2022 4E5N E Math Prelim Paper 2 (Marking Scheme)

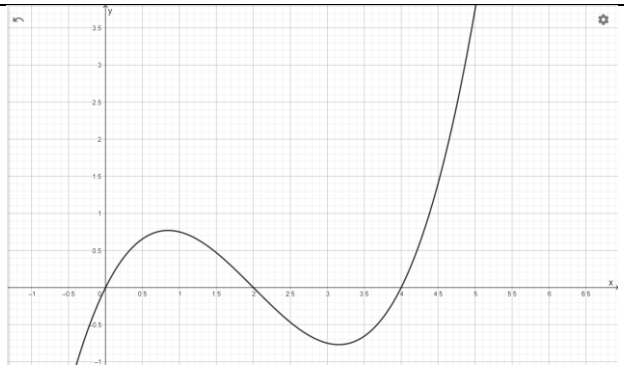
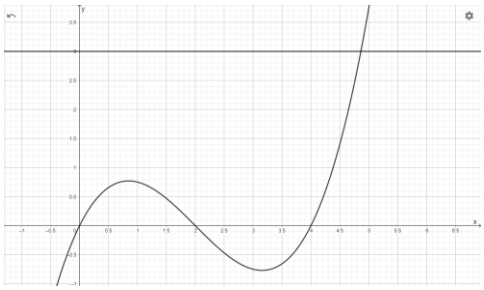
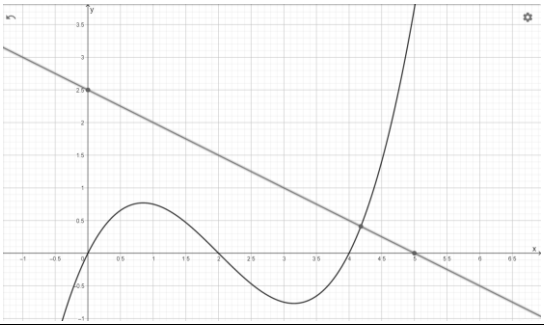
Qn	Solution	Remarks
1(a)	Total= $5000\left(1+\frac{\left(\frac{4}{12}\right)}{100}\right)^{(3\times12)}$ = \$5636 (nearest dollar)	
1(b)	$\frac{100}{107}\times 82500 = \$77102.80$ (nearest cent)	
1(c)	$7mb = 3b + 2m$ $b(7m-3) = 2m$ $b = \frac{2m}{7m-3}$	all the terms with *m* on 1 side of the equation
1(d)	$21x^2 = 2 - 11x$ $21x^2 + 11x - 2 = 0$ $(7x-1)(3x+2) = 0$ x = $\frac{1}{7}$ or x = $-\frac{2}{3}$	
2(a)	$\left(\frac{500}{n}\right)$ jars of cookies	$\left(\frac{500}{n}\right)$
2(b)	$\$\left(\frac{500}{n}-2\right)(n+3)$ or $\$\left(500+\frac{1500}{n}-2n-6\right)$	No need \$ sign
2(c)	$\left(\frac{500}{n}-2\right)(n+3)-500=92$ $500+\frac{1500}{n}-2n-6-500=92$ $\frac{1500}{n}-2n-98=0$ $1500-2n^2-98n=0$ $n^2+49n-750=0$ (Shown)	Simplify until RHS=0 then shown to given equation
2(d)	$n=\frac{-49\pm\sqrt{(49)^2-4(1)(-750)}}{2(1)}$ *n* = 12.246 (3dp) or *n* = – 61.246 (3dp)	Correct working with quadratic formula with correct values substituted in
2(e)	***n* = – 61.246 needs to be rejected** because n represents cost and **cost cannot be negative**	
2(f)	(Reject $n=-61.246$ as $n>0$) Selling price = 12.246 + 3=\$15 (nearest dollar)	

