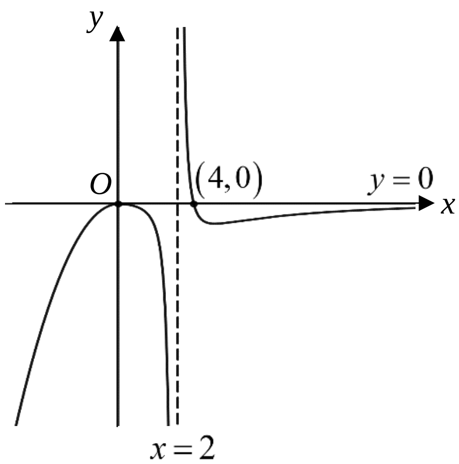
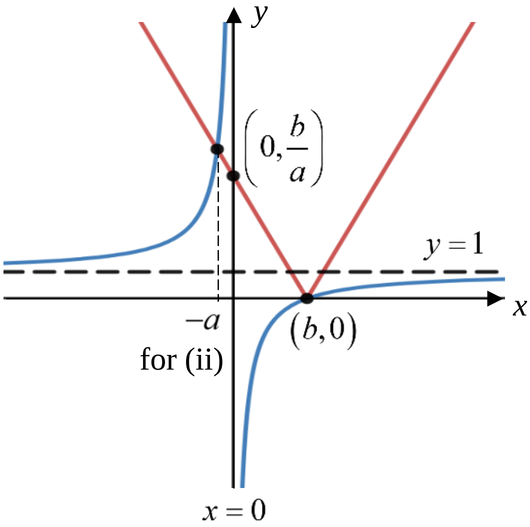
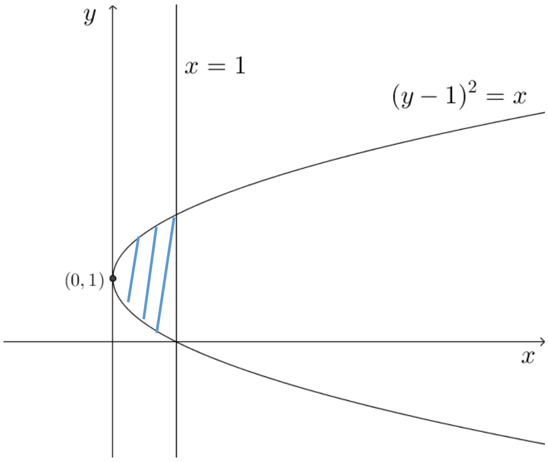
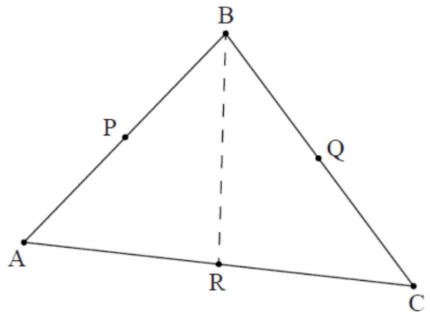
Qn	Suggested Solutions
1	Let \$$A$, \$$B$ and \$$C$ be the selling price of 1 kg of salmon, 1 kg of tuna and 1 kg of swordfish respectively before discount. $2A+B+3C=246 \quad \cdots(1)$ $A+C=B+18$ $\therefore A-B+C=18 \quad \cdots(2)$ $0.85A+0.9B+0.95C=123.3 \quad \cdots(3)$ By G.C., $A=48, B=60, C=30$ The selling price of 1 kg of salmon, tuna and swordfish before discount is \$48, \$60 and \$30 respectively.

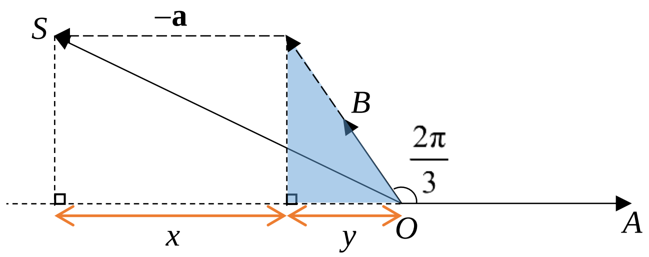
Qn	Suggested Solutions
2(a)	
(b)	

$y=\mathrm{h}(x)$	$y=\dfrac{1}{\mathrm{h}(x)}$	$y=\dfrac{1}{\mathrm{h}(x-p)}$	$y=\dfrac{1}{\mathrm{h}(-x-p)}$	$y=\dfrac{1}{\mathrm{h}(-x-p)}+q$
$y=-2$	$y=-0.5$	$y=-0.5$	$y=-0.5$	$y=-0.5+q=1$ $\Rightarrow q=1.5$
$(-1,0)$	$x=-1$	$x=-1+p$	$x=1-p$	$x=1-p=-2$ $\Rightarrow p=3$

Qn	**Suggested Solutions**
3(i)	
(ii)	Solve for intersection between the curves: $\frac{x-b}{x} = \frac{1}{a}\lvert x-b \rvert$ $\frac{x-b}{x} = \frac{1}{a}(x-b)$ or $\frac{x-b}{x} = -\frac{1}{a}(x-b)$ $(x-b)\left(\frac{1}{x} - \frac{1}{a}\right) = 0$ or $(x-b)\left(\frac{1}{x} + \frac{1}{a}\right) = 0$ $x = b$ or $\frac{1}{x} = \frac{1}{a} \Rightarrow x = a$ or $x = b$ or $\frac{1}{x} = -\frac{1}{a} \Rightarrow x = -a$ Therefore, we have $x = b$ or $x = a$ or $x = -a$ From graph in part **(i)**, $x \le -a$ or $x > 0$

