Raffles Institution
H2 Mathematics (9758)
Solution for 2023 A-Level Paper 1

Question 1

No.	Suggested Solution	Remarks for Student
	$\ln y = (11-5x)^2$ Differentiate with respect to x, $\frac{1}{y}\frac{\mathrm{d}y}{\mathrm{d}x} = 2(11-5x)(-5) = -10(11-5x)$ $\frac{\mathrm{d}y}{\mathrm{d}x} = -10y(11-5x)$ When $x = 2$, $\ln y = (11-5(2))^2 = 1 \Rightarrow \quad y = \mathrm{e}$ $\frac{\mathrm{d}y}{\mathrm{d}x} = -10y(11-5x) = -10(\mathrm{e})(11-10) = -10\mathrm{e}$ The equation of tangent is $y - \mathrm{e} = -10\mathrm{e}(x-2)$ $y = -10\mathrm{e}x + 21\mathrm{e}$	

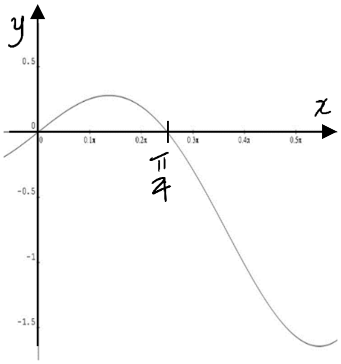
Question 2

No.	Suggested Solution	Remarks for Student
(a)	$u_n = an^3 + an^2 + cn + d$ $u_1 = a+b+c+d = 10$ $u_2 = 8a+4b+2c+d = 61$ $u_3 = 27a+9b+3c+d = 206$ $u_4 = 64a+16b+4c+d = 469$ Solving, $a = 4, b = 23, c = -46, d = 29$ So, $u_n = 4n^3 + 23n^2 - 46n + 29$	
(b)	$u_n = 4n^3 + 23n^2 - 46n + 29 > 25000$ When n =16, $u_n = 21565 < 25000$ When n =17, $u_n = 25546 > 25000$ Range of values of n is $\{n \in \mathbb{Z} : n \geq 17\}$	Those who gave the answer as $n > 16.9$ will not get the full marks for not recognizing that n has be an integer.

Question 3

No.	Suggested Solution	Remarks for Student
(a)	$(\mathbf{a}\times\mathbf{b}+\mathbf{a})\perp(\mathbf{a}\times\mathbf{b}+\mathbf{b})$ $(\mathbf{a}\times\mathbf{b}+\mathbf{a}).(\mathbf{a}\times\mathbf{b}+\mathbf{b})=0$ $(\mathbf{a}\times\mathbf{b}).(\mathbf{a}\times\mathbf{b})+(\mathbf{a}\times\mathbf{b}).\mathbf{b}+\mathbf{a}.(\mathbf{a}\times\mathbf{b})+\mathbf{a}.\mathbf{b}=0$ $\lvert\mathbf{a}\times\mathbf{b}\rvert^2-1=0$, since $(\mathbf{a}\times\mathbf{b}).\mathbf{b}=\mathbf{a}.(\mathbf{a}\times\mathbf{b})=0$ and $\mathbf{a}.\mathbf{b}=-\mathbf{1}$ $\lvert\mathbf{a}\times\mathbf{b}\rvert^2=1$ $\lvert\mathbf{a}\times\mathbf{b}\rvert=1$, since $\lvert\mathbf{a}\times\mathbf{b}\rvert\geq 0$	$(\mathbf{a}\times\mathbf{b}).\mathbf{b}=\mathbf{a}.(\mathbf{a}\times\mathbf{b})=0$ because $\mathbf{a}\times\mathbf{b}$ is perpendicular to both **a** and **b**. The reason needs to be given.
(b)	Let θ be the angle between the direction of **a** and the direction of **b**. Then, $\mathbf{a}.\mathbf{b}=\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\cos\theta=-1$ ---- (1) $\lvert\mathbf{a}\times\mathbf{b}\rvert=\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\sin\theta=1$ ---- (2) $(2)\div(1)$, $\dfrac{\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\sin\theta}{\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\cos\theta}=-1$ $\tan\theta=-1$ So, $\theta=135^\circ$	

