$.

**(iii)** Determine whether $\mathrm{gf}^{-1}$ exists. [2]

**5.** [It is given that the surface area and the volume of a sphere of radius $r$ is $4\pi r^2$ and $\frac{4}{3}\pi r^3$ respectively.]
A manufacturer produces barriers which consist of a hemisphere of radius $r$ cm joined to a cylinder of base radius $r$ cm and height $h$ cm (see diagram).




The manufacturer decides that the external surface area, excluding the base, of each barrier is $16200\,\text{cm}^2$.
Using differentiation, find the exact value of the maximum volume of a barrier. [8]

**6.** The curves $C_1$ and $C_2$ have equations $y = \frac{2x^2+9}{x^2-4}$ and $\frac{y^2}{25} - \frac{x^2}{9} = 1$ respectively.

**(i)** Sketch $C_1$ and $C_2$ on the same diagram, including the coordinates of the points where the curves cross the axes, the equations of any asymptotes and the coordinates of the points of intersection of the curves. [6]

**(ii)** Hence, solve the inequality $5\sqrt{1+\frac{1}{9}x^2} \geq \frac{2x^2+9}{x^2-4}$. [2]

**7.** **(i)** It is given that $y^3 + y^2 + 2y = x^2 - 3x$, find the Maclaurin series for $y$ up to and including the term in $x^2$. [6]

**(ii)** Hence find the Maclaurin series for $\frac{1}{2+y}$ up to and including the term in $x^2$. [3]

**8.** Curve $C$ has parametric equations $x = 3t^2$, $y = -6t$.

**(i)** Point $P$ on $C$ has parameter $p$, where $0 < p < 1$.

Find the equations of the tangent and the normal to $C$ at $P$. [4]

**(ii)** The tangent to $C$ at $P$ meets the $x$-axis at point $T$, while the normal to $C$ at $P$ meets the $x$-axis at point $N$. Show that the coordinates of the midpoint $M$ of $TN$ is independent of $p$. [3]

**(iii)** Prove that the acute angle $PM$ makes with the $x$-axis is always twice the acute angle $PN$ makes with the $x$-axis. [3]

**9.** **(a)** **(i)** Show that, for $-a < x < a$, $\frac{\mathrm{d}}{\mathrm{d}x}\left[x\sqrt{a^2-x^2}+a^2\sin^{-1}\left(\frac{x}{a}\right)\right]=2\sqrt{a^2-x^2}$. [3]

The region $R$ in the first quadrant is bounded by the $x$-axis, $y$-axis, the line $x=2$ and the curve $x^2+16y^2=16$. Find

**(ii)** the exact area of $R$, [3]

**(iii)** the exact volume generated when $R$ is rotated through $360°$ about the $x$-axis. [3]

**(b)** The region $S$ is bounded by the curves $y=x^2$ and $x^2+4y^2=4$ for $y \geq 0$. Find the volume of the solid generated when the region $S$ is rotated through $180°$ about the $y$-axis. [3]

**10.** Mr Lim is considering an education endowment plan for his child. EduPlan Awesome allows him to contribute \$300 into the account on the first day of every month. At the end of each month, the total in the account is increased by 0.3%.

**(i)** Mr Lim contributes \$300 on 1 January 2020 and continues to contribute \$300 on the first day of each subsequent month.

**(a)** Show that the total amount in the account at the start of the $n$th month (where January 2020 was the 1st month, February 2020 was the 2nd month, and so on) is \$ $A\left(1.003^n-1\right)$, where $A$ is an integer to be determined. [3]

**(b)** Hence, find the total amount in the account at the start of 1 January 2021, giving your answer correct to 2 decimal places. [1]

EduPlan Blessing allows him to contribute \$$k$ into the account on the first day of every month. At the end of each month, the account earns a bonus and is added to the account. This bonus is \$$0.01k$ for the first month, and for each subsequent month, the bonus is \$$0.01k$ more than the bonus in the previous month.

**(ii)** State the amount of bonus earned for the 20th month, giving your answer in terms of $k$. [1]

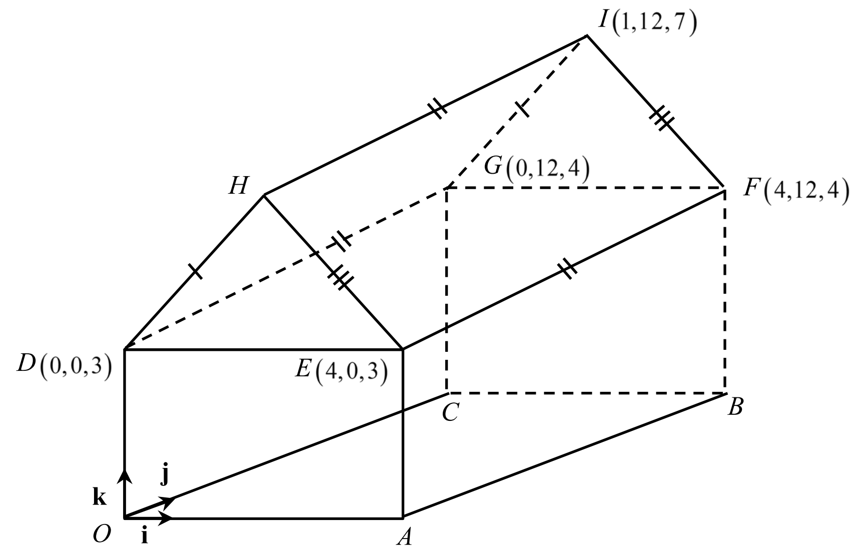
**(iii)** Find the total amount in the account at the start of the $n$th month, giving your answer in the form $0.005nk\left(n+B\right)$, where $B$ is an integer to be determined. [3]

Mr Lim wishes to cash out the education endowment plan on 1 January 2040 when he makes his first monthly contribution on 1 January 2020.

**(iv)** Find the value of $k$ such that both EduPlans have equal value when the education endowment plan is cashed out, giving your answer correct to the nearest whole number. [3]

**(v)** Given that Mr Lim has other financial commitments, explain which EduPlan is the better option for him. [2]

**11.** A temporary isolation centre is built to manage the increasing number of COVID-19 cases. The roof takes the shape of a triangular prism. Points $(x, y, z)$ are defined relative to an origin, *O,* with unit vectors **i** along $\overrightarrow{OA}$, j along $\overrightarrow{OC}$, and **k** along $\overrightarrow{OD}$ (see diagram). The coordinates of *D*, *E*, *F*, *G* and *I* are (0,0,3), (4,0,3), (4,12,4), (0,12,4) and (1,12,7) respectively. The units are measured in metres.




**(i)** Find a cartesian equation of the plane that contains the roof section *EFIH*. [3]

It is given that the roof section *DGIH* is part of the plane with equation $36x + y - 12z = -36$.

**(ii)** Find a cartesian equation of the line that contains the roof ridge *HI*. [3]

Steel cables are used to hold the isolation centre in place. Cables are laid in straight lines and the widths of cables can be neglected. It is given that cable 1 passes through *F* and *D* and cable 2 passes through *G* and *M*, where *M* is the mid point of *EF*.

A builder needs to locate the point *J* where both cables meet.

**(iii)** Find the coordinates of *J*. [4]

To strengthen the structure, it is recommended that another steel cable should be extended from *J* to the closest point on the roof ridge *HI*. The builder is left with 3.2 metres of steel cable.

**(iv)** Determine whether the remaining steel cable is long enough to connect *J* to the roof ridge *HI*. [4]

**End of Paper**