Qn	Suggested Solutions
4(a)	$\int \frac{x+1}{4+3x^2}\ \mathrm{d}x.$ $= \int \frac{x}{4+3x^2}\ \mathrm{d}x + \int \frac{1}{4+3x^2}\ \mathrm{d}x$ $= \frac{1}{6}\int \frac{6x}{4+3x^2}\ \mathrm{d}x + \frac{1}{\sqrt{3}}\int \frac{\sqrt{3}}{2^2+\left(\sqrt{3}x\right)^2}\ \mathrm{d}x$ $= \frac{1}{6}\ln\left(4+3x^2\right) + \frac{1}{2\sqrt{3}}\tan^{-1}\left(\frac{\sqrt{3}x}{2}\right) + A$
(b)	 $\left(y-1\right)^2 = x$ $y-1 = \pm\sqrt{x}$ $y = \sqrt{x}+1$ or $y = -\sqrt{x}+1$ Volume required $= \pi\int_0^1 \left(\sqrt{x}+1\right)^2\ \mathrm{d}x - \pi\int_0^1 \left(-\sqrt{x}+1\right)^2\ \mathrm{d}x$ $= 8.3776$ (4 d.p.) by GC
	Alternatively (Shell Method) – For Teacher's Ref only Volume required $= 2\pi\int_0^2 y\left[1-\left(y-1\right)^2\right]\mathrm{d}y$ $= 8.3776$ (4 d.p.) by GC

Qn	Suggested Solutions
5(i)	 By Ratio Theorem, $\overrightarrow{BR}=\frac{\overrightarrow{BA}+\overrightarrow{BC}}{2}=\frac{\mathbf{a}-\mathbf{b}+\mathbf{c}-\mathbf{b}}{2}=\frac{1}{2}(\mathbf{a}-2\mathbf{b}+\mathbf{c})$ $l_{BR}:\ \mathbf{r}=\mathbf{b}+\lambda\overrightarrow{BR}=\mathbf{b}+\frac{\lambda}{2}(\mathbf{a}-2\mathbf{b}+\mathbf{c}),\lambda\in\mathbb{R}$
(ii)	Given that $l_{AQ}:\ \mathbf{r}=\mathbf{a}+\mu(-2\mathbf{a}+\mathbf{b}+\mathbf{c}),\mu\in\mathbb{R}$ To show that lines *BR* and *AQ* meet, we let $\mathbf{b}+\frac{\lambda}{2}(\mathbf{a}-2\mathbf{b}+\mathbf{c})=\mathbf{a}+\mu(-2\mathbf{a}+\mathbf{b}+\mathbf{c})$ $\frac{\lambda}{2}\mathbf{a}+(1-\lambda)\mathbf{b}+\frac{\lambda}{2}\mathbf{c}=(1-2\mu)\mathbf{a}+\mu\mathbf{b}+\mu\mathbf{c}$ Since the points *O*, *A*, *B* and *C* do not lie on the same plane, we can compare the coefficients of **a**, **b** and **c** to obtain $\frac{\lambda}{2}=1-2\mu \quad \cdots(1)$ $1-\lambda=\mu \quad \cdots(2)$ $\frac{\lambda}{2}=\mu \quad \cdots(3)$ Sub (3) into (2), we obtain $1-\lambda=\frac{\lambda}{2}\Rightarrow\lambda=\frac{2}{3}$. Hence $\overrightarrow{OG}=\mathbf{b}+\frac{1}{3}(\mathbf{a}-\mathbf{2b}+\mathbf{c})=\frac{1}{3}(\mathbf{a}+\mathbf{b}+\mathbf{c})$
(iii)	Length of projection of $\overrightarrow{OS}$ onto $\overrightarrow{OA}$ $=\left\lvert\overrightarrow{OS}\cdot\frac{\overrightarrow{OA}}{\lvert\overrightarrow{OA}\rvert}\right\rvert$

	$=\left\lvert\dfrac{(2\mathbf{b}-\mathbf{a})\cdot\mathbf{a}}{\lvert\mathbf{a}\rvert}\right\rvert$ $=\left\lvert\dfrac{2\mathbf{b}\cdot\mathbf{a}-\mathbf{a}\cdot\mathbf{a}}{\lvert\mathbf{a}\rvert}\right\rvert$ $=\left\lvert\dfrac{2\lvert\mathbf{b}\rvert\lvert\mathbf{a}\rvert\cos\frac{2\pi}{3}-\lvert\mathbf{a}\rvert^2}{\lvert\mathbf{a}\rvert}\right\rvert$ $=\left\lvert 2\lvert\mathbf{b}\rvert\left(-\dfrac{1}{2}\right)-\lvert\mathbf{a}\rvert\right\rvert$ $=\left\lvert -\lvert\mathbf{b}\rvert-\lvert\mathbf{a}\rvert\right\rvert$ $=\lvert -1\rvert\left\lvert \lvert\mathbf{b}\rvert+\lvert\mathbf{a}\rvert\right\rvert$ $=\lvert\mathbf{a}\rvert+\lvert\mathbf{b}\rvert$
	Geometrical Method  The length of projection of $\overrightarrow{OS}$ onto $\overrightarrow{OA}=x+y$ $x=\lvert-\mathbf{a}\rvert=\lvert\mathbf{a}\rvert$ To find y: Consider the shaded right-angled triangle, $\cos\dfrac{\pi}{3}=\dfrac{y}{2\lvert\mathbf{b}\rvert}$ $y=\lvert\mathbf{b}\rvert$ Therefore, length of projection of $\overrightarrow{OS}$ onto $\overrightarrow{OA}$ $=x+y=\lvert\mathbf{a}\rvert+\lvert\mathbf{b}\rvert$