Question 4

No.	Suggested Solution	Remarks for Student
(a)	$\int \cos px \cos qx \ \mathrm{d}x = \frac{1}{2}\int \cos(p+q)x + \cos(p-q)x \ \mathrm{d}x$ $= \frac{1}{2}\left[\frac{1}{p+q}\sin(p+q)x + \frac{1}{p-q}\sin(p-q)x\right] + C$	Refer to the list of factor formulae in MF26 for assistance.
(b)	$\int x\cos nx \ \mathrm{d}x = \frac{1}{n}x\sin nx - \frac{1}{n}\int \sin nx \ \mathrm{d}x$ $= \frac{1}{n}x\sin nx + \frac{1}{n^2}\cos nx + D$	Integration by parts. $u = x, \qquad \frac{\mathrm{d}v}{\mathrm{d}x} = \cos nx$ $\frac{\mathrm{d}u}{\mathrm{d}x} = 1, \qquad v = \frac{\sin nx}{n}$
(c)	Using the results in part **(b)**, $\int_0^{\pi} x\cos nx \ \mathrm{d}x$ $= \left[\frac{1}{n}x\sin nx + \frac{1}{n^2}\cos nx\right]_0^{\pi}$ $= \frac{1}{n}\pi\sin n\pi + \frac{1}{n^2}\cos n\pi - \frac{1}{n}(0)\sin n(0) + \frac{1}{n^2}\cos n(0)$ $= \frac{1}{n^2}\cos n\pi - \frac{1}{n^2}$, since $\sin n\pi = 0$ for $n \in \mathbb{Z}$ and $\cos 0 = 1$ When n is even, $\cos n\pi = 1$ and $\int_0^{\pi} x\cos nx \ \mathrm{d}x = \frac{1-1}{n^2} = \frac{0}{n^2} = 0$ When n is odd, $\cos n\pi = -1$ and $\int_0^{\pi} x\cos nx \ \mathrm{d}x = \frac{-1-1}{n^2} = \frac{-2}{n^2}$ Therefore, $\int_0^{\pi} x\cos nx \ \mathrm{d}x = \frac{k}{n^2}$, where $k = \begin{cases} 0 & \text{when } n \text{ is even} \\ -2 & \text{when } n \text{ is odd} \end{cases}$	
(d)	Since $x\cos 2x \geq 0$ when $0 \leq x \leq \frac{\pi}{4}$ and $x\cos 2x \leq 0$ when $\frac{\pi}{4} \leq x \leq \frac{\pi}{2}$, so in the interval $0 \leq x \leq \frac{\pi}{2}$, $\lvert x\cos 2x\rvert = \begin{cases} x\cos 2x, & \text{when } 0 \leq x \leq \frac{\pi}{4}; \\ -x\cos 2x, & \text{when } \frac{\pi}{4} \leq x \leq \frac{\pi}{2}. \end{cases}$	Again, using your GC, sketch $y = x\cos 2x$. 

$$\int_0^{\frac{\pi}{2}} \left|x\cos 2x\right|\ \mathrm{d}x$$

$$= \int_0^{\frac{\pi}{4}} x\cos 2x\ \mathrm{d}x + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} -x\cos 2x\ \mathrm{d}x$$

$$= \int_0^{\frac{\pi}{4}} x\cos 2x\ \mathrm{d}x - \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} x\cos 2x\ \mathrm{d}x$$

$$= \left[\frac{x\sin 2x}{2}+\frac{\cos 2x}{4}\right]_0^{\frac{\pi}{4}} - \left[\frac{x\sin 2x}{2}+\frac{\cos 2x}{4}\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

$$= \left[\frac{\frac{\pi}{4}\sin\frac{\pi}{2}}{2}+\frac{\cos\frac{\pi}{2}}{4}-\frac{0}{2}-\frac{\cos 0}{4}\right]$$