3(a)	$\mathbf{P} = \begin{pmatrix} 218 \\ 138 \\ 66 \\ 40 \end{pmatrix}$	
3(b)	$\begin{pmatrix} 0.75 & 0 & 0 & 0 \\ 0 & 0.75 & 0 & 0 \\ 0 & 0 & 0.85 & 0 \\ 0 & 0 & 0 & 0.85 \end{pmatrix}\begin{pmatrix} 218 \\ 138 \\ 66 \\ 40 \end{pmatrix} = \begin{pmatrix} 163.5 \\ 103.5 \\ 56.1 \\ 34 \end{pmatrix}$	
3(c)	$\mathbf{T} = \begin{pmatrix} 35 & 120 & 75 & 16 \\ 60 & 93 & 112 & 27 \end{pmatrix}$	
3(d)	$\mathbf{S} = \mathbf{TP} = \begin{pmatrix} 35 & 120 & 75 & 16 \\ 60 & 93 & 112 & 27 \end{pmatrix}\begin{pmatrix} 218 \\ 138 \\ 66 \\ 40 \end{pmatrix} = \begin{pmatrix} 29780 \\ 34386 \end{pmatrix}$	
3(e)	29780, 34386 represent the amount collected from the sales of tickets for the afternoon show and evening show **respectively**. **OR** 29780, 34386 represent the amount collected from thesales of tickets for **each** show. **OR** 29780 represents the amount collected from the sales of tickets for the afternoon show. 34386 represents the amount collected from the sales of tickets for the evening show.	Accept any answer with equivalent meaning e.g. ticket sales, amount obtained, revenue, Do **not** accept: profit, price, cost, amount of tickets sold Must quote values from (c).
4(a)(i)	angle BDC = 180° – 62° = 118° (∠s in opp. segments)	
4(a)(ii)	angle OCB = $\frac{118°}{2} = 59°$ (OC bisects ∠BCD) angle OCB = angle OBC = 59° (base ∠s of isosceles triangle) angle BOC = 180° – 59° – 59° = 62° (∠ sum of isosceles triangle)	
4(b)	Area of minor segment BC $= \left[\frac{62°}{360°} \times \pi(6)^2\right] - \left[\frac{1}{2} \times 6 \times 6 \times \sin 62°\right] = 3.58\text{cm}^2$ (3sf)	Find area of minor sector. Find area of triangle.

5(a)	Total surface area of frustum $=(40\times 25)+(48\times 30)$ $+2\left(\frac{1}{2}(40+48)\left(\sqrt{2.5^2+27^2}\right)\right)$ $+2\left(\frac{1}{2}(25+30)\left(\sqrt{27^2+4^2}\right)\right)$ $=6330cm^2$	Area of either bases Any correct trapezium area or adding 6 faces
5(b)(i)	Let height of original pyramid be h $\frac{h-27}{h}=\frac{40}{48}$ $48(h-27)=40h$ $48h-1296=40h$ $8h=1296$ $h=\frac{1296}{8}=162$ cm (Shown)	
5(b)(ii)	Volume $=\frac{1}{3}(30\times 48)(135+27)-\frac{1}{3}(40\times 25)(135)$ $=32760$ cm^3	
6(a)	$\frac{-4-8}{x-(-2)}=-1.5$ $-1.5(x+2)=-12$ $x=6$ **OR** Equation of line *AB*: $y = -1.5x + c$ $8 = -1.5(-2) + c \rightarrow c = 5$ Hence, equation of line *AB*: $y = -1.5x + 5$ $-4 = -1.5x + 5$ $x = 6$	
6(b)	Length of *AB* $=\sqrt{(-2-6)^2+\left(8-(-4)\right)^2}$ $=\sqrt{208}=14.4$ units (3sf)	
6(c)	$6(-b)+9(4b)=5 \rightarrow b=\frac{1}{6}$	
6(d)	Line *CD*: $6y+9x=5 \rightarrow y=-1.5x+\frac{5}{6}$ Gradient of line *CD* = – 1.5 Since the gradients of line *CD* and line *AB* are equal, line *CD* is parallel to line *AB*. Thus, line *CD* does not intersect line *AB*.	Reason: Gradients equal **OR** lines parallel

