H2 PHYSICS · NOTES · TOPIC 6

Motion in a Circle

Checklist:

Motion in a Circle	Check (✓)
1. I am able to express angular displacement in radians and use its respective formula.	
2. I am able to show an understanding of and use the concept of angular velocity to solve problems.	
3. I am able to recall and use $v = r\omega$ to solve problems.	
4. I can qualitatively describe the motion due to a constant perpendicular force.	
5. I am able to recall and use $a = r\omega^2$ and $a = \frac{v^2}{r}$ to solve problems.	
6. I am able to recall and use $F = mr\omega^2$ and $F = \frac{mv^2}{r}$ to solve related problems.	
7. I am able to distinguish between uniform and non-uniform circular motion by a. Identifying the respective forces acting on an object in non-unform circular motion b. Resolving relevant forces in the intended directions.	

Introduction

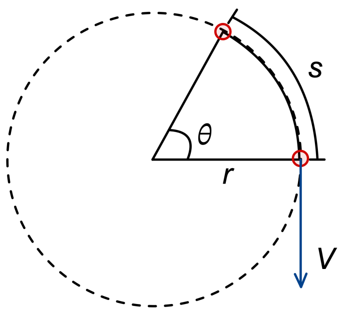
Why does water stay at the bottom of a container when it is swung in a vertical circle? Why do we feel weightless in orbit in space? Why are we slightly heavier in the north pole than on the equator? Many phenomenon in our lives can be explained through the topic of circular motion, the study of objects moving in a circular paths, and the conditions required to stay in circular motion.

Angular Displacement

Angular Displacement is the angle swept out by the radius *r* joining the body to the centre of a circle.

Following the formula for arc length, the angular displacement θ, is given by the ratio of the arc length *s,* and the radius of the circle, *r.*

$$\theta = \frac{s}{r}$$



One radian is defined as the angle subtended at the centre of the circle by an arc equal in length to the radius of the circle.

Conversion from radians(rad) to degrees: $Angle\ in\ rad\ \times 180° = Angle\ in\ ° \times \pi$

Standard conversion formula:

$$Angle\ in\ unit\ x \times \frac{quantity\ in\ y}{equivalent\ quanity\ in\ unit\ x} = Angle\ in\ unit\ y$$

Active recall: What is the definition of **one joule**?

Angular Velocity, ω

Angular velocity, ω is the **rate of change** of **angular displacement.**

$$\omega = \frac{d\theta}{dt}$$

This equation represents the *instantaneous angular velocity.* If the object has a **constant** angular velocity, the instantaneous value and the average value will be **the same.**

Angular Velocity, ω, in terms of period *T*

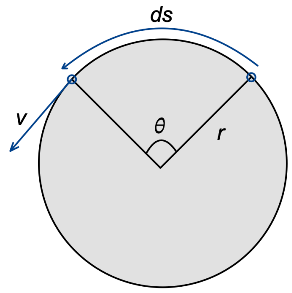
Period(*T*), is the time it takes for an object in circular motion to make one complete revolution, or cycle. **Frequency *(f)*** is the number of revolutions, or cycles, made per unit time. The unit of frequency is s^{-1}, also known as the hertz(*Hz*).

Given the definition: **Angular velocity, ω** is the **rate of change** of **angular displacement, f**or *uniform* circular motion, the particle in motion has the same **tangential** speed throughout the motion. Hence, angular velocity is a constant and the time taken for 1 revolution also known as *period T* can be expressed as:

$$\omega = \frac{d\theta}{dt} = \frac{2\pi}{T} = 2\pi f$$

Relationship between Angular Velocity and Tangential(Linear) Speed

A particle moving in a circle has an instantaneous velocity tangential to its circular path. For a constant angular speed, the particle's orbital or tangential speed v is also constant. At any instant, it is directed tangentially to the circular path at the specific point.

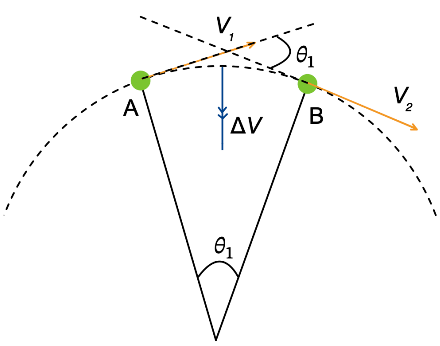


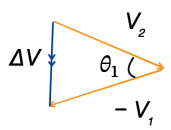
From $\omega = \frac{d\theta}{dt} = \frac{d\frac{s}{r}}{dt}$, since radius *r* is a constant, hence we can get $\omega = \frac{1}{r}(\frac{ds}{dt})$, and since speed is the rate of change of distance, we can finally get $v = r\omega$.