$$-\left[\frac{\frac{\pi}{2}\sin\pi}{2}+\frac{\cos\pi}{4}-\frac{\frac{\pi}{4}\sin\frac{\pi}{2}}{2}-\frac{\cos\frac{\pi}{2}}{4}\right]$$

$$= \frac{\pi}{8}+0-0-\frac{1}{4}-0+\frac{1}{4}+\frac{\pi}{8}+0$$

$$= \frac{\pi}{4}$$

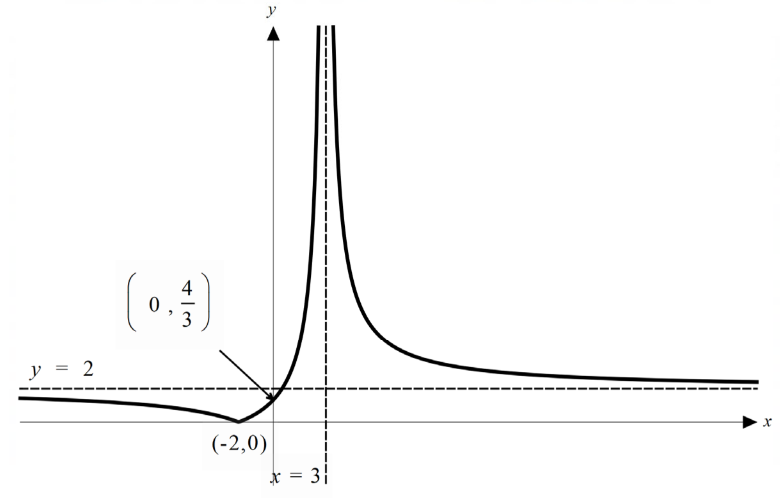
Question 5

No.	Suggested Solution	Remarks for Student
(a)	$\sum_{r=2}^{n} \ln\left[\frac{(r-1)(r+1)}{r^2}\right] = \sum_{r=2}^{n}\left[\ln(r-1) - 2\ln r + \ln(r+1)\right]$ $= \ln(1) \quad - \quad 2\ln 2 \quad + \quad \cancel{\ln 3}$ $+\ln(2) \quad - \quad \cancel{2\ln 3} \quad + \quad \cancel{\ln 4}$ $+\cancel{\ln(3)} \quad - \quad \cancel{2\ln 4} \quad + \quad \cancel{\ln 5}$ $+\cancel{\ln(4)} \quad - \quad \cancel{2\ln 5} \quad + \quad \cancel{\ln 6}$ $\vdots$ $+ \cancel{\ln(n-3)} \quad -\cancel{2\ln(n-2)} + \cancel{\ln(n-1)}$ $+ \cancel{\ln(n-2)} \quad -\cancel{2\ln(n-1)} + \ln(n)$ $+\cancel{\ln(n-1)} \quad -2\ln(n) \quad +\ln(n+1)$ $= \ln(1) - 2\ln 2 + \ln(2) + \ln(n) - 2\ln(n) + \ln(n+1)$ $= -\ln 2 - \ln(n) + \ln(n+1)$ $= \ln\left(\frac{n+1}{n}\right) - \ln 2$ (Shown)	
(b)	As $n \to \infty$, $\frac{n+1}{n} = 1 + \frac{1}{n} \to 1$, therefore $\sum_{r=2}^{n} \ln\left[\frac{(r-1)(r+1)}{r^2}\right] = \ln\left(\frac{n+1}{n}\right) - \ln 2 \to \ln 1 - \ln 2 = -\ln 2$ (a finite number) So, the corresponding infinite series in convergent and the sum to infinity is $-\ln 2$.	
(c)	$\sum_{r=10}^{20} \ln\left[\frac{(r-1)(r+1)}{r^2}\right] = \sum_{r=2}^{20} \ln\left[\frac{(r-1)(r+1)}{r^2}\right] - \sum_{r=2}^{9} \ln\left[\frac{(r-1)(r+1)}{r^2}\right]$ $= \ln\left(\frac{21}{20}\right) - \ln 2 - \ln\left(\frac{10}{9}\right) + \ln 2$ $= \ln\left(\frac{21\times 9}{20\times 10}\right) = \ln\left(\frac{189}{200}\right)$ Therefore, $a = 189$ and $b = 200$.	