<table>
<tr><th>Qn</th><th>Suggested Solutions</th></tr>
<tr><td>6(i)</td><td>

$x = m\tan t \Rightarrow \dfrac{dx}{dt} = m\sec^2 t$

$\displaystyle\int \frac{1}{\sqrt{m^2+x^2}}\ dx$

$\displaystyle= \int \frac{1}{\sqrt{m^2+m^2\tan^2 t}}\, m\sec^2 t\ dt$

$\displaystyle= \frac{1}{m} m \int \frac{1}{\sqrt{1+\tan^2 t}} \sec^2 t\ dt \quad \because \sqrt{m^2} = |m| = m \text{ since } m>0$

$\displaystyle= \int \frac{1}{\sqrt{\sec^2 t}} \sec^2 t\ dt$

$\displaystyle= \int \sec t\ dt \quad \because \sqrt{\sec^2 t} = |\sec t| = \sec t \text{ since } 0<t<\frac{\pi}{2}$

$= \ln|\sec t + \tan t| + C$

$\displaystyle= \ln\left|\frac{\sqrt{x^2+m^2}+x}{m}\right| + C$

OR $\ln\left|\sqrt{x^2+m^2}+x\right| + A$

</td></tr>
<tr><td>(ii)</td><td>

$\displaystyle\int \frac{x}{\sqrt{m^2+x^2}}\ dx = \frac{1}{2}\frac{\left(m^2+x^2\right)^{\frac{1}{2}}}{\frac{1}{2}} + C$

$= \sqrt{m^2+x^2} + C$

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<tr><td>(iii)</td><td>

$\displaystyle\int \frac{x}{\sqrt{m^2+x^2}} \tan^{-1}\left(\frac{x}{m}\right)\ dx$

$u = \tan^{-1}\left(\dfrac{x}{m}\right), \quad \dfrac{dv}{dx} = \dfrac{x}{\sqrt{m^2+x^2}}$

$u = \tan^{-1}\left(\dfrac{x}{m}\right) \Rightarrow \dfrac{du}{dx} = \dfrac{\frac{1}{m}}{1+\left(\frac{x}{m}\right)^2} = \dfrac{m}{m^2+x^2}$

From part (ii),

$\dfrac{dv}{dx} = \dfrac{1}{2}(2x)\left(m^2+x^2\right)^{-\frac{1}{2}} \Rightarrow v = \dfrac{1}{2}\dfrac{\left(m^2+x^2\right)^{\frac{1}{2}}}{\frac{1}{2}} = \sqrt{m^2+x^2}$

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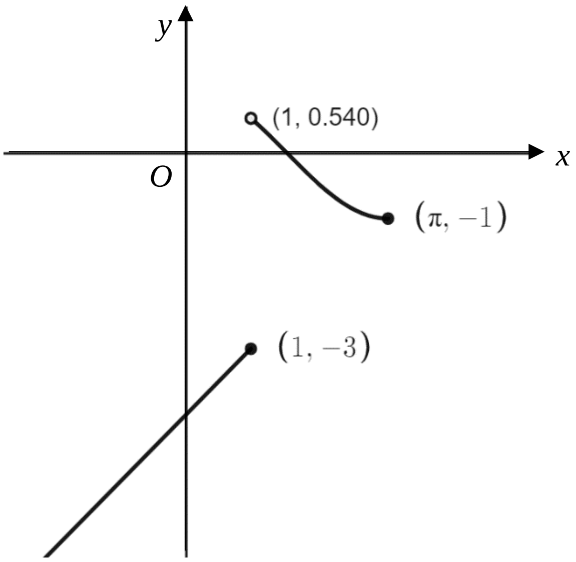
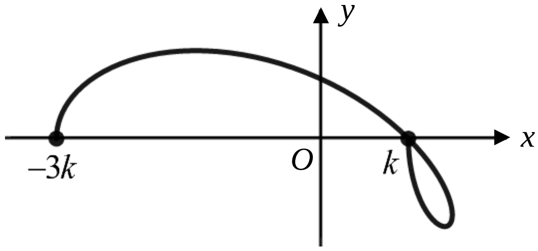
Qn	Suggested Solutions
	$\int \frac{x}{\sqrt{m^2+x^2}} \tan^{-1}\left(\frac{x}{m}\right) \, \mathrm{d}x$ $= \left[\tan^{-1}\left(\frac{x}{m}\right)\right]\sqrt{m^2+x^2} - \int \sqrt{m^2+x^2} \, \frac{m}{m^2+x^2} \, \mathrm{d}x$ $= \left[\tan^{-1}\left(\frac{x}{m}\right)\right]\sqrt{m^2+x^2} - m\int \frac{1}{\sqrt{m^2+x^2}} \, \mathrm{d}x$ $= \sqrt{m^2+x^2} \tan^{-1}\left(\frac{x}{m}\right) - m\ln\left\|\frac{\sqrt{x^2+m^2}+x}{m}\right\| + D$ from (i) OR $\sqrt{m^2+x^2} \tan^{-1}\left(\frac{x}{m}\right) - m\ln\left\|\sqrt{x^2+m^2}+x\right\| + K$ from (i)