7(a)	Trapezium	
7(b)(i)	$\overrightarrow{ZX} = \overrightarrow{ZW} + \overrightarrow{WX} = -\mathbf{b} + \mathbf{a}$ **OR** $\overrightarrow{ZX} = \overrightarrow{WX} - \overrightarrow{WZ} = \mathbf{a} - \mathbf{b}$	
7(b)(ii)	$\overrightarrow{WY} = \overrightarrow{WX} + \overrightarrow{XY} = \mathbf{a} + \frac{3}{2}\mathbf{b}$	
7(b)(iii)	$5\overrightarrow{TX} = 3\overrightarrow{ZX}$ $\overrightarrow{TX} = \frac{3}{5}\overrightarrow{ZX}$ $\overrightarrow{ZT} = \frac{2}{5}\overrightarrow{ZX}$ $\overrightarrow{WT} = \overrightarrow{WZ} + \overrightarrow{ZT} = \overrightarrow{WZ} + \frac{2}{5}\overrightarrow{ZX} = \mathbf{b} + \frac{2}{5}(\mathbf{a} - \mathbf{b})$ $\overrightarrow{WT} = \frac{2}{5}\mathbf{a} + \frac{3}{5}\mathbf{b}$ **OR** $\frac{1}{5}(2\mathbf{a} + 3\mathbf{b})$ **OR** $\frac{2}{5}(\mathbf{a} + \frac{3}{2}\mathbf{b})$	
	OR	
	$\overrightarrow{WT} = \overrightarrow{WX} + \overrightarrow{XT}$ $\overrightarrow{WT} = \overrightarrow{WX} + \frac{3}{5}(-\overrightarrow{ZX})$ $\overrightarrow{WT} = \mathbf{a} + \frac{3}{5}(\mathbf{b} - \mathbf{a})$ $\overrightarrow{WT} = \frac{2}{5}\mathbf{a} + \frac{3}{5}\mathbf{b}$ **OR** $\frac{1}{5}(2\mathbf{a} + 3\mathbf{b})$ **OR** $\frac{2}{5}(\mathbf{a} + \frac{3}{2}\mathbf{b})$	
7(c)	$\overrightarrow{WY} = \mathbf{a} + \frac{3}{2}\mathbf{b} = \frac{1}{2}(2\mathbf{a} + 3\mathbf{b})$ $\overrightarrow{WT} = \frac{2}{5}\mathbf{a} + \frac{3}{5}\mathbf{b} = \frac{1}{5}(2\mathbf{a} + 3\mathbf{b})$ $\frac{WT}{WY} = \frac{(1/5)}{(1/2)} = \frac{2}{5}$ $\overrightarrow{WT} = \frac{2}{5}\overrightarrow{WY}$ $\overrightarrow{WT}$ and $\overrightarrow{WY}$ are parallel. W is a common point. Hence W, T and Y lie on a straight line.	$\overrightarrow{WT} = \frac{2}{5}\overrightarrow{WY}$ $\overrightarrow{WT}$ and $\overrightarrow{WY}$ are parallel **AND** W is a common point.
7(d)(i)	Triangle WZT and Triangle YXT are similar $\frac{\text{Area of triangle } WZT}{\text{Area of triangle } YXT} = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$	

7(d)(ii)	$\dfrac{\text{Area of triangle } YZT}{\text{Area of triangle } YXT} = \dfrac{\left(\frac{1}{2}\times ZT\times h\right)}{\left(\frac{1}{2}\times XT\times h\right)} = \dfrac{2}{3}$	
8(a)(i)	angle $SPR = 18°$ $\dfrac{\sin P\hat{S}R}{90} = \dfrac{\sin 18°}{28}$ $\angle PSR = \sin^{-1}\left(\dfrac{90\sin 18°}{28}\right)$ or $\angle PSR = 180° - \sin^{-1}\left(\dfrac{90\sin 18°}{28}\right)$ $\angle PSR = 83.348°(3dp)$ or $\angle PSR = 96.652°(3dp)$ (reject $\angle PSR = 83.348°(3dp)$ as $\angle PSR$ is obtuse) Hence, obtuse $\angle PSR = 96.652°(3dp)$ obtuse $\angle PSR = 96.7°(1dp)$	angle $SPR = 18°$ seen or implied correct use of sine rule
8(a)(ii)	bearing of P from R = 180° + 18° = 198°	
8(b)	$90^2 = 72^2 + 45^2 - 2(72)(45)\left(\cos P\hat{Q}R\right)$ $\cos P\hat{Q}R = \dfrac{72^2 + 45^2 - 90^2}{2(72)(45)}$ $\angle PQR = 97.9032° = 97.903°(3dp)$	Find value to at least 4 decimal places then show to given answer
8(c)(i)	area of triangle PQR $= \dfrac{1}{2}\times 72\times 45\times \sin 97.903° = 1604.613 = 1600\text{m}^2(3\text{sf})$	
8(c)(ii)	Let shortest distance be BN. $\dfrac{1}{2}\times 90\times BN = 1604.613$ $BN = 35.658\text{m} = 35.7\text{m}(3\text{ sf})$	
8(d)	$\tan 8° = \dfrac{\text{height}}{35.658}$ height $= 35.658\tan 8° = 5.01\text{m}$ (3sf)	

9(a)	$h = 0.66$ (2dp)	c.o.
9(b)		Deduct 1 mark for every incorrect and/or missing point. Allow ecf for answer in (a). Smooth curve passing through all plotted points.
9(c)	$x^3 - 6x^2 + 8x = 12$ $x\left(x^2 - 6x + 8\right) = 12$ $\frac{x}{4}\left(x^2 - 6x + 8\right) = 3$ $\frac{x}{4}\left(8 - 6x + x^2\right) = 3$ Hence, draw line $y = 3$. See graph in (b).  $x = 4.9$	Manipulate given equation to $\frac{x}{4}\left(8 - 6x + x^2\right) = -3$ Accept intersection of line $y = 3$ with their graph
9(d)(i)	$y = -\frac{1}{2}x + c$ $1 = -\frac{1}{2}(3) + c \rightarrow c = \frac{5}{2}$ $y = -\frac{1}{2}x + \frac{5}{2}$ 	Line of negative gradient. Correct line $y = -\frac{1}{2}x + \frac{5}{2}$ drawn for $-1 \leq x \leq 5$: Passes through (–1, 3), (3, 1) and (5, 0)
9(d)(ii)	$x = 4.15$	Read up to 0.05