Question 6

No.	**Suggested Solution**	**Remarks for Student**
(a)	Use double angle formula $\cos^2\theta = \frac{1}{2}\left(\cos 2\theta + 1\right)$, we have $\cos^4\theta = \left[\cos^2\theta\right]^2$ $= \left[\frac{1}{2}\left(\cos 2\theta + 1\right)\right]^2$ $= \frac{1}{4}\left[\cos^2 2\theta + 2\cos 2\theta + 1\right]$ $= \frac{1}{4}\left[\frac{1}{2}\left(\cos 4\theta + 1\right) + 2\cos 2\theta + 1\right]$ since $\cos^2 2\theta = \frac{1}{2}\left(\cos 4\theta + 1\right)$ $= \frac{1}{4}\left[\frac{1}{2}\cos 4\theta + 2\cos 2\theta + \frac{3}{2}\right]$ $= \frac{1}{8}\left(\cos 4\theta + 4\cos 2\theta + 3\right)$ (Shown)	
(b)	Volume generated $=\pi\int_{1.5}^{3} y^2 \ \mathrm{d}x = \pi\int_{1.5}^{3} \left(9 - x^2\right)^{\frac{3}{2}} \ \mathrm{d}x$ $x = 3\sin\theta \Rightarrow \frac{\mathrm{d}x}{\mathrm{d}\theta} = 3\cos\theta$ When $x = 1.5$, $3\sin\theta = 1.5 \Rightarrow \sin\theta = 0.5 \Rightarrow \theta = \frac{\pi}{6}$ When $x = 3$, $3\sin\theta = 3 \Rightarrow \sin\theta = 1 \Rightarrow \theta = \frac{\pi}{2}$ Therefore, volume generated $=\pi\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \left(9 - 9\sin^2\theta\right)^{\frac{3}{2}} \ 3\cos\theta \,\mathrm{d}\theta$ $= \pi\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} 9^{\frac{3}{2}} \left(1 - \sin^2\theta\right)^{\frac{3}{2}} \ 3\cos\theta \,\mathrm{d}\theta$ $= \pi\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} 27\left(\cos^3\theta\right) \ 3\cos\theta \,\mathrm{d}\theta$ $= 81\pi\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cos^4\theta \,\mathrm{d}\theta$ $= 81\pi\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{1}{8}\left(\cos 4\theta + 4\cos 2\theta + 3\right)\mathrm{d}\theta$ $= \frac{81\pi}{8}\left[\frac{\sin 4\theta}{4} + 2\sin 2\theta + 3\theta\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$ $= \frac{81\pi}{8}\left[\frac{3\pi}{2} - \frac{\sqrt{3}}{8} - \sqrt{3} - \frac{\pi}{2}\right]$ $= \frac{81\pi}{8}\left[\pi - \frac{9\sqrt{3}}{8}\right] = \frac{81\pi}{64}\left[8\pi - 9\sqrt{3}\right]$	

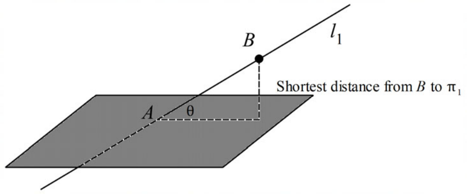
Question 7

No.	Suggested Solution	Remarks for Student
(a)	$y=\left\|\frac{2x+4}{3-x}\right\|$ 	
(b)	Range of f = $R_f = [0, \infty)$	
(c)	Since $3 \in R_f$ but $3 \notin D_f$. So, $R_f \not\subset D_f$, therefore the function f^2 does not exist.	Since answer is known (f^2 does not exist), it is important to write down how come $R_f \not\subset D_f$ by giving a specific value that inside R_f but not inside D_f.
(d)	For the function f^{-1} to exits, f has to be one-one, so the greatest value of a is -2.	
(e)	Let $y=\left\|\frac{2x+4}{3-x}\right\|=-\frac{2x+4}{3-x}$, since $\frac{2x+4}{3-x}\le 0$ for $x\le -2$ $y=-\frac{2x+4}{3-x}$ $3y-xy=-2x-4$ $x=\frac{3y+4}{y-2}$ Therefore $f^{-1}(x)=\frac{3x+4}{x-2}$. Its domain is $D_{f^{-1}}=R_f=[0, 2)$	