Qn	Suggested Solutions
7(i)	$3x^2+2xy-y^2+k=0$ Differentiating with respect to x: $6x+(2x)\frac{dy}{dx}+y(2)-2y\frac{dy}{dx}=0$ $6x+2y=(2y-2x)\frac{dy}{dx}$ $\frac{dy}{dx}=\frac{3x+y}{y-x}$
(ii)	At stationary points, $\frac{dy}{dx}=\frac{3x+y}{y-x}=0$. $\Rightarrow 3x+y=0 \Rightarrow y=-3x$ Sub $y=-3x$ into equation of C: $3x^2+2x(-3x)-(-3x)^2+k=0$ $3x^2-6x^2-9x^2+k=0$ $12x^2=k$ $x^2=\frac{k}{12}$ For C to have no stationary points, $x^2=\frac{k}{12}$ has no solution. Since $x^2\geq 0$, therefore $k<0$.
(iii)	Tangents parallel to the y-axis $\Rightarrow \frac{dy}{dx}$ is undefined (or $\frac{dx}{dy}=0$). Thus, we have $y-x=0\Rightarrow y=x$. Sub $y=x$ into equation of C: $3x^2+2x^2-x^2+k=0$ $4x^2=k$ $x^2=-\frac{k}{4}$ $x=\sqrt{\frac{-k}{4}}$ or $x=-\sqrt{\frac{-k}{4}}$ $x=\frac{\sqrt{-k}}{2}$ or $x=-\frac{\sqrt{-k}}{2}$ $\therefore \beta=\frac{\sqrt{-k}}{2}, \alpha=-\frac{\sqrt{-k}}{2}$ (Given that $k<-1$ so $-k>1$. Thus $\sqrt{-k}$ is well-defined.)
	Sub $x=\frac{\sqrt{-k}}{2}$ into C:

Qn	Suggested Solutions
	$3\left(\frac{\sqrt{-k}}{2}\right)^2+2\left(\frac{\sqrt{-k}}{2}\right)y-y^2+k=0$ $3\left(-\frac{k}{4}\right)+\sqrt{-k}\,y-y^2+k=0$ $y^2-\sqrt{-k}\,y-\frac{k}{4}=0$ $\left(y-\frac{\sqrt{-k}}{2}\right)^2=0\Rightarrow y=\frac{\sqrt{-k}}{2}$ OR Discriminant $=\left(-\sqrt{-k}\right)^2-4\left(-\frac{k}{4}\right)=0$, thus only one solution. Since there is only one solution, the tangent meets C at exactly one point. Therefore, the tangent does not intersect C again.

Qn	Suggested Solutions
8(i)	$\mathrm{f}(x)=\begin{cases} x-4 & \text{for} \quad x\le 1, \\ \cos x & \text{for} \quad 1<x\le \pi. \end{cases}$  Since every horizontal line $y=k$, $k\in\mathbb{R}$ cuts the graph of $y=\mathrm{f}(x)$ **<u>at most once</u>**, f is **<u>one-one</u>**. Therefore f has an inverse. OR Since every horizontal line $y=k$, $\underline{k\in(-\infty,-3]\cup[-1,0.540)}$ cuts the graph of $y=\mathrm{f}(x)$ **<u>exactly once</u>**, f is **<u>one-one</u>**. Therefore f has an inverse.
(ii)	$\mathrm{f}(x)=\begin{cases} x-4 & \text{for} \quad x\le 1, \\ \cos x & \text{for} \quad 1<x\le \pi. \end{cases}$ When $x\le 1$, $y=x-4\Rightarrow x=y+4$. From graph of $y=\mathrm{f}(x)$, range is $(-\infty,-3]$. When $1<x\le\pi$, $y=\cos x\Rightarrow x=\cos^{-1}y$. From graph of $y=\mathrm{f}(x)$, range is $[-1,0.540)$. $\therefore \mathrm{f}^{-1}(x)=\begin{cases} x+4 & \text{for} \quad x\le -3, \\ \cos^{-1}x & \text{for} \quad -1\le x<0.540 \end{cases}$
(iii)	$\mathrm{f}^2\left(\frac{\pi}{2}\right)=\mathrm{ff}\left(\frac{\pi}{2}\right)=\mathrm{f}\left(\cos\frac{\pi}{2}\right)=\mathrm{f}(0)=-4$