Uniform Circular Motion

In uniform circular motion, although speed is constant, why would there be centripetal acceleration?

- Although the magnitude of velocity in the circular motion remains constant, its direction changes. Since velocity is a vector quantity, hence there will be changing velocity and a value for acceleration will be present.
- In addition, by Newton's Second Law, there must be a resultant force in the direction of acceleration, which is also in the direction of the change in velocity. (Recall: acceleration is the rate of change of velocity) This direction is towards the centre of the circle.

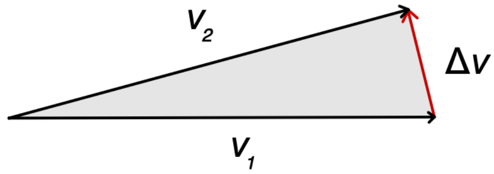




vector diagram for ΔV

Derivation of Centripetal Force and Centripetal Acceleration

We need to consider the change in velocity of an object before we derive an equation for acceleration. Hence, from the diagram below we can tell that the **change in velocity** is given by $v_2 - v_1$.



$$\Delta v \approx v \times \Delta\theta$$

$$\frac{\Delta v}{\Delta t} \approx \frac{v \times \Delta\theta}{\Delta t}$$

$$a = v \times \frac{\Delta\theta}{\Delta t}$$

$$a = v\omega = r\omega^2 = \frac{v^2}{r}$$

$$F_c = \frac{mv^2}{r} = mr\omega^2$$

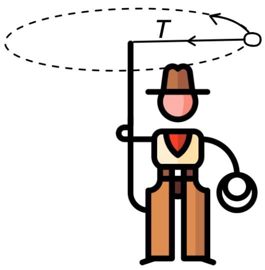
Dynamics of uniform circular motion

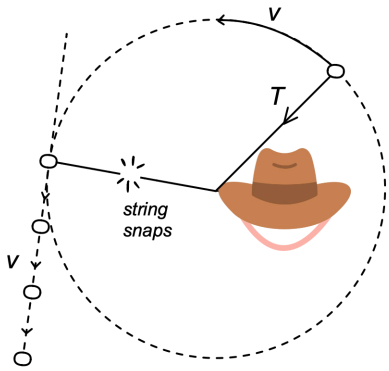
Let's take a look at common examples of exam questions that ask us to describe why a force towards the centre of the circular motion needs to be present in order for uniform circular motion to take place.

Why must the object experience a force that is directed towards the centre of the circle, perpendicular to the motion of the object?

- Recap **Newton's First Law**: In accordance to Newton's First Law, the object will continue to move in uniform speed in the same direction unless there is a **net force** acting on it. For a net force to **modify the magnitude of velocity**, it needs to have a component along the velocity. Hence, if the net force is to only change the **direction** of velocity and allow the object to remain in uniform motion, it needs to be perpendicular to the velocity.

- By **Newton's Second Law**, there must be a corresponding resultant force acting along the same direction as the **change in momentum**, which is the same direction as the **change in velocity**.



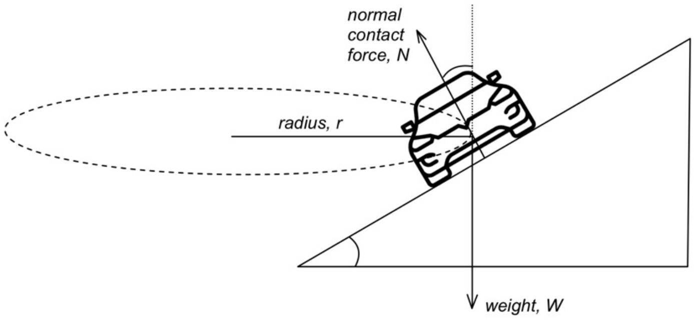


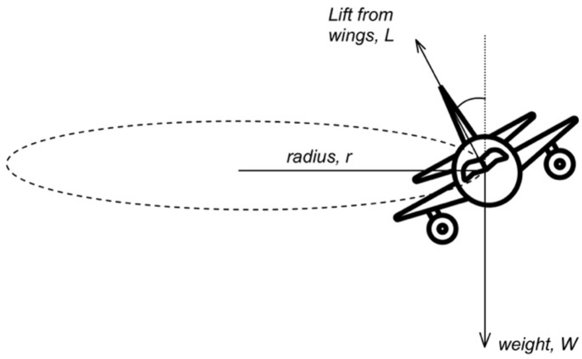
The equation for centripetal force must be understood as a way to calculate centripetal force **needed** for uniform circular motion for a mass m moving with speed v in a circle of radius r.