Question 8

No.	Suggested Solution	Remarks for Student
(a)(i)	$\lvert z\rvert=\sqrt{1+3^2}$ and $\arg z=\pi-\tan^{-1}\sqrt{3}=\frac{2\pi}{3}$ (z is in the 2nd quadrant) Therefore $z=-1+\sqrt{3}\,\mathrm{i}=2\mathrm{e}^{\mathrm{i}\frac{2\pi}{3}}$	
(ii)	$\frac{z^n}{\mathrm{i}z*}=\frac{2^n\mathrm{e}^{\mathrm{i}\frac{2n\pi}{3}}}{\mathrm{e}^{\mathrm{i}\frac{\pi}{2}}\times 2\mathrm{e}^{-\mathrm{i}\frac{2\pi}{3}}}=2^{n-1}\mathrm{e}^{\mathrm{i}\left(\frac{2n\pi}{3}+\frac{2\pi}{3}-\frac{\pi}{2}\right)}=2^{n-1}\mathrm{e}^{\mathrm{i}\left(\frac{2n\pi}{3}+\frac{\pi}{6}\right)}$ For $\frac{z^n}{\mathrm{i}z*}$ to be purely imaginary, $\cos\left(\frac{2n\pi}{3}+\frac{\pi}{6}\right)=0\Rightarrow\frac{2n\pi}{3}+\frac{\pi}{6}=(2k+1)\frac{\pi}{2}$ where $k\in\mathbb{Z}$ $\Rightarrow n=\frac{3}{2}k+\frac{1}{2}=\frac{1}{2}(3k+1)$ [Observe the graph of $y=\cos x$ to get the expression $(2k+1)\frac{\pi}{2}$] It is clear that the required smallest positive n occurs when $k=1$. Therefore, the required smallest positive $n=2$.	Recall that $\mathrm{i}=\mathrm{e}^{\mathrm{i}\frac{\pi}{2}}$ Alternatively, $\frac{2n\pi}{3}+\frac{\pi}{6}=\pm\frac{\pi}{2},\pm\frac{3\pi}{2},\pm\frac{5\pi}{2},\dots$ $n=\frac{1}{2},-1,2,-\frac{5}{2},\frac{7}{2},-4,\dots$
(b)	$2v+\lvert w\rvert=1 \quad ----(1)$ $3v-\mathrm{i}w=-3+4\mathrm{i} \quad ----(2)$ Let $v=a+\mathrm{i}b$ and $w=x+\mathrm{i}y$, where $a,b,x,y\in\mathbb{R}$ From (1), $2v+\lvert w\rvert=1 \quad\Rightarrow\quad v=\frac{1}{2}(1-\lvert w\rvert)\in\mathbb{R}$ Thus, we can say that $v=a\in\mathbb{R}$ (with $b = 0$) From (2), $3a-\mathrm{i}(x+\mathrm{i}y)=-3+4\mathrm{i}$ $(3a+y)-\mathrm{i}x=-3+4\mathrm{i}$ Comparing imaginary parts, $x=-4$ Comparing real parts, $3a+y=-3$ From (1), $2a+\sqrt{x^2+y^2}=1$ $2a+\sqrt{16+(-3-3a)^2}=1$ $16+9(1+a)^2=(1-2a)^2$ $16+9(1+2a+a^2)=1-4a+4a^2$ $5a^2+22a+24=0$ $a=-2,\ -\frac{12}{5}$ then, $y=3,\ \frac{21}{5}$ So, possible solutions are $v=-2,\ w=-4+3\mathrm{i}$ and $v=-\frac{12}{5},\ w=-4+\frac{21}{5}\mathrm{i}$	Note that in this case $\lvert w\rvert$ does **not** mean w or $-w$. You can only say this when w is a real number. Since w is a complex number, then $\lvert w\rvert$ means the magnitude of the complex number w (i.e. $\sqrt{(\mathrm{Re}(w))^2+(\mathrm{Im}(w))^2}$)