Qn	**Suggested Solutions**
	For $x \le 1$, the range for $y = x - 4$ is $(-\infty, -3]$. Since $(-\infty,-3] \subseteq (-\infty,-1]$, $\mathrm{f}^2(x) = \mathrm{f}(x-4) = (x-4)-4$ for $x \le 1$. For $1 < x \le \pi$, the range for $y = \cos x - 4$ is $[-1, 0.540)$. Since $[-1, 0.540) \subseteq (-\infty, -1]$, $\mathrm{f}^2(x) = \mathrm{f}(\cos x - 4) = (\cos x - 4) - 4 = \cos x - 8$ for $1 < x \le \pi$. Hence, $\mathrm{f}^2(x) = \begin{cases} x-8 & \text{for } x \le 1, \\ \cos x - 4 & \text{for } 1 < x \le \pi \end{cases}$
(iv)	When $x \le 1$, we apply the rule $\mathrm{f}(x) = x - 4$. By observing pattern: $\mathrm{f}^{2022}(0) = \mathrm{f}^{2021}(0-4)$ $= \mathrm{f}^{2020}(-4-4)$. $= -4 \times 2022$ $= -8088$

Qn	Suggested Solutions
9(i)	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{d}y}{\mathrm{d}t}\div\frac{\mathrm{d}x}{\mathrm{d}t}=\frac{-2k\cos 2\theta}{k\left(-2\sin\theta+2\sin 2\theta\right)}$ Let $\frac{\mathrm{d}y}{\mathrm{d}x}=0$. We have $\cos 2\theta=0$. $2\theta=\frac{\pi}{2}$ or $\frac{3\pi}{2}\Rightarrow\theta=\frac{\pi}{4}$ or $\frac{3\pi}{4}$ For $\theta=\frac{\pi}{4}$, $x=k\left(2\cos\frac{\pi}{4}-\cos\frac{\pi}{2}\right)=\sqrt{2}k, y=-k\sin\frac{\pi}{2}=-k$ For $\theta=\frac{3\pi}{4}$, $x=k\left(2\cos\frac{3\pi}{4}-\cos\frac{3\pi}{2}\right)=-\sqrt{2}k, y=-k\sin\frac{3\pi}{2}=k$ Therefore the stationary points are $\left(\sqrt{2}k,-k\right)$ and-$\left(-\sqrt{2}k,k\right)$.
(ii)	
(iii)	Area of required region is $\int_{-3k}^{k} y\ \mathrm{d}x=\int_{\pi}^{\frac{\pi}{2}}-k\sin 2\theta\left[k\left(-2\sin\theta+2\sin 2\theta\right)\right]\ \mathrm{d}\theta$ $=\int_{\frac{\pi}{2}}^{\pi}k\sin 2\theta\left[k\left(-2\sin\theta+2\sin 2\theta\right)\right]\ \mathrm{d}\theta$ $=k^2\int_{\frac{\pi}{2}}^{\pi}\left(2\sin^2 2\theta-2\sin\theta\sin 2\theta\right)\ \mathrm{d}\theta$
(iv)	From part **(iii)**, area of required region $=k^2\int_{\frac{\pi}{2}}^{\pi}2\sin^2 2\theta-2\sin\theta\sin 2\theta\ \mathrm{d}\theta$ $=k^2\int_{\frac{\pi}{2}}^{\pi}\left(1-\cos 4\theta\right)-2\sin\theta\left(2\sin\theta\cos\theta\right)\ \mathrm{d}\theta$ $=k^2\int_{\frac{\pi}{2}}^{\pi}1-\cos 4\theta-4\cos\theta\sin^2\theta\ \mathrm{d}\theta$ $=k^2\left[\theta-\frac{\sin 4\theta}{4}-4\left(\frac{\sin^3\theta}{3}\right)\right]_{\frac{\pi}{2}}^{\pi}$ $=k^2\left[\left(\pi-0-0\right)-\left(\frac{\pi}{2}-0-\frac{4}{3}\right)\right]$ $=k^2\left(\frac{4}{3}+\frac{\pi}{2}\right)$

<table>
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<tr><td>10(i)</td><td>
Let the plane representing the ground be p_g.

$$p_g : \mathbf{r} \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = 0, \quad p_1 : \mathbf{r} \cdot \begin{pmatrix} 1 \\ 0 \\ -5 \end{pmatrix} = -2, \quad p_2 : \mathbf{r} \cdot \begin{pmatrix} 4 \\ 1 \\ -n \end{pmatrix} = 30.$$

Let θ_{gp_1} be the acute angle between the ground and p_1.
Let θ_{gp_2} be the acute angle between the ground and p_2.

Since p_2 is steeper than p_1 relative to the ground,

$$\theta_{gp_2} > \theta_{gp_1}$$
$$\cos\theta_{gp_2} < \cos\theta_{gp_1}$$

(Note the change in inequality sign, because $\cos\theta$ is a decreasing function for acute angle θ.)

$$\frac{\left|\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 1 \\ -n \end{pmatrix}\right|}{\sqrt{1}\sqrt{4^2+1^2+(-n)^2}} < \frac{\left|\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ -5 \end{pmatrix}\right|}{\sqrt{1}\sqrt{1^2+(-5)^2}}$$
$$\frac{|n|}{\sqrt{17+n^2}} < \frac{|-5|}{\sqrt{26}}$$
$$\left(\frac{|n|}{\sqrt{17+n^2}}\right)^2 < \left(\frac{|-5|}{\sqrt{26}}\right)^2$$
$$\frac{n^2}{17+n^2} < \frac{25}{26}$$
$$26n^2 < 25\left(17+n^2\right)$$
$$n^2 < 425$$
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<u>Alternative Method</u> (consider $\sin\theta$)

$$\theta_{gp_2} > \theta_{gp_1}$$
$$\sin\theta_{gp_2} > \sin\theta_{gp_1}$$

(Note that there is NO change in inequality sign, because $\sin\theta$ is a increasing function for acute angle θ.)