***For an object to move in a circular path, a physical force must **provide** the centripetal force.

<u>Question:</u> Is there any work done by the centripetal force in *uniform circular motion*?

Providing centripetal force using banking





- For the object to remain at **constant height** and continue to move in **circular path of radius *r*,** we need to introduce **banking** – tilting the plane of force so that there will be a **component of force** acting towards the center of a circular path. This force will then **provide** for the centripetal force.

$$\theta = \text{banking angle}$$

- Horizontally, since the horizontal component of *Normal contact force/Lift* provides centripetal force,

$$L sin\theta = \frac{mv^2}{r}$$

- At constant height,

$$L cos\theta = mg$$

- Considering the question does not provide a value for *N or L,* or a value for mass *m,* we can divide both equations to get,

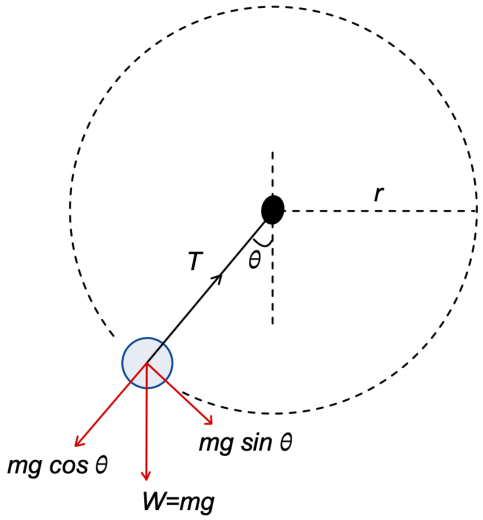
$$tan\theta = \frac{v^2}{rg}$$

- This also indicates that the relationship between the **baking angle,** the **speed** and the **radius** is independent of mass *m.*

Non-uniform Circular Motion

When an object is set to travel in a vertical circular path its speed will not be constant as some kinetic energy will be lost to account for gravitational potential energy the object gains as it climbs the circle.

- This is also known as **non-uniform circular motion,** where speed is no longer constant. For non-uniform motion, the net force is **no longer equal** to the centripetal force. We can still resolve the net force into its tangential and radial components.
- The **tangential component** of net force changes only the **speed** of the motion, while the radial component changes only the direction of the motion.
- Centripetal Acceleration & Force must now vary as the **speed changes**.



- Tangential acceleration is given by,

$$mgsin\theta = ma$$

- Centripetal acceleration is given by,

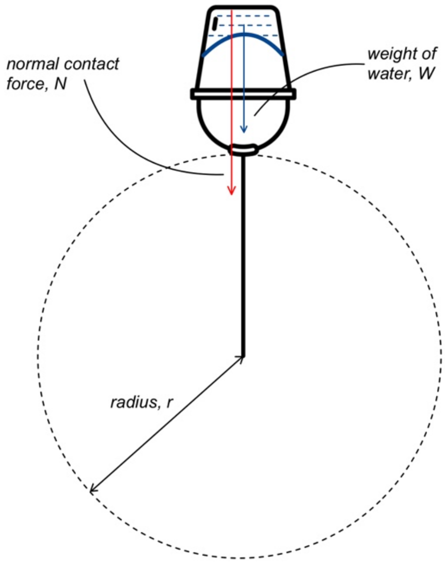
$$a = \frac{v^2}{r}$$

- Considering forces acting towards the centre of the circle,

$$T - mgcos\theta = \frac{mv^2}{r}$$

Water in a bucket – Centrifugal force

Question: *A pail containing water of mass m, is whirled in a vertical circle of radius r. Calculate the minimum speed at which the pail must be whirled such that the water is able to remain in the pail.*



- At the top,

$$N + mg = \frac{mv^2}{r}$$

- Water will remain in the pail if *N>0*

$$m\left(\frac{v^2}{r} - g\right) \geq 0$$

$$v \geq \sqrt{rg}$$