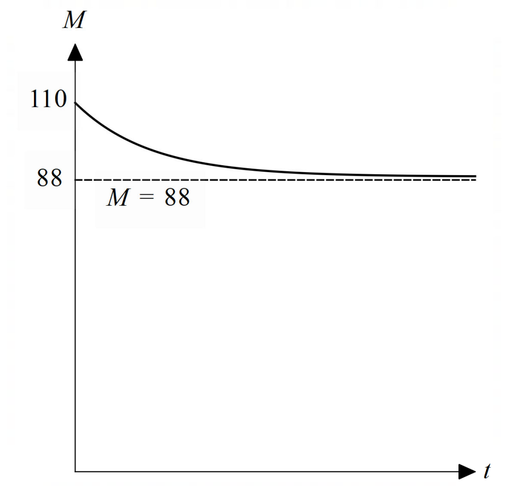
Question 9

No.	Suggested Solution	Remarks for Student
(a)	$l_1 : \mathbf{r} = \begin{pmatrix}-3\\1\\2\end{pmatrix} + \lambda\begin{pmatrix}2\\1\\a\end{pmatrix}, \ \lambda \in \mathbb{R}$ and $l_2 : \mathbf{r} = \begin{pmatrix}-2\\1\\5\end{pmatrix} + \mu\begin{pmatrix}3\\2\\1\end{pmatrix}, \ \mu \in \mathbb{R}$ Since l_1 and l_2 cross at the point *B*, $\begin{pmatrix}-3\\1\\2\end{pmatrix} + \lambda\begin{pmatrix}2\\1\\a\end{pmatrix} = \begin{pmatrix}-2\\1\\5\end{pmatrix} + \mu\begin{pmatrix}3\\2\\1\end{pmatrix}$, for some $\lambda, \mu \in \mathbb{R}$ $\Rightarrow \begin{cases} 2\lambda - 3\mu = 1 & -(1) \\ \lambda - 2\mu = 0 & -(2) \\ \lambda a - \mu = 3 & -(3) \end{cases}$ Using (1) and (2), $\lambda = 2$ and $\mu = 1$, hence using (3), $a = 2$ Using $\mu = 1$, $\mathbf{r} = \begin{pmatrix}-2\\1\\5\end{pmatrix} + \begin{pmatrix}3\\2\\1\end{pmatrix} = \begin{pmatrix}1\\3\\6\end{pmatrix}$ The coordinates of *B* is $(1, 3, 6)$.	
(b)(i)	Since π_1 is perpendicular to l_2, then a normal vector of π_1 is $\begin{pmatrix}3\\2\\1\end{pmatrix}$. $\overrightarrow{AB} = \begin{pmatrix}1\\3\\6\end{pmatrix} - \begin{pmatrix}-3\\1\\2\end{pmatrix} = \begin{pmatrix}4\\2\\4\end{pmatrix}$ The shortest distance from *B* to π_1 is $\frac{\left\lvert \begin{pmatrix}4\\2\\4\end{pmatrix} \cdot \begin{pmatrix}3\\2\\1\end{pmatrix} \right\rvert}{\sqrt{3^2+2^2+1^2}} = \frac{20}{\sqrt{14}}$	
(ii)	Let the required angle be θ. Then, $\sin\theta = \frac{\frac{20}{\sqrt{14}}}{\lvert\overrightarrow{AB}\rvert} = \frac{\frac{20}{\sqrt{14}}}{6} = \frac{10}{3\sqrt{14}}$ $\theta = 62.98^\circ = 63.0^\circ$ (correct to 1 d.p.) 	
(c)	Direction vectors of π_2 are $\begin{pmatrix}4\\2\\4\end{pmatrix}$ and $\begin{pmatrix}3\\2\\1\end{pmatrix}$	