$$\frac{\left|\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \times \begin{pmatrix} 4 \\ 1 \\ -n \end{pmatrix}\right|}{\sqrt{1}\sqrt{4^2+1^2+(-n)^2}} > \frac{\left|\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \times \begin{pmatrix} 1 \\ 0 \\ -5 \end{pmatrix}\right|}{\sqrt{1}\sqrt{1^2+(-5)^2}}$$
</td></tr>
<tr><td>(ii)</td><td>
$$\begin{pmatrix} 30 \\ 30 \\ 40 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 1 \\ -n \end{pmatrix} = 30 \Rightarrow 120 + 30 - 40n = 30 \Rightarrow n = 3$$
</td></tr>
</table>

Qn	Suggested Solutions
(iii)	$p_1: \mathbf{r}\cdot\begin{pmatrix}1\\0\\-5\end{pmatrix}=-2, \quad p_2: \mathbf{r}\cdot\begin{pmatrix}4\\1\\-3\end{pmatrix}=30$ **<u>VERIFY (Method 1)</u>** $\begin{pmatrix}8+5\lambda\\4-17\lambda\\2+\lambda\end{pmatrix}\cdot\begin{pmatrix}1\\0\\-5\end{pmatrix}=8+5\lambda-10-5\lambda=-2$ $\begin{pmatrix}8+5\lambda\\4-17\lambda\\2+\lambda\end{pmatrix}\cdot\begin{pmatrix}4\\1\\-3\end{pmatrix}=32+20\lambda+4-17\lambda-6-3\lambda=30$ Hence *l* lies in both p_1 and p_2. **<u>VERIFY (Method 2)</u>** $\begin{pmatrix}5\\-17\\1\end{pmatrix}\cdot\begin{pmatrix}1\\0\\-5\end{pmatrix}=5-5=0$ and $\begin{pmatrix}5\\-17\\1\end{pmatrix}\cdot\begin{pmatrix}4\\1\\-3\end{pmatrix}=20-17-3=0$ $\therefore \begin{pmatrix}5\\-17\\1\end{pmatrix}$ is parallel to both p_1 and p_2. $\begin{pmatrix}8\\4\\2\end{pmatrix}\cdot\begin{pmatrix}1\\0\\-5\end{pmatrix}=8-10=-2$ and $\begin{pmatrix}8\\4\\2\end{pmatrix}\cdot\begin{pmatrix}4\\1\\-3\end{pmatrix}=32+4-6=30$ $\therefore \begin{pmatrix}8\\4\\2\end{pmatrix}$ is on both p_1 and p_2. Hence *l* lies in both p_1 and p_2. **<u>SHOW</u>** To find a direction vector of line *l*: $\begin{pmatrix}1\\0\\-5\end{pmatrix}\times\begin{pmatrix}4\\1\\-3\end{pmatrix}=\begin{pmatrix}(0)(-3)-(-5)(1)\\(-5)(4)-(1)(-3)\\(1)(1)-(0)(4)\end{pmatrix}=\begin{pmatrix}5\\-17\\1\end{pmatrix}$ To find a position vector of a point on *l*: $x-5z=-2$ --- (1) $4x+y-3z=30$ --- (2) Let $x=8$ From (1): $8-5z=-2 \Rightarrow z=2$ From (2): $4(8)+y-3(2)=30 \Rightarrow y=30-32+6=4$ $\therefore (8, 4, 2)$ is a point of intersection between p_1 and p_2. Hence, $l: \mathbf{r}=\begin{pmatrix}8\\4\\2\end{pmatrix}+\lambda\begin{pmatrix}5\\-17\\1\end{pmatrix}, \lambda\in\mathbb{R}.$

Qn	Suggested Solutions
(iv)	For BT to be minimum, BT must be perpendicular to boundary i.e. B is the foot of perpendicular from T to l. $\overrightarrow{OB} = \begin{pmatrix} 8+5\mu \\ 4-17\mu \\ 2+\mu \end{pmatrix}$ for some $\mu \in \mathbb{R}$ $\overrightarrow{BT} = \overrightarrow{OT} - \overrightarrow{OB} = \begin{pmatrix} 30 \\ 30 \\ 40 \end{pmatrix} - \begin{pmatrix} 8+5\mu \\ 4-17\mu \\ 2+\mu \end{pmatrix} = \begin{pmatrix} 22-5\mu \\ 26+17\mu \\ 38-\mu \end{pmatrix}$ $\overrightarrow{BT} \cdot \begin{pmatrix} 5 \\ -17 \\ 1 \end{pmatrix} = 0$ $\begin{pmatrix} 22-5\mu \\ 26+17\mu \\ 38-\mu \end{pmatrix} \cdot \begin{pmatrix} 5 \\ -17 \\ 1 \end{pmatrix} = 0$ $110 - 25\mu - 442 - 289\mu + 38 - \mu = 0$ $315\mu = -294 \Rightarrow \mu = -\frac{14}{15}$ $\overrightarrow{OB} = \begin{pmatrix} 8+5\left(-\frac{14}{15}\right) \\ 4-17\left(-\frac{14}{15}\right) \\ 2-\frac{14}{15} \end{pmatrix} = \begin{pmatrix} \frac{10}{3} \\ \frac{298}{15} \\ \frac{16}{15} \end{pmatrix}$ Total distance travelled from B to T $= BT$ $= \sqrt{\left(\frac{10}{3}-30\right)^2 + \left(\frac{298}{15}-30\right)^2 + \left(\frac{16}{15}-40\right)^2}$ $= \sqrt{2329.6}$ Stamina expended $= \frac{\sqrt{2329.6}}{2} = 24.1$ (3 s.f.)
(v)	Let l_{TR} be the line representing journey taken from T to R.