9(d)(iii)	$\frac{x}{4}\left(8-6x+x^2\right)=-\frac{1}{2}x+\frac{5}{2}$ $x\left(8-6x+x^2\right)=-2x+10$ $8x-6x^2+x^3=-2x+10$ $x^3-6x^2+10x-10=0$	
10(a)(i)	$\frac{50}{100}\times 80=40$ eggs → Median = 53g	
10(a) (ii)	$\frac{30}{100}\times 80=24$ eggs → 30th percentile = 48g	
10(a) (iii)	$\frac{25}{100}\times 80=20$ eggs → Lower quartile = 46g $\frac{75}{100}\times 80=60$ eggs → Upper quartile = 58g Interquartile range = 58 – 46 = 12g	
10(b)	Number of eggs less than or equal 60 g = 69 Number of eggs more than 60g = 80 – 69 = 11	(accept 70) (accept 10)
10 (c)(i)	Frequency for mass, $x \leq 45$ = 18 Frequency for mass, $x \leq 35$ = 6 $m = 18 - 6 = 12$ (Shown) **OR** $6+m+30+32=80$ $m = 80 - 6 - 30 - 32 = 12$(Shown)	
10 (c)(ii)	P(both eggs more than 45g $=\frac{62}{80}\times\frac{61}{79}=\frac{1891}{3160}=0.5984(4sf)$	
10 (c)(iii)	<table><tr><td>Mid value (x)</td><td>30</td><td>40</td><td>50</td><td>60</td></tr><tr><td>Mass (x g)</td><td>$25 < x \leq 35$</td><td>$35 < x \leq 45$</td><td>$45 < x \leq 55$</td><td></td></tr><tr><td>Frequency</td><td>6</td><td>12</td><td>30</td><td>32</td></tr></table> Mean of masses of eggs at Farm A = 51g Standard deviation of masses of eggs at Farm A = 9.16515139=9.17g (3sf)	
10(d)	1. The eggs at Farm B are heavier because the mean of the massess of eggs at Farm B, 53g, is more than that of Farm A, 51 g. 2. The masses of the eggs at Farm B are more consistent because the standard deviation of the massess of eggs at Farm B, 9g, is less than that of Farm A, 9.17g.	

11(a)

$\frac{420}{60} \times 145 = 1015$ calories

11(b)

2000 calorie diet need 28g dietary fiber

1 calorie diet need $\frac{28}{2000}$ g dietary fiber

2800 calorie diet need $\frac{28}{2000} \times 2800$g

= 39.2g dietary fiber

% Daily Value of dietary fiber per serving of bread for average male teenager

$= \frac{3.6}{39.2} \times 100\% = 9.183673469$ **= 9.1 %** (2sf)

= 9% (nearest whole number) (Shown)

Calculate the amount of dietary fiber for 2800 calorie diet

Calculated value must be shown to at least 1 more significant figure than given in the question

11 (c)(i)

Nutrient	**DV for male adult**	**Recommended Limits**
Saturated fat (g)	$\frac{20}{2000} \times 2500 = \mathbf{25}$	Less than
Dietary fiber (g)	$\frac{28}{2000} \times 2500 = \mathbf{35}$	At least
Sodium (mg)	$\frac{2300}{2000} \times 2500 = \mathbf{2875}$	Less than

11 (c)(ii)

Johan's meal plan

2 slices of wholemeal bread + 2 servings of peanut spread + 1 hard boiled egg + 1 turkey ham sandwich + 1 package chicken and mushroom soup + 2 avocado

Johan's meal plan (based on servings)

1 serving of wholemeal bread + 2 servings of peanut spread + 1 serving of hard boiled egg + 1 serving turkey ham sandwich + 2 servings chicken and mushroom soup + 2 avocado

Total calories and nutrients for Johan's meal plans:

Energy (calories)	145 + 2(486) + 78 + 270 + 2(213.3) + 2(320) = 2531.6
Saturated fat (g)	0.7 + 2(1.3) + 2 + 1 + 2(3) + 2(4.2) = 20.7
Dietary fiber (g)	3.6 + 2(1.6) + 0 + 4 + 2(3) + 2(13.4) = 43.6
Sodium (mg)	206.4 + 2(51) + 62 + 800 + 2(718.8) + 2(14) = 2636

Johan's meal plan on this particular day **meet the basic daily meal plan requirements** because

- the **total calorie intake**, 2531.6 calories is **more than the recommended minimum calorie intake**, 2500 calories.
- the **saturated fat**, 20.7g, is **less than the recommended limit**, 25g
- the **total dietary fiber**, 43.8g, is **more than the recommended limit**, 35g
- the **total sodium**, 2636g, is **less than the recommended limit** for sodium, 2875g