	Normal vector to $\pi_2 = \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} \times \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} -3 \\ 4 \\ 1 \end{pmatrix}$ The equation of the plane π_2 is $\mathbf{r} . \begin{pmatrix} -3 \\ 4 \\ 1 \end{pmatrix} = \begin{pmatrix} -3 \\ 1 \\ 2 \end{pmatrix} . \begin{pmatrix} -3 \\ 4 \\ 1 \end{pmatrix} = 15$ So, cartesian equation of π_2 is $-3x + 4y + z = 15$	

Question 10

No.	Suggested Solution	Remarks for Student
(a)	Energy intake by Andrew per day = C Calories Energy expenditure by Andrew per day = $30M$ Calories Difference between energy intake and energy expenditure $= C-30M$ Since $\frac{\mathrm{d}M}{\mathrm{d}t} \propto C-30M$, then $\frac{\mathrm{d}M}{\mathrm{d}t} = k\left(C-30M\right)$, where k is the constant of proportionality.	
(b)	$\frac{\mathrm{d}M}{\mathrm{d}t} = 0$ when $M = 110$. So, $k\left(C-30(110)\right) = 0$ $C = 3300$ Calories per day	Note that mass is constant means $\frac{\mathrm{d}M}{\mathrm{d}t} = 0$
(c)	New $C = 0.8 \times 3300 = 2640$ Calories per day $\frac{\mathrm{d}M}{\mathrm{d}t} = k\left(2640-30M\right)$ $\int \frac{1}{2640-30M}\ \mathrm{d}M = \int k\ \mathrm{d}t$ $-\frac{1}{30}\ln\lvert 2640-30M \rvert = kt + D$ where D is an arbitrary constant $\lvert 2640-30M \rvert = \mathrm{e}^{-30(kt+D)}$ $2640-30M = A\mathrm{e}^{-30kt}$ where $A = \pm\mathrm{e}^{-30D}$ $30M = 2640 - A\mathrm{e}^{-30kt}$ $M = 88 + B\mathrm{e}^{-30kt}$ where $B = -\frac{A}{30}$ When $t = 0$, $M = 110$, $110 = 88 + B \Rightarrow B = 22$ Therefore, $M = 88 + 22\mathrm{e}^{-30kt}$ (Shown)	Remember to include the modulus Recall how to remove the modulus
(d)	When $t = 75$, $M = 100$, $100 = 88 + 22\mathrm{e}^{-30k(75)}$ $k = -\frac{1}{2250}\ln\left(\frac{6}{11}\right)$ Note that $k = -\frac{1}{2250}\ln\left(\frac{6}{11}\right) > 0$ For $M < 96$, $88 + 22\mathrm{e}^{-30kt} < 96$ $\mathrm{e}^{-30kt} < \frac{4}{11}$ $-30kt < \ln\left(\frac{4}{11}\right)$ $t > \frac{\ln\left(\frac{4}{11}\right)}{-30k} = \frac{75\ln\left(\frac{4}{11}\right)}{\ln\left(\frac{6}{11}\right)} = 125.17$	The variable t is measured from the time when Andrew's mass is initially 110 kg. But the question is asking for the number of

	Number of additional days needed is $126-75=51$	**additional** days to fall from 100 kg to 96 kg. Thus, there is a need to subtract by 75 days.
(e) **(i)**	 Andrew's mass is reducing and approaching 88 kg in the long run. Thus, Andrew cannot achieve a mass of 80 kg using this plan.	Note that t starts from 0, so do not draw with negative t-values. It is **important** to label the intercepts and the equation of asymptotes.
(ii)	From part **(c)**, it can be observed that for Andrew could achieve a mass of 80 kg, $0<\frac{C}{30}<80 \Rightarrow 0<C<2400$ The range of possible values of energy intake for which Andrew could achieve a mass of 80 kg is $\left(0,\ 2400\right)$	