Qn	Suggested Solutions
	$l_{TR}: \mathbf{r} = \begin{pmatrix} 30 \\ 30 \\ 40 \end{pmatrix} + \gamma \begin{pmatrix} 10 \\ 3a \\ 9a \end{pmatrix}, \gamma \in \mathbb{R}$ $\begin{pmatrix} 30+10\gamma \\ 30+3a\gamma \\ 40+9a\gamma \end{pmatrix} = \begin{pmatrix} 90 \\ 0 \\ 45 \end{pmatrix}$ $30+10\gamma = 90$ ---(1) $30+3a\gamma = 0$ ---(2) $40+9a\gamma = 45$ ---(3) From (1): $10\gamma = 60 \Rightarrow \gamma = 6$ From (2): $3a\gamma = -30 \Rightarrow a = \frac{-30}{3\gamma} = -\frac{5}{3}$ From (3): $9a\gamma = 5 \Rightarrow a = \frac{5}{9\gamma} = \frac{5}{54} \neq -\frac{5}{3}$ Since there is no consistent solution for *a*, avatar will not be able to reach the treasure. **Alternative Method** Find value of *a* such that $\begin{pmatrix} 10 \\ 3a \\ 9a \end{pmatrix}$ is parallel to p_3: $\begin{pmatrix} 10 \\ 3a \\ 9a \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ -12 \end{pmatrix} = 0$ $10 - 108a = 0$ $a = \frac{5}{54}$ Using this value of *a*, determine if *R* lies in the path taken:

Qn	Suggested Solutions
	$\begin{pmatrix} 30+10\gamma \\ 30+\frac{15\gamma}{54} \\ 40+\frac{45\gamma}{54} \end{pmatrix} = \begin{pmatrix} 90 \\ 0 \\ 45 \end{pmatrix}$ $30+10\gamma=90$ ---(1) $30+\frac{15\gamma}{54}=0$ ---(2) $40+\frac{45\gamma}{54}=45$ ---(3) From (1): $10\gamma=60 \Rightarrow \gamma=6$ From (2): $\frac{15\gamma}{54}=-30 \Rightarrow \gamma=-108 \neq 6$ Since there is no consistent solution for γ, avatar will not be able to reach the treasure.

<table>
<tr><th>Qn</th><th>Suggested Solutions</th></tr>
<tr><td>11(i)</td><td>

$$\begin{aligned}C &= 5(180)(AX)+180(XB)\\ &= 900\sqrt{480^2+x^2}+180(1000-x)\\ &= 900\sqrt{230400+x^2}+180(1000-x)\end{aligned}$$

</td></tr>
<tr><td>(ii)</td><td>

$$C = 900\sqrt{230400+x^2}+180(1000-x)$$

$$\begin{aligned}\frac{\mathrm{d}C}{\mathrm{d}x} &= 180\left(\frac{10x}{2\sqrt{230400+x^2}}-1\right)\\ &= 180\left(\frac{5x}{\sqrt{230400+x^2}}-1\right)\end{aligned}$$

$$\frac{\mathrm{d}C}{\mathrm{d}x} > 0$$

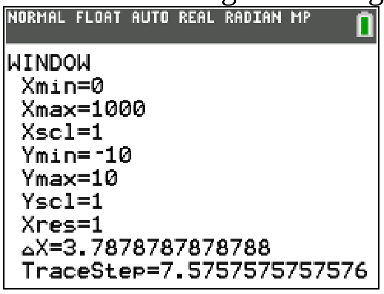
$$180\left(\frac{5x}{\sqrt{230400+x^2}}-1\right) > 0$$

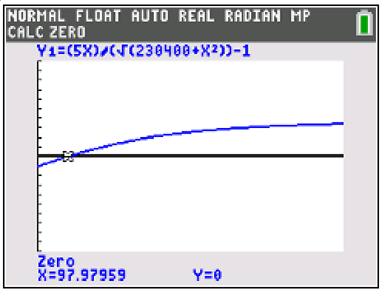
$$\frac{5x}{\sqrt{230400+x^2}}-1 > 0$$

<u>Graphical Method</u>

We sketch the graph of $y = \frac{5x}{\sqrt{230400+x^2}}-1$.

In the context of the question, $0 < x < 1000$, thus we adjust the WINDOW settings accordingly.





Since $x < 1000$ due to the context, we have $98.0 < x < 1000$.

<u>Algebraic Method</u>

$$\frac{5x}{\sqrt{230400+x^2}} > 1$$

$$5x > \sqrt{230400+x^2} \quad \text{since } \sqrt{230400+x^2} > 0$$

$$25x^2 > 230400+x^2 \quad \text{since both sides positive}$$

$$24x^2 > 230400$$

$$x^2 > 9600$$

$$\therefore x > \sqrt{9600} \quad \left(\text{or } 40\sqrt{6}\right) \quad \text{since } x > 0$$

Since $x < 1000$ due to the context, we have $\sqrt{9600} < x < 1000$.

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<table>
<tr><th>Qn</th><th>Suggested Solutions</th></tr>
<tr><td>(iii)</td><td>

$\tan\angle AXE = \dfrac{480}{x}$

Since $\angle AXE + \theta = \pi$,

$\tan(\pi-\theta) = \dfrac{480}{x}$

$x = \dfrac{480}{\tan(\pi-\theta)} = 480\cot(\pi-\theta)$

<u>Algebraic Method</u>

$\dfrac{2\pi}{3} \le \theta \le \dfrac{3\pi}{4}$

$\dfrac{\pi}{4} \le \pi-\theta \le \dfrac{\pi}{3}$

$1 \le \tan(\pi-\theta) \le \sqrt{3}$

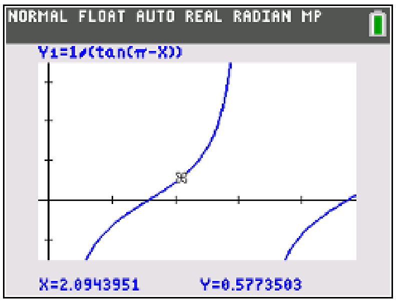
$\dfrac{1}{\sqrt{3}} \le \tan(\pi-\theta) \le 1$

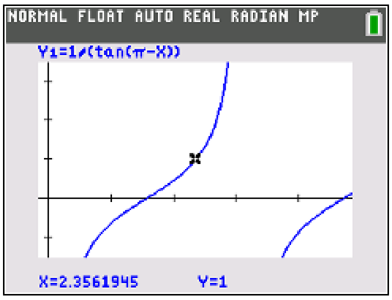
$\dfrac{480}{\sqrt{3}} \le \dfrac{480}{\tan(\pi-\theta)} \le 480$

Hence, $\dfrac{480}{\sqrt{3}} \le x \le 480$.

<u>Graphical Method</u>

We can use the GC to sketch the graph of $x = \dfrac{1}{\tan(\pi-\theta)}$, then apply scale factor of 480 to obtain the answer.





The function is increasing, thus we can substitute end points.

From graph, when $\dfrac{2\pi}{3} \le \theta \le \dfrac{3\pi}{4}$,

$0.5773503 \le \dfrac{1}{\tan(\pi-\theta)} \le 1$

$277 \le \dfrac{480}{\tan(\pi-\theta)} \le 480$ (3 s.f.)

Hence, $277 \le x \le 480$ (3 s.f.)

</td></tr>
</table>

Qn	**Suggested Solutions**
(iv)	$\dfrac{\mathrm{d}C}{\mathrm{d}\theta}=\dfrac{\mathrm{d}C}{\mathrm{d}x}\times\dfrac{\mathrm{d}x}{\mathrm{d}\theta}$ To explain $\dfrac{\mathrm{d}x}{\mathrm{d}\theta}>0$: From part (iii), $x=\dfrac{480}{\tan(\pi-\theta)}=480\cot(\pi-\theta)$. $\dfrac{\mathrm{d}x}{\mathrm{d}\theta}=-480\left[-\operatorname{cosec}^2(\pi-\theta)\right]=480\operatorname{cosec}^2(\pi-\theta)>0$ OR $\dfrac{\mathrm{d}x}{\mathrm{d}\theta}=\dfrac{\mathrm{d}}{\mathrm{d}\theta}480\left[\tan(\pi-\theta)\right]^{-1}=480\left[\tan(\pi-\theta)\right]^{-2}\sec^2(\pi-\theta)>0$ To explain $\dfrac{\mathrm{d}C}{\mathrm{d}x}>0$: From part **(ii)**, we know that when $x>\sqrt{9600}$, $\dfrac{\mathrm{d}C}{\mathrm{d}x}>0$. From part **(iii)**, for $\dfrac{2\pi}{3}\le\theta\le\dfrac{3\pi}{4}$, we have $\dfrac{480}{\sqrt{3}}\le x\le 480$. Therefore, for $\dfrac{2\pi}{3}\le\theta\le\dfrac{3\pi}{4}$, since $\left[\dfrac{480}{\sqrt{3}},480\right]\subseteq\left(\sqrt{9600},\infty\right)$, then we have $\dfrac{\mathrm{d}C}{\mathrm{d}x}>0$. Hence for $\dfrac{2\pi}{3}\le\theta\le\dfrac{3\pi}{4}$, since $\dfrac{\mathrm{d}C}{\mathrm{d}x}>0$ and $\dfrac{\mathrm{d}x}{\mathrm{d}\theta}>0$, we have $\dfrac{\mathrm{d}C}{\mathrm{d}\theta}=\dfrac{\mathrm{d}C}{\mathrm{d}x}\times\dfrac{\mathrm{d}x}{\mathrm{d}\theta}>0$.
(v)	The value of θ that gives the minimum total cost is $\dfrac{2\pi}{3}$.