Question 11

<table>
<tr><th>No.</th><th>Suggested Solution</th><th>Remarks for Student</th></tr>
<tr><td>(a)</td><td>Total amount saved
$=\$\left[a+\left(a+50\right)+\left(a+2(50)\right)+\left(a+3(50)\right)+...+\left(a+35(50)\right)\right]$
$=\$\left[\frac{36}{2}\left(a+\left(a+35(50)\right)\right)\right]$ OR $\$\left[\frac{36}{2}\left(2a+35(50)\right)\right]$
$=\$\left[18\left(2a+1750\right)\right]$
To save at least $50000.
$18\left(2a+1750\right)\geq 50000$
$\Rightarrow a\geq\frac{1}{2}\left(\frac{50000}{18}-1750\right)=513.888889$

The smallest value of <i>a</i> so that she saves at least $50000 is $513.89</td><td>From Jan 2021 to Dec 2023, there are a total of 36 months.

It is mentioned in the question, answer is to be rounded to the nearest cent, and not 3 s.f. Also, $513.90 is not correct as this to the nearest ten cents.</td></tr>
<tr><td>(b)</td><td>
<table>
<tr><th>Month</th><th>Amount owed at the end of <i>n</i>th month ($)</th></tr>
<tr><td>1</td><td>$400000\times1.001-x$</td></tr>
<tr><td>2</td><td>$\left(400000\times1.001-x\right)\times1.001-x$
$=\ 400000\times1.001^2-1.001x-x$</td></tr>
<tr><td>3</td><td>$\left(400000\times1.001^2-1.001x-x\right)\times1.001-x$
$=\ 400000\times1.001^3-1.001^2x-1.001x-x$</td></tr>
<tr><td>4</td><td>$\left(400000\times1.001^3-1.001^2x-1.001x-x\right)\times1.001-x$
$=\ 400000\times1.001^4-1.001^3x-1.001^2x-1.001x-x$</td></tr>
</table>
The amount, in dollars, that Wei owes at the end of <i>n</i>th months
$=\left(400000\times1.001^n\right)-1.001^{n-1}x-1.001^{n-2}x-...-1.001^2x-1.001x-x$
$=400000\times1.001^n-\frac{x\left(1.001^n-1\right)}{1.001-1}$
$=400000\times1.001^n-1000x\left(1.001^n-1\right)$ (Shown)</td><td>Note that 0.1% is equivalent to $\frac{0.1}{100}=0.001$.</td></tr>
<tr><td>(c)(i)</td><td>Since the loan is repaid in 360 monthly repayments, then
$400\,000\times1.001^{360}-1000x\left(1.001^{360}-1\right)\leq0$
$1000x\left(1.001^{360}-1\right)\geq400\,000\times1.001^{360}$
$x\geq\frac{400\,000\times1.001^{360}}{1000\left(1.001^{360}-1\right)}$
$=1323.634776$
The value of the monthly repayment to the nearest cent is $1323.64</td><td>As this is a contextual question and stated that the loan was repaid in 360 months, rounding $1323.634776 down to $1323.63 does not actually repay the loan.</td></tr>
</table>

(c)(ii)	Total interest paid on the loan is $\$\left(1323.64\times360-400000\right)=\$76510.40$	Instead of using a more accurate answer in **(c)(i),** should follow the instruction in the question to use the answer from **(c)(i)** which is more appropriate in the context of the question
(d)(i)	Let $L=400\,000\times1.001^{n}-1000\left(1600\right)\left(1.001^{n}-1\right)$ If repayment of the loan is at the rate of $\$1600$ per month, then for $L\geq0$, we get $n\leq287.82589$ So, Wei repays the loan at the rate of $\$1600$ per month for 287 months. When n = 287, L = 1320.21799 The final monthly repayment amount is $\$1320.21799\times1.001$ $=\$1321.5382$ $\approx\$1321.54$ (to the nearest cent) So, k = 287 and y = 1321.54	
(d)(ii)	Total interest paid on the loan in **(d)(i)** is $\$\left(1600\times287+1321.54-400000\right)=\$60521.54$ Total saving to Wei $=\ \$76510.40-\$60521.54=\$15988.86\approx\$15990$ (to 4 s.f.)	The question asked for the answer to 4 